Steady-State Optimality Analysis of MPC Controllers

Jozef Vargan [1] Jakub Puk [2] Karol Ľubušký [3] Miroslav Fikar [1]

[1]Slovak University of Technology in Bratislava, Bratislava, Slovakia

[2]ATP spol. s.r.o., Bratislava, Slovakia

[3]Slovnaft a.s., Bratislava, Slovakia

ESCAPE 34 – PSE 24
June 2–6, 2024

Steady-State Optimality Analysis of MPC Controllers

Jozef Vargan [1] Jakub Puk [2] Karol Ľubušký [3] Miroslav Fikar [1]

[1]Slovak University of Technology in Bratislava, Bratislava, Slovakia

[2]ATP spol. s.r.o., Bratislava, Slovakia

[3]Slovnaft a.s., Bratislava, Slovakia

ESCAPE 34 – PSE 24
June 2–6, 2024

Motivation

- Distillation column
- Processing heavy oil distillates to oil fractions
- Honeywell APC controllers

Motivation

- Distillation column
- Processing heavy oil distillates to oil fractions
- Honeywell APC controllers
- Economical aspects – constraints manipulation alerts

Process Variable	Label	No.
Controlled	*CV*	23
Manipulated	*MV*	11
Disturbance	*DV*	13
Total		47

Process Constraint	No.
Controlled	46
Manipulated	22
Total	68

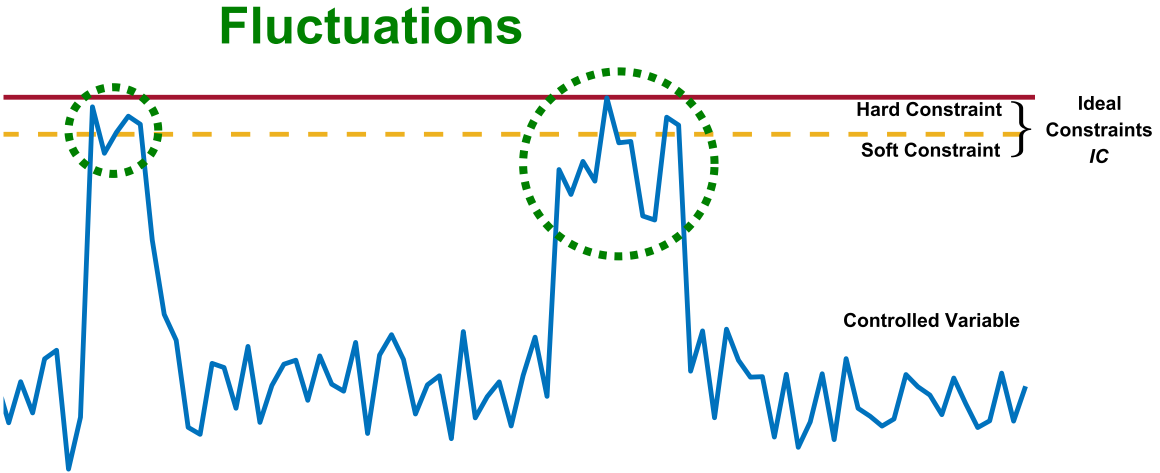
Motivation

- Distillation column
- Processing heavy oil distillates to oil fractions
- Honeywell APC controllers
- **Economical aspects – constraints manipulation alerts**

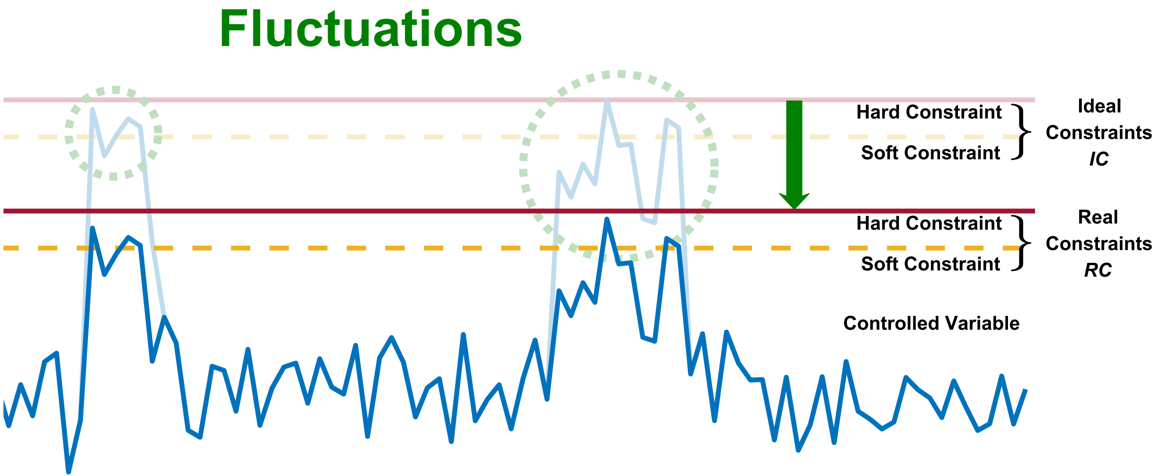
Process Variable	Label	No.
Controlled	CV	23
Manipulated	MV	11
Disturbance	DV	13
Total		47

Process Constraint	No.
Controlled	46
Manipulated	22
Total	**68**

Example of Problem

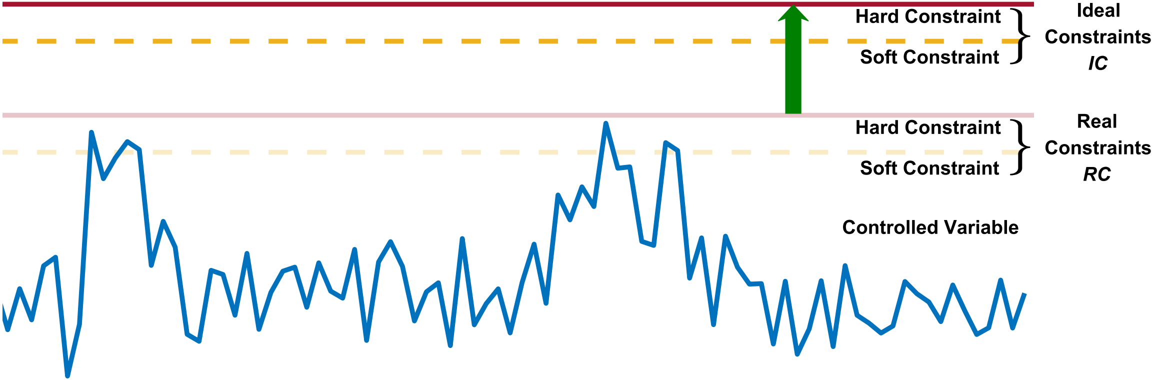


Example of Problem

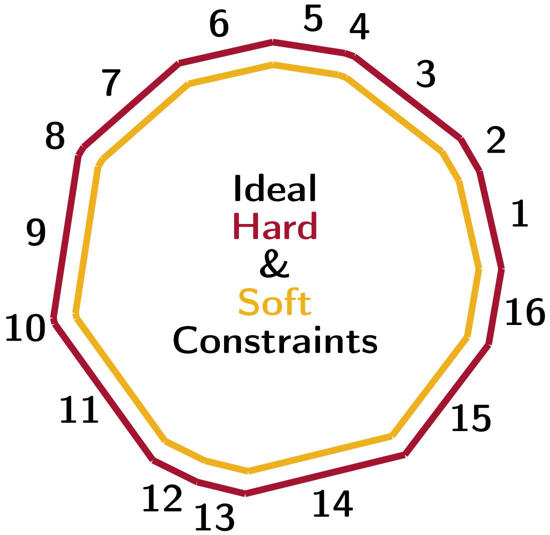


Example of Problem

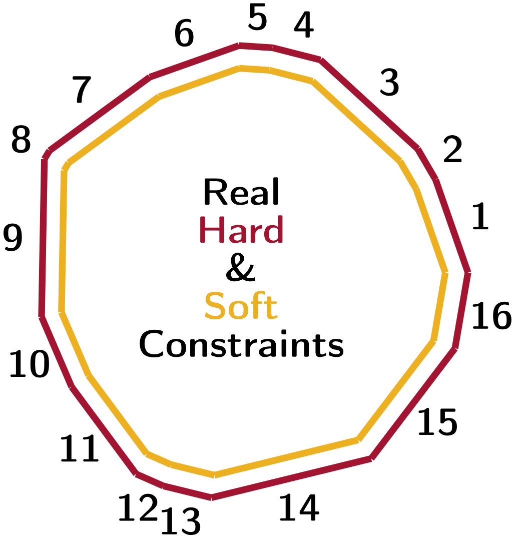
Fluctuations



Example of Problem



Example of Problem



Steady-State Problem Formulation

- Optimization problem – MPC analysis in steady-state
- Objective function J
  - Quadratic formulation
  - Soft constraints implementation
- Constraints
  - Equality (steady-state model of the distillation column)
  - Inequality
    - Hard & soft constraints
    - Slack constraints

Steady-State Problem Formulation

- Optimization problem – MPC analysis in steady-state
- **Objective function *J***
  - Quadratic formulation
  - Soft constraints implementation
- Constraints
  - Equality (steady-state model of the distillation column)
  - Inequality
    - **Hard & soft constraints**
    - Slack constraints

Constraint change

?

=

Objective function change

=

Profit generation change

Sensitivity Analysis

Local sensitivity analysis
(Ideal Lagrange multiplier $\lambda_i^\star$)

$$\lambda_i^\star = -\frac{\partial J_{Ideal}(x^\star)}{\partial IC_i}$$

for i-th constraint

Sensitivity Analysis

Local sensitivity analysis
(Ideal Lagrange multiplier $\lambda_i^\star$)

$$\lambda_i^\star = -\frac{\partial J_{Ideal}(x^\star)}{\partial IC_i}$$

for i-th constraint

Non-local sensitivity analysis
(Real Lagrange multiplier $\widetilde{\lambda}_i^\star$)

$$\widetilde{\lambda}_i^\star = \frac{J_{Real,i}(x^\star) - J_{Ideal}(x^\star)}{RC_i - IC_i}$$

for each $RC_i \neq IC_i$

Sensitivity Analysis

Local sensitivity analysis
(Ideal Lagrange multiplier $\lambda_i^\star$)

$$\lambda_i^\star = -\frac{\partial J_{Ideal}(x^\star)}{\partial IC_i}$$

for i-th constraint

Pro Only 1 optimization

Con Works for infinitesimal constraint change

Non-local sensitivity analysis
(Real Lagrange multiplier $\widetilde{\lambda}_i^\star$)

$$\widetilde{\lambda}_i^\star = \frac{J_{Real,i}(x^\star) - J_{Ideal}(x^\star)}{RC_i - IC_i}$$

for each $RC_i \neq IC_i$

Pro Works for any size of constraint change

Con $N+1$ optimizations

One Operating Point Recommendations

- **Create a list of Lagrange multipliers.**
- Check – constraints order (edit distance)

Priority	Constraint name	Lagrange multiplier	Verbal recommendation
1	MV 2 HL	24,000,000	Keep the constraint in place
2	CV 10 LL	21,000,000	Keep the constraint in place
3	MV 4 LL	850,000	Keep the constraint in place
4	CV 8 HL	9,999	Move the real constraint to the ideal
5	CV 20 HL	9,999	Move the real constraint to the ideal
6	CV 14 LL	1,380	Keep the constraint in place
7	CV 1 HL	100	Move the real constraint to the ideal

One Operating Point Recommendations

- **Create a list of Lagrange multipliers.**
- Check – constraints order (edit distance)

Priority	Constraint name	Lagrange multiplier	Verbal recommendation
1	MV 2 HL	24,000,000	Keep the constraint in place
2	CV 10 LL	21,000,000	Keep the constraint in place
3	MV 4 LL	850,000	Keep the constraint in place
4	**CV 8 HL**	**9,999**	**Move the real constraint to the ideal**
5	**CV 20 HL**	**9,999**	**Move the real constraint to the ideal**
6	CV 14 LL	1,380	Keep the constraint in place
7	**CV 1 HL**	**100**	**Move the real constraint to the ideal**

One Operating Point Recommendations

- Create a list of Lagrange multipliers.
- **Check – constraints order (edit distance)**

Priority	Lagrange multiplier				Constraint change [rel. %]
	Constraint	LSA	NSA	Constraint	
1	CV 8 HL	9,999	9,999	CV 8 HL	0.28
2	CV 20 HL	9,999	9,999	CV 20 HL	0.57
3	CV 1 HL	100	93	CV 1 HL	1.83

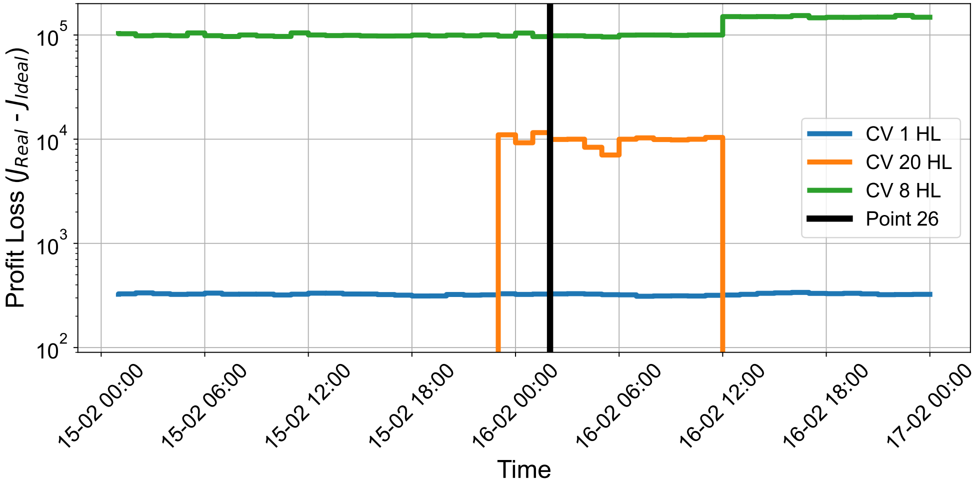
One Operating Point Recommendations

- Create a list of Lagrange multipliers.
- **Check – constraints order (edit distance)**

Priority	Lagrange multiplier				Constraint change [rel. %]
	Constraint	LSA	NSA	Constraint	
1	**CV 8 HL**	9,999	9,999	**CV 8 HL**	0.28
2	**CV 20 HL**	9,999	9,999	**CV 20 HL**	0.57
3	**CV 1 HL**	100	93	**CV 1 HL**	1.83

Edit distance $= \mathbf{0}$

Long-Term Profit Loss



Results

- Distillation column plant
- MPC analysis in steady-state
- Constraints sensitivity
- Recommendations list
- Open-source application

Python source

Acknowledgments: APVV-21-0019, VEGA 1/0691/21, EC 101079342 (FrontSeat)

Optimization Problem

$$\min_{CV,MV,\epsilon,E} J = \sum_{i=1}^{s} b_{CV,i} CV_i + \sum_{i=1}^{s} a_{CV,i}^2 (CV_i - CV_{0,i})^2 + \sum_{j=1}^{t} b_{MV,j} MV_j + \sum_{j=1}^{t} a_{MV,j}^2 (MV_j - MV_{0,j})^2$$
$$+ \sum_{i=1}^{s} Q_{CV,H,i} E_{CV,H,i} + \sum_{i=1}^{s} Q_{CV,L,i} E_{CV,L,i} + \sum_{j=1}^{t} Q_{MV,H,j} E_{MV,H,j} + \sum_{j=1}^{t} Q_{MV,L,j} E_{MV,L,j}$$

$$\text{s.t.} \quad CV_i - CV_i^{SS} = \sum_{j=1}^{t} K_{i,j} \left(MV_j - MV_j^{SS}\right),$$
$$\boldsymbol{\lambda_i} : \boldsymbol{CV_{L,i} + \Delta CV_{L,i} - \epsilon_{CV,L,i} \leq CV_i \leq CV_{H,i} - \Delta CV_{H,i} + \epsilon_{CV,H,i}},$$
$$\boldsymbol{\lambda_j} : \boldsymbol{MV_{L,j} + \Delta MV_{L,j} - \epsilon_{MV,L,j} \leq MV_j \leq MV_{H,j} - \Delta MV_{H,j} + \epsilon_{MV,H,j}},$$
$$0 \leq \epsilon_{CV,H,i}, \quad 0 \leq \epsilon_{CV,L,i},$$
$$0 \leq \epsilon_{MV,H,j} \leq \Delta MV_{H,j}, \quad 0 \leq \epsilon_{MV,L,j} \leq \Delta MV_{L,j},$$
$$\epsilon_{CV,H,i} \leq E_{CV,H,i}, \quad \epsilon_{CV,L,i} \leq E_{CV,L,i},$$
$$\epsilon_{MV,H,j} \leq E_{MV,H,j}, \quad \epsilon_{MV,L,j} \leq E_{MV,L,j}.$$