GO-VIKING

Flow Induced Vibration

The numerical approach

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TU Delft

GO-VIKING

Gathering expertise On Vibration ImpaKt In Nuclear power Generation

- Follow-up of the VIKING initiative (2020)
- Understanding and prediction of Flow-Induced Vibration
- Focus on nuclear power generation

Today: focus on the numerical approach of FIV

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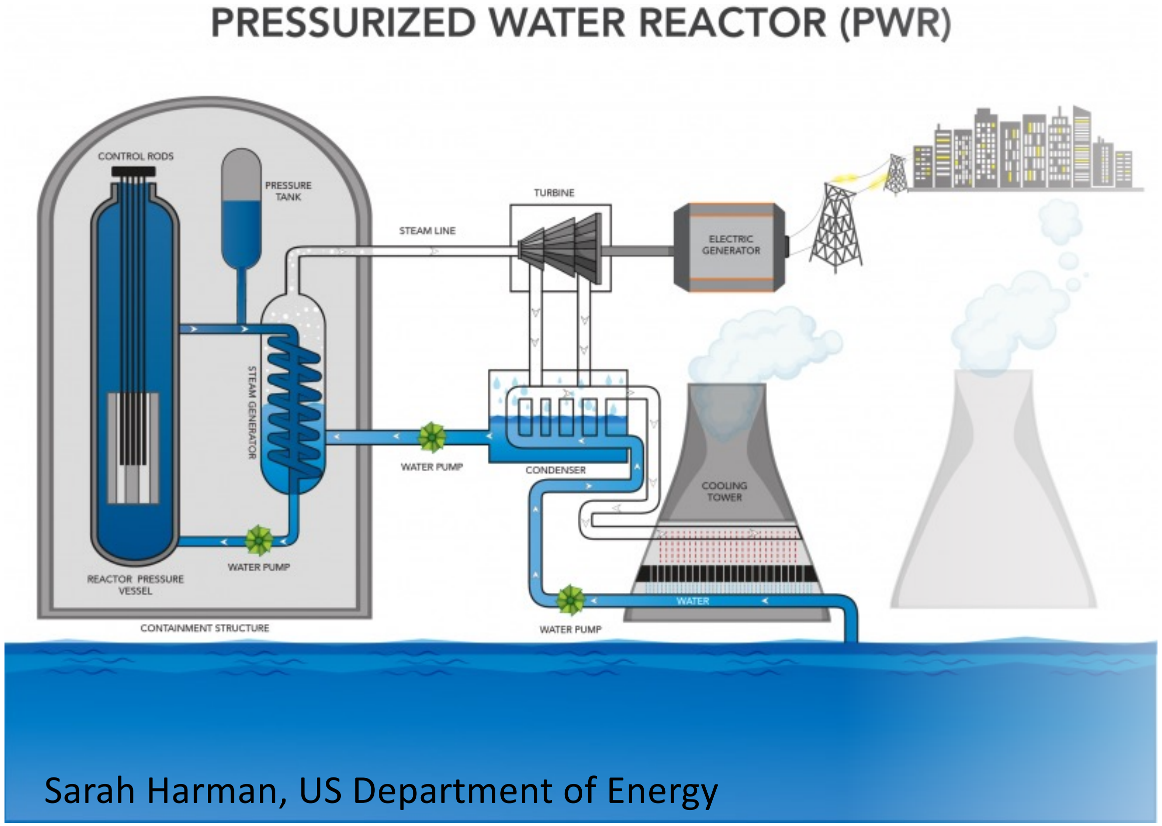
FIV for nuclear power



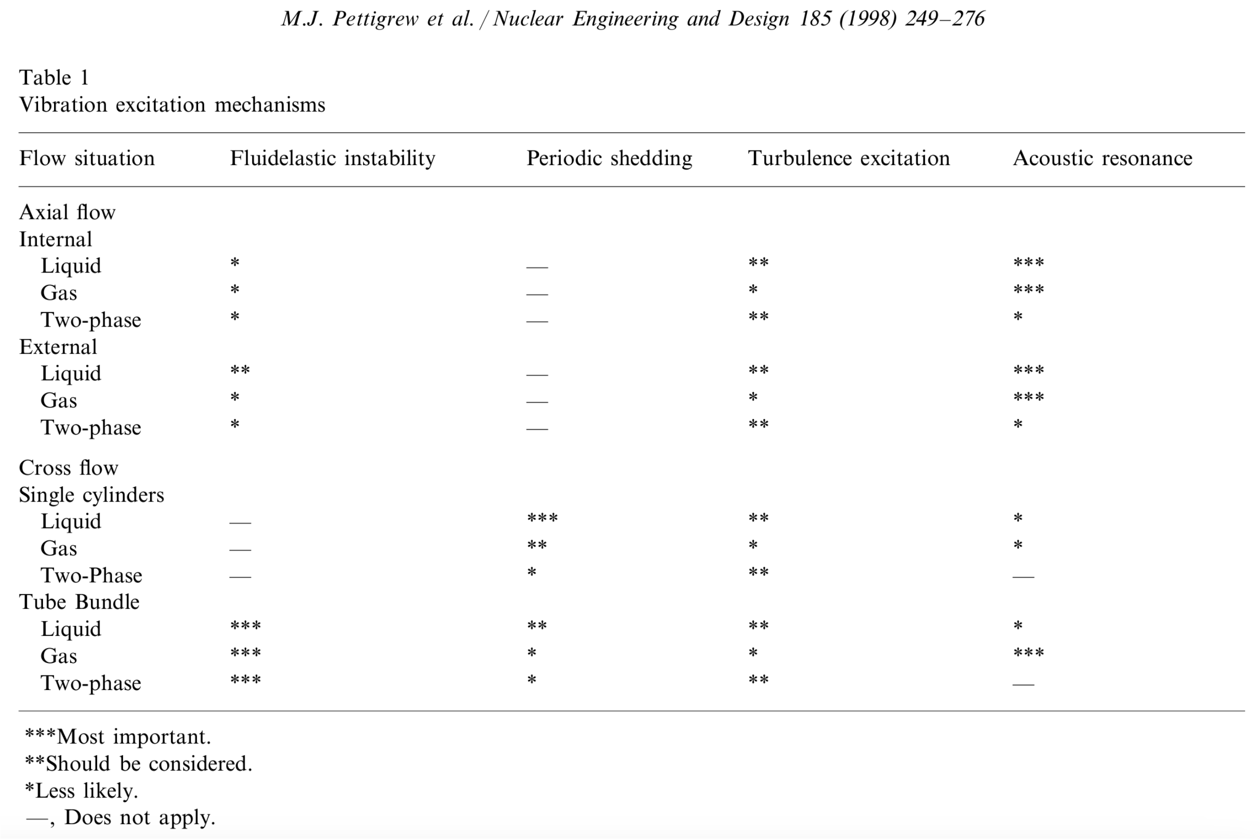
Sarah Harman, US Department of Energy

FIV in pressure vessel and steam generator

Vibration is sustained by feeding energy from the flow into the structure

There are tubes/rods (bundles), with axial/cross flow

Types of Flow Induced Vibrations



M.J. Pettigrew et al. / Nuclear Engineering and Design 185 (1998) 249–276

Table 1
Vibration excitation mechanisms

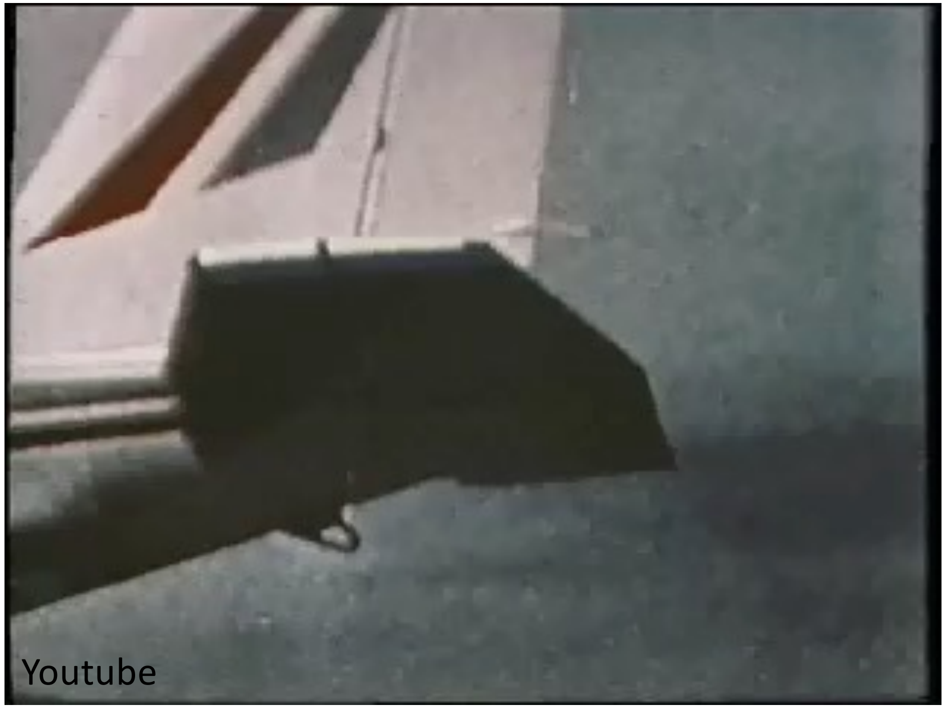
Flow situation	Fluidelastic instability	Periodic shedding	Turbulence excitation	Acoustic resonance
Axial flow				
Internal				
Liquid	*	—	**	***
Gas	*	—	*	***
Two-phase	*	—	**	*
External				
Liquid	**	—	**	***
Gas	*	—	*	***
Two-phase	*	—	**	*
Cross flow				
Single cylinders				
Liquid	—	***	**	*
Gas	—	**	*	*
Two-Phase	—	*	**	—
Tube Bundle				
Liquid	***	**	**	*
Gas	***	*	*	***
Two-phase	***	*	**	—

***Most important.
**Should be considered.
*Less likely.
—, Does not apply.

Contents

- Examples of FIV / FSI
- Numerical modeling of FSI
  - Fluid dynamics
  - Solid dynamics
  - Mesh motion
  - Spatial & temporal interface coupling
- Academic examples
  - Vortex Induced Vibration
  - Turbulence Induced Vibration
- Challenges for validation

Examples of possibly dangerous FSI


Youtube


Youtube

Examples of modelling FSI

Youtube

Examples of modelling FSI

(rbf-morph)

ANSYS®

12 CYLINDERS
TRANSIENT FSI

Youtube

FSI: (Dynamic) interaction between flows and deforming structures

- How to model this numerically?
- Which physics do we need to model?
- Where does the interaction between flow and structure occur?
- Which conditions should be satisfied?

FSI: (Dynamic) interaction between flows and deforming structures

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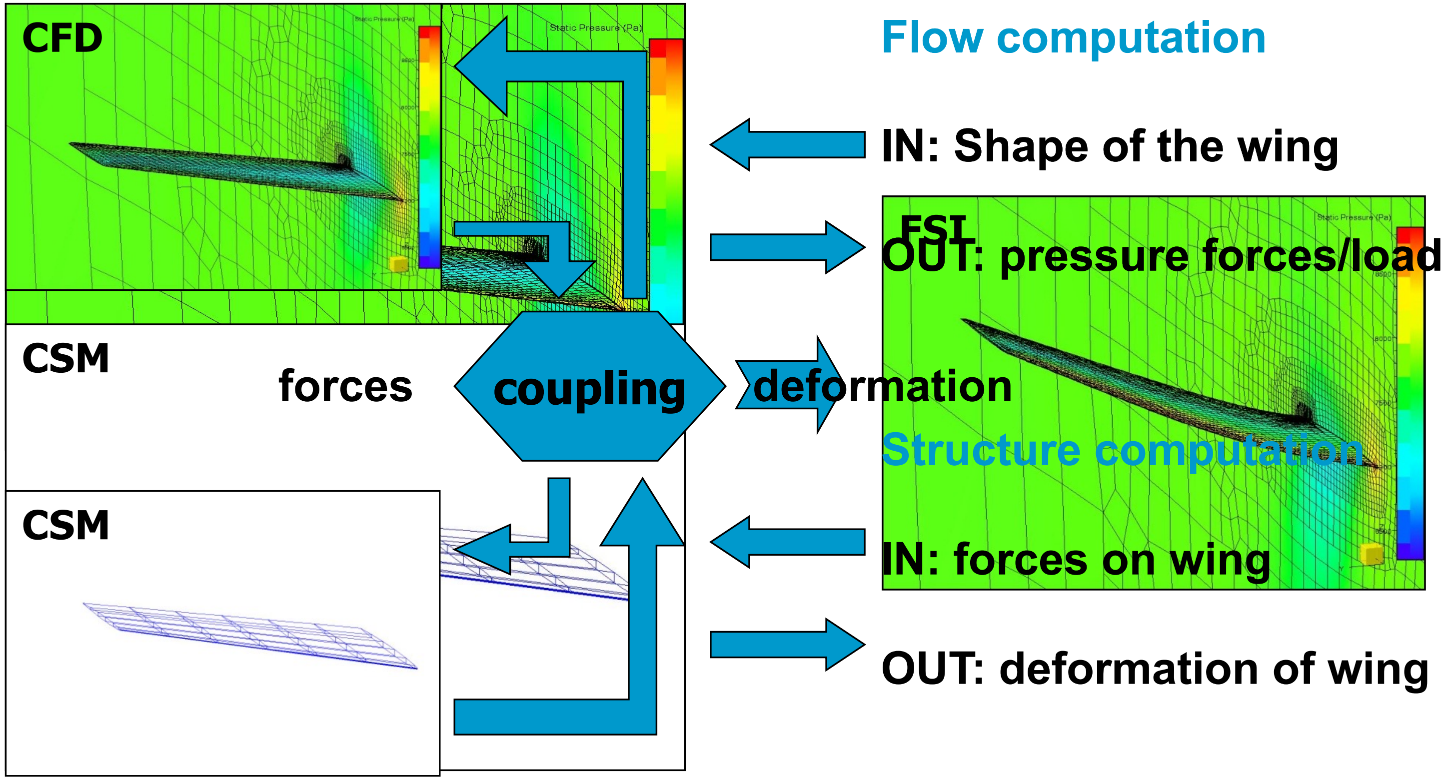
- In all examples there is a dynamic, possibly dangerous, interaction between the flow around a deforming structure
- Multi-physics are involved (solid mechanics & fluid mechanics)
- Aerodynamic loads on the structure cause a deformation of the structure
- Deformation of the aerodynamic surface results in a change in aerodynamic loads
- Coupling between flow and structure at the fluid-structure interface:
  - Equality of velocity at (and location of) the interface
  - Equilibrium in stress on the interface

FSI: solving a coupled system



Flow computation

IN: Shape of the wing

OUT: pressure forces/loads

Structure computation

IN: forces on wing

OUT: deformation of wing

Coupled fluid and structure

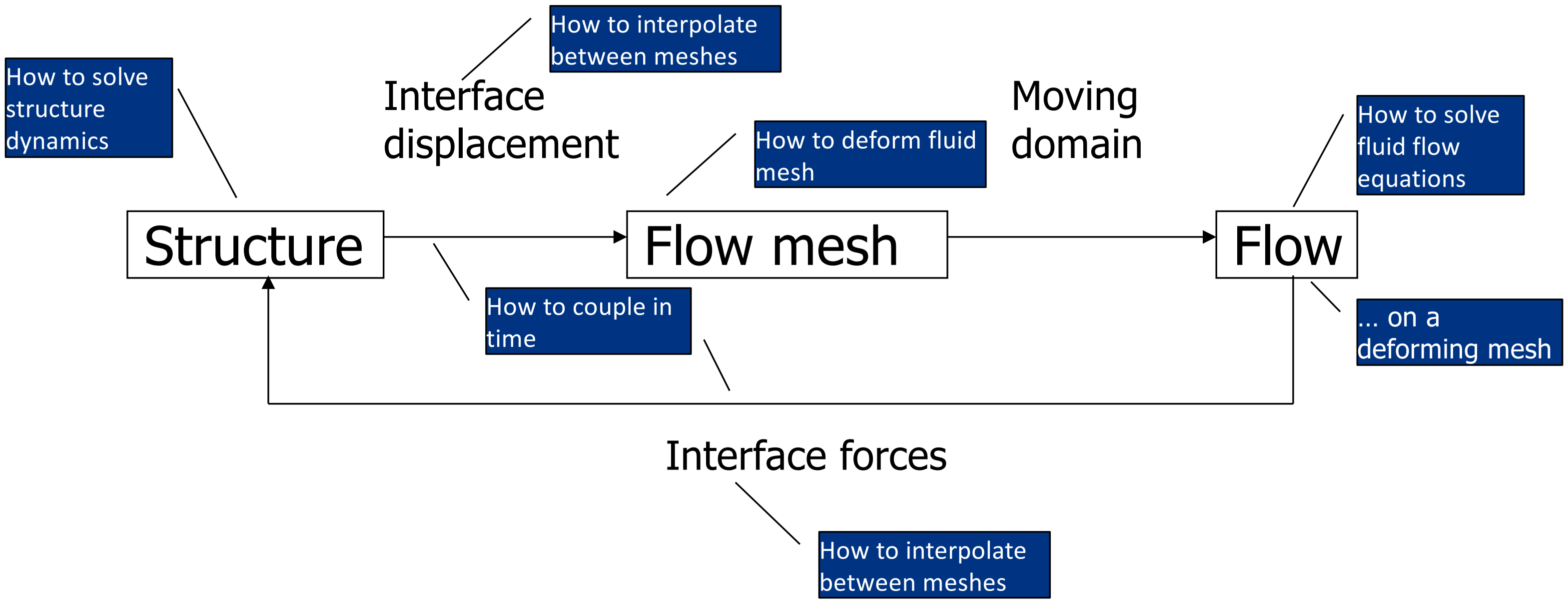
- Structural system: $M\frac{\partial^2 q}{\partial t^2} + D\frac{\partial q}{\partial t} + Kq = F_{\text{interface}}$

- Fluid system: $\frac{d}{dt}\int_{V(t)} W \; dV + \oint_{S(t)} (\vec{F}(W) - W\frac{d\vec{x}}{dt}) \cdot \vec{n} \; dS = 0$

- Moving fluid mesh

- Boundary conditions at the interface:
    - flow speed = time derivative of displacement of the structure
    - stress in structure at interface = stress (incl. pressure) of flow at interface

Coupling diagram of flow and structure



How to solve structure dynamics

Interface displacement

How to interpolate between meshes

How to deform fluid mesh

Moving domain

How to solve fluid flow equations

Structure

Flow mesh

Flow

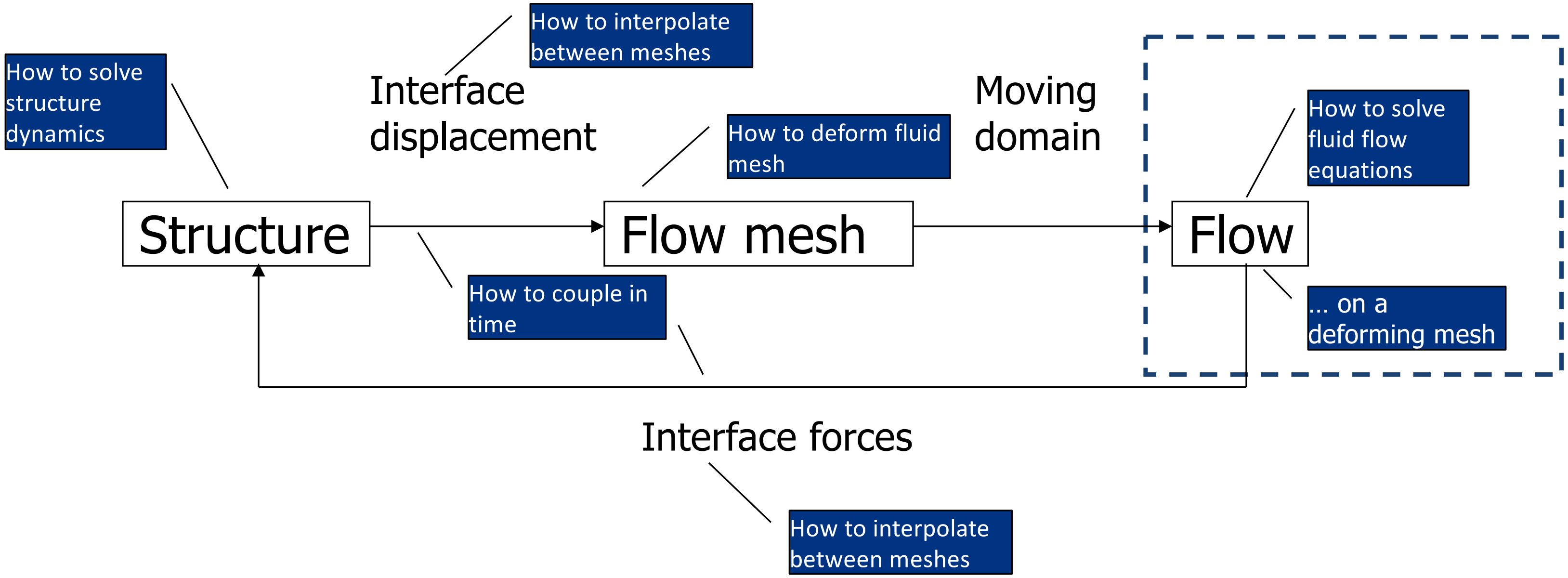
How to couple in time

... on a deforming mesh

Interface forces

How to interpolate between meshes

Coupling diagram of flow and structure



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Fluid dynamics

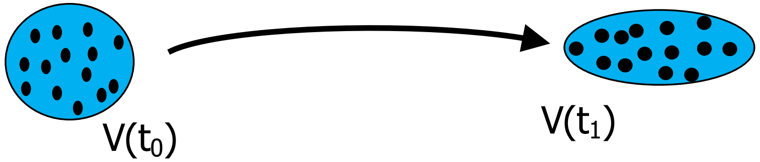
- Conservation laws:
    - Conservation of mass
    - Conservation of momentum
    - Conservation of energy

- Navier-Stokes equations

- Finite volume discretisation

Lagrangian versus Eulerian approach

- Lagrangian: reference frame "attached" to the particles / control mass



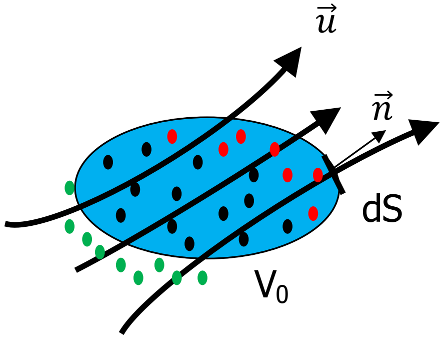
  - Mass is constant:

$$\frac{Dm}{Dt} = 0$$

$$m = \int_V \rho dV$$

  - Volume V varies in space and time

- Eulerian: Inertial (fixed) reference frame



  - Volume is constant (in space and time)
  - Mass inside volume varies in time

$$\frac{d}{dt}\int_{V_0} \rho dV + \oint_{\partial V_0} \rho\vec{u}\cdot\vec{n}dS = 0$$

The Navier-Stokes equations on a fixed mesh in conserved variables in 2D

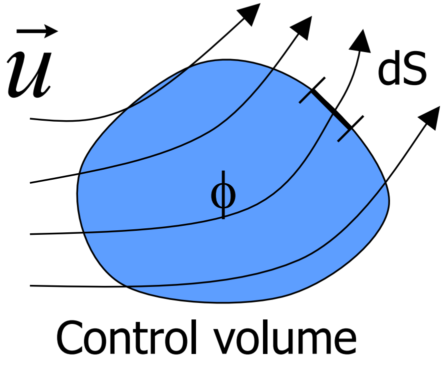
$$\frac{d}{dt}\int_{\Omega_{CV}} \vec{W}\; d\Omega + \int_{S_{CV}} [\vec{E}(\vec{W}),\vec{F}(\vec{W})]\cdot\; \vec{n}\; dS = \int_{\Omega_{CV}} \vec{J}\; d\Omega$$

$$\vec{W} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{bmatrix} \qquad \vec{E}(\vec{W}) = \begin{bmatrix} \rho u \\ \rho u^2 + p - \tau_{xx} \\ \rho u v - \tau_{xy} \\ (\rho E + p) u - u\tau_{xx} - v\tau_{xy} + q_x \end{bmatrix}$$

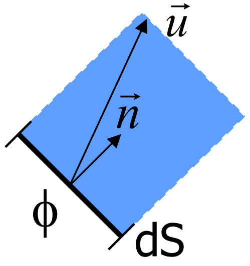
$$\text{convection} = \vec{W}\vec{u}$$

$$\vec{J} = \begin{bmatrix} 0 \\ \rho f_x \\ \rho f_y \\ \rho\vec{f}\cdot\vec{v} \end{bmatrix} \qquad \vec{F}(\vec{W}) = \begin{bmatrix} \rho v \\ \rho u v - \tau_{xy} \\ \rho v^2 + p - \tau_{yy} \\ (\rho E + p) v - u\tau_{xy} - v\tau_{yy} + q_y \end{bmatrix}$$

Convection term changes for moving meshes



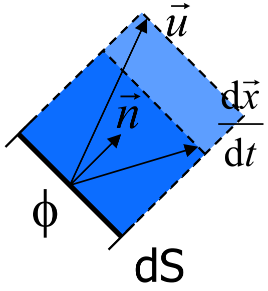
Amount of ϕ that is flowing out of the control volume through dS:



$$\phi \vec{u} \cdot \vec{n}\, \mathrm{d}S$$

Net: $\oint \phi \vec{u} \cdot \vec{n}\, \mathrm{d}S$

Suppose the boundary dS moves at a velocity



$$\phi \vec{u} \cdot \vec{n}\, \mathrm{d}S$$

$$-\phi \tfrac{\mathrm{d}\vec{x}}{\mathrm{d}t} \cdot \vec{n}\, \mathrm{d}S$$

Net: $\oint \phi\left(\vec{u} - \tfrac{\mathrm{d}\vec{x}}{\mathrm{d}t}\right) \cdot \vec{n}\, \mathrm{d}S$

Arbitrary Lagrangian-Eulerian Formulation: Satisfaction of the GCL/DGCL required

$$\frac{d}{dt}\int_{V(t)} \vec{W}\ dV + \int_{S(t)} \left([\vec{E},\vec{F}]\cdot\ \vec{n}\ - \vec{W}\frac{d\vec{x}}{dt}\cdot\vec{n} \right) dS = \int_{V(t)} \vec{J}\ dV$$

- Moving mesh introduces changes the effective convection : $\vec{W}\left(\vec{u}-\frac{d\vec{x}}{dt}\right)$
  - Eulerian: $\frac{d\vec{x}}{dt}=0$
  - Lagrangian: $\frac{d\vec{x}}{dt}=\vec{u}$
- Velocity of the mesh generally unknown (“arbitrary”)
- A geometric conservation law (GCL) exists for moving meshes
- GCL / Discrete GCL should be satisfied (numerical stability and ensures preservation of uniform flow on moving mesh)

Obtaining the GCL: assume uniform flow

$$\frac{d}{dt}\int_V \vec{W}\ dV + \int_S \left([\vec{E},\vec{F}]\cdot\ \vec{n}\ - \vec{W}\frac{d\vec{x}}{dt}\cdot\vec{n} \right) dS = \int_V \vec{J}\ dV$$

Suppose we have a uniform flow $\vec{W}_0$ and no body forces

Closed surface *S*: $\int_S \mathrm{constant}\cdot\vec{n}\ dS = 0$

$$\frac{d}{dt}\int_V \vec{W}_0\ dV + \int_S \left(\underbrace{[\vec{E}_0,\vec{F}_0]\cdot\ \vec{n}}_{0}\ - \vec{W}_0\frac{d\vec{x}}{dt}\cdot\vec{n} \right) dS = 0$$

$V, S, \vec{n}$ and $\vec{x}$ are not constant in time.

$$\vec{W}_0\frac{d}{dt}\int_V dV - \vec{W}_0\int_S \frac{d\vec{x}}{dt}\cdot\vec{n}\ \ dS = 0$$

$$\boxed{\Rightarrow\ \ \mathrm{GCL:}\ \frac{d}{dt}\int_V dV = \int_S \frac{d\vec{x}}{dt}\cdot\vec{n}\ \ dS}$$

The Discrete Geometric Conservation Law for Backward Euler

GCL continuous in space and time: $\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{V(t)} dV = \int\limits_{S(t)} \frac{d\vec{x}}{dt}\cdot\vec{n}\;\, dS$

Spatial discretization for a control volume V_i consisting of a number of discrete faces:

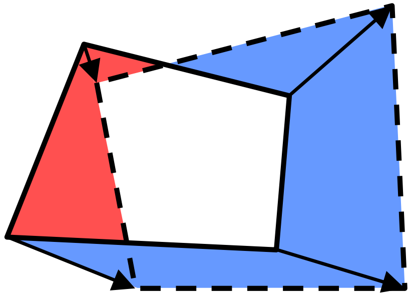
$$\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{V_i} dV = \sum_{j=1}^{\text{faces}}\left(\frac{d\vec{x}}{dt}\cdot\vec{n}\;\Delta S\right)_{i,j}$$

Discretization in time by Backward Euler scheme to obtain DGCL:

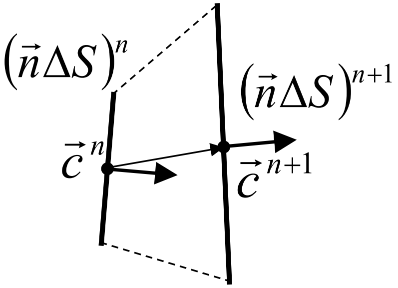
$$\frac{V_i^{n+1}-V_i^{n}}{\Delta t} = \sum_{j=1}^{\text{faces}}\left(\frac{d\vec{x}}{dt}\cdot\vec{n}\;\Delta S\right)_{i,j}^{n+1}$$

- How to define $\left(\frac{d\vec{x}}{dt}\cdot\vec{n}\Delta S\right)$ for each face individually?

Example: swept "volume" in 2D



- Control volume moves from t_n to t_{n+1}
- Four faces with four swept volumes ΔV_j
- Note that: $V^{n+1} - V^{n} = \sum_{j=1}^{4} \Delta V_j$



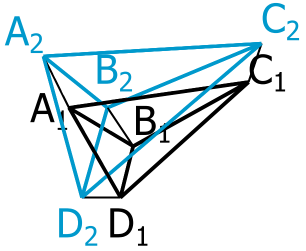
Face with surface ΔS, normal $\vec{n}$ and center $\vec{c}$

Swept volume: $\Delta V = \left(\vec{c}^{\,n+1} - \vec{c}^{\,n}\right) \cdot \left(\frac{(\vec{n}\Delta S)^n + (\vec{n}\Delta S)^{n+1}}{2} \right)$

Mesh velocity condition: $\left(\frac{d\vec{x}}{dt} \cdot \vec{n}\Delta S \right)_j^{n+1} = \frac{\Delta V_j^{n+1}}{\Delta t}$

Verifies that: $\frac{V_i^{n+1} - V_i^{n}}{\Delta t} - \sum_{j=1}^{\text{faces}} \left(\frac{d\vec{x}}{dt} \cdot \vec{n}\Delta S \right)_{i,j}^{n+1} = 0$

Swept volume in 3D: example tetrahedron



- Control volume moves from t_n to t_{n+1}
- Four faces with four swept volumes ΔV_j

$$V_{A_1B_1C_1D_1} : \{S_{A_1B_1C_1}, S_{A_1D_1B_1}, S_{B_1D_1C_1}, S_{C_1D_1A_1}\}$$

$$V_{A_2B_2C_2D_2} : \{S_{A_2B_2C_2}, S_{A_2D_2B_2}, S_{B_2D_2C_2}, S_{C_2D_2A_2}\}$$

A2
C2
B2
A2
C1
A1 B2 B1
A1 B1

D2 D1

$$\begin{aligned}
&\Delta V_{ABC} : \{S_{C_1B_1A_1}, S_{A_2B_2C_2}, \cancel{S_{A_1B_1B_2A_2}}, \cancel{S_{B_1C_1C_2B_2}}, \cancel{S_{C_1A_1A_2C_2}}\} \\
&\Delta V_{ADB} : \{S_{B_1D_1A_1}, S_{A_2D_2B_2}, \cancel{S_{A_1D_1D_2A_2}}, \cancel{S_{D_1B_1B_2D_2}}, \cancel{S_{A_2B_2B_1A_1}}\} \\
&\Delta V_{CDA} : \{S_{A_1D_1C_1}, S_{C_2D_2A_2}, \cancel{S_{D_2C_2C_1D_1}}, \cancel{S_{A_2D_2D_1A_1}}, \cancel{S_{C_2A_2A_1C_1}}\} \\
+ \quad &\Delta V_{BDC} : \{S_{C_1D_1B_1}, S_{B_2D_2C_2}, \cancel{S_{D_2B_2B_1D_1}}, \cancel{S_{D_1C_1C_2D_2}}, \cancel{S_{B_2C_2C_1B_1}}\}
\end{aligned}$$

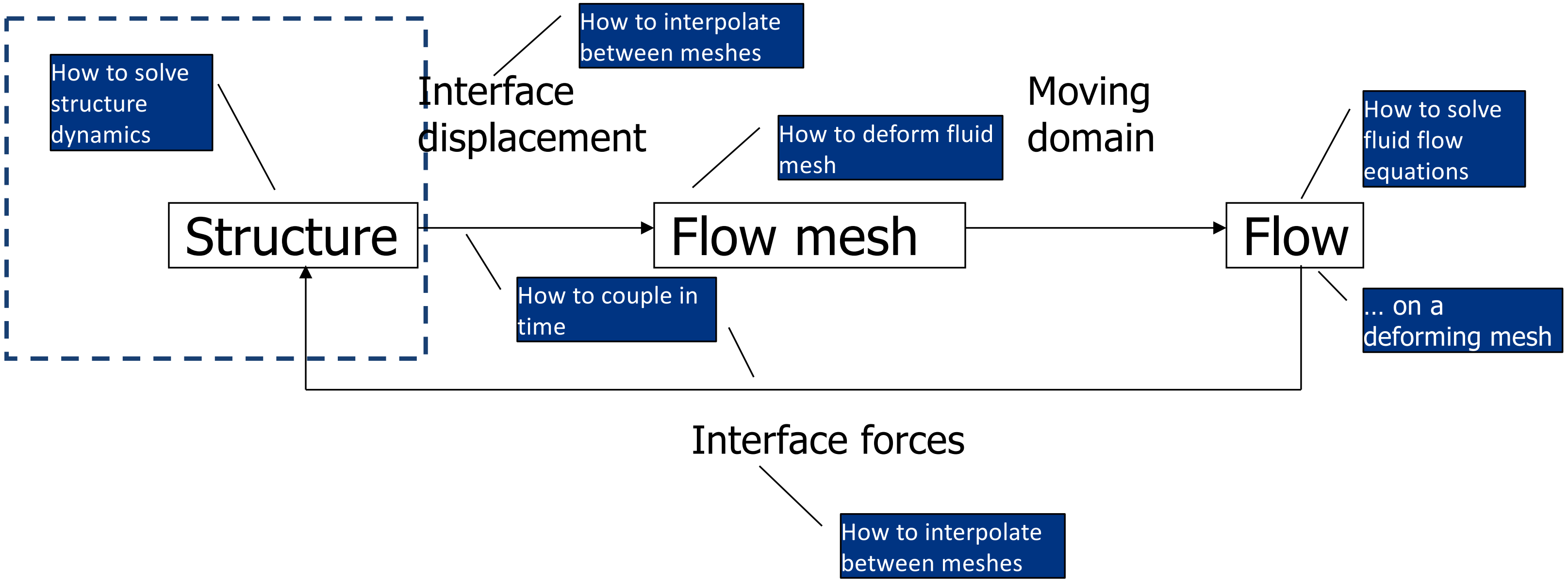
$$\sum \Delta V = V_{A_2B_2C_2D_2} - V_{A_1B_1C_1D_1}$$

No gaps or overlaps between faces!

DGCL for 1D/2D/3D problems

- For every time integration scheme the DGCL results in a different constraint for the moving mesh contribution:
    - Backward Euler: $\left(\frac{d\vec{x}}{dt}\cdot\vec{n}\Delta S\right)_{i,j}^{n+1} = \frac{\Delta V_{i,j}^{n+1}}{\Delta t}$
    - Multi-step: $\left(\frac{d\vec{x}}{dt}\cdot\vec{n}\Delta S\right)_{i,j}^{n+1} = \frac{3}{2}\frac{\Delta V_{i,j}^{n+1}}{\Delta t} - \frac{1}{2}\frac{\Delta V_{i,j}^{n}}{\Delta t}$
    - Multi-stage: $\left(\frac{d\vec{x}}{dt}\cdot\vec{n}\Delta S\right)_{i,j}^{(k)} = \frac{1}{a_{kk}}\left[\frac{\Delta V_{i,j}^{(k)}}{\Delta t} - \sum_{m=1}^{k-1} a_{km}\left(\frac{d\vec{x}}{dt}\cdot\vec{n}\Delta S\right)_{i,j}^{(m)}\right]$
- Constraint depends on the swept volumes for the faces $\Delta V_{i,j}$
- DGCL satisfied when $\sum_j \Delta V_{i,j}^{\alpha} = V_i^{\alpha} - V_i^{n}$

Coupling diagram of flow and structure



How to solve structure dynamics

How to interpolate between meshes

Interface displacement

How to deform fluid mesh

Moving domain

How to solve fluid flow equations

Structure

Flow mesh

Flow

How to couple in time

... on a deforming mesh

Interface forces

How to interpolate between meshes

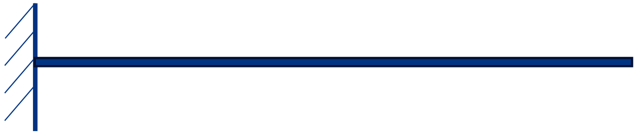
Structure dynamics

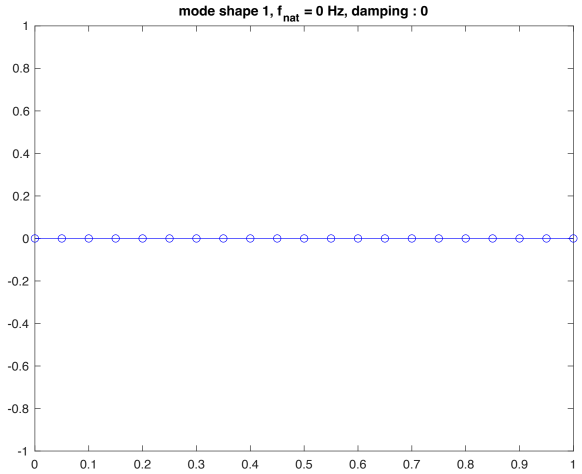
- Structure dynamics: $M\ddot{q} + D\dot{q} + Kq = F(t)$
- The system can be decomposed into modes: $q(x,t) = \sum_{i=1}^{N} \varphi_i(x) a_i(t)$
- $\varphi_i(x)$ is the i-th mode shape, and $a_i(t)$ the amplitude of that mode
- The system can be decoupled by projecting onto $\vec{\varphi}_i$ (spatial filter):

$$\vec{\varphi}_i^T M \vec{\varphi}_i \ddot{a}_i + \vec{\varphi}_i^T D \vec{\varphi}_i \dot{a}_i + \vec{\varphi}_i^T K \vec{\varphi}_i a_i = \vec{\varphi}_i^T F(t) \boxed{\begin{array}{c} \Longrightarrow \ddot{a}_i + c_i \dot{a}_i + \omega_i^2 a_i = f_i(t) \\ \text{Harmonic oscillator} \end{array}}$$

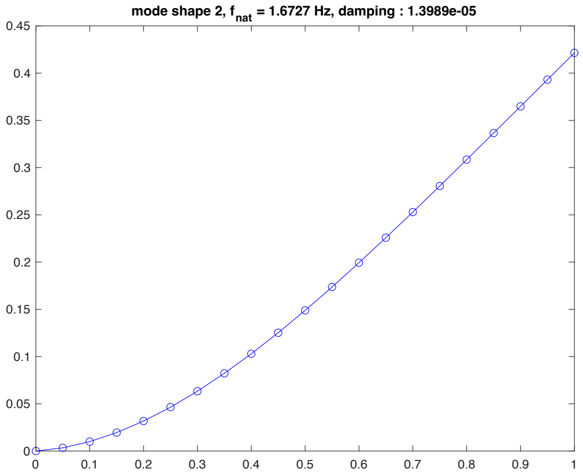
- Properties defined by damping and natural frequency

Example of modes

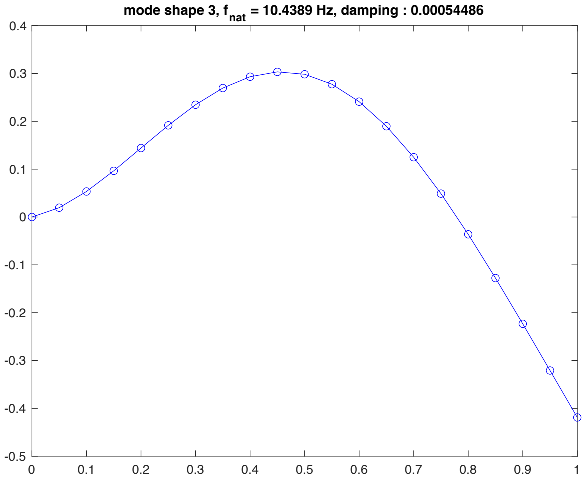




Mode 1



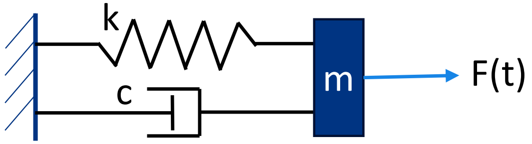
Mode 2



Mode 3

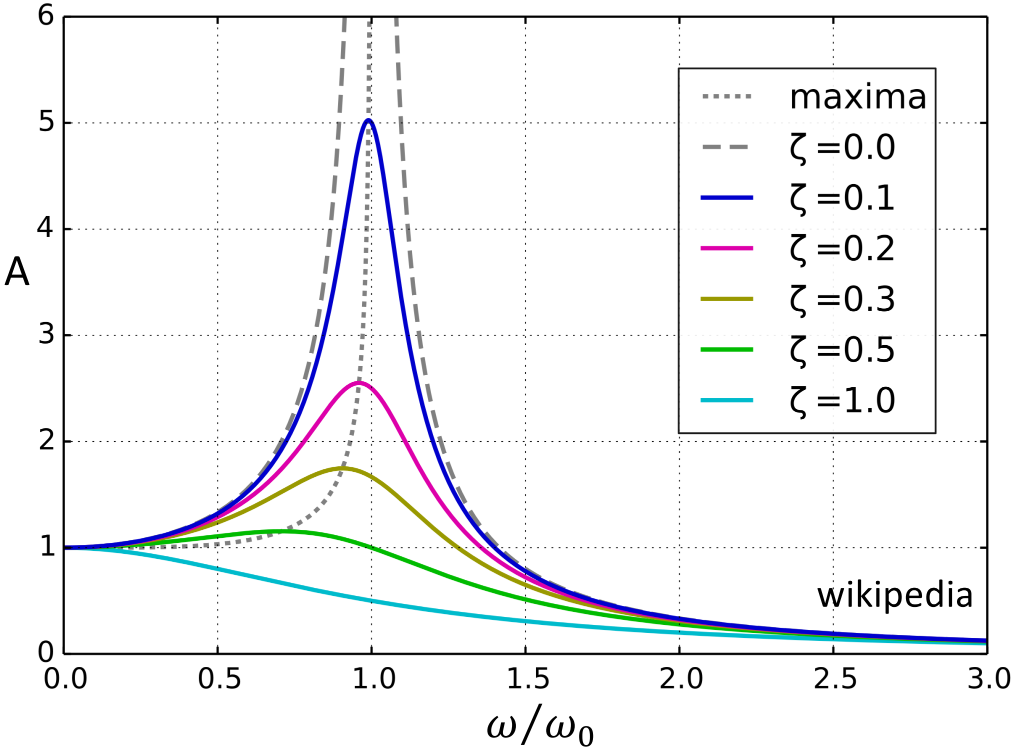
Increasing natural frequency and damping

Harmonic oscillator with external forcing



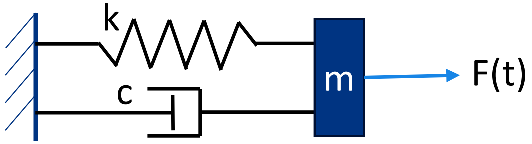
$$\ddot{a} + 2\zeta\omega_0\dot{a} + \omega_0^2 a = sin(\omega t)$$

- Vibrational response to a harmonic forcing
- Resonance when $\omega \approx \omega_0$
- Any forcing $f(t)$ can be decomposed in Fourier modes
- The structure acts as a temporal filter



wikipedia

Energy transfer to structure



- Consider a single d.o.f. undamped system: $m\ddot{x} + kx = F(t)$
- If $F(t) = 0$ energy (and amplitude of the vibration) remain constant

- Change in energy: $\int_{t_0}^{t_1} (m\ddot{x} + kx)\dot{x}dt = \int_{t_0}^{t_1} F(t)\dot{x}dt$

or: $$\left[\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right]_{t_0}^{t_1} = \int_{t_0}^{t_1} F(t)\frac{dx}{dt}dt = \int_{x_0}^{x_1} F(x)\,dx$$

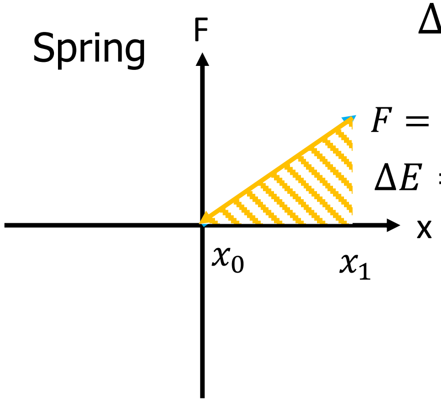
- Work done by external force is integration of force * displacement

Energy transfer from the fluid to the structure

Energy fed into the structure is the work done over the interfaces:

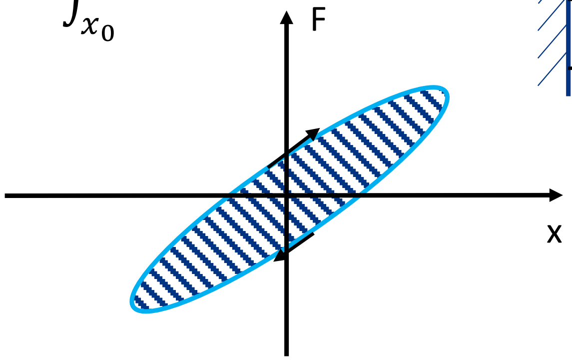
$$\Delta E = \int_{t_0}^{t_1} F \frac{dx}{dt} dt = \int_{x_0}^{x_1} F dx$$

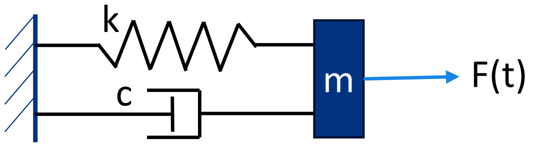
Spring



$F = kx$

$\Delta E = \frac{1}{2} k x_1^2$



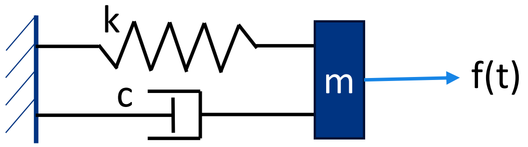


Note: moving back from x1 to x0 releases exact same amount of energy

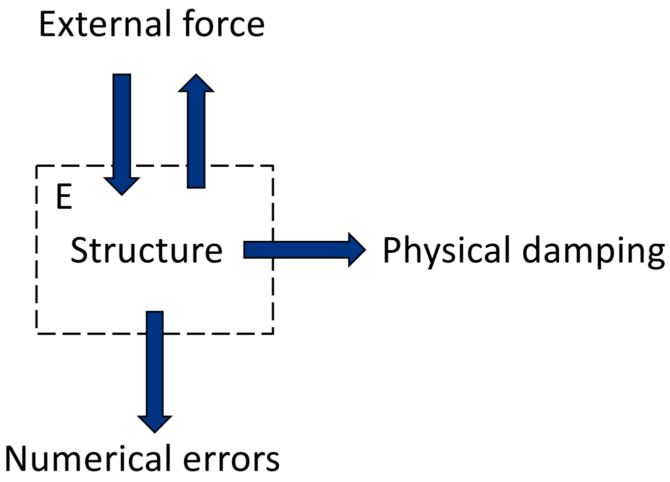
If there is a clockwise motion in the F-x diagram, energy is added to the structure

Energy balance of the structure

- Energy in a vibration mode determines its amplitude



- Energy balance:
    - Physical damping (c)
    - Numerical damping (e.g. time discretization)
    - External forcing


External force

E

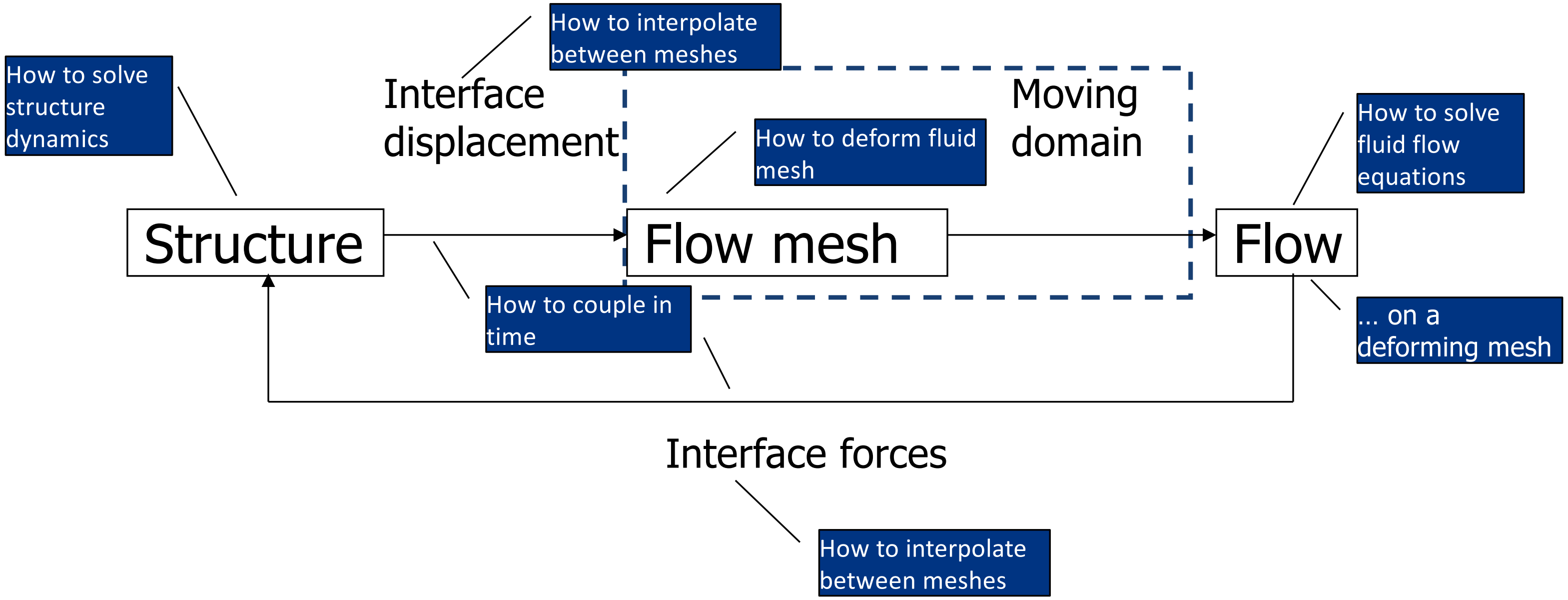
Structure

Physical damping

Numerical errors

- Net addition: amplitude increases
- Net removal: amplitude decreases

Coupling diagram of flow and structure



How to solve structure dynamics

Interface displacement

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... on a deforming mesh

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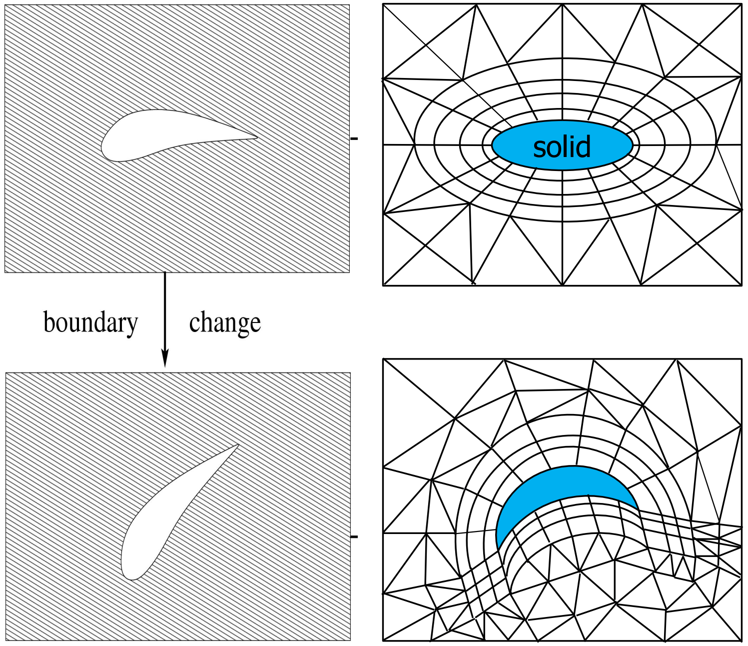
Mesh regeneration

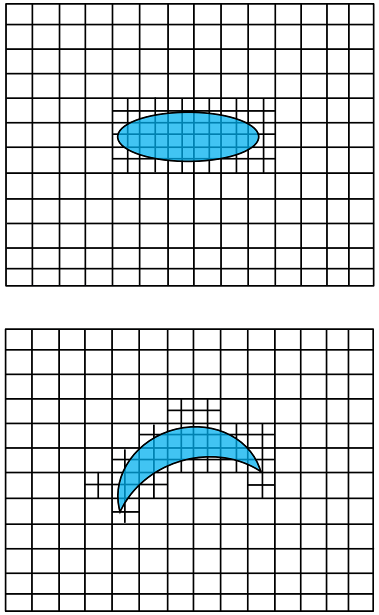


Regenerating the grid

- Time consuming
- Non-trivial:
  - generation
  - solution interpolation

+ Robust (mesh quality)
+ Account for topology changes
+ Large displacements

Immersed/embedded boundaries

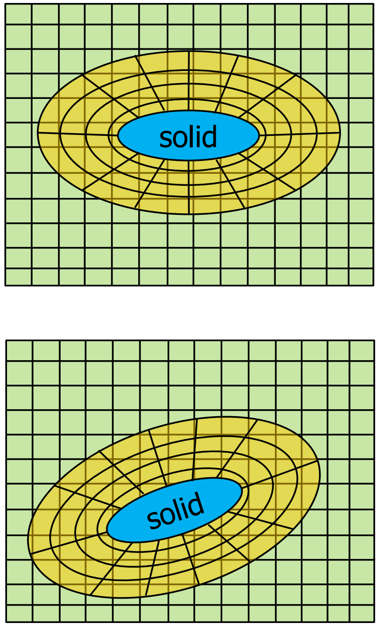


Immersed boundary treatment

+ Large deformations/displacements possible
+ Account for topology changes

- Non-trivial solution interpolation:
  - Conservation
  - Temporal relation
- Difficult to capture anisotropy in boundary layers – adaptive mesh refinement can be necessary

Overlay meshes

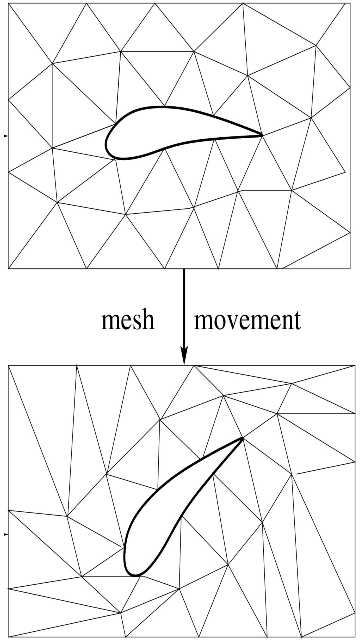


Static background mesh + moving body-conformal mesh

+ Good boundary layer quality
+ Maneuvers (large displacements/rotations)

- Interpolation between meshes:
  - Conservation errors
  - Can be expensive (time consuming)
- Need to combine with other method to account for solid shape deformation

Arbitray Langrangian-Eulerian



Mesh deformation

+ Good boundary layer quality
+ Conservative

- Limited deformation possible (mesh quality)
- Topology changes
- Can be time consuming

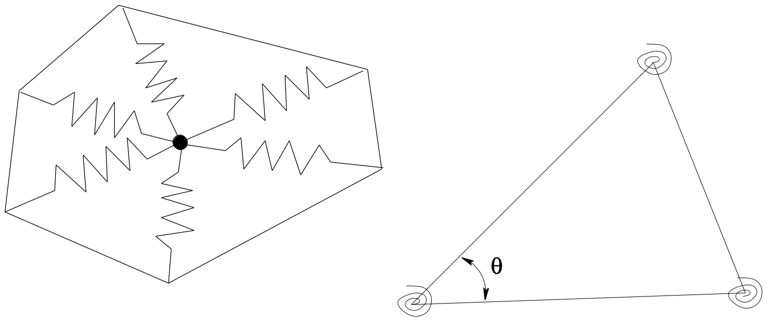
Mesh deformation

Structured meshes:

- Transfinite Interpolation: interpolating along gridlines

Unstructured meshes:

- Structure analogy: Spring analogy, solid body elasticity
- Solving a PDE: Laplace smoothing, Biharmonic operator
- Using interpolation functions (e.g. radial basis functions)



Typical “pseudo-structure” representation: $\vec{d}_{in} = K(\vec{x}_{in}, \vec{x}_b)\vec{d}_b$

$\vec{d}_{in}$: Internal node displacements

$\vec{d}_b$: Boundary node displacements

$\vec{x}_{in}$: Internal node location

$\vec{x}_b$: Boundary node location

K : Pseudo stiffness matrix

Absolute vs. relative displacement

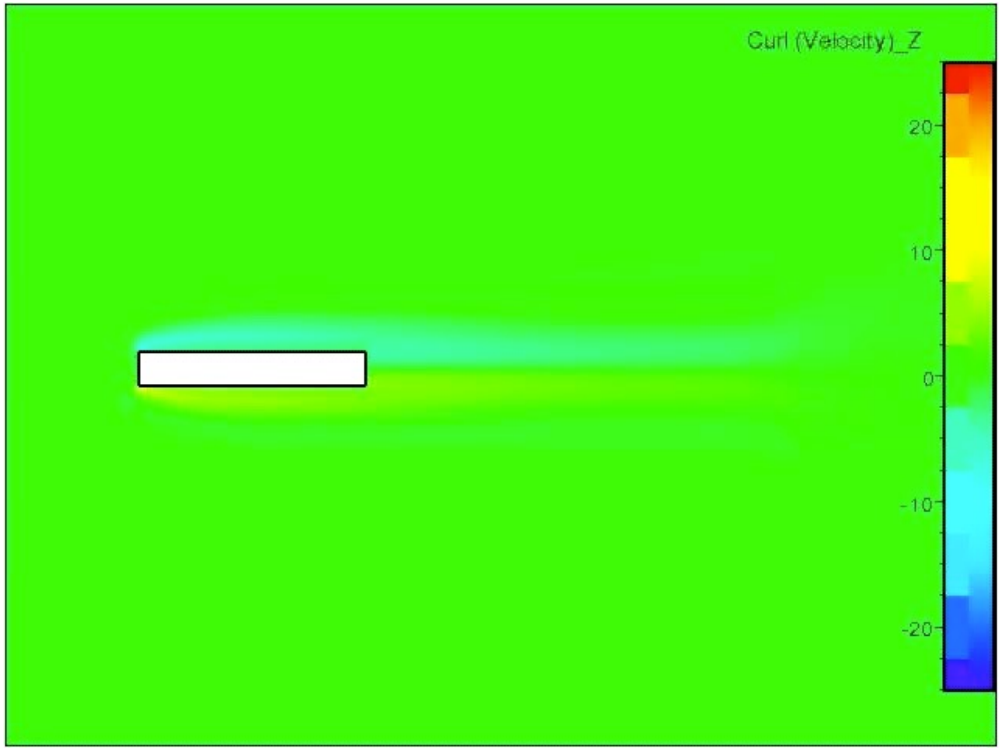
- Deformation of the mesh can be defined with respect to the previous mesh location (relative displacement $\vec{\delta} = \vec{x}^{n+1} - \vec{x}^{n}$) or the initial mesh location (absolute displacement $\vec{d} = \vec{x}^{n+1} - \vec{x}^{0}$)

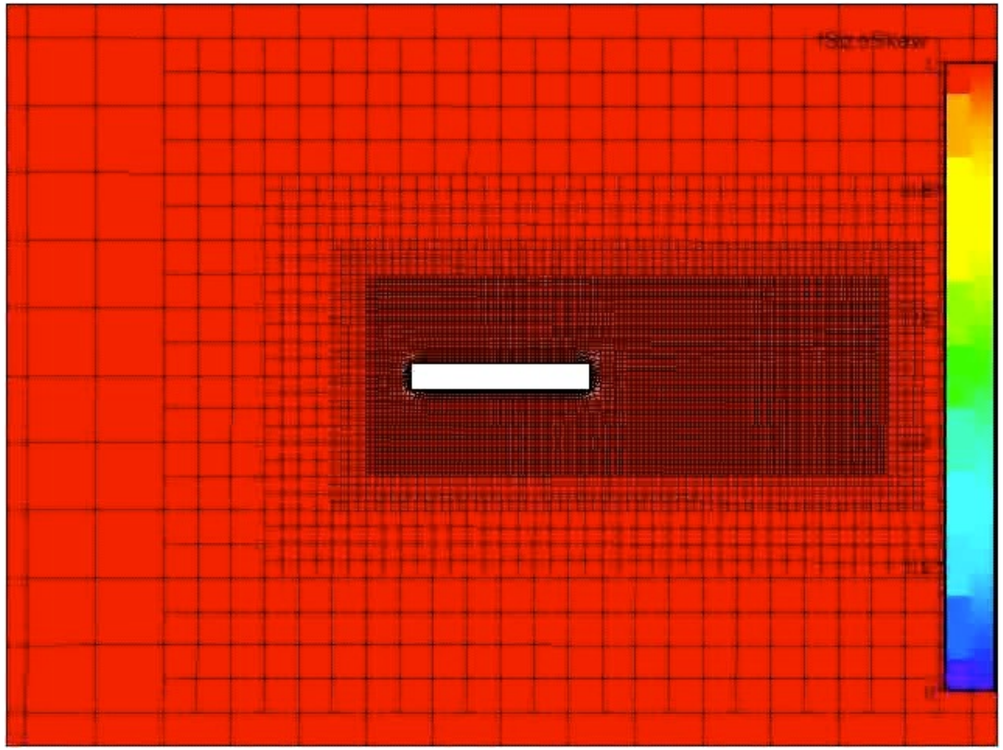
- Absolute: $\vec{d}_{in}^{n+1} = K\left(\vec{x}_{in}^{0}, \vec{x}_{b}^{0}\right)\vec{d}_{b}^{n+1}$ Note: *K* is constant
- Relative: $\vec{\delta}_{in}^{n+1} = K(\vec{x}_{in}^{n}, \vec{x}_{b}^{n})\vec{\delta}_{b}^{n+1}$ Note: *K* changes

- Relative mesh deformation can handle large displacements better
- Absolute mesh deformation preserves original mesh when returning to initial position

Example mesh motion using relative displacement (deform from previous mesh)

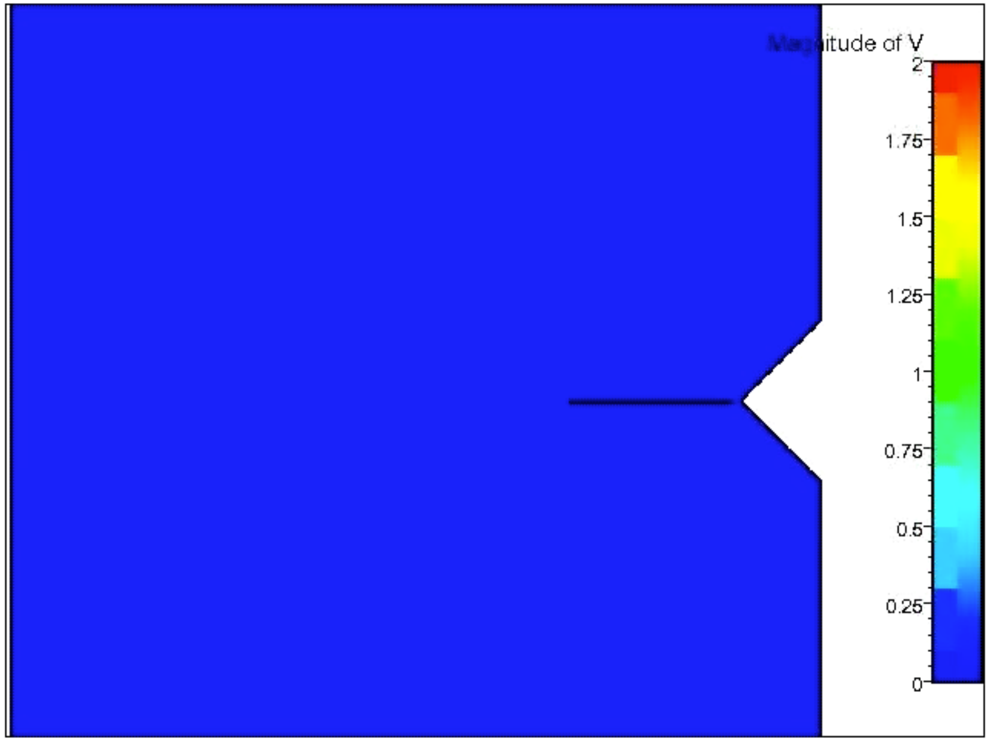
Mesh quality shows a continuous deterioration in time for this oscillating motion

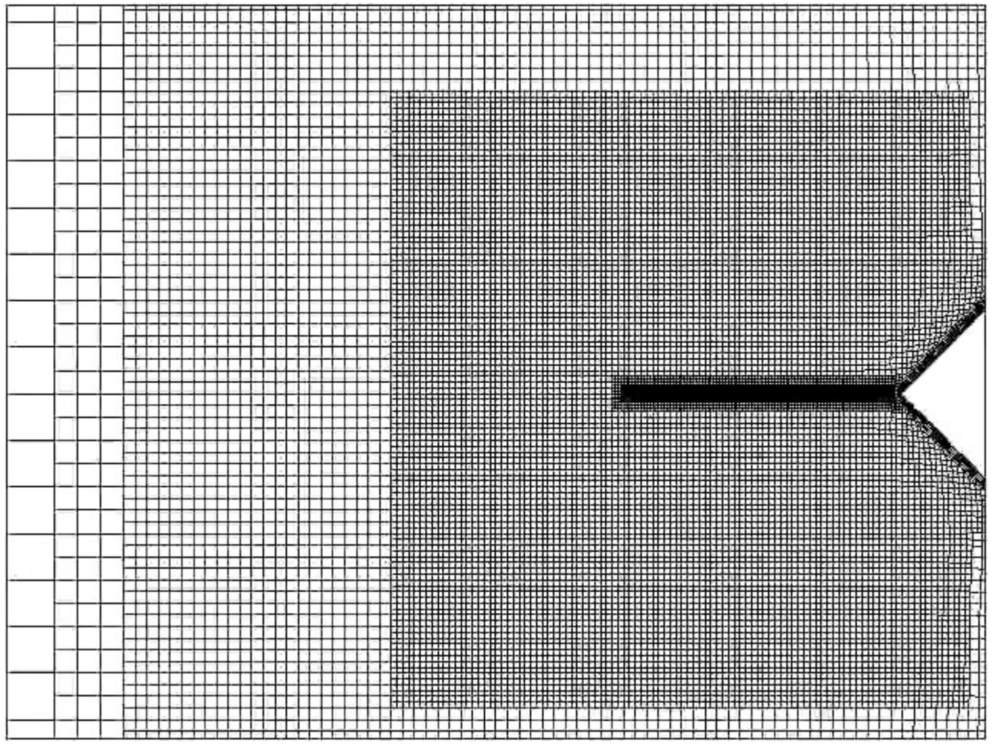




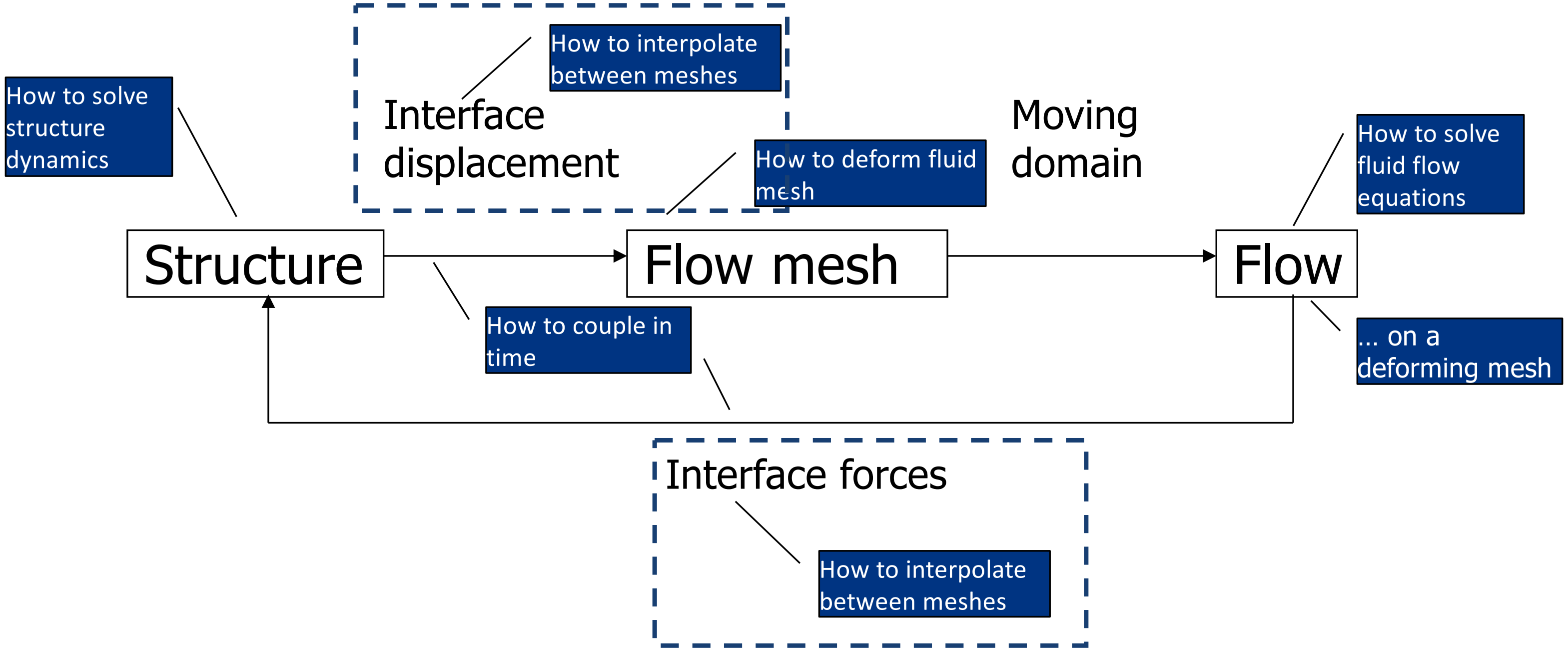
Example mesh motion using absolute displacement (deform from initial mesh)

Mesh shows a constant mesh quality variation for this oscillating motion





Coupling diagram of flow and structure



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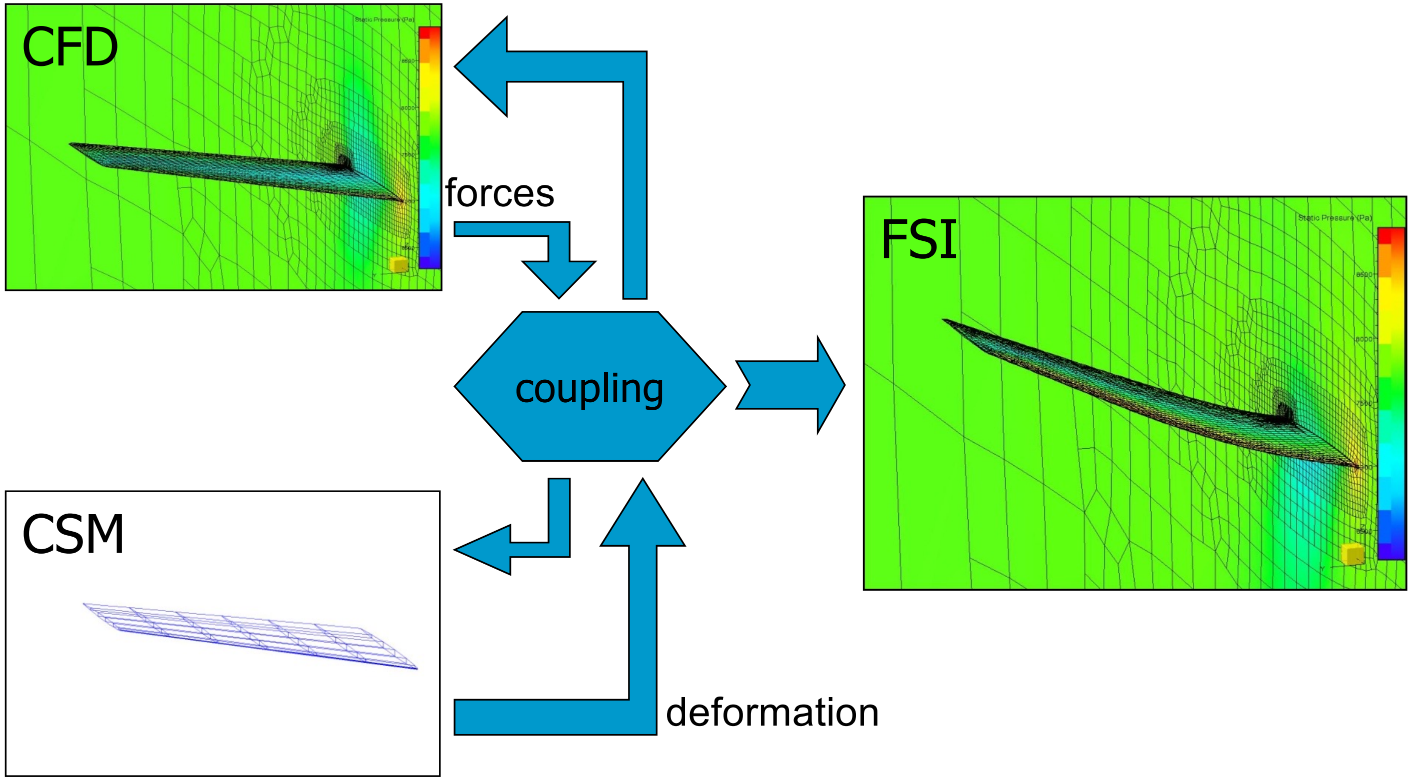
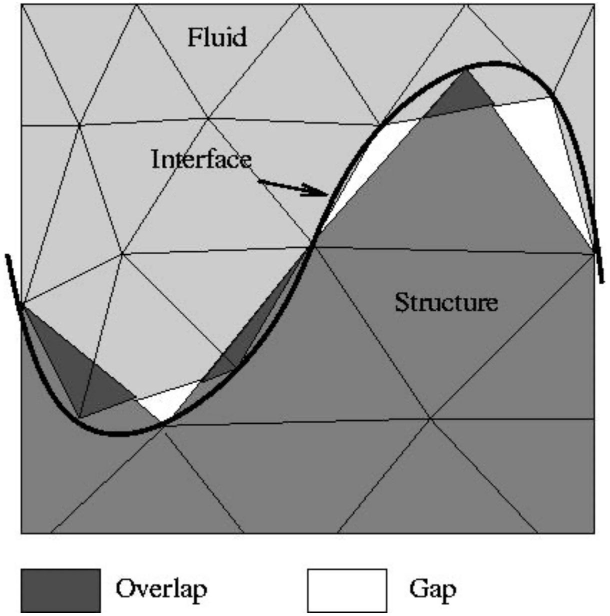
Different solvers may use different meshes



Non-matching meshes

Problem: Grids do not have to match at the interface



Exchange of stresses:
fluid ⇒ structure

Exchange of displacements:
structure ⇒ fluid

↓

Interpolation/projection needed

Consistent and conservative interpolation

Kinematic and dynamic interface conditions

$$\mathbf{u}_f = \mathbf{u}_s \qquad \xrightarrow{\text{Discretization}} \qquad \mathbf{U}_f = H_{sf}\mathbf{U}_s.$$

$$p_s\mathbf{n}_s = p_f\mathbf{n}_f \qquad\qquad\qquad \mathbf{P}_s = H_{fs}\mathbf{P}_f$$

Different possibilities for the set up of the transformation matrices: Nearest Neighbor, Weighted Residual, Radial Basis Function Interpolation.

Consistent interpolation when constant displacement and constant pressure are exactly recovered $\longrightarrow$ rowsum of H is equal to one.

Consistent and conservative interpolation

Exchange of displacements with a transformation matrix

$$\mathbf{U}_f = H_{sf}\mathbf{U}_s.$$

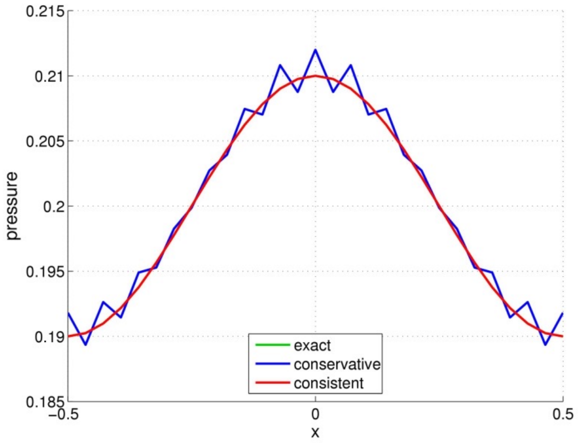
Conservation of the change in work at the interface

$$\partial W_f = \partial W_s \qquad \text{with} \qquad \partial W = \mathbf{F}^T\mathbf{U}.$$

This gives the following exchange of pressure forces:

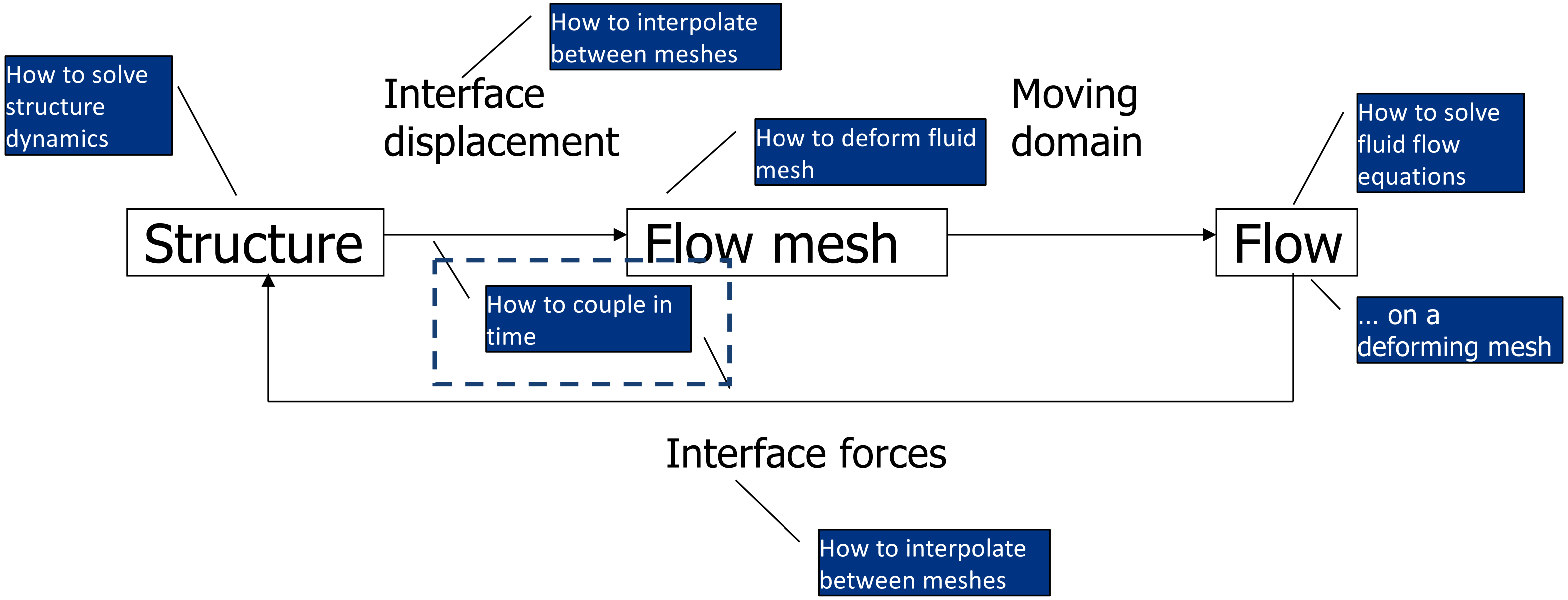
$$\mathbf{F}_s = H_{sf}^T\mathbf{F}_f \qquad \text{with} \qquad \mathbf{F} = M^T\mathbf{P}$$

Then for the pressure yields: $\mathbf{P}_s = \underbrace{\left[M_f H_{sf} M_s^{-1}\right]^T}_{H_{fs}} \mathbf{P}_f.$ $\rightarrow$ Rowsum generally not equal to one!



- Non-physical oscillations in pressure received by structure with conservative approach.

Coupling diagram of flow and structure



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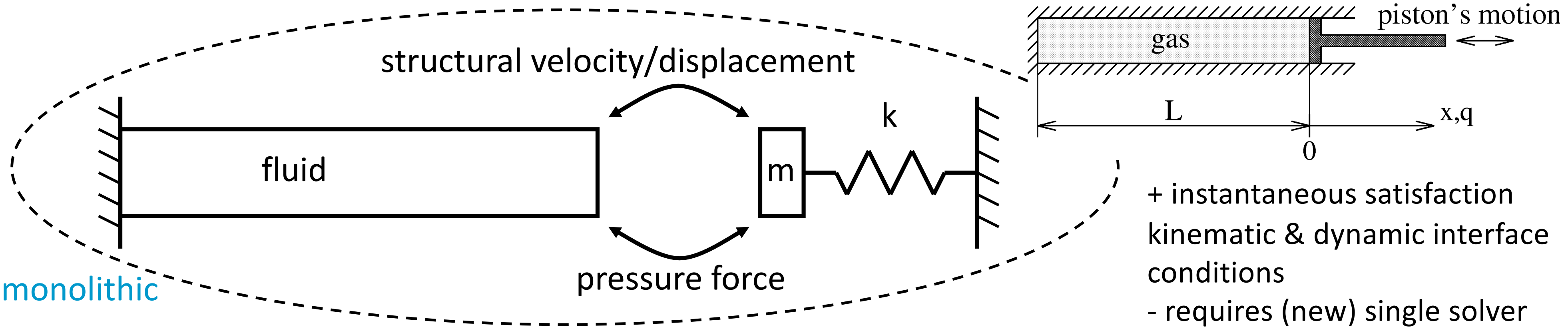
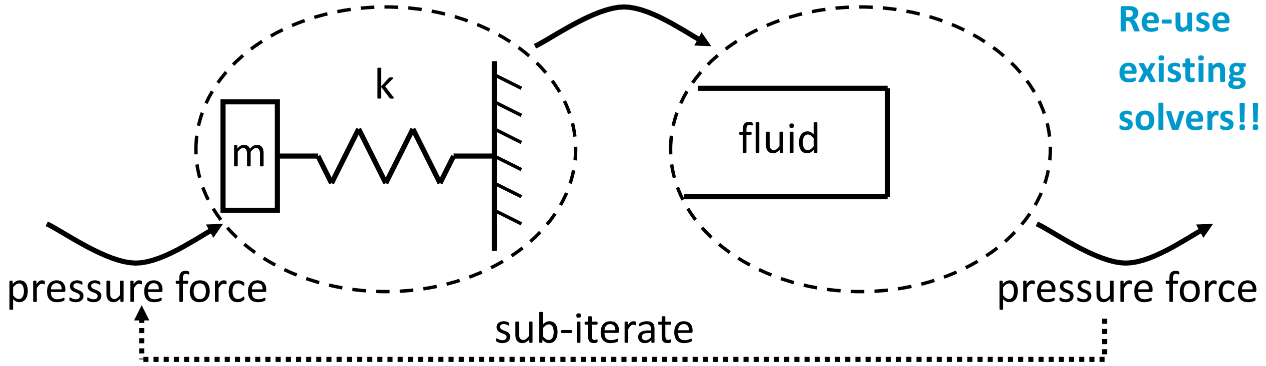
Monolithic vs. partitioned coupling



+ instantaneous satisfaction kinematic & dynamic interface conditions

- requires (new) single solver

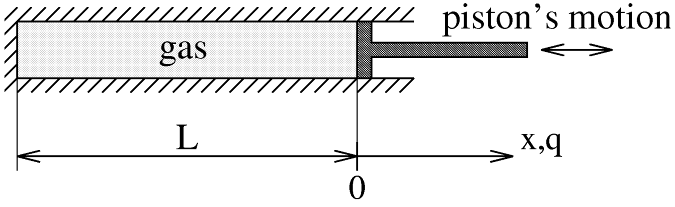


Re-use existing solvers!!!

+ reuse existing solvers

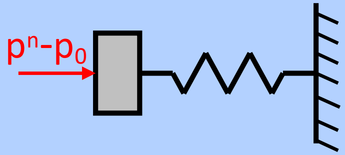
- Interface conditions not instantaneously satisfied (partitioning error)

Partitioning error



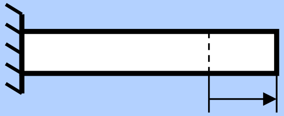
- Error in the equilibrium of forces

a) solve structure



$m\ a^{n+1} + k\ q^{n+1} = p^n - p_0$

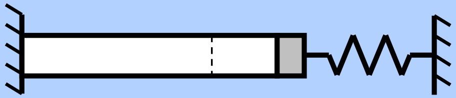
b) solve fluid



$u^{n+1} = v^{n+1}$

$x^{n+1} = q^{n+1}$

Evaluate coupling at t^{n+1}



$u^{n+1} = v^{n+1}\ ,\ x^{n+1} = q^{n+1}$

$m\ a^{n+1} + k\ q^{n+1} \neq p^{n+1} - p_0$

Black-box solver approach

- In partitioned coupling, flow and structure solver are considered as black-boxes

- The structure and flow solver are given by

$$\mathbf{d} = S(\mathbf{p})$$

$$\mathbf{p} = F(\mathbf{d})$$

d : fluid-structure interface displacement

p : fluid-structure interface stress/pressure

Fully coupled: satisfy all interface conditions simultaneously

- Ensure equilibrium on the fluid-structure interface

- For black-box solvers would require:

$$\mathbf{d}^{n+1} = S\left(\mathbf{p}^{n+1}\right)$$

$$\mathbf{p}^{n+1} = F\left(\mathbf{d}^{n+1}\right)$$

Chicken-and-Egg problem

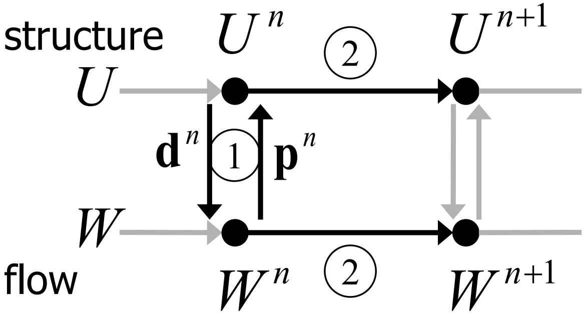
d : fluid-structure interface displacement
p : fluid-structure interface stress/pressure

- **Cannot be solved directly: need coupling iterations!**

Loosely coupled methods

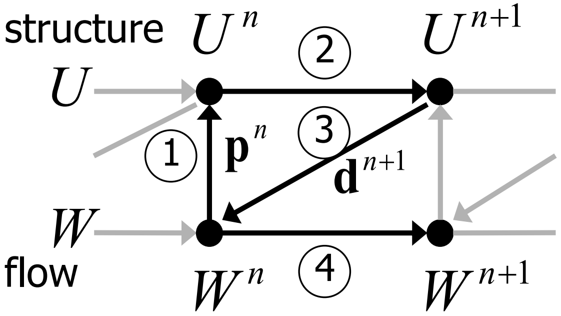
Jacobi iteration (parallel)

$$\mathbf{d}^{n+1} = S(\mathbf{p}^{n})$$
$$\mathbf{p}^{n+1} = F(\mathbf{d}^{n})$$



Gauss-Seidel (serial)

$$\mathbf{d}^{n+1} = S(\mathbf{p}^{n})$$
$$\mathbf{p}^{n+1} = F(\mathbf{d}^{n+1})$$



Time-lag in
the interface
conditions

Basic sub-iteration methods

Jacobi iteration (parallel)

$$\mathbf{d}^{k+1} = S\left(\mathbf{p}^{k}\right)$$

$$\mathbf{p}^{k+1} = F\left(\mathbf{d}^{k}\right)$$

Gauss-Seidel (serial)

$$\mathbf{d}^{k+1} = S\left(\mathbf{p}^{k}\right)$$

$$\mathbf{p}^{k+1} = F\left(\mathbf{d}^{k+1}\right)$$

Can be written as a fixed-point iteration, e.g. Gauss-Seidel:

$$\tilde{\mathbf{p}}^{k} = F \circ S\left(\mathbf{p}^{k}\right)$$

with an interface residual

$$\mathbf{r}^{k} = F \circ S\left(\mathbf{p}^{k}\right) - \mathbf{p}^{k} = \tilde{\mathbf{p}}^{k} - \mathbf{p}^{k}$$

Or as minimization problem for the interface residual operator:

$$R(\mathbf{p}) = F \circ S(\mathbf{p}) - \mathbf{p}$$

Increase stability of coupling iterations: underrelaxation

Coupling iteration: $\tilde{\mathbf{p}}^k = F \circ S\left(\mathbf{p}^k\right)$ $\mathbf{r}^k = \tilde{\mathbf{p}}^k - \mathbf{p}^k$

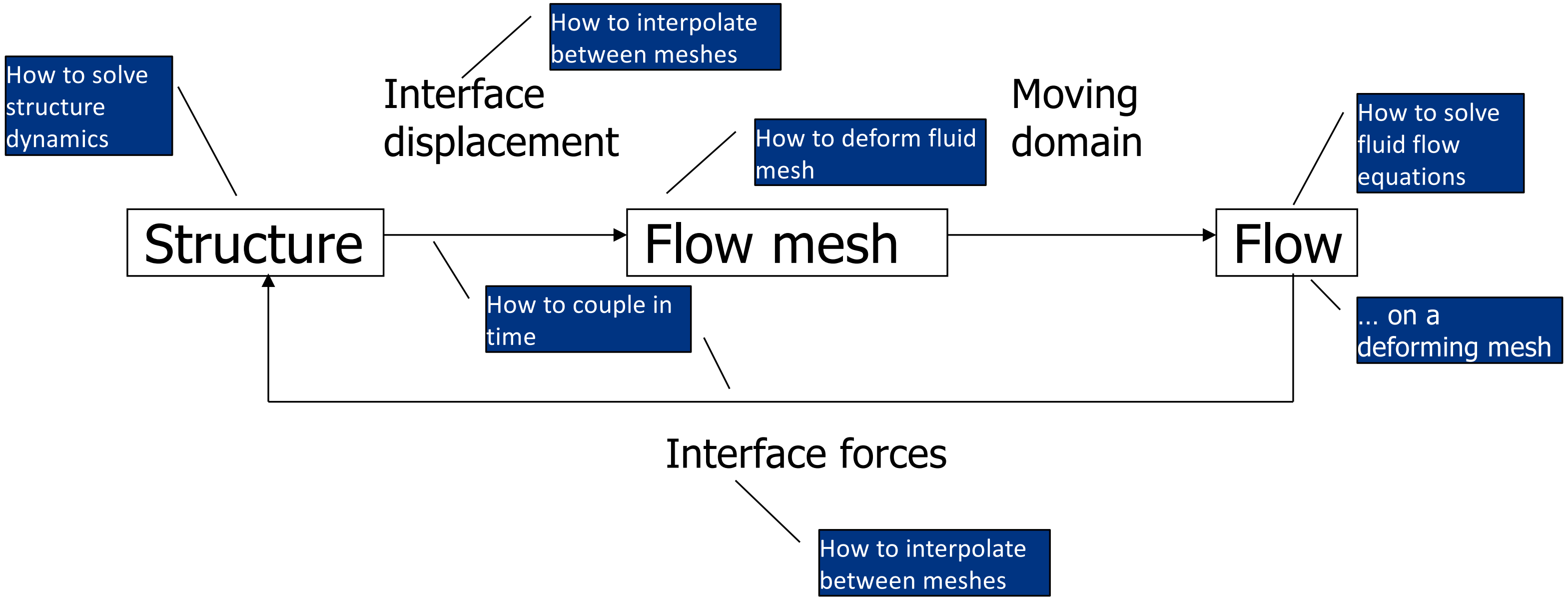
Gauss-Seidel: $\mathbf{p}^{k+1} = \tilde{\mathbf{p}}^k$

Under-relaxation: $\mathbf{p}^{k+1} = \mathbf{p}^k + \omega\left(\tilde{\mathbf{p}}^k - \mathbf{p}^k\right)$

Adaptive under-relaxation (Aitken's method):

$$\omega^k = -\omega^{k-1} \frac{\left\langle \left(\mathbf{r}^{k-1}\right), \left(\mathbf{r}^k - \mathbf{r}^{k-1}\right) \right\rangle}{\left\langle \left(\mathbf{r}^k - \mathbf{r}^{k-1}\right), \left(\mathbf{r}^k - \mathbf{r}^{k-1}\right) \right\rangle}$$

Coupling diagram of flow and structure



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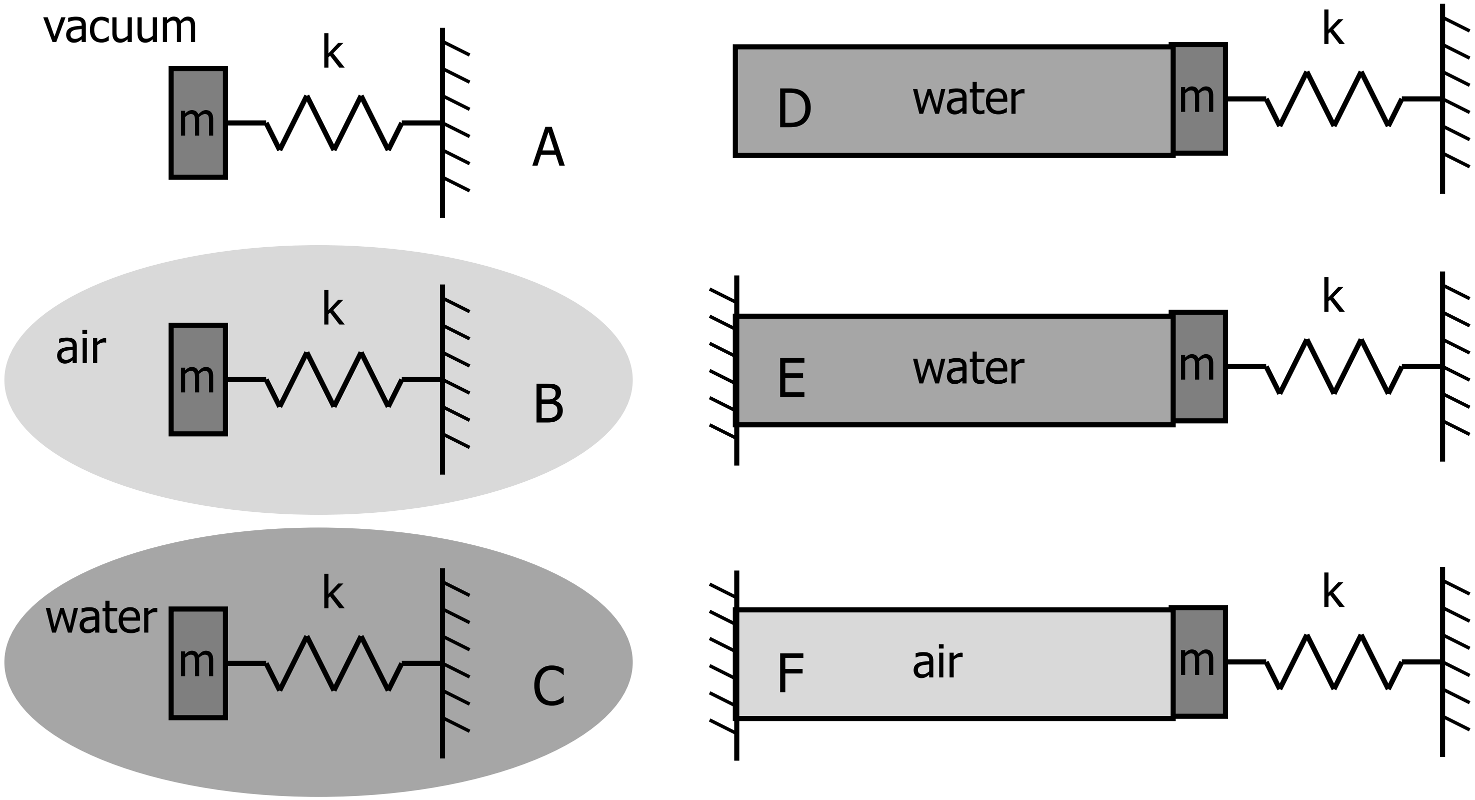
Summary

- Three field problem: Flow, Structure, Mesh:
  - Flow is solved on a moving/deforming mesh in ALE formulation
  - Structure vibration behaviour similar to harmonic oscillator
  - Mesh can be deformed with repect to its previous or initial state
- Using interpolation between meshes to transfer loads and displacements
- Satisfying the kinematic and dynamic interface conditions using partitioned black-box approach:
  - Loosely coupled – flow and structure only solved once: partitioning error
  - Strongly coupled – sub-iterations with(out) underrelaxation

Physics of fluid-structure interaction

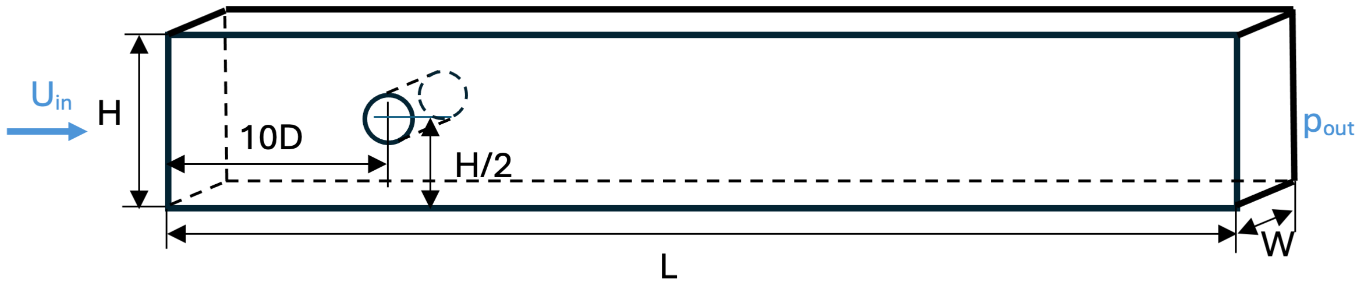
- Added mass, damping, stiffness effects determines the strength of the interaction between flow and structure
  - Added mass: fluid exerts a force (opposite to and) relative to the structural acceleration
  - Added damping: fluid exerts a force (opposite to and) relative to the structural velocity
  - Added stiffness: fluid exerts a force (opposite to and) relative to the structural displacement
- Negative (aero/fluiddynamic) damping can result in a physical instability
- Added mass and stiffness change the vibration frequency of the structure and can result in a numerical instability
- Strong interaction effects require strong coupling algorithms

Intuitive examples of coupling effects



Example: Vortex Induced Vibration

Fluid



- Air at 25°C
- U_{in} = 0.03 m/s
- P_{out} = 0 Pa

- H = 10D
- L = 60D
- W = 1D
- D = 0.1m

$$\rho_f = 1.185 \text{ kg/m}^3$$

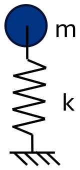
$$\boxed{\frac{\rho_s}{\rho_f} = 1.07 \text{ or } \frac{\rho_f}{\rho_s} = 0.93}$$

Strouhal: $St = \frac{fD}{U}$

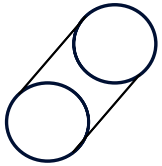
$$\Longrightarrow f_f = 0.2\frac{U}{D} = 0.06\text{Hz}$$

$$\boxed{\frac{f_s}{f_f} \approx \frac{f_f}{f_s} \approx 1.0}$$

Structure



- m = 0.001 kg
- k = 1.42e-4 N/m



$$V_s = \frac{\pi}{4}D^2D = 7.85\text{e-4 m}^3$$

$$\rho_s = \frac{m}{V} = 1.273 \text{ kg/m}^3$$

Natural frequency: $\omega = \sqrt{\frac{k}{m}}$

$$\Longrightarrow \omega = 0.377\text{rad/s}$$

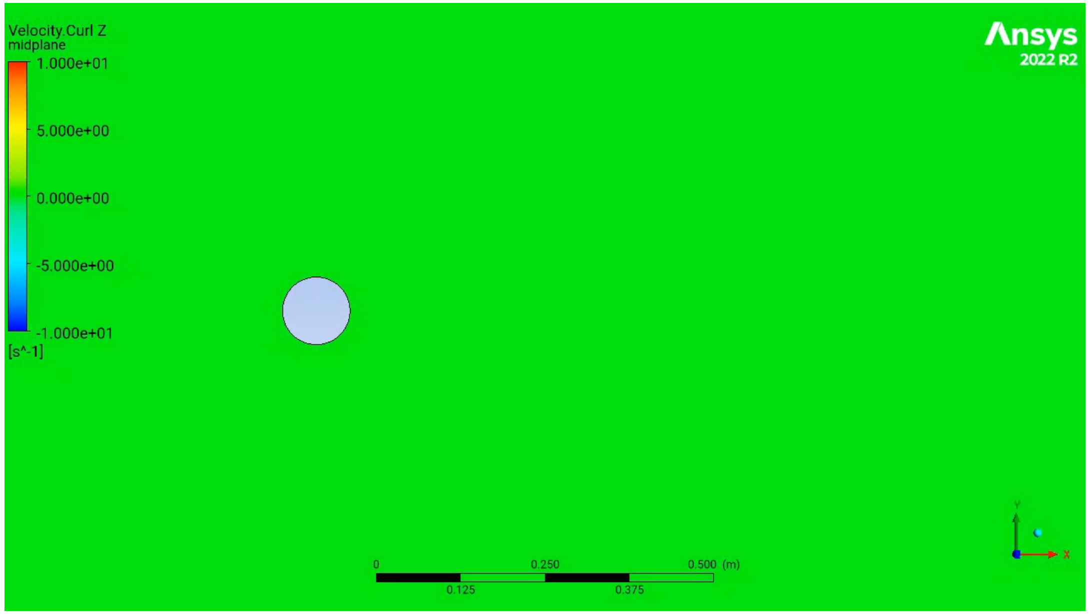
$$\Longrightarrow f_s = \frac{\omega}{2\pi} = 0.0599\text{Hz}$$

Fluid shedding frequency related to Strouhal

Rigid cylinder – Vortex shedding with frequency f

D: Cylinder diameter

Uinf: Incoming flow velocity



Fluid

Strouhal:

St = (f D) / Uinf

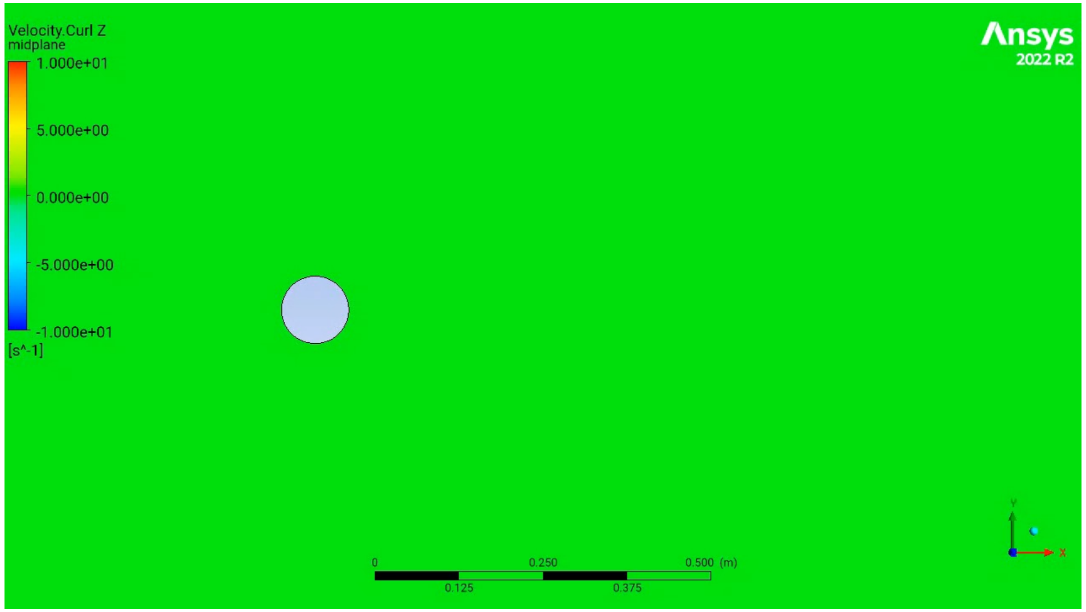
St ≈ 0.22

Heavy structure, shedding frequency >> natural frequency

Cylinder of mass m suspended with spring of stiffness k

D: Cylinder diameter

Uinf: Incoming flow velocity



Fluid

Strouhal:

St = (f D) / Uinf

St ≈ 0.22

Structure

Mass:

$m >> \rho_f \pi/4\, D^2$

$\omega = \sqrt{(k/m)}$

k such that:

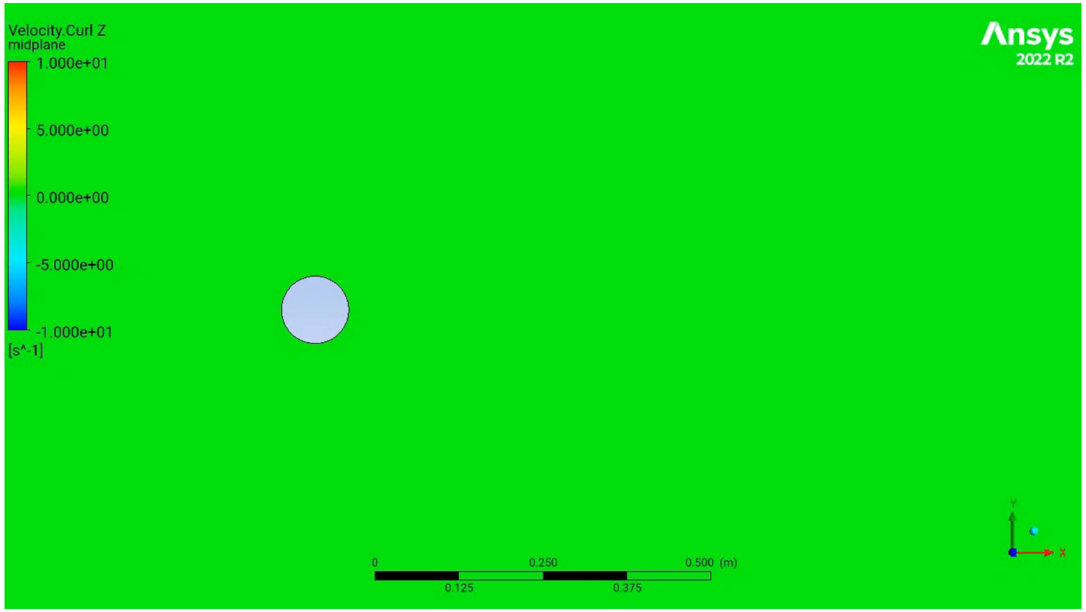
$\omega << 2\pi f$

Heavy structure, shedding frequency ≈ natural frequency

Cylinder of mass m suspended with spring of stiffness k

D: Cylinder diameter

Uinf: Incoming flow velocity



Fluid

Strouhal:

St = (f D) / Uinf

St ≈ 0.22

Structure

Mass:

$m >> \rho_f \pi/4 D^2$

$\omega = \sqrt{(k/m)}$

k such that:

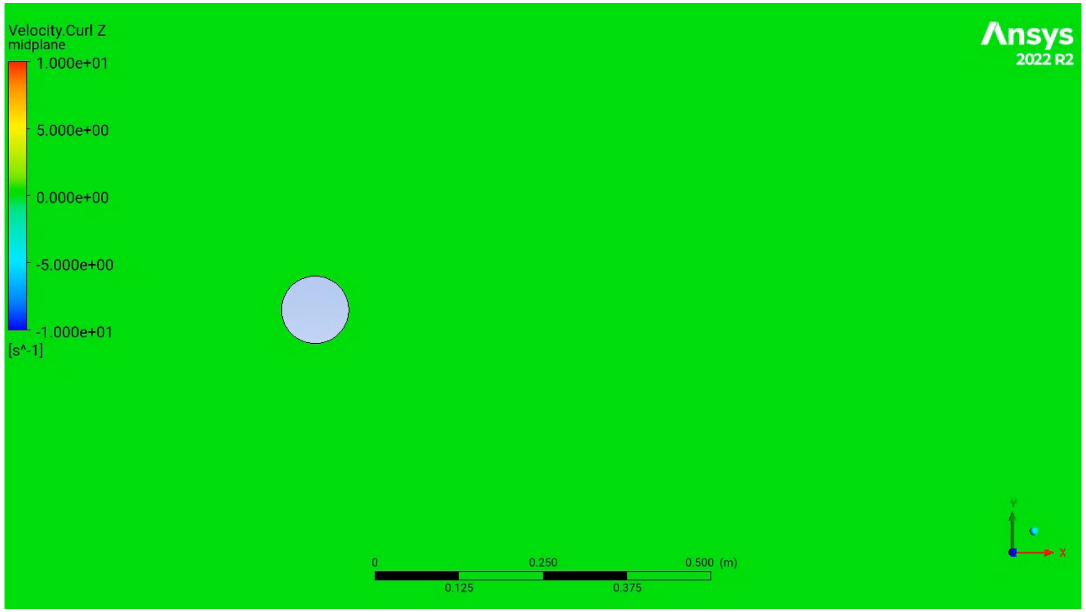
$\omega \approx 2 \pi f$

Lightweight structure, shedding frequency ≈ natural frequency

Cylinder of mass m suspended with spring of stiffness k

D: Cylinder diameter

Uinf: Incoming flow velocity



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St = (f D) / Uinf

St ≈ 0.22

Structure

Mass:

$m \approx \rho_f \, \pi/4 \, D^2$

$\omega = \sqrt{(k/m)}$

k such that:

$\omega \approx 2 \, \pi \, f$

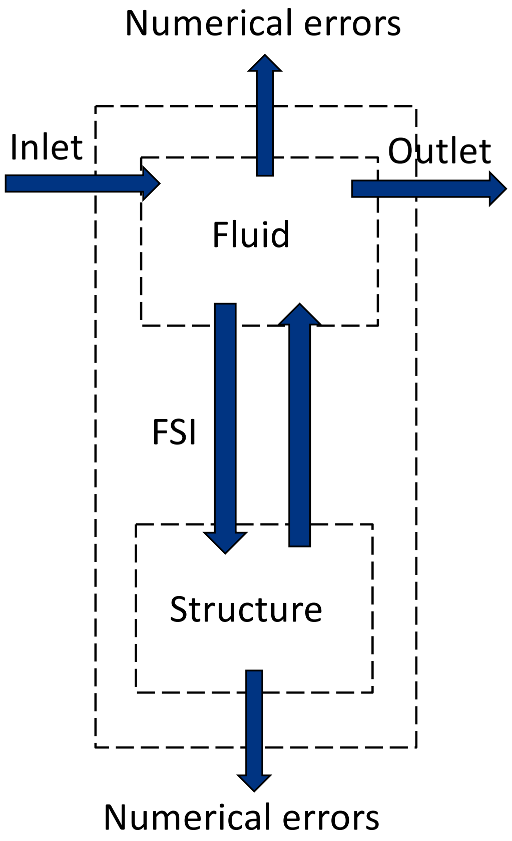
Energy analysis of the FIV system

- Exchange of energy over the fluid-structure interface

- The fluid has an inlet b.c. that allows new energy to enter, and an outlet b.c. that removes energy from the system.
- The fluid can also dissipate energy because of numerical discretization errors.

- The structure has no physical damper, therefore would not remove energy from the system, apart from numerical discretization errors

- The motion/amplitude of the structure is therefore governed by the work done by the fluid on the structure: if the fluid adds energy to the structure, its amplitude increases, and when the fluid extracts energy from the structure its amplitude decreases.



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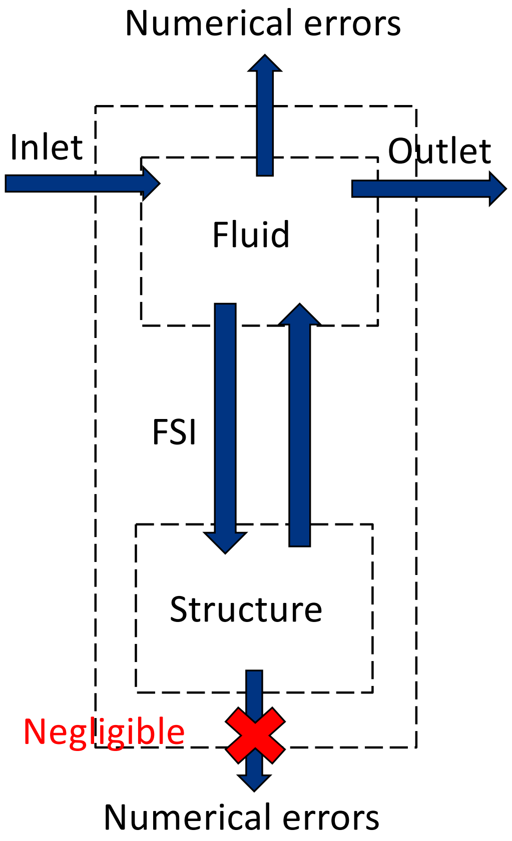
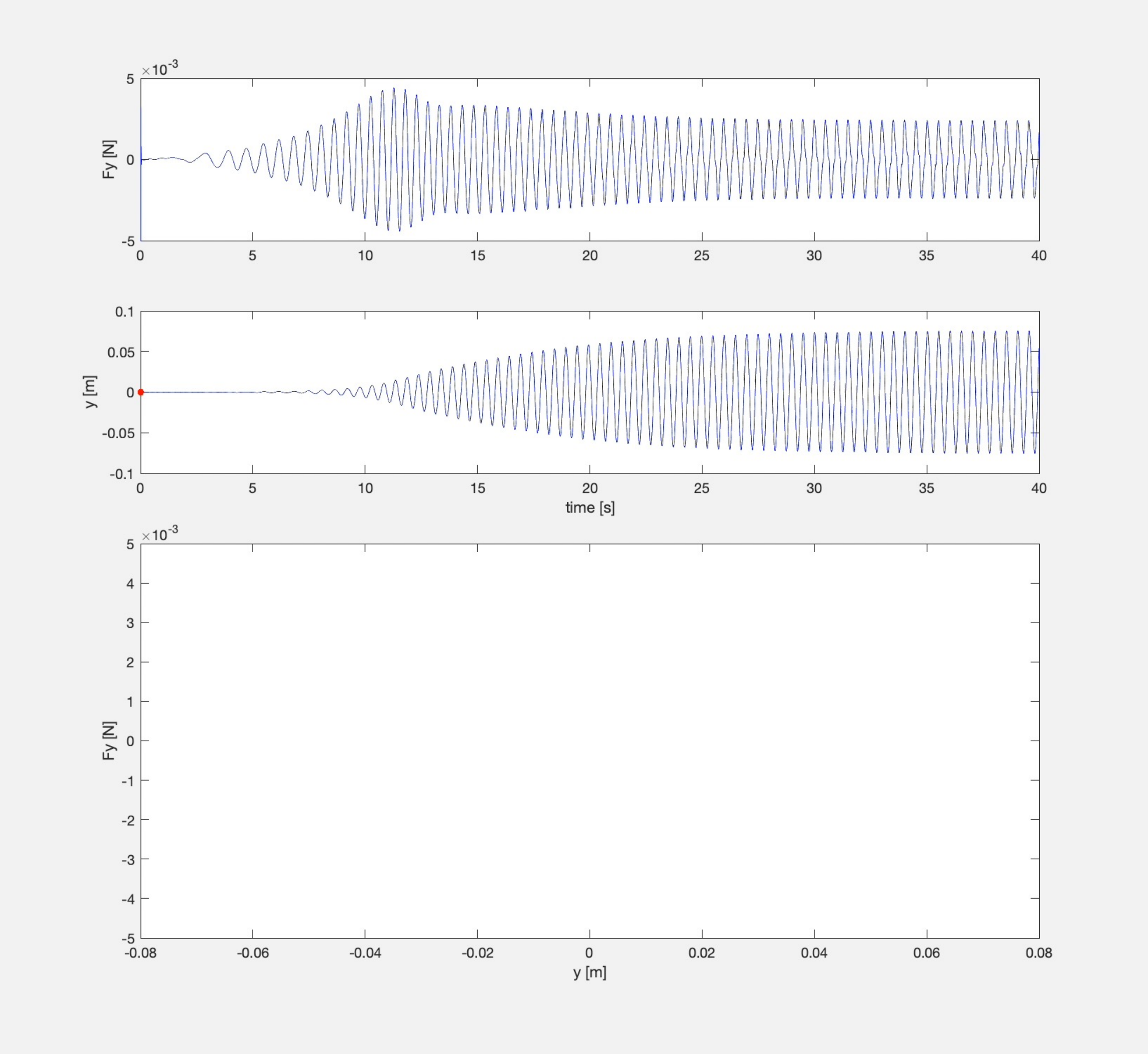
- The motion/amplitude of the structure is therefore governed by the work done by the fluid on the structure: if the fluid adds energy to the structure, its amplitude increases, and when the fluid extracts energy from the structure its amplitude decreases. Only when the net work over one period is equal to zero, the structure amplitude remains constant





Fluid

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St ≈ 0.22

Structure

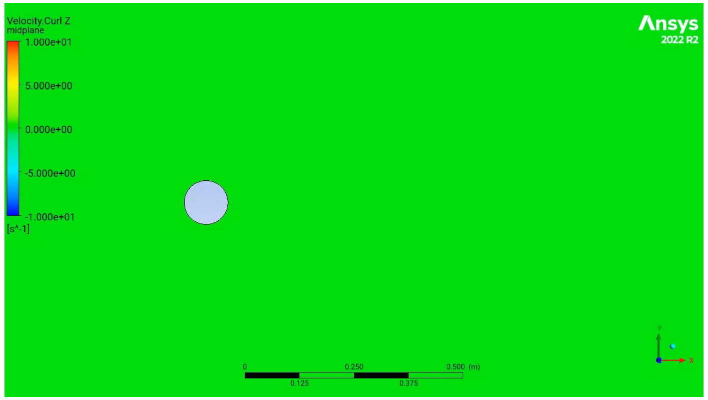
Mass:

$m \approx \rho_f \, \pi/4 \, D^2$

$\omega = \sqrt{(k/m)}$

k such that:

$\omega \approx 2 \, \pi \, f$

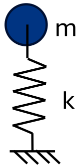


Free vibration in still flow

- Same domain, but no in/outflow condition
- Structure is at an initial displacement
- Observe the structur vibration when released
- Medium is either vacuum, air or water
- Structure mass and stiffness varied (keeping natural frequency constant), but changing the mass (density) ratio:

$$m^* = \frac{\rho_f}{\rho_s}$$

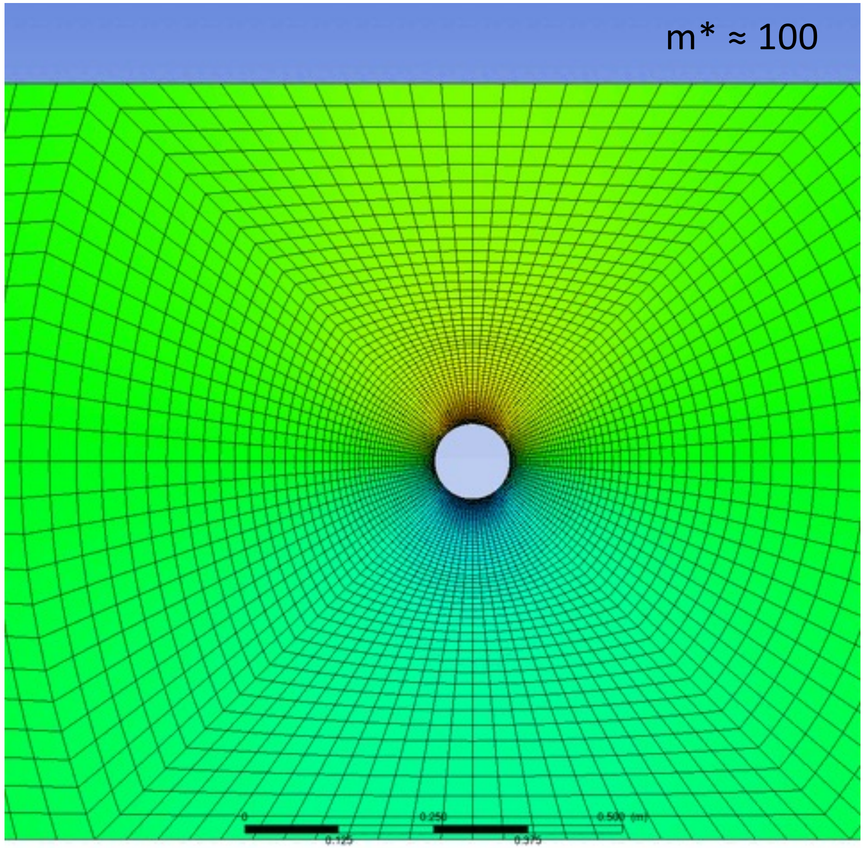
Structure



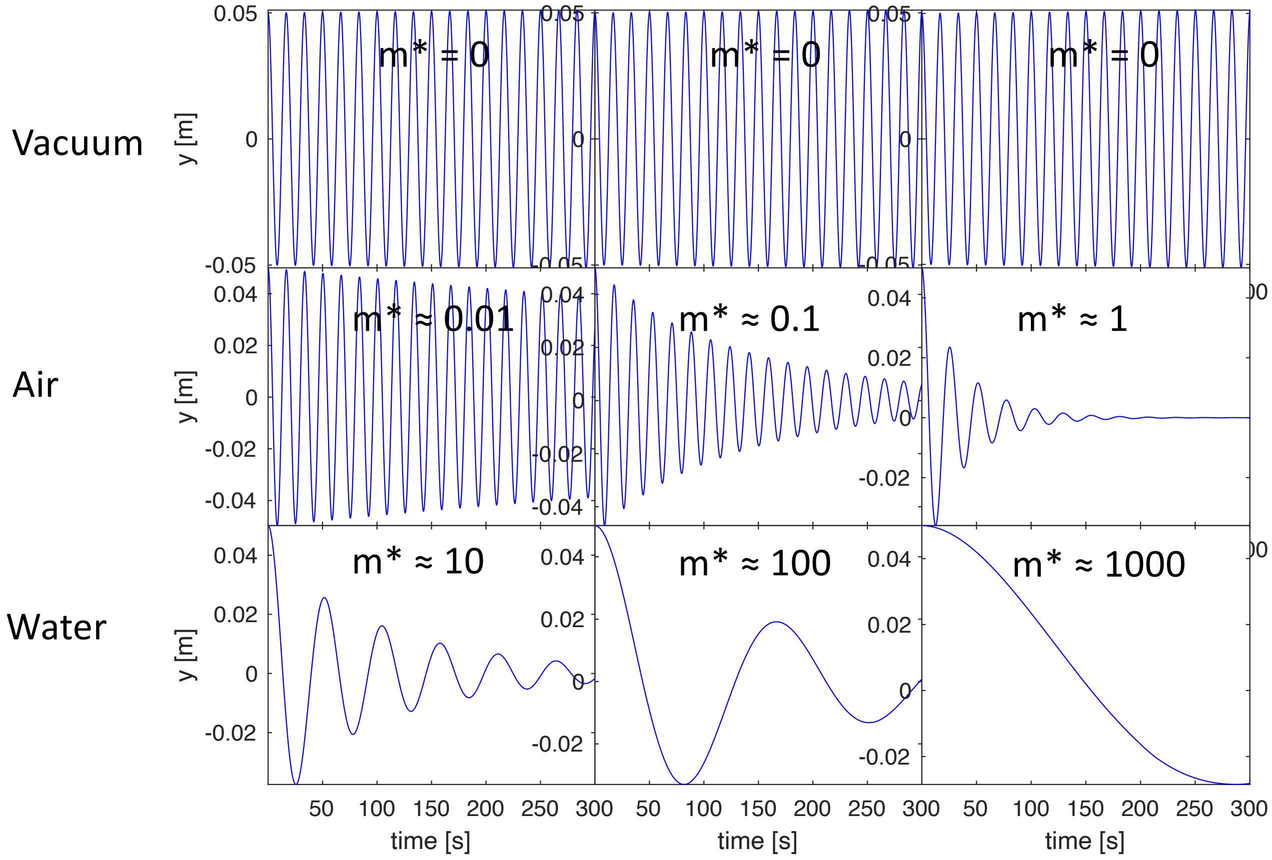
- m = 0.001 kg
- k = 1.42e-4 N/m

Instability for high m* values

- Added mass effect creates a numerical instability:
  - Structure accelerates based on the spring force initially
  - The acceleration creates a larger opposing force from the fluid
  - This results in an even larger opposite acceleration
- Need strong coupling (sub-iterations)
- For very high m*, we need underrelaxation as well



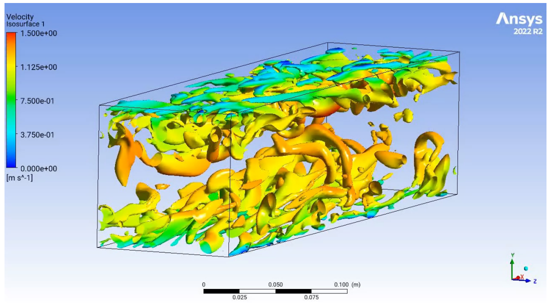
Effect of added mass and damping clearly visible in response frequency and decay

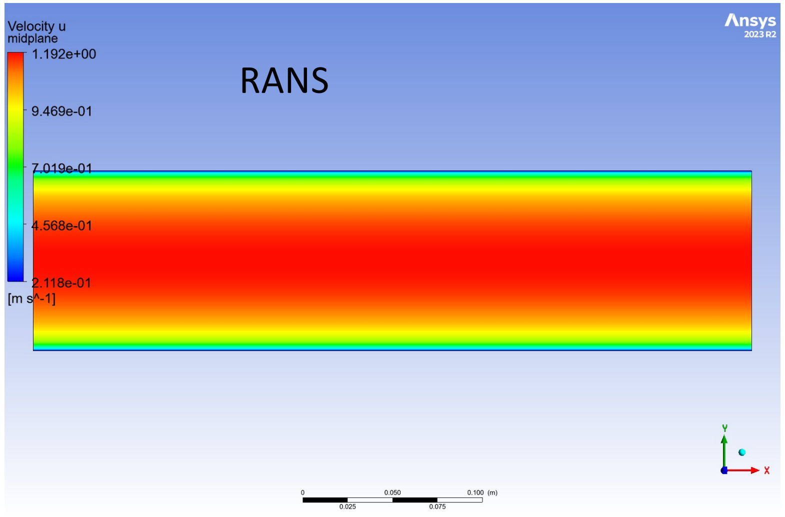


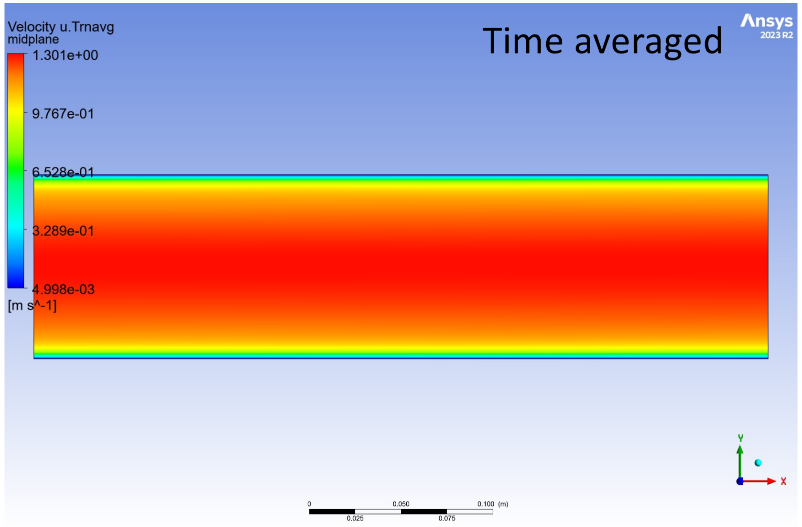
- Structure response is according to harmonic oscillator in vacuum
- For small mass ratios the simulation can be solved loosely coupled
- For high mass ratios the strong coupling (with underrelaxation) is required

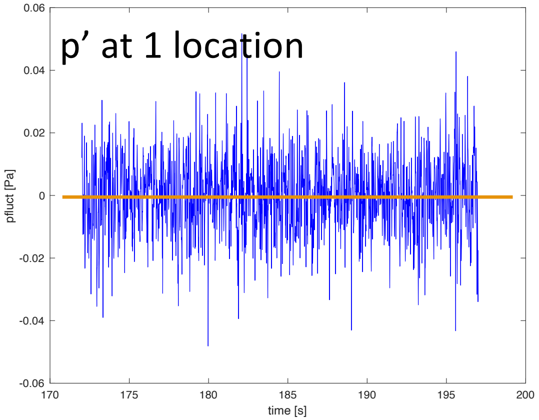
Example: Turbulence Induced Vibration

- When solving turbulence using a RANS approach, we get good mean flow proprties, but we loose the turbulent pressure fluctuations
- If the structure eigenfrequency is close to some spectral content of p’, this may feed very effectively energy into the structure that (U)RANS fails to predict




RANS


Time averaged


p’ at 1 location

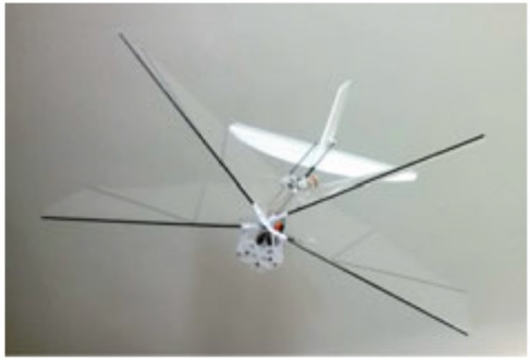
Challenges for validation

- Structure model is in vacuum: fluid added mass / damping / stiffness
- Determining structure parameters: mass, damping, stiffness
- Determining inflow conditions: turbulence intensity / turb forcing

Validation issues: obtaining structure properties

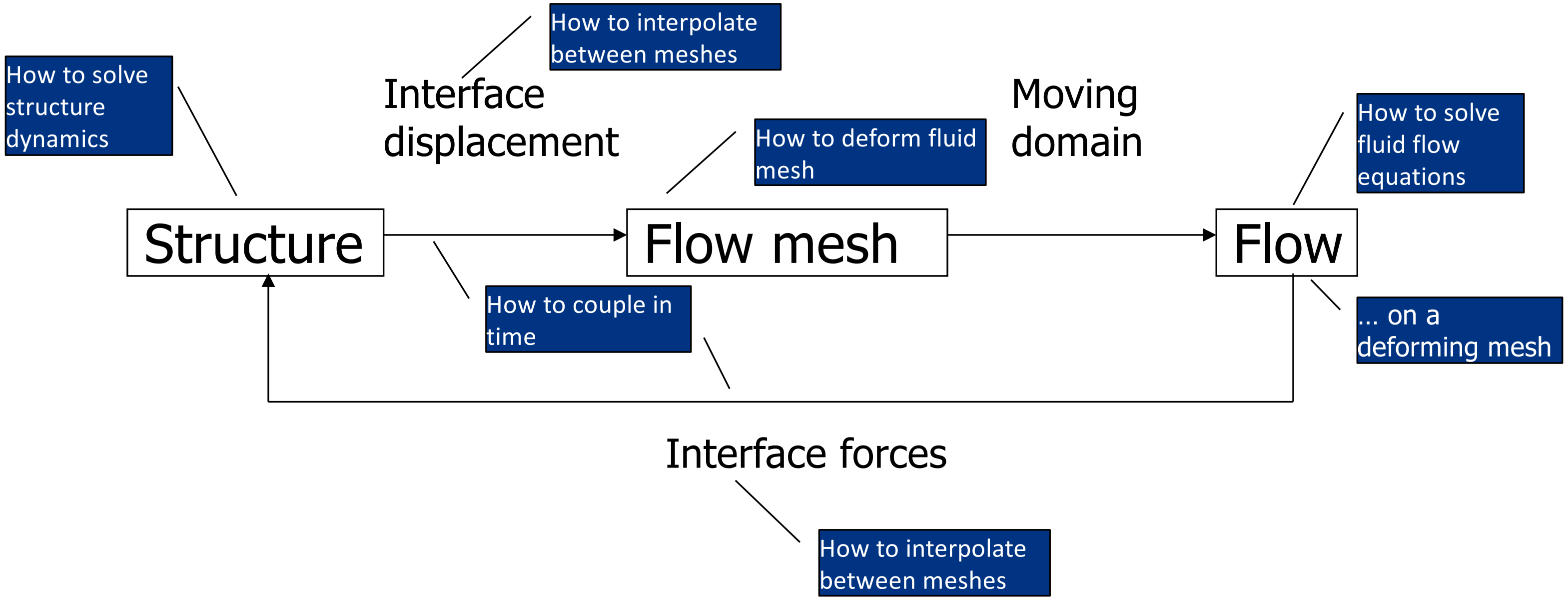
To measure structure properties (mass, stiffness, damping) often vibration tests are performed. From the vibration response, one can obtain the objects stiffness / structural damping

Example: DelfFly II : a micro-aerial vehicle that uses flapping wings for lift/propulsion



=> How can you measure the structure-only response of e.g. the wings?

Coupling diagram of flow and structure



How to solve structure dynamics

Interface displacement

How to interpolate between meshes

How to deform fluid mesh

Moving domain

How to solve fluid flow equations

Structure

Flow mesh

Flow

How to couple in time

... on a deforming mesh

Interface forces

How to interpolate between meshes

Summary

- Three field problem: Flow, Structure, Mesh:
    - Flow is solved on a moving/deforming mesh in ALE formulation
    - Structure vibration behaviour similar to harmonic oscillator
    - Mesh can be deformed with repect to its previous or initial state
- Using interpolation between meshes to transfer loads and displacements
- Satisfying the kinematic and dynamic interface conditions using partitioned black-box approach:
    - Loosely coupled – flow and structure only solved once: partitioning error
    - Strongly coupled – sub-iterations with(out) underrelaxation
- Structure model requires “in vacuum” properties, which may be difficult to determine in the presence of added mass/damping/stiffness effects

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Thank you!

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