Particle properties

Thomas Schindelbeck, Mainz, Germany
schindelbeck.thomas@gmail.com

Abstract

The standard model of physics classifies particles into elementary leptons and hadrons composed of quarks. In this article the existence of an alternate ordering principle will be demonstrated giving particle energies to be quantized as a function of the fine-structure constant, α. The quantization can be derived using an appropriate wave function that acts as a probability amplitude on the electric field. Necessary input parameters are elementary charge and electric constant only. The value of α itself can be approximated numerically by the gamma functions of the integrals involved.
The model may be used to calculate other particle properties. The magnetic moment can be calculated directly from the electromagnetic fields. In the range of femtometer the wave function overlap provides a mechanism for strong interaction. Gravitational force can be calculated quantitatively with model parameters. The model gives some indication for a common base of strong, Coulomb and gravitational force.

1.1 Introduction

Particle zoo is the informal though fairly common nickname to describe what was formerly known as "elementary particles". The standard model of physics [1] divides these particles into leptons, considered to be the fundamental "elementary particles" and the hadrons, composed of two (mesons) or three (baryons) quarks. Well hidden in the data of particle energies lies another ordering principle which can be derived by interpreting particles as electromagnetic objects subject to some general principles of quantum mechanics [2]. The concept of expressing mass in electromagnetic terms is almost as old as Maxwell´s equation, going back as far as 1881 with the work of J.J.Thomson [3]. W.Wien was a prominent advocate of reducing mass and gravitation to electromagnetism and based on the works of O.Heaviside [4] and others in 1900 presented a mass-energy relation for charged particles in a form that is still in use today with minor modifications, $\mathcal{E} = 3/4\ mc_0^2$ [5][1].
In the model presented here, the particles are interpreted as some kind of standing electromagnetic wave originating from a rotating electromagnetic field with the E-vector pointing towards the origin. Neutral particles are supposed to exhibit nodes[2] separating corresponding equal volume elements of opposite polarity. To obtain quantifiable results, the electromagnetic field will be modified with an appropriate exponential function, $\Psi(r, \vartheta, \varphi, e, \varepsilon)$ [3], serving as probability amplitude of the field. The two integrals needed to calculate energy in point charge and photon representation exhibit the following two relations:
1) Their product - resulting from energy conservation - is characterized by containing the product of the two gamma functions $\Gamma(1/3)|\Gamma(-1/3)| \approx \alpha^{-1}/(4\pi)$,
2) their ratio features a quantization of energy states with powers of $1/3^n$ over some base α_0, a relation that can be found in the particle data with $\alpha_0 = \alpha$ as:

$$W_n/W_e = 1.509\left(y_l^m\right)^{-1/3}\Pi_{k=0}^{n}\alpha\text{^}(-1/3^k) \qquad n = \{0;1;2;..\} \tag{1}$$

with W_e = energy of electron, W_n = energy of particle n and y_l^m representing the angular part of $\Psi(r, \vartheta, \varphi)$. For spherical symmetry $y_0^0 = 1$ holds, corresponding particles are e, μ, η, p/n, Λ, Σ and Δ [4]. The factor 1.509 is related to angular momentum |J| = 1/2, see 2.2, 2.5, 2.8.

1 Here $\mathcal{E}$ denotes energy - in all other parts of this article energy is identified by the letter W while E is for electric field; m = mass ; c_0 = speed of light in vacuum;
2 nodes of positive and negative charge regions will have to coincide with nodes of the wave function but not necessarily vice versa.
3 r = distance from origin, ϑ, φ = angular coordinates, e = elementary charge, ε = electric constant
4 The relation of the masses e, μ, π with α was noted in 1952 by Y.Nambu [6]. M.MacGregor calculated particle mass and constituent quark mass as <u>multiples</u> of α and related parameters [7].

The terms for calculating energy do not distinguish between charged and neutral particles and have to be considered a first approximation, accurate only within order of magnitude of the spread of energies of particle families, typically in a range of ± 0.01.
Apart from calculating particle energies the model may be used to describe other particle properties. The magnetic moment of particles can be calculated directly from the electromagnetic fields modified by Ψ. At distances comparable to particle size, typically femtometer for hadrons, direct interaction of particle wave functions (“overlap”) has to be expected. Interpreting this interaction as strong interaction and considering the basic spatial characteristics of the functions may provide a possible explanation why leptons, in particular the tauon, are not subject to this interaction. Gravitational force can be calculated quantitatively with model parameters i.e. as function of elementary charge and electric constant.
Terms for potential energy of strong and Coulomb interaction as well as particle energy (mass) can be attributed to the terms of the expansion for r −> 0 of the incomplete gamma function of the integrals for calculating particle energy and a quadratic relationship between a characteristic parameter of strong, Coulomb and gravitational force can be found, indicating a common base for all three forces.
The model is an electrostatic approximation of an electromagnetic object implying some asymmetry in its terms, e.g. the electromagnetic units used.
This is a preliminary working paper intended to provide food for thought.

1.2 Unit System

The unit system used in this work is SI with the exception of electromagnetic units that are required to be based on their relation to c_0, in the simplest case using a symmetric split of electric and magnetic constant, ε and μ, such as given in Planck units. In this work SI units are kept with the modification:

$$c_0^2 = (\varepsilon_0\, \mu_0)^{-1} \tag{2}$$

being replaced by

$$c_0^2 = (\varepsilon_c\, \mu_c)^{-1} \tag{3}$$

with
$\varepsilon_c = (2.998\text{E+8}\ [\text{m}^2/\text{Jm}]\ )^{-1} = (2.998\text{E+8})^{-1}\ [\text{J/m}]$
$\mu_c = (2.998\text{E+8}\ [\text{Jm/s}^2]\ )^{-1} = (2.998\text{E+8})^{-1}\ [\text{s}^2/\text{Jm}]$
i.e. the numerical values for c_0, $1/\varepsilon_c$, $1/\mu_c$ are identical, the units of ε_c, μ_c are expanded by [Jm] for the convenience of this model.
In the following the abbreviation b_0 is used for the Coulomb term $b_0 = e^2/(4\pi\ \varepsilon_0) = e_c^2\ /(4\pi\varepsilon_c) = 2{,}307\text{E-}28$ [Jm] which is identical in both unit systems, thus all calculations concerning particle energy are not affected except for the definition of τ_e, equ. (32).
From b_0 follows for the square of the elementary charge: $e_c^2 = 9{,}67\text{E-}36\ [\text{J}^2]$.

2 Energy levels of elementary particles

2.1 Calculation of energy - point charge

Particle energy is expected to be equally divided into electric and magnetic part, $W_n = 2W_{n,el} = 2W_{n,mag}$ [5]. To calculate energy the integral over the electrical field E of a point charge is used as a first approximation. However, it can not be expected that the expression derived from Coulomb's law for two interacting particles can be used unaltered and it will be demonstrated in chpt 2.2 that a factor 4π is needed [6] as modification for W_n to yield a half integral angular momentum, giving:

$$W_{pc,n} = 4\pi \int_0^\infty \varepsilon_0 E(r)^2 d^3 r \;=\; 4\pi \int_0^\infty \frac{e^2}{4\pi\varepsilon_0 r^2} dr \;=\; 4\pi b_0 \int_0^\infty r^{-2} dr \tag{4}$$

5 Alternatively $W_n/2$ may be interpreted as $W_n/2 = W_{pot} = W_{kin}$ of a harmonic vibration. Terms of $W_n/2$ will be used in chpt. 2.2, 4.1, etc.
6 The parameters σ, τ, introduced below may be adapted to allow for the factor 4π to be omitted in (4). However not all relationships given in the following may be recovered with such a parameter set.

The field E is modified with a function [7]

$\Psi(r) = \exp(-\{(\sigma\ \tau\ b_0^2\ r^{-3}) + [(\sigma\ \tau\ b_0^2\ r^{-3})^2 - 4\ \tau\ b_0^2\ r^{-3}]^{0.5}\}/2)$ (5)

The first term, $\exp(-\sigma\ \tau\ b_0^2\ r^{-3})$, avoids ultraviolet divergence of E(r) for r −> 0, the part in square brackets provides an integration limit, r_l, where the root term equals zero [8]. r_l of particle n can be given by:

$r_{l,n} = (\sigma^2\ \tau_n\ b_0^2/4)^{1/3}$ (6)

providing a boundary condition for the problem.
Coefficient σ is a constant (σ = 1.756E+8[-]) related to constant angular momentum J (see below), τ is a parameter representing particle energy, $\tau_n \sim W_n^{-3}$. The coefficient τ_{n+1} of a particle can always be expressed by a term multiplying the coefficient of its predecessor n (defined in this work by $W_n < W_{n+1}$) with a parameter $\alpha_{\tau,n+1}$: $\tau_{n+1} = \tau_n\ \alpha_{\tau,n+1}$. In general for the coefficient of particle n a partial product is formed relative to a reference particle, chosen here to be the electron, τ_e:

$\tau_n = \tau_e \Pi_{k=0}^{n} \alpha_{\tau,k} = \tau_e\ \Pi_{\tau,n}$ (7)

$$\tau_e = \frac{(2/3)^3 \alpha^9}{e_c \varepsilon_c} = 1.676\text{E+6 [m/J}^2] \quad (8)$$ [9]

In all integrals over Ψ(r) given below equ. (9) may be used as approximation for (5) up to $r = r_l$ with relative error << 0.01:

$\Psi_n(r < r_l) \approx \exp(-\sigma\ \tau_n\ b_0^2\ r^{-3}) = \exp(-\beta_n/2\ r^{-3})$ (9)

where $\beta_n = 2\ \sigma\ \tau_n\ b_0^2$ is used for brevity. The factor 2 takes into account that Ψ(r) appears squared in all integrals.
The integrals over the approximation of Ψ(r) according to equ. (9) are closely related:

$$\int_0^{r_l} \Psi(r)^2 r^{-(m+1)} dr = \Gamma(m/3,\ \beta/r_l^3)\ \beta^{-m/3}/3 \quad (10)$$

with m = {...;-1;0;1;2;...}. The term $\Gamma(m/3, \beta/r_l^3)$ denotes the upper incomplete gamma function, given by the Euler integral of the second kind:

$$\Gamma(m/3,\ \beta/r_l^3) = \int_{\beta/r_l^3}^{\infty} t^{m/3\ -1} e^{-t} dt \quad (11)$$

It follows from the boundary condition (6) that the integration limit, β/r_l^3, has to be a constant for all particles:

$\beta_n/r_{l,n}^3 = 2\sigma\tau_n b_0^2/\ r_{l,n}^3 = 8/\sigma$ (12)

For m ≥ 1 the term $\Gamma(m/3, \beta/r_l^3)$ may be approximated by Γ(m/3) [10], for m ≤ 0 the integrals (10), (11) depend critically on the integration limit and have to be integrated numerically.
The integral for m = 1 is needed to calculate $W_{pc,n}$. Inserting (9) and (10) in equ. (4) will turn out:

$$W_{pc,n} = 4\pi\varepsilon_0 \int_0^{\infty} E(r)^2 \Psi_n(r)^2 d^3 r = 4\pi b_0 \int_0^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr = 4\pi\ b_0\ \Gamma_{1/3}\ \beta_n^{-1/3}/3 \quad (13)$$

Equation (13) is the source of $\tau_n \sim W_n^{-3}$. From (7) and (13) follows:

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7 Phase of wave function ignored on this approximation level, Ψ(r) appears only squared in all equations.
8 In chapter 7.1 some additional reasoning for the form of (5) will be given.
9 see 2.5
10 The complete Γ-function Γ(m) will be abbreviated to Γ_m. Most relations use $\Gamma_{1/3}$ and $|\Gamma_{-1/3}|$. The sign of the latter arises from the relation between Γ-functions, the relevant integrals, (10), (11), give the positive value.

$\tau_n/\tau_e = \Pi^{n}_{k=0}\,\alpha_{\tau,k} = \Pi_{\tau,n} = \Pi^{n}_{k=0}\,\alpha^{-3}_{W,k}$ (14)

with $\alpha_{W,k}$ being the coefficients for the general case of a partial product $\Pi_{W,n}$ for particle energies [11]. Through equ. (6) the relations $\tau_n \sim r_{l,n}^3$ and $W_n \sim r_{l,n}^{-1}$ hold.

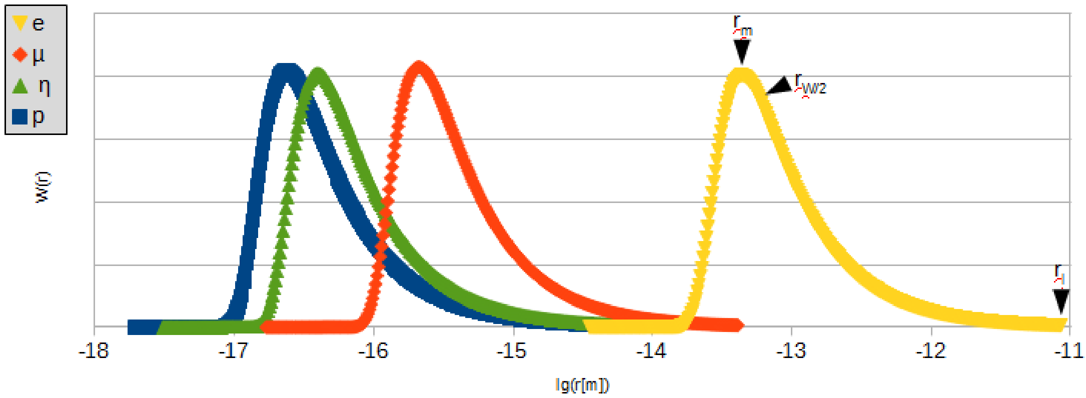

Figure 1: Example for particle energy $W_{n\,calc}$ (r) (normalized) vs lg(r[m]) according to equ. (13); $r_{m,n}$: see (15); $r_{W/2}$ => radius at which integrals of (13) attain half their final value; r_l see (6);

$$r_{m,n} = \left|\Gamma_{-1/3}\right|\ \beta_n^{1/3}/3\ \approx r_{max,n} \tag{15}$$

2.2 Angular momentum

The factor 4π added in equ. (4) may be derived by applying a semi-classical approach for angular momentum J, using

$$J = r_2\, x\, p(r_1) = r\, W_n(r)/c_0 \tag{16}$$

with $W_{kin,n} = 1/2\ W_n$ and $|r_2| = |r_1|$ (for e and μ). This gives the integral:

$$|J| = \int_0^{r_{l,n}} J_n(r)\,dr = 4\pi\frac{b_0}{c_0}\int_0^{r_{l,n}} \Psi_n(r)^2 r^{-1}\,dr \tag{17}$$

From (10), (11) follows for m = 0:

$$\int_0^{r_{l,n}} \Psi(r)^2 r^{-1}\,dr = 1/3\int_{8/\sigma}^{\infty} t^{-1}e^{-t}\,dt = 5.442 \approx \alpha^{-1}/8\pi \;^{12} \tag{18}$$

yielding the constant $\alpha^{-1}/8\pi$. Inserting (18) in (17) provides a half integer angular momentum, |J| = 1/2:

$$|J| = 4\pi\frac{b_0}{c_0}\,\frac{\alpha^{-1}}{8\pi} = 1/2\ [\hbar] \tag{19}$$

Analogous to the postulate for neutral particles to be composed of volume elements of opposite charge, particles with J = 0, J ≥ 1 are supposed to be composed of a combination of half integer contributions of angular momentum J = ± 1/2, adding up accordingly, implying appropriate multiples for the ratio of $|r_2|\ /\ |r_1|$ in (16) [13].

2.3 Calculation of energy - photon

For m = -1 equations (10), (11) give a relation between radii and Euler-integral:

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11 Not used in the following. However, coefficients in Ψ, α_τ, should not be confused with those in energy terms, α_W .
12 Exact value = 0.9981 $\alpha^{-1}/8\pi$
13 E.g. in case of the proton a contribution of J = 3 |1/2] is needed, i.e. 3 contributions of J = |1/2] each, one with opposite sign resulting in total spin of J = 1/2. For this formally $|r_2| = 3\,|r_1|$ in (16) has to hold, see also 3.1.

$$r_{x,n} = \int_0^{r_{x,n}} \Psi_n(r)^2 dr = \beta_n^{1/3}/3 \int_{\beta/r_{x,n}^3}^{\infty} t^{-4/3} e^{-t} dt \tag{20}$$

Using the value of the Compton wavelength, λ_C, in the term for the energy of a photon gives hc_0/λ_C. With equ. (20) λ_C can be given by:

$$\lambda_{C,n} = \int_0^{\lambda_{C,n}} \Psi_n(r)^2 dr = \beta_n^{1/3}/3 \int_{\beta/\lambda_{C,n}^3}^{\infty} t^{-4/3} e^{-t} dt \approx \beta_n^{1/3}/3 \;\; 18\pi \,|\, \Gamma_{-1/3} \,| \tag{21}$$

According to (13) particle energy is proportional to $\beta_n^{-1/3}$ and $\lambda_{C,n} \sim \beta_n^{1/3}$ has to hold, requiring the lower integration limit of the Euler integral and the factor ≈ 18π to be a constant for all particles. Energy of a photon can be expressed by:

$$W_{Phot,n} = hc_0/\lambda_{C,n} = \frac{hc_0}{\int^{\lambda_{C,n}} \Psi_n(r)^2 dr} = \frac{3hc_0}{18\pi|\Gamma_{-1/3}|\beta_n^{1/3}} \tag{22}$$

2.4 Relation of integrals for $W_{pc,n}$ and $W_{Phot,n}$ with fine-structure constant α

The energy of a particle has to be the same in both photon and point charge description. From (13) and (22) follows:

$$W_{pc,n} = W_{Phot,n} = 4\pi b_0\, \Gamma_{1/3}\, \beta_n^{-1/3}/3 = \frac{3hc_0}{18\pi|\Gamma_{-1/3}|\beta_n^{1/3}} \tag{23}$$

which may be rearranged to emphasize the relationship of the gamma functions ($\Gamma_{1/3}$ = 2.679; $|\Gamma_{-1/3}|$ = 4.062) with α, $4\pi\, \Gamma_{1/3}\, |\Gamma_{-1/3}| = 0.998\, \alpha^{-1}$, giving (note: h => ħ):

$$4\pi\, \Gamma_{1/3}\, |\Gamma_{-1/3}| \approx \frac{9hc_0}{18\pi b_0} = \frac{\hbar c_0}{b_0} = \alpha^{-1} \text{ [14] [15]} \tag{24}$$

Factor k_a = 0.998 will be used in equations below to indicate the deviation from the exact value.

Equation (24) uses three approximations:
1) $\Gamma_{1/3}$ is used in place of the incomplete Γ-function $\Gamma(1/3, \beta/r_l^3) = 0.9960\,\Gamma_{1/3}$
2) the approximation for $\alpha^{-1}/(8\pi)$ in equ. (18) requires a correction factor of 0.9981 for 4π in the equation for $W_{Coul,n}$ if the experimental value of α is used.
3) For the integration limit $\beta_n/r_{x,n}^3 << 0$ the result of the Euler integral in (20) is approximated by

$$\int_{\beta_n/r_{x,n}^3}^{\infty} t^{-4/3} e^{-t} dt \approx 3\, (\beta_n/r_{x,n}^3)^{-1/3} \tag{25}$$

yielding $3\, \lambda_{C,n}/(\beta_n^{1/3}\, \Gamma_{1/3}) = 56.87 = 1.0057\, (18\pi)$ as approximation for 18π.
All three factors add up to change the remaining inequality of (24) from 0.9980 to 0.9978. Calculation errors, approximation residuals as well as possible higher order correction terms of e.g. QED type have to be considered to contribute to the remaining discrepancy.

2.5 Relation of σ and τ with α

According to equation (20) $r_{l,n}$ may be given by :

$$r_{l,n} = \int_0^{r_{l,n}} \Psi_n(r)^2 dr = \beta_n^{1/3}/3 \int_{8/\sigma}^{\infty} t^{-4/3} e^{-t} dt \approx 1.5\, \alpha^{-1} \,|\, \Gamma_{-1/3}|\, \beta_n^{1/3}/3 \tag{26}$$

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14 $\Gamma_{1/3}\, |\Gamma_{-1/3}| = 3^{0.5}\, 2\pi$

15 With the unit system of 1.1 follows: $4\pi\, \Gamma_{1/3}\, |\Gamma_{-1/3}| \approx \hbar c_0 4\pi \varepsilon_c / e_c^2$ => $\hbar c_0 \varepsilon_c = \Gamma_{1/3} |\Gamma_{-1/3}| e_c^2$ => $\hbar[J^2] \approx \Gamma_{1/3}\, |\Gamma_{-1/3}|\, e^2[J^2]$

The term ≈ 1.5 α^{-1} is within the accuracy of the calculations [16] identically to W_μ/W_e = 206.8 = 1.509 α^{-1}, the latter coefficient will be used in all calculations if not indicated otherwise. Thus the coefficient σ is related to factor ≈ 1.509 α^{-1} by equ. (12) and (26) to be:

$$\sigma = 8\, r_{l,n}^3 / \beta_n = (1.509\, \alpha^{-1} \,|\, \Gamma_{-1/3} \,|\, 2/3)^3 = (k_s\, \alpha^{-1} \,|\, \Gamma_{-1/3} \,|\,)^3 = 1.756\text{E+8}[-] \quad [17] \tag{27}$$

Coefficients 1.509 α^{-1} and σ are part of the terms setting the integration limits in equ. (18), determining the value of J=1/2.

Since the term 1.509 α^{-1} from (1) is approximately equal to the factor in r_l, ≈ 1.5 α^{-1}, these terms will cancel in the expression for $r_{l,\mu}$ (note: $W_n \sim 1/r_{l,n}$) :

$$r_{l,e} \approx 1.5\, \alpha^{-1} \,|\, \Gamma_{-1/3} \,|\, \beta_e^{1/3}/3 \tag{28}$$

$$r_{l,\mu} \approx 1.5^{-1}\, \alpha^{+1} [1.5\, \alpha^{-1} \,|\, \Gamma_{-1/3} \,|\, \beta_e^{1/3}/3\,] = \,|\, \Gamma_{-1/3} \,|\, \beta_e^{1/3}/3 = r_{m,e} = 1.5\, \alpha^{-1} \,|\, \Gamma_{-1/3} \,|\, \beta_\mu^{1/3}/3 \tag{29}$$

Relation (29), though empirically, requires $\alpha_{\tau,\mu}$ to be $\alpha_{\tau,\mu} \approx (\alpha/1.5)^3$.

The coefficient $\tau_e = (2/3)^3\, \alpha^9\, e_c^{-1}\, \varepsilon_c^{-1}$ has been introduced ad hoc in (8). It may be calculated from a relation between two terms used in 5.2:

$$\rho_1 = \frac{4\pi\, b_0\, \Gamma_{1/3}}{|\Gamma_{-1/3}|\ \beta_e^{2/3}} \approx \frac{W_e}{r_{m,e}} = 1.919\,[\text{J/m}]\,. \tag{30}$$

and

$$\sigma\, \varepsilon_c = 0.5859\ [\text{J/m}] = 0.3055\, \rho_1 = \rho_1' \tag{31}$$

Using the expression (27) to replace σ and (30) for ρ_1 in (31) and solving for τ_e gives a complicated expression that can be neatly simplified to give the right term in (32)

$$\tau_{e,calc} = \left[\left(\frac{2}{3}\right)^6 \frac{1}{e_c^2 \varepsilon_c^2}\ \frac{0.3055^3 3^{21} (4\pi)^4 \Gamma_{1/3}^3}{2^{23}\, 1.509^{15}\, \Gamma_{-1/3}^{18}}\ \alpha^{15} \right]^{0.5} = 1.004 \left(\frac{2}{3}\right)^3 \frac{\alpha^9}{e_c\, \varepsilon_c} = 1.004\, \tau_e \tag{32}$$

in which the factor $(2/3)^3$ can be traced back to equ. (27). The term $\alpha_{\tau,e} = \alpha^9$ is the expected extension of the partial product term for the electron, see 2.6.

2.6 Quantization with powers of 1/3^n over α

In general a relation between coefficients such as given by equ. (7) is arbitrary. The special form of expression (1) may be derived from the ratio of the integrals used in (13) and (22) for the point charge and photon representation of energy.

$$Q(\psi_n) = \frac{\int^{r_{l,n}} \Psi_n(r)^2 r^{-2}\, dr}{\int^{\lambda_{C,n}} \Psi_n(r)^2\, dr} = \frac{\Gamma_{1/3}}{18\pi\ |\Gamma_{-1/3}|\beta_n^{2/3}} \sim \frac{\alpha_{\tau,0}^{1/3}\alpha_{\tau,1}^{1/3}\ldots\ldots\alpha_{\tau,n}^{1/3}}{\alpha_{\tau,0}\alpha_{\tau,1}\ldots\ldots\alpha_{\tau,n}} \qquad n = \{0;1;2;..\} \tag{33}$$

The term given by (33) is related to the boundary condition (12) (see 7.3.2) and via (13) and (22) to the square of particle energy $W_n^2 \sim \tau_n^{-2/3}$. The last expression of (33) is obtained by expanding the product $\Pi_{\tau,n}^{-2/3}$ included in $\beta_n^{-2/3}$ with $\Pi_{\tau,n}^{1/3}$ From this term it is obvious that a relation $\alpha_{n+1} = \alpha_n^{1/3}$ such as given by equation (1) yields a distinct solution for $Q(\psi_n)$, $Q(\psi_n)$ being a function of coefficient α_n and α_0 only. By comparison with experimental data $\alpha_{\tau,0}$ may be identified as $\alpha_{\tau,0} = \alpha_{\tau,e} = \alpha^9$ and $Q(\Psi_n)$ can in general be given by:

$$Q(\psi_n) \sim \frac{\alpha^3 \alpha^1 \alpha^{1/3} \ldots\ldots \alpha \wedge (3/3^n)}{\alpha^9 \alpha^3 \alpha^1 \ldots\ldots \alpha \wedge (9/3^n)} = \alpha \wedge (3/3^n)/\alpha^9 \qquad n = \{0;1;2;..\} \tag{34}$$

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16 Calculating factor ≈ 1.5 numerically via the Euler integral of (26) with a value of σ ~ 1.509 gives 1.501, numerical fits of particle energy give values in a range of ~ 1.515 .

17 Factor k_s = 1.509 * 2/3 = 1.006 used as abbreviation in the following.

where all intermediate particle coefficients cancel out. [18]
The corresponding term for particle energies will be given by:

$$W_n = (4\pi b_0 h c_0 Q(\psi_n))^{0.5} = \left(\frac{(4\pi b_0)^2}{2\alpha} Q(\psi_n)\right)^{0.5} = \left(\frac{(4\pi b_0)^2 \Gamma_{1/3}^2}{9\ [\alpha\, 4\pi\ |\Gamma_{-1/3}| \Gamma_{1/3}] \beta_n^{2/3}}\right)^{0.5} =$$
$$= 4\pi b_0 \frac{\Gamma_{1/3}}{3} \frac{3\pi^{2/3}}{k_a^{0.5}\sigma^{1/3}} \left(\frac{\varepsilon_c}{e_c}\right)\ \alpha \wedge (1.5/3^n)/\alpha^{4.5} \qquad n = \{0;1;2;..\} \tag{35}$$

The term $\alpha\wedge(1.5/3^n)/\alpha^{4.5}$ is the partial product of equ. (1) times α^{-3} as coefficient for W_e. The coefficients of the partial product for $\Pi_{\tau,n}$ of (7) are given by (34) as their 3rd root:

$$\tau_n = \tau_e\, 0.291 \quad \Pi_{k=0}^{n} \alpha\wedge(3/3^k) \approx \frac{0.29}{e_c \varepsilon_c}\ \Pi_{k=0}^{n} \alpha\wedge(9/3^k) = \frac{0.29}{e_c \varepsilon_c}\ \Pi_n \qquad n = \{0;1;2;..\} \tag{36}$$

with Π_n used as abbreviation. Details of factor $0.291 = 1.509^{-3}$ are discussed in 2.8.

2.7 Extension to non-spherical symmetry

Up to here only spherical symmetry and Ψ(r) is considered, introduced through equ. (4), (13). For non-spherically symmetric states an appropriate angular term, y_l^m, will be introduced:

$$y_l^m = \int\int \Psi(\varphi,\vartheta)^2 \sin(\vartheta) d\varphi\, d\vartheta\ /\ 4\pi \tag{37}$$

Assuming a term corresponding to the spherical harmonic Y_1^0 to be a sufficient approximation for the next angular term, gives $y_1^0 = 1/3$ [19]. Assuming $W_n \sim 1/r_n \sim 1/V_n^{1/3}$ (V = volume) may be applicable for non-spherically symmetric states will give $W_1^0/W_0^0 = 3^{1/3} = 1.44$ [20].
Relation (36) will turn into:

$$\tau_n = y_l^m \quad \frac{0.29}{e_c \varepsilon_c}\ \Pi_n \tag{38}$$

A change in angular momentum is expected for this transition which is actually observed with ΔJ = ± 1 except for the pair μ/π with Δ J = 1/2.
Extending the model to energies below the electron with a coefficient of α^3 in equ. (1): $W_\nu / W_e = 1.509\ \alpha^3$ gives a state with energy 0.3eV which is in a range expected for a neutrino [9].
Results for particles assigned to y_0^0, y_1^0 are presented in table 1.

2.8 Accuracy of energy calculation

The values calculated for y_0^0 agree within ± 0.01 with experimental data. There are three major causes preventing a significant improvement of accuracy.
1) Especially in the case of particle families [21] effects on top of the relations given in this work have to play a role to explain different energy levels of differently charged particles. This limits accuracy and the possibility to precisely identify candidates for calculated energies (e.g. both ρ^0 and ω^0 are given for $1.44\ \alpha^{-1}\alpha^{-1/3}$ in tab. 1). If possible, particles chosen for y_0^0 in table 1 are of charge ± 1. In cases such as Σ with three energy levels, the intermediate energy level is chosen. For the y_1^0 series particles of the same charge as their y_0^0 equivalent are preferred in table 1.

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18 $Q(\psi_{n+1}) / Q(\psi_n) = \alpha_{\tau,n+1}^{1/3} / \alpha_{\tau,n+1} \Rightarrow Q(\psi_n) \sim \alpha_{\tau,n+1}$
19 Note: the wave function over the E-field will not be normalized to 1.
20 An equivalent to the ratio of two terms (33) $[\ Q(\Psi_{n+1})/Q(\Psi_n) \sim W_{n+1}^2/ W_n^2\ ]^2$ can be expressed by simple geometric terms, using a characteristic radius, e.g. r_l. The volume integral in (33) has to be replaced by the inverse relative to the reference n, $\int^{r,l,n} d^3r\ /\ \int^{r,l,n+1} d^3r = V_n/V_{n+1}$ reflecting the fact that energy increases with decreasing volume. The 1-D integral of (33) originating from calculating λ_C has to be replaced by $V_{n+1}^{1/3}/V_n^{1/3}$. This gives in the general case:
$(W_{n+1}/ W_n)^4 \sim V_n^{4/3}/V_{n+1}^{4/3}$.
21 Particle families, defined here as possessing the same exponent n in (38) but being different in charge or other properties, show a typical spread in energies of 3-4MeV and no dependence on total particle energy.

	n	$W_{n,Lit}$ [MeV]	$\Pi_{k=0}^{n}$ α^(-1/3^k) equ (1)	Π_n equ (38)	W_{calc} equ(1)	W_{calc}/W_{Lit} equ(13)	J	r_l [fm]	uds
ν	-1	3E-7 *	α^{+3}			-	1/2	1.5E+10	O
e^{+-}	0	0.51	Reference	α^9	Ref	1.003	1/2	8877	O
μ^{+-}	1	105.66	α^{-1}	$\alpha^9\alpha^3$	0.998	1.001	1/2	42.9	O
π^{+-}	1	139.57	1.44 α^{-1}	$\alpha^9\alpha^3/3$	1.090	1.093	0	29.8	uds
K		495					0		uds
η^0	2	547.86	$\alpha^{-1}\alpha^{-1/3}$	$\alpha^9\alpha^3\alpha^1$	0.992	0.995	0	8.3	
ρ^0	2	775.26	1.44 $(\alpha^{-1}\alpha^{-1/3})$	$\alpha^9\alpha^3\alpha^1/3$	1.011	1.014	1	5.8	uds
ω^0	2	782.65	1.44 $(\alpha^{-1}\alpha^{-1/3})$	$\alpha^9\alpha^3\alpha^1/3$	1.002	1.005	1	5.8	
K*		894					1		uds
p^{+-}	3	938.27	$\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}$	1.001	1.004	1/2	4.8	uds
n	3	939.57	$\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}$	1.000	1.003	1/2	4.8	uds
η'		958					0		
Φ^0		1019					1		uds
Λ^0	4	1115.68	$\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}$	1.010	1.013	1/2	4.0	uds
Σ^0	5	1192.62	$\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}$	1.004	1.007	1/2	3.8	uds
Δ	∞	1232.00	$\alpha^{-3/2}$	$\alpha^{27/2}$	1.002	1.005	3/2	3.7	uds
Ξ		1318					1/2		uds
Σ^{*0}	3	1383.70	1.44 $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}/3$	0.979	0.982	3/2	3.3	uds
Ω^-	4	1672.45	1.44 $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}/3$	0.972	0.975	3/2	2.8	uds
N(1720)	5	1720.00	1.44 $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}/3$	1.004	1.007	3/2	2.7	
tau^{+-}	∞	1776.82	1.44 $(\alpha^{-3/2})$	$\alpha^{27/2}/3$	1.002	1.005	1/2	2.5	O

Table 1: Particles up to tauon energy [22]; values for $\mathbf{y_0^0}$, y_1^0; col. 3: energy values from literature [8] except *: calculated from model; "uds" in col. 3 indicates particles covered by the quark model, linear combination states excluded; leptons indicated as O;

2) Some uncertainty exists concerning the factor 1.5x and its inverse ~2/3 as well as their 3rd powers appearing in many equations. In (32) the coefficient of the electron τ_e is proportional to $(2/3)^3$ which corresponds to 3/2 in the energy expression. The experimental ratio of μ and e energy features almost the same factor, 1.509, which accordingly has to show up in the partial product (1) while this factor is absent [23] in the additional components of the partial product. According to (26) this factor is related to the integration limit r_l which affects the precise value of the angular moment. Both values 1.5 and 1.509 are close to the ratio of $|\Gamma_{-1/3}| / \Gamma_{1/3} = 1.516$ [24]. It is not possible to distinguish clearly between these terms. The factor 1.509 of the energy ratio μ/e gives a good compromise and is used in all calculations of energy except for the electron coefficient of (32) which is left to be 2/3. This is considered to be accurate to 1.509 +/- 0.09 i.e. resulting in a relative error of+/- 0.006.

3) The accuracy of the calculations is already in the order of magnitude of expectable QED corrections. Since these originate from the interaction of particles with the vacuum they are not included in the equations of this model. Thus it is not advisable to use a factor from a fit to the experimental energy of the electron which would result in a value of 1.5135 [25].

As for comparing accuracy of the energy calculation with results from the standard model, the quark model provides no authoritative data set to do so. Calculations of simplicity comparable to the model presented here, using constituent quarks, yield approximately the same accuracy for the proton and heavier particles, yet accuracy decreases dramatically for lighter particles. However, the prevailing QCD calculations for particle mass use the mass of current quarks as input parameter. For u,d,s quarks, relevant in the energy range dealt with here, this mass is only vaguely defined, e.g. in the case of the u, W_u = 1.8 - 2.8 MeV [10].

22 up to Σ'^0 all resonance states given in [8] as **** included; Exponent of -3/2, 27/2 for Δ and tau is equal to the limit of the partial products in (1) and (36); r_l calculated with equ. (6);

23 or cancelled by 2/3

24 There exists an additional relation with factor $\Gamma_{2/3}$ of the integral $\int\Psi(r)^2 r\, dr$: $|\Gamma_{2/3}| = 3/2\ \Gamma_{1/3}$

25 The deviation of the calculated value of W_e (based on 1.509) from the exact value corresponds to ~ g_a^2 (g_a = anomalous g-factor of the electron).

3 Other properties

3.1 Magnetic moment [26]

Within this model particles are treated as electromagnetic objects principally enabling a direct calculation of the magnetic moment M from the electromagnetic fields.

The magnetic moment M_e of the electron is given as product of the anomalous g-factor, g_a = 1,00116, Dirac-g-factor, g_D = 2, and the Bohr magneton, $\mu_B = e\,\hbar/(2m_e)$, times the quantum number for angular momentum J = 1/2:

$$M_e = g_a g_D \mu_B / 2 = g_a \frac{2 e c_0^2}{2 W_e} \frac{\hbar}{2} = g_a 9.274\text{E-}24\,[\mathrm{Am}^2] \tag{39}$$

The factor g_a arises from the interaction of the electron with virtual photons as calculated in quantum electrodynamics and should not be part of a calculation of the magnetic moment from the field of the electron itself. Within this model the factor 2 of g_D originates from the fact that particle energy is supposed to be equally divided into contributions of the electric and magnetic field, $W_{el} = W_{mag} = W_n/2$ and only the magnetic field, i.e. W_{mag} contributes to the magnetic moment.

Inserting the term for particle energy of (13) in (39) gives:

$$\frac{M_e}{g_a} = \frac{e\hbar c_0^2}{2W_e} = \frac{e\hbar c_0^2}{2} \frac{3\beta_e^{1/3}}{4\pi b_0 \Gamma_{1/3}} = \frac{e c_0 \beta_e^{1/3}}{2} \left(\frac{|\Gamma_{-1/3}|}{3} \frac{3}{|\Gamma_{-1/3}|} \right) \frac{3[\hbar c_0 4\pi\varepsilon]}{\Gamma_{1/3} 4\pi [e^2]} = \frac{e c_0 \beta_e^{1/3}}{2} \frac{|\Gamma_{-1/3}|}{3} \left[\frac{9[\alpha^{-1}]}{4\pi \Gamma_{1/3} |\Gamma_{-1/3}|} \right] \tag{40}$$

The term on the right is expanded by $|\Gamma_{-1/3}|/3$ and 4π and turned into a form that will be needed for comparison with a calculation starting directly from the fields as explained in the following.

The relation of the values of E and B in an electromagnetic wave is given by $B = E/c_0$. With factor 4π of (4) this gives for the value of M_n :

$$M_n \approx \frac{4\pi}{2\mu} \int_0^{r_l} B(r) \Psi_n(r)^2 d^3 r = \frac{4\pi\varepsilon c_0}{2} \int_0^{r_l} E(r) \Psi_n(r)^2 d^3 r = \frac{4\pi \; e c_0 \beta'^{1/3}_n}{2} \frac{|\Gamma_{-1/3}|}{3} \tag{41}$$

The term β_n is replaced by $\beta_n' = W_{n,exp} 3/(\Gamma_{1/3}\, 4\pi b_o)$, recalculated from experimental energy values to avoid transferring errors from the energy calculation (in the order of ≤ 0.01). Magnetic moments calculated with this approximation are given in table 2.

	|M|_Calc [Am²]	M_Lit [Am²]	|M|_Calc/|M|_Lit
e^{+-}	1.29E-23	-9.28E-24	1.392
μ^{+-}	6.25E-26	-4.49E-26	1.392
p^{+-}	7.04E-27	1.41E-26	0.499
n	7.04E-27	-9.66E-27	0.728
Λ^0	5.92E-27	-3.10E-27	1.912

Table 2: Absolute values calculated for magnetic moment with (41) compared to literature [8]

Equation (41) neglects contributions to B(r) from other parts of the standing wave and requires an appropriate integration of those. The term in brackets of (40) contains integral terms over $\Psi(r)^2$ that might provide suitable contributions since

$$\frac{9\alpha^{-1}}{4\pi \Gamma_{1/3} |\Gamma_{-1/3}|} = \frac{3\beta_e^{1/3}}{\Gamma_{1/3}} \frac{3}{\beta_e^{1/3} |\Gamma_{-1/3}|} \frac{2\alpha^{-1}}{8\pi} = \frac{2\int^{r_l} \Psi(r)^2 r^{-1} dr}{\int^{r_l} \Psi(r)^2 r^{-2} dr \int^{r_l} \Psi(r)^2 dr} \tag{42}$$

holds. Some more assumption about symmetry (in the presence of a magnetic field) is required. In the simplest case the electron may be modelled by contributions to the field via current loops with $B(r) = \mu_0 I/(2r_2)$ as illustrated in fig.2 [27]. The radius r_2 of the loops increases proportional to distance from the origin, r_1 [28].

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26 Note: to allow for comparison with tabulated values of M in units of [Am²] the calculations in this chapter use e [C] not e_c [J], conversion factor: [m²C/s] /[m² J/s] = e/e_c = 1/19.4 [C/J].

27 Relation between r_1 and r_2 equivalent to that of angular momentum discussed in 2.2.

28 $|r_1| = |r_2|$ for e, μ

The current I is not constant but a function of $r_2 \sim r_1$ and e(r), $I = e(r)c_0/(2\pi r_2)$. These 3 factors might be attributed to integrals over $\Psi(r)^2$:

1.) Integral $\int\Psi(r)^2 r^{-1}dr$ may be assigned to the integral over contributions of $B(r) \sim I/(2r_2)$.

2.) The radius dependence of $I = e\, c_0 /(2\pi r_2)$ might be given by $\int\Psi(r)^2\, 2\pi\, dr$ in the denominator.

3.) e as identified as $e = \varepsilon\int E(r)dA$ is <u>not</u> a constant but a function of radius: $e(r) = \varepsilon\int E(r)\Psi(r)^2\, dA$ and has to be integrated accordingly. Since (41) contains such a term, this leaves the remaining term $\int\Psi(r)^2\, r^{-2}dr$ in the denominator of (42) to represent a correction due to the contributions given above being expressed as integrated values, e.g. $\int e(r)\Psi(r)^2\, dr$ in place of e.

The term for electron and muon would be:

$$M_{e,\mu} = \frac{4\pi e c_0 \beta'^{1/3}_{e,\mu}}{2}\frac{\Gamma_{-1/3}}{3}\frac{2\int^{r_1}\Psi(r)^2 r^{-1}dr/2}{\int^{r_1}\Psi(r)^2 r^{-2}dr\ \ 2\pi\int^{r_1}\Psi(r)^2 dr} = \frac{4\pi e c_0 \beta'^{1/3}_{e,\mu}}{2}\frac{\Gamma_{-1/3}}{3}\frac{\alpha^{-1}}{8\pi}\frac{3}{2\pi\ \Gamma_{-1/3}}\frac{3}{\Gamma_{1/3}} = \tag{43}$$

$$= \left[\frac{4\pi\ e c_0 \beta'^{1/3}_{e,\mu}}{2}\frac{\Gamma_{-1/3}}{3}\right]\left[\frac{\alpha^{-1}}{4\pi}\frac{3}{\Gamma_{-1/3}}\frac{3}{\Gamma_{1/3}}\right]\left\{\frac{1}{4\pi}\right\}$$

The term in the first bracket corresponds to (41) while the term of the second bracket gives the correction factor of (40). The term in curled brackets of (43) is supposed to be a particle specific structure factor representing geometry and in case of e and μ stands for the distribution of the E-vector over a sphere.

Equation (40) will be recovered by cancelling the first and last 4π terms which is not done since it is the two terms in square brackets that will remain constant when proceeding with proton and neutron magnetic moment.

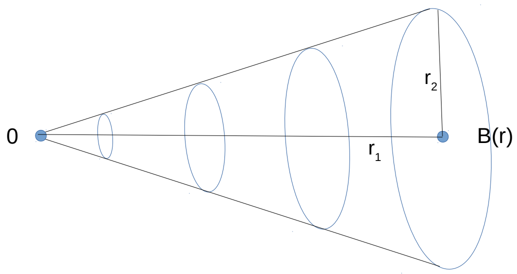

Figure 2: Field of a particle modelled -only for illustration purpose- as consisting of circular threads of current of value $I = ec_0/(2\pi r_2)$ forming a cone with tip at the origin.

In case of the proton this structure factor has to be replaced by $1/2\ (2/3)^2$ to yield the exact value of M_p. Part of this may be explained within this simple model. Assuming the mesons to consist of two cones [29] as depicted in fig. 2 with cancelling angular momentum, the proton should be modelled with three such cones which in the most symmetric case might form a planar orientation of 120° angles covering the φ-plane, fig.3,

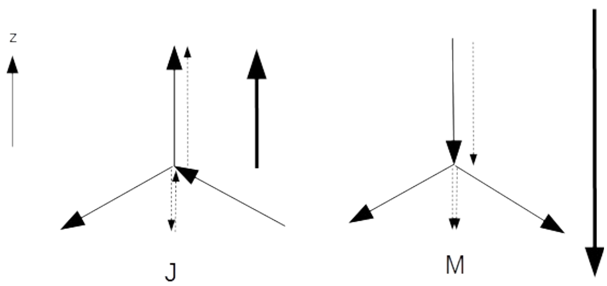

Figure 3: Illustration for orientation of the proton J and M components; direction of arrow indicating direction of rotation with respect to center;

thus resulting in a multiplication of the factor 1/4π of the structure factor by 2π. $|r_2| = 3\,|r_1|$ has to hold for the proton (see note 13) giving a factor $1/3^2$ via the requirements 1.) and 2.) above. Number and orientation of J

29 "Cone" does not refer to a separate entity but depicts local maxima in the probability density.

= 1/2 contributions have not been accounted for yet. A contribution as illustrated in fig. 3 would account for a factor 2, i.e. in summary a term 4π/9 would result, cancelling the last two terms in brackets of (43) and recovering term (41) which is a factor 2 off the experimental value. Including the missing factor 2 in the structure factor term would give M_p as:

$$M_p = \left[\frac{4\pi \; e\, c_0\, \beta'^{1/3}_p}{2}\,\frac{\Gamma_{-1/3}}{3}\right]\left[\frac{\alpha^{-1}}{4\pi}\,\frac{3}{\Gamma_{-1/3}}\,\frac{3}{\Gamma_{1/3}}\right]\left\{\frac{1}{2}\left(\frac{2}{3}\right)^2\right\} \tag{44}$$

For the neutron the situation is expected to be more complicated [30]. Without further speculation it might be noted, that raising the exponential of 2/3 in (44) to 3 fits the value for M_n quite well. Table 3 gives magnetic moments calculated with according structure factor (curled bracket) of (43)f indicated:

	Structure -factor { }	|M|_Calc [Am²]	|M|_Lit [Am²]	|M|_Calc/|M|_Lit
e^-	1/4π	9.284765E-24	9.284765E-24	1.000000
μ^-	1/4π	4.490478E-26	4.490448E-26	1.000007
p^+	1/2 (2/3)²	1.410463E-26	1.410607E-26	0.999898
n	1/2 (2/3)³	9.403086E-27	9.662365E-27	0.973166

Table 3: Absolute values calculated for magnetic moment with (43)f compared to literature [8]; Values of e, μ are corrected for g_a.[31]

3.2 Particle decay / mean lifetime

To check if the model yields any information about mean lifetimes (MLT) the particles attributed to y_0^0 and y_1^0 are arranged according to their α-exponent index n and indicated for different types of particle families in fig. 4. There seems to be a tendency for charged particles to be significantly more stable than neutral ones and for y_1^0- lifetimes to be lower than y_0^0- lifetimes. [32]

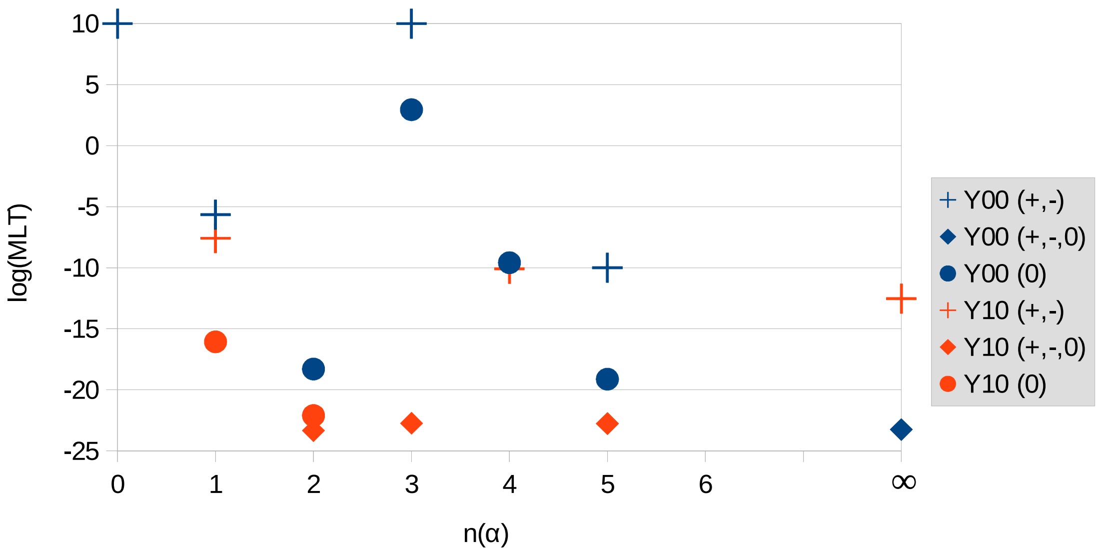

Figure 4: Mean lifetime for y_0^0 (blue) and y_1^0 (red) particles; charged only (+,-), neutral only (0), charged and neutral particle families with near identical MLT (+,-,0).

30 Within this model, nearly identical energy requires nearly identical extent of electromagnetic fields. C3-symmetry does not fit to a neutral particle in a static model, suggesting to assume some kind of switching of E-vector orientation and thus charge in time, requiring far more complicated schemes than for the proton.
31 For both e and μ g_a of the electron is used;
32 In [7] a dependence of MLT on α is given, however, there seems not to be a direct relation to the α-coefficients of this work.

	MLT [s]	log(MLT)	n(alpha)
e	∞		0
μ	2.20E-06	-5,7	1
η	5.00E-19	-18,3	2
p	∞		3
n	8.80E+02	2,9	3
$Λ^0$	2.60E-10	-9,6	4
$Σ^0$	7.40E-20	-19,1	5
$Σ^{+-}$	8.00E-11	-10,1	5
Δ	5.60E-24	-23,3	∞
$π^{+-}$	2.60E-08	-7,6	1
$π^0$	8.50E-17	-16,1	1
ρ+-0	4.50E-24	-23,3	2
ω0	7.80E-23	-22,1	2
$Σ^*0^{+-}$	1.80E-23	-22,7	3
$Ω^-$	8.20E-11	-10,1	4
N(1720)	1.70E-23	-22,8	5
tau^{+-}	2.90E-13	-12,5	∞

Table 4: Values for mean lifetime [8] used in figure 4

4 Differential equation

4.1 Radial part

The approximation $\Psi(r<r_l)$ of equation (9) provides a solution to a differential equation of type

$$-\frac{r}{6\sigma\tau b_0}\frac{d^2\Psi(r)}{dr^2} + \frac{b_0}{2r^3}\frac{d\Psi(r)}{dr} - \frac{b_0}{r^4}\Psi(r) = 0 \quad [33] \qquad (45)$$

However the correct discriminant form of Ψ(r) of equ. (5) would be provided by a slightly different equation (revised by 6 in 2nd, 2 in 1st and σ in 0th order term) :

$$-\frac{r}{\sigma\tau b_0}\frac{d^2\Psi(r)}{dr^2} + \frac{b_0}{r^3}\frac{d\Psi(r)}{dr} - \frac{b_0}{\sigma r^4\Psi(r)} = 0 \qquad (46)$$

To proceed from the heuristic mathematical approach of equation (45) to one based more on physics the second order term is expected to represent a quantum mechanical term for kinetic energy including the impulse operator. Based on (13) mass may be replaced by the term $W_e/(2\,c_0^2)$ [34] giving

$$W_{kin} = \left(\frac{2\hbar^2 c_0^2}{2\ W_e}\right)\frac{d^2\Psi(r)}{dr^2} \qquad (47)$$

To recover the r-dependence of (45) the following procedures are used as approximation

1.) $W_e \Rightarrow |\Gamma_{-1/3}|\,\Gamma_{1/3}\,4\pi\,b_0/(9\,r)$ which is an approximation for $r \approx r_m$ [35];

2.) Using the first derivative of Ψ(r), $[3\,\sigma\,\tau\,b_0^2\,r^{-4}]$ (and $[3\,\sigma\,\tau\,b_0^2\,r^{-3}]$) to modify the 0th (and 1st order term), i.e. effectively turning them into the next higher derivative, allows for canceling the 2nd order term. Since this term is almost identical to the expression for the supposed term of the strong force, the last term in equ. (53) below, the latter term including 4π/4 is preferred [36], i.e. $[\pi\,\sigma\,\tau\,b_0^2\,r^{-4}]$ and $[\pi\,\sigma\,\tau\,b_0^2\,r^{-3}]$ will be chosen for the

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33 [N15.1] $d\psi(r)/dr = 3\,\sigma\,\tau\,b_0^2\,r^{-4}\,\Psi(r)$

[N15.2] $d^2\psi(k)/dk^2 = 9\,(\sigma\,\tau\,b_0^2)^2\,r^{-8}\,\Psi(r) - 12\,\sigma\,\tau\,b_0^2\,r^{-5}\,\Psi(r) + 6\,\sigma\,\tau\,b_0^2\,r^{-5}\,\Psi(r)$ (polar coordinates)

[N15.1] -[N15.2] inserted in (34) gives:

[N15.3] $r(6\,\sigma\,\tau\,b_0)^{-1}\,\{-9\,(\sigma\,\tau\,b_0^2)^2\,r^{-8} + 6\,\sigma\,\tau\,b_0^2\,r^{-5}\} + 3/2\,\sigma\,\tau\,b_0^3\,r^{-7} - b_0\,r^{-4} = 0$

[N15.4] $-3/2\,\sigma\,\tau\,b_0^3\,r^{-7} + b_0\,r^{-4} + 3/2\,\sigma\,\tau\,b_0^3\,r^{-7} - b_0\,r^{-4} = 0$

34 Using $W_{pot,n} = W_{kin,n} = W_n/2$

35 β_n in (13) replaced via term of (15)

36 factor 2 of β not included

terms of the differential equation.
3.) Inserting σ in the denominator of the last term in accordance with (46);
4.) Since στ, technically $\sigma\tau_e$, has to match the resulting expression, τ_e will have to be redefined as τ_e'.
This gives:

$$-\left(\frac{9\hbar^2 c_0^2 r}{|\Gamma_{-1/3}|\Gamma_{1/3} 4\pi b_0}\right)\frac{d^2\Psi(r)}{dr^2} + \frac{\pi b_0(\sigma\tau_e' b_0^2)}{r^3}\frac{d\Psi(r)}{dr} - \frac{\pi b_0(\sigma\tau_e' b_0^2)}{\sigma\ r^4}\Psi(r) = 0 \tag{48}$$

as differential equation. Equation (5) will turn into:

$$\Psi(r)=\exp-\left(\left(\left(\frac{\pi|\Gamma_{-1/3}|\Gamma_{1/3} 4\pi\sigma\tau_e' b_0^4}{9\hbar^2 c_0^2\ r^4}\right)+\left[\left(\frac{\pi|\Gamma_{-1/3}|\Gamma_{1/3} 4\pi\sigma\tau_e' b_0^4}{9\hbar^2 c_0^2\ r^4}\right)^2-\frac{4\pi|\Gamma_{-1/3}|\Gamma_{1/3} 4\pi\tau_e' b_0^4}{9\hbar^2 c_0^2\ r^5}\right]^{0.5}\right)\frac{r}{2}\right) \tag{49}$$

which may be rewritten, using (24), as

$$\Psi(r) = \exp-\left(\left(\left(\frac{\pi k_a\alpha\sigma\tau_e' b_0^2}{9r^3}\right) + \left[\left(\frac{\pi k_a\alpha\sigma\tau_e' b_0^2}{9r^3}\right)^2 - \frac{4\pi k_a\alpha\tau_e' b_0^2}{9r^3}\right]^{0.5}\right)\frac{1}{2}\right) \tag{50}$$

According to (50) τ_e' has to be defined as:

$$\tau_e' = \tau_e\frac{9}{\pi k_a\alpha} = \tau_e\, 393.4 = 6.594\text{E+8 [m/J}^2] \tag{51}$$

The conversion of τ_e' into τ_e depends on the assumptions given above, expected to be valid to approximately one order of magnitude.

4.2 Complete solution / angular part

For the type of differential equation (45)ff a separation of variables will in general not be possible, the spherical harmonics such as Y_1^0 will not be a solution for the differential equation of type (45). However, any wave function corresponding to a rough equivalent of an atomic p-orbital will have to feature a coefficient from the integration over φ, ϑ close to 3 and be accessible to the reasoning in 2.7.
Applying the same qualitative reasoning to d-orbital equivalents would result in a factor of $5^{1/3}$ as energy ratio relative to spherical symmetric terms, giving the start of an additional partial product series to be at
$5^{1/3}$ W(η) = 937MeV = 0.978 W(η').

5 Particle-particle interaction

5.1 Relationship between particle energy and strong, Coulomb potential energy

The series expansion of $\Gamma(1/3,\beta_n/r^3)$ in the equation for calculating particle energy (13) gives [11]:

$$\Gamma(1/3,\ \beta_n/(r^3)) \approx \Gamma_{1/3} - 3\left(\frac{\beta_n}{r^3}\right)^{1/3} + \frac{3}{4}\left(\frac{\beta_n}{r^3}\right)^{4/3} = \Gamma_{1/3} - 3\frac{\beta_n^{1/3}}{r} + \frac{3}{4}\frac{\beta_n^{4/3}}{r^4} \tag{52}$$

and for $W_n(r)$:

$$W_n(r) \approx W_n - 4\pi b_0\frac{3\beta_n^{1/3}}{3\beta_n^{1/3} r} + 4\pi b_0\frac{3}{4}\frac{\beta_n^{4/3}}{3\beta_n^{1/3} r^4} = W_n - \frac{4\pi b_0}{r} + 4\pi b_0\frac{\beta_n}{4r^4} \tag{53}$$

The 2nd term in (53) drops the particle specific factor β_n and gives the electrostatic energy of two elementary charges at distance r multiplied by the factor 4π introduced in (4). The 3rd term is closely related to the terms of the differential equation given in 4.1. The 0^{th} order term in the differential equation is supposed to represent a potential energy which, though being composed of coefficients originating from electrodynamics, does not represent an electrodynamic or gravitational term but a term which has a high dependence on r and is obviously responsible for the localized character of an electromagnetic object. In 5.4 some arguments are

given that demonstrate a relationship of the properties of the wave functions used in this model with the "strong force" of the standard model. It may be assumed that the 3rd term of (53) represents this strong force.
Equation (52)f is a series expansion for r –> 0. In the next chapter it will be tried to derive an expression for F_G, starting from the term with the lowest r-dependence, i.e. the Coulomb term. The resulting term for F_G will not be part of a series expansion for $\Gamma(1/3,\beta_n/r^3)$ –> ∞. The incomplete gamma function seems to be useful to describe short range interaction. It has to be tested to what extent the approximations of this function are appropriate for long range interaction and if related functions provide better results. A close relationship of strong and Coulomb interaction with gravitational interaction is supported by the results of 5.2.2.

5.2 Gravitation

In the following it will be tried to express gravitational attraction in terms of parameters of this model, i.e. essentially as function of elementary charge and electric constant. For this task coefficients to provide correct units are needed, involving basically a combination of [J] and [m] units. The model provides several suitable coefficients establishing a unit set of acceptable consistence [37]. Coefficients of various sources might vary, in particular contributions to the conversion of τ_e and τ_e' of chpt. 4.1 resulting in one order of magnitude or more. However, this has to be judged in respect of the 42 orders of magnitude that separate electric and gravitational forces in the case of the electron. The examples below are based on the expressions for energy calculation. The necessary coefficients for unit correction are chosen with as few assumptions and input parameters as possible.
In general the derivation of the expression for gravitational force is speculative in some of the terms used, yet the equations applied are quite conservative with no need for any unconventional physics.

5.2.1 Relation of Ψ(r > r$_l$) with gravitational force

The extension of (53) wil be used in the form:

$$W_n(r) \approx 4\pi b_0 \Gamma_{1/3} \beta_n^{-1/3}/3 \ - \ 4\pi b_0 \frac{3\beta_n^{1/3}}{3\beta_n^{1/3} r} = W_n\left[1 \ - \ \frac{3\beta_n^{1/3}}{\Gamma_{1/3}\ r}\right] \tag{54}$$

The r-dependent part contains $W_n = m_n c_0^2$ and $3\beta_n^{1/3}/\Gamma_{1/3}$.
Using (54), in the following a term equivalent to Newton's law, $F_G = G\ m_m m_n$ will be derived for two electrons, $m_m = m_n = m_e$. In this simplified electrostatic approach $W_e = m_e c_0^2$ will be replaced by $W_e = m_e' \varepsilon_c^{-2}$ [38] [39], $3\beta_e^{1/3}/\Gamma_{1/3}$ will be considered to be part of $G^{1/2}$, this gives with the 2nd term of (54) squared:

$$F_{G,ee} \sim -\frac{9\,\varepsilon_c^4\ W_e^2 \beta_e^{2/3}}{\Gamma_{1/3}^2\ r^2} \tag{55}$$

The term $\beta_e^{2/3}$ will be replaced via the relation (30) by

$$\beta_e^{2/3} = \frac{4\pi b_0 \Gamma_{1/3}}{\rho_1 |\Gamma_{-1/3}|} \tag{56}$$

giving for F_G:

$$F_{G,ee} \sim -\frac{9\,\varepsilon_c^4\ W_e^2}{r^2} \frac{4\pi b_0}{\rho_1 \Gamma_{1/3} |\Gamma_{-1/3}|} \tag{57}$$

The units are not correct yet. It seems reasonable to include the coefficient responsible for the absolute energy scale of particles, i.e. τ_e explicitly in this equation. Including both τ_e' and ρ_1 squared in the equation gives the final result with correct units :

37 Compare ρ_0, ρ_1, of 5.2. with ρ_2 of 7.3.2.
38 units of m' would have to be adjusted appropriately, not relevant for the following;
39 The need to use the 4th power of ε_c is the reason to drop ρ_0 of the earlier versions of this work.

$$F_{G,ee} = -b_0 \frac{36\pi\tau_e'^2 \varepsilon_c^4 \; W_e^2}{\rho_1^2 \Gamma_{1/3} |\Gamma_{-1/3}| \; r^2} = -b_0 \frac{36\pi \; \tau_e'^2 \varepsilon_c^4}{\rho_1^2 \Gamma_{1/3} |\Gamma_{-1/3}| \; r^2} (4\pi b_0)^2 \left(\int_0^{r_{l,e}} \Psi_e(r)^2 r^{-2} dr \right)^2 \quad = 4.24 \; F_{G,ee,exp}{}^{40} \tag{58}$$

The alternate gravitation constant, γ_0, corrected by factor 4.24 is given by

$$\gamma_0 = \frac{36\pi \;\; \tau_e'^2}{4.24 \rho_1^2 \;\; \Gamma_{1/3} |\Gamma_{-1/3}|} \quad = 2.893E+17 \; [m^4/J^6] \tag{59}$$

which is essentially a function of the electron energy, $\gamma = f(\tau_e, \rho_1) = f(b_0, W_e)$, implying the electron to represent a reference state. For interaction between other particles W_e may be replaced in (58)ff according to equ. (1). or the integrals over $\Psi_n(r)^2 \, r^{-2}$.

In the derivation for F_G of equation (58) an asymmetry is introduced by using (56) and the minor factors included in it are somewhat arbitrary. Using the related simpler ansatz for γ,

$$\gamma_1' = [\tau_e'/\sigma]^2 = \left(\frac{8\alpha^{11}}{3\pi k_a k_s^3 |\Gamma_{-1/3}|^3 e_c \varepsilon_c} \right)^2 \quad = 3.754^2 \;\; [m^2/J^4] \tag{60}$$

gives a result for $F_{G,ee,calc}$ of almost the same accuracy as given by (58) with less assumptions as well as less justification for this term:

$$F_{G,nn,calc} = \frac{b_0}{r^2} \; \gamma_1' \left[e_c^2 \int_0^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr \right]^2 \quad = 4.4 \; F_{G,nn,exp} \quad {}^{41} \tag{61}$$

In the following the constant, $\gamma_1 = \gamma_1'/4.4$ will be used.

The error of the calculations compared to $F_{G,exp}$ is well within the uncertainties of the assumptions made above. Inserting the expression for τ_e' (32), (51) in (60)f gives a result where all electromagnetic terms except for b_0 cancel:

$$\begin{aligned} \frac{F_{G,ee}}{2.098^2} &= -b_0 \left(\frac{9\tau_e \;\; \varepsilon_c}{\pi k_a \alpha \sigma} \; \frac{4\pi b_0 \Gamma_{1/3}}{3(2\sigma\tau_e b_0)^{1/3}} \right)^2 = -b_0 \left(\frac{8\alpha^{11}}{3\pi k_a k_s^3 |\Gamma_{-1/3}|^3 e_c \varepsilon_c^2} \frac{e_c^2 \varepsilon_c \pi^{2/3} \Gamma_{1/3} \varepsilon_c}{k_s |\Gamma_{-1/3}| \alpha^2 e_c} \right)^2 = \quad {}^{42} \\ &= -b_0 \left(\frac{8\pi^{2/3} \Gamma_{1/3} \;\; \alpha^9}{3\pi k_a k_s^4 |\Gamma_{-1/3}|^4} \right)^2 \end{aligned} \tag{62}$$

5.2.2 Comparison of particle interaction terms

Comparing electrostatic and gravitational force between two identical particles n gives:

$$F_{n\text{-}n} = F_{Cn\text{-}n} + F_{Gn\text{-}n} = \frac{1}{4\pi\varepsilon_c r^2} \left[e_c^2 \; - \; \gamma_1 (e_c^2)^3 \left(\int_0^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr \right)^2 \right] \tag{63}$$

which may be rearranged (with units indicated accordingly) as

$$F_{n\text{-}n} = \left(\left\{ \frac{e_c^2}{4\pi\varepsilon_c} \right\} [Jm] - \left\{ \frac{1}{4\pi\varepsilon_c} \left(\int_0^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr \right)^2 \right\} [\frac{1}{Jm}] \;\; \{\gamma_1 (e_c^2)^3\} [Jm]^2 \right) \; r^{-2} \tag{64}$$

with the following values (electron):

$F_{e\text{-}e}$ = {1/(4π) 2,90E-27}[Jm] r^{-2}-{1,90E+34}[1/(Jm)] $\{5,4E\text{-}53\}^2$ $[Jm]^2 \, r^{-2}$.

---

40 The term b_0 is supposed to be unsigned. Since neutral particles are supposed to be composed of charged volume elements of equal size and opposite sign a more detailed mechanism to describe this type of interaction might be possible.

41 Factor 4.378 $F_{G,nn,exp} = 2.092^2$ if the numerical result of the integral over $\Psi(r)^2$ is used, 4.402 $F_{G,nn,exp} = 2.098^2 \, F_{G,nn,exp}$ if coefficients of (62) are used; compare note 25;

42 Better numerical results can be obtained easily by altering a few terms, e.g. using factor 2 of β (note 36) gives: 1.05^2 $F_{G,ee,exp}$, dropping the term $\pi^{2/3}$ and k_s in (62) gives: $F_{G,ee} = -b_0 \left[\left(8/3 \Gamma_{1/3} \;\; \alpha^9 \right) / \left(k_a \pi |\Gamma_{-1/3}|^4 \right) \right]^2$ = 1.002 $F_{G,ee,exp}$.

Coefficient $\rho_1' = \sigma\, \varepsilon_c = 0.5859$ [J/m] of (31) and the following coefficients will be used to compare strong-, Coulomb- and gravitational potential energy terms in dimensionless parameters:

$$\rho_0 = \tau_e'/\sigma = 3.754 \text{ [m/J}^2\text{]} \ (= \gamma_1'^{\,0.5}) \tag{65}$$

$$\rho_2 = \rho_0^2 \rho_1^3{}' = \tau_e'^2 \sigma \varepsilon_c^3 = 2.834 \text{ [1/(Jm)]} \tag{66}$$

To represent the strong interaction the term $\sigma\tau_e b_0 /\rho_0$ = 1.809 E-14 [Jm] will be used [43]. Table 5 compares the three [Jm] terms for gravitational and electrostatic potential energy (4π excluded) and $\sigma\tau_e b_0 /\rho_0$ in SI units and converted to dimensionless terms.

Force	Unit SI [Jm]	Unit dimension-less [-]	Gravitation, Strong value [-] relative to Coulomb^x term		
Gravitation	5.40E-53	1.53E-52	Coulomb ^{2}	6.75E-53	2.27
Coulomb	2.90E-27	8.22E-27	Coulomb		-
Strong	1.81E-14	5.13E-14	Coulomb ^{0,5}	9.07E-14	0.57

Tab. 5 Comparison of potential energy "[Jm]" terms in SI and dimensionless;

The quadratic relationship of the [Jm] terms demonstrated in tab. 5 gives a second indication, in addition to the series expansion of $\Gamma(1/3,\beta_n/r^3)$ of chpt. 5.1. that terms for strong and Coulomb interaction and energy / mass / gravitation might be connected.

5.2.3 Comparison with classical constant of gravitation

The classical constant $G = F_G/(m_1 m_2)$ may be expressed in terms of this model as

$$G = \gamma\, c_0^4\, \varepsilon_c^4\, b_0 = \gamma\, c_0^4\, \varepsilon_c^3\, e_c^2 /(4\pi) = 6.67408 \pm 0.00031 \text{ E-11[m}^5/(\text{Js}^4)] \tag{67}$$

5.3 Relation to nonlinear effects

The terms of the model exhibit some resemblance with concepts of nonlinear electromagnetic effects where the polarization density P is given by

$$P = \varepsilon_c \sum \chi^{(n)} E^n \tag{68}$$

with $\chi^{(n)}$ representing the nonlinear susceptibility of n-th order. The lowest nonlinear contribution in centro-symmetric potentials is of 3rd order, $P = \varepsilon_c\, \chi^{(3)}\, E^3$. The exponential of the function Ψ(r) as given by (9) contains a related term $\chi^{(3)}V(r)^3$, with V(r) being the electric potential:

$$\frac{\sigma\tau_e b_0^2}{r^3} = \left(\frac{2^3 k_s^3 |\Gamma_{-1/3}|^3 \alpha^6}{3^3 (4\pi)^2} \right) \left[\frac{e_c}{\varepsilon_c\ r} \right]^3 \tag{69}$$

The term in square brackets of (69) is equivalent to $V(r)^3$ while the term in round brackets, or parts of it, might represent an equivalent of $\chi^{(3)}$ [44]. Consequently this nonlinear term is included in several related expressions, in particular the 1st order term of the differential equation (45) as well as in the 0th order term and the 3rd term of the Γ-function extension (53) though with an additional term r^{-1} each.

5.4 Short range interaction - strong force

In this model, on the length scale of particle radius, the wave functions of two particles should start to

43 The strong force might be reflected in appropriate cosmological parameters (compare [12]). The basic term $\sigma\tau_e b_0 /\rho_0$ = 1.81 E-14[m²/J] from the function Ψ can be approximated (using (13)) by the ratio of the square of $r_{m,e}$ and energy of the electron ~ $r_{m,e}^2/W_e$ = 2,2E-14[m²/J]. Comparing this with estimated values of cosmological parameters of similar unit such as the square of the radius of the universe divided by its energy, r_{uni}^2/W_{uni} ~ 2E-23 [m²/J] (ordinary matter) leaves plenty of space for an additional expansion of the universe. - r(univ) ~ 4.5E+7ly [J. R. Gott III, et. al., " Astro. Jour., vol. 624, pp. 463–484, 2005); m(univ) ~1E+53kg [wikipedia7/17];

44 Both in dimensionless units

overlap and exert some kind of direct interaction. As demonstrated in table 1, col.9, for hadrons the model yields particle radius in the range of femtometer, the characteristic scale for strong interaction and it seems likely to identify strong interaction with the interaction of wave functions. Interaction via overlapping of wave functions constitutes the basis of chemical bonding and has been examined extensively [5]. In general wave functions are signed (not to be confused with electrical charge), for particles above the ground state regions of different sign exist, separated by nodes. There are two major requirements for effective interaction:
1) Comparable size and energy of wave functions,
2) sufficient net overlap: In the overlap region of two interacting wave functions sign should be the same (bonding) or opposite (antibonding) in all overlapping regions. If regions with same and opposite sign balance to give zero net overlap, no interaction results.
From condition 1) and the data of table 1 it is obvious that the wave functions of neutrino and electron/positron will not show effective interaction with hadrons due to mismatch of size and energy. In the case of the tauon the second rule is crucial. According to this model the tauon is at the end of the partial product series for y_1^0 and should consequently exhibit a high, potentially infinite number of nodes, separating densely spaced volume elements of alternating wave function sign. Though having particle size and energy in the same order of magnitude as other hadrons, such as the proton, the frequent change of sign of the tauon wave function will prohibit net overlap and effective interaction.
Overlap of wave functions should provide a possible description of nuclear bonding as well.

6 Other aspects of the model

6.1 Free particle

Omitting the 0th order term in the differential equations might produce the equation of a free particle. Using the following version of equ. (45) for the electron gives:

$$\frac{r}{6\sigma\tau_e b_0}\frac{d^2\Psi(r)}{dr^2} - \frac{b_0}{2r^3}\frac{d\Psi(r)}{dr} = 0 \tag{70}$$

$$\frac{d^2\Psi(r)}{dr^2} \approx \frac{3\sigma\tau_e b_0^2}{r^4}\frac{d\Psi(r)}{dr} + ... \tag{71}$$

indicating there could exist a function in the general form of (9) for a photon, maybe describing the decrease of the electromagnetic fields perpendicular to wave propagation.

$$\Psi(r) \approx \exp\left(\frac{-\sigma\tau_e b_0^2}{r^3}\right) + ... \tag{72}$$

6.2 Elementary charge

6.2.1 Electrical charge

As $\Psi(r)$ approaches 1 for $r \rightarrow r_l$ the Gauss integral $\varepsilon_0 \int E(r)\Psi(r)^2\, dA$ approaches the limit of the elementary charge e. Since for $r \rightarrow 0$ the term $E(r)\Psi(r)^2$ goes to zero, there is no 'point charge' at the origin.
At a distance of r_m, (see equ. (15)), marking the approximate maximum of W(r), $\Psi(r)^2$ attains a value of 0.667 yielding a calculated charge of 2/3 e and a value of W_n of $W_n = W_n/4$ [45].

6.2.2 Magnetic charge

The model outlined above should principally be suited to calculate the energy of particles with magnetic charge e_m, i.e. magnetic monopoles. Using the equations above to calculate energies of Dirac magnetic monopoles [13] is straightforward. Replacing e by the magnetic charge e_m

$$e_m = e/(2\alpha) \tag{73}$$

45 For the pair e, μ the value of r_m is also distinguished by the relation $r_{l,\mu} = r_{m,e}$, see 2.5.

turns b_0 into b_m. The integral (18) yields only minor variations even when changing input parameters by several orders of magnitude. This indicates the product $4\pi b_0 = xb_m$ has to be essentially a constant to provide half integer spin. The proportionality $\lambda_{C,n} \sim \beta_n^{1/3}$ has to be applicable for magnetic monopoles as well, yielding the same factor 18π in (21). As a result equ. (24) should hold for both electric and magnetic monopoles. Using the same coefficients τ_n according to equ. (36) as for electric monopoles in equ. (13) would leave $(2\alpha)^{4/3} = 1/280$ as ratio between electric and magnetic particle energies. Assuming $\tau_{0,magn} \sim 1/e_m$ (see (32)) would reduce this ratio to $2\alpha = 2/137$. Both versions place magnetic monopole particles approximately in the same energy range as their electric counterparts.

7 Discussion

7.1 Basic model

The basic idea behind this work is that elementary particles can be considered to be a standing electromagnetic wave, allowing for angular momentum, with the **E**-vector pointing towards the origin and **B** and $\mathbf{V_{rot}}$ [46] being orthogonal to each other, at least on a local scale. Neutral particles are supposed to exhibit appropriate nodes and corresponding equal volume elements of opposite polarity. Switching direction of the fields will result in the corresponding antiparticles.

Whatever the detailed mechanism of this might be, there are two basic problems to overcome:

1. Since energy of the particle as calculated from electrostatics increases infinitely for r –> 0 a function that serves as a damping term is needed to prevent this. [47]

2. $\mathbf{V_{rot}}$ which is considered to be some kind of wave propagation velocity i.e. speed of light c in its broadest sense, has to approach 0 for r –> 0 .

The function to be modified in this way is of the form

$$W_n(r) \sim b_0\, r^{-2} = \frac{e^2}{4\pi\varepsilon r^2} \sim e^2 c_0 r^{-2} \tag{74}$$

Thus the function used to modify this, Ψ(r), has to act on terms that contain r, e, c (or related electromagnetic parameters). Decreasing the value of c_0 obviously is sufficient to meet both requirements.

The term Ψ(r) of (5) is based on 3 assumptions:

1.) A term of general form $\Psi(r) \sim \exp(\frac{-\beta}{r^y})$ is used to avoid UV-divergence of the fields

2.) Ψ(r) has to be the solution of a 2nd order differential equation with 1st order term

$$\Psi(r) = \exp\left(-\left(\frac{\beta/2}{r^y} + \left[\left(\frac{\beta/2}{r^y}\right)^2 - 4\frac{\beta/2}{\sigma r^y}\right]^{0.5}\right)/2\right)$$

3.) Energy of a particle can be given in a point charge and a photon expression. This gives a correct relation for the fine-structure constant α, equ. (24) only if the power y of r is given by y = 3, resulting in (5) with β_e given as reference term for the electron.

The exponential term β can be expressed in electromagnetic terms only and since there are several indications to identify effects within the range of the wave function with the strong force, it seems possible to express this force in electromagnetic terms as well.

7.2 Relation to standard model

The standard model classifies particles into leptons, considered to be the fundamental "elementary particles" and hadrons, composed of two (mesons) or three (baryons) quarks. In the model presented the y_0^0 and y_1^0 groups each include all three particle types. The possibility to calculate particle energies with a single model using a uniform set of parameters does not support to identify a special set of particles as more “elementary” than others. However, the classification into the three groups may be reproduced.

46 tangential velocity, not **ω**

47 The 1st order term of the differential equation (48) which can be given in the form of a 3rd order nonlinear term (see 5.3) formally corresponds to the damping term of a damped oscillator equation.

Mesons constitute a distinct group of particles due to their integer angular momentum which is considered to be a combination of half-integer contributions in both models. In the standard model leptons are characterized by being essentially point like particles not subject to strong interaction. Neutrinos, electron and muon are the particles of lowest mass which in itself might provide an explanation for this quality. The tauon however is outstanding in possessing a mass almost twice that of the proton and major decay channels involving hadrons. The considerations in chpt. 5.4 about overlap and wave function symmetry might provide a consistent explanation for all leptons not to be subject to strong interaction with hadrons which in turn should prohibit detection of internal structure of these particles. However, this model suggests a smooth transition in the effects of strong interaction. For the pair muon / pion this seems not to be obvious [48]. The same reasoning as for the tauon would have to apply e.g. for Δ-particles, for which scattering data are not available. The supporting assumption of the Δ being subject to the strong force based on its short lifetime is not a general distinctive feature of both particle groups and in this model the presence of the strong force, i.e. the wave function character of particle states, is considered a constituent element of all particles anyway. The existence of a tau-neutrino as a special particle is not expected in this model. However, effects causing the distinction of the leptons from the hadrons might reflect in a distinctive interaction with neutrinos as well.
Except for the reasoning given for “lepton” particles the description of particles as electromagnetic wave structured by nodes implies some kind of measurable substructure though it goes without saying that this substructure does not provide any possibility for a division into smaller entities.

7.3 Relation to classical quantum mechanics

7.3.1 General

The relation of this model to classical quantum mechanics may be given by interpreting Ψ(r) as probability amplitude applied to a field instead of a particle. Inverting the usual interpretation it is the wave that acquires particle character. Ψ(r) will be the solution of a corresponding 2nd order differential equation. The derivation of this model started from the function Ψ since it is considered easier to develop terms for this function than to guess a term for the differential equation and in particular the term representing potential energy.
The differential equation may be given as:

$$\left(\frac{\hbar^2 c_0^2}{2W_{kin}}\right)\Delta\Psi(r) - f(W_{pot})\nabla\Psi(r) + W_{pot}\Psi(r) = 0 \qquad (75)$$

This gives no eigenvalue for energy, since energy as well as other properties has to be calculated by the integral over $\Psi(r)^2$. This implies that concepts such as orthonormalization may not be applicable on the level of the differential equation[49].
As for the number of parameters needed to calculate energy states, the model resembles the simplicity of ab initio quantum mechanical models, relying essentially on $4\pi\, b_0 = e^2/\varepsilon$ and J = 1/2 to yield the expression (1) [50]. Parameter τ_e or more generally β_e is needed to transform the relative energy scale of (1) into an absolute one and may be itself reduced to the elementary form (32) allowing all calculations to be based on e_c and ε_c as sole input parameters.

7.3.2 Quantization condition

The quantization condition given in 2.6 is not exclusive. The solution of (34)f relates to the rest mass of particles of sufficiently high mean lifetime to be experimentally observable but does not prohibit the existence of particles with any other mass. Other approaches have been tried to obtain a more definite derivation for the quantization.

A particular simple interpretation may be given using (6) and considering that the ratio $r_{l,n} / r_{l,n+1}^3$ is constant:

$$r_{l,n} / r_{l,n+1}^3 = (\sigma\, \beta_e\, \Pi_{\tau,n} / 8)^{1/3}) / (\sigma\, \beta_e\, \Pi_{\tau,n+1} / 8) = \text{const} \qquad (76)$$

---

48 With the data of tab.1 the ratio of the energy density w is $w_\mu/w_\pi \sim 1/4$; spatial distribution may add some factor;
49 The equations might be considered to be “normalized” to yield the elementary charge for $r > r_l$.
50 J = 1/2 and the values ~1.5 and σ are closely related. Factor 1.509 from the energy ratio μ/e might be considered an additional parameter, yet applies only to this particle pair.

To be valid for all n this implies $\Pi_{\tau,n} \in \Pi_{\tau,n+1}$ and $\Pi_{\tau,n}^{1/3} \in \Pi_{\tau,n+1}$ requiring $\alpha_{\tau,n+1} = \alpha_{\tau,n}^{1/3}$. Since $W_{n+1}^3 / W_n \sim \lambda_{C,n} / \lambda_{C,n+1}^3 \sim r_{l,n} / r_{l,n+1}^3$ this result is a restatement of the relations given above though suggesting that some geometrical interpretation in r- or k-space might be conceivable.

Some relationships may be obtained from the integral over the boundary condition. Using the boundary condition (12) in the form $\sigma\ \beta_n/(8r^3) = 1$, multiplying with $\Psi_n(r)^2$, integrating and using the coefficient $\rho_3\ \rho_1'^2\rho_0 = 1.289[1/m]$, for cancelling the unit [m], directly yields the partial series of particle coefficients of the next particle, n+1 [51]:

$$\rho_3 \frac{\sigma}{8} \beta_n \int_0^\infty \Psi_n(r)^2 r^{-3} dr \;=\; \rho_3 \frac{\sigma}{8} \frac{\Gamma_{2/3}\beta_n}{3(\beta_n)^{2/3}} \;=\; \rho_3 \Pi_n \alpha_{n+1} \;=\; \rho_3 \Pi_{n+1} \tag{77}$$

The integral $\int\Psi(r)^2 r^{-3}$ dr of (77) is directly proportional to $Q(\Psi_n)$, equ. (33), via the term $\beta_n^{-2/3}$. Since the value of σ is a constant this approach works for all τ_n only if the electron provides the starting value τ_0. The calculation of the particle coefficients may start directly from the value of τ_e. In equation (78) τ_e is replaced by the expression (32) giving the first coefficient of the series, $\alpha_{\tau,\mu} = \alpha^3$.

$$\frac{\sigma|\Gamma_{-1/3}|\beta_e^{1/3}}{8*9} \frac{\sigma^2 \tau_e' \varepsilon_c^2}{\sigma} = \frac{2^{1/3}|\Gamma_{-1/3}| e_c^{4/3} \sigma^{7/3} \tau_e^{4/3} \varepsilon_c^2}{8\pi(4\pi)^{2/3} \alpha\, \varepsilon_c^{2/3}} = \frac{k_s^7 \Gamma_{-1/3}^8 \alpha^4}{3^4 \pi^{5/3}} \approx 0.993\, \alpha^3 \tag{78}$$

Taking the ratio of the two integrals for the particle energy (note ε_0 replaced by ε_c)

$$W_{pc,n} = 4\pi\varepsilon_c \int_0^\infty E(r)^2 \Psi_n(r)^2 d^3r = 4\pi\, b_0\, \Gamma_{1/3}\, \beta_n^{-1/3}/3 \tag{79}$$

$$W_{pc,n} = 4\pi e_c \int_0^\infty E(r) \Psi_n(r)^2 dr = 4\pi\, b_0\, \Gamma_{1/3}\, \beta_n^{-1/3}/3 \tag{80}$$

gives:

$$\frac{e_c}{\varepsilon_c} = \int_0^\infty E(r)^2 \Psi_n(r)^2 d^3r \;/ \int_0^\infty E(r) \Psi_n(r)^2 dr \tag{81}$$

and suggests that solutions for E(r) other than the point charge may be used.
As indicated above it is the angular momentum that most clearly requires some sort of quantization. The term e_c/ε_c suggests that it might be replaced by $e_c c_0 \sim I$ in a term equivalent to (81), maybe providing a more accessible approach to quantization via phase of the wave function, an aspect which has been totally neglected on this level of approximation. To model this a mathematical approach using quaternions might be promising.

7.4 Particles

7.4.1 Ground state

The results, in particular of chpt. 2.5, 3.1 and 5.2 strongly suggest that the electron, the charged particle of lowest mass, constitutes a kind of reference state. However, an indication against the electron being a ground state might be that going to lower states seems to be possible, see 7.4.2.

7.4.2 Lower limit

For extending this model to energies below the electron a coefficient of α^3 is used in equ. (1): $W_\nu / W_e = 1.509\ \alpha^3$. This gives a state with energy 0.3eV which is in a range expected for a neutrino [9].
Yet the final lower limit should be reached soon. While r_l of the hypothetical neutrino is $r_l = 1{,}5E\text{-}5$ [m], the next lower state would be the last one to fit into the universe, with $r_l \sim 1E{+}13$[m] ~ 0,001 light year.

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51 Integrating over both sides of (12) requires an appropriate integration limit for the integral over $\psi(r)^2$dr in order to get two matching expressions.

7.4.3 Upper limit

The partial product of each symmetry group has an upper limit though it is not clear if there is an absolute upper limit for the sum of all symmetry groups. 7.4.4 discusses other possibilities for higher energy states.

7.4.4 Particle states not in y_0^0 and y_1^0

On the present level the y_0^0 and y_1^0 states of this model cover the 13 particle families of table 1 {e, μ, π, η, ρ/ω, p/n, Λ, Σ, Δ, Σ*, Ω, N(1720), τ} (excluding ν). This may be compared with the number of particle families given by the multipletts of u, d, s quarks in roughly the same energy range which is 13 as well, {π, K, ρ, K*, p/n, Φ, Λ, Σ, Δ, Ξ, Σ*, Ω, Ξ*}(excluding linear combinations).
Apart from particles attributed to y_0^0 and y_1^0 symmetry, assignment of more particle states will be not obvious. The following gives some possible approaches.

7.4.4.1 Partial products

Additional partial product series will have to start with higher exponents n in α^(-1/3^n) giving smaller differences in energy while density of experimentally detected states gets higher. There might be a tendency of particles to exhibit a lower MLT making experimental detection of particles difficult [52]. To determine the factor y_l^m requires the complete solution of the differential equation yet to be done. All these factors will impede the identification of additional partial product series.
One more partial product might be inferred from the fact that η' or Φ^0 are the first particles available as starting point (considered to be an equivalent of a 3d state, i.e. following η) while Δ(2420) with a spin of 11/2, indicating a high number of nodes, might be close to an end point. The difference in energy fits a series, some candidates for intermediate particles exist. However, in general it is not expected that partial products can explain all values of particle energies.

7.4.4.2 Linear combinations and particle compounds

The first particle family that does not fit to the partial product series scheme are the kaons at ~ 495MeV. They might be considered to be an equivalent to linear combination states of classical quantum mechanics. The π-states of the y_1^0 series are expected to be similar to p-orbitals of the H-atom, giving a charge distribution of +|+, -|- and +|-. A linear combination of two π-states would yield the basic symmetry properties of the 4 kaons as:

	+	-	-	+
	K^+ + +	K^- - -	K_S^o + +	K_L^o + -	(+/- = charge)
	+	-	-	-

providing two neutral kaons of different structure and parity, implying a decay with two different MLT values. For the charged Kaons, K^+, K^-, a configuration for wave function sign equal to the configuration for charge of K_S^o and K_L^o might be possible, giving two variants of P+ and P- parity of otherwise identical particles and corresponding decay modes not violating parity conservation.

-	+	
$K^{+/-}$ + +	$K^{+/-}$ + -	(+/- = wave function sign)
-	-	

The general formalism of such linear combinations might be different from classical quantum mechanics. At least the normalization condition would have to be altered or entirely dropped, which might result in a simple addition of particle energies. This is not the case for two pions adding up to one kaon. However, it has been noted for a long time that simple multiple-mass relations can be found among particle masses [7], [14]. Easy identifiable examples of near integer multiples can be found in particular among mesons, e.g. K, K* or η', η_c and η_b; among baryons e.g. the doubly charged particles stand out.
The latter particles draw attention to another possibility to explain particle resonances. A particle like Δ^{++} (from the reaction of p and π^+) is not expected within this model. Replacing elementary charge e in the equations by 2e would give energies not compatible with other single charged or neutral Δ particles and a whole series of doubly charged particles should exist. A particle of charge 2+ in this energy range would be

52 Which might explain missing particles of higher n in the y_0^0 and y_1^0 series as well.

rather considered to be a compound of n and two π^+, giving an equivalent of the ^{3}He nucleus [53]. In general compounds of strongly interacting particles might be a source for experimentally observed resonances.

8 Summary

The main results obtained by applying the function Ψ(r) to E(r)will be summarized here in terms of α, e_c, ε_c as well as the minor terms k_s, k_a. $\Gamma_{1/3} = \Gamma_+$, $|\Gamma_{-1/3}| = \Gamma_-$.

● wave function

$$\Psi_n(r < r_l) = \exp\left(-\left(\frac{2}{3}\right)^3 \frac{k_s^3 \Gamma_-^3 \alpha^6}{(4\pi)^2} \left(\frac{e_c}{\varepsilon_c}\right)^3 \frac{\Pi_{\tau,n}}{r^3}\right) \tag{82}$$

● fine structure constant:

$$4\pi\, \Gamma_{1/3}|\Gamma_{-1/3}| \approx \frac{\hbar c_0}{b_0} = \alpha^{-1} \tag{83}$$

● particle energy
absolute:

$$W_n = \frac{e_c^2}{\varepsilon_c} \int_0^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr = \frac{\pi^{2/3} \Gamma_+}{k_s \Gamma_-} \frac{e_c}{\alpha^2} \frac{1}{\Pi_{\tau,n}^{1/3}} \tag{84}$$

relative :

$$W_n / W_e = \left(y_l^m\right)^{-1/3} 1{,}509\, \Pi_{k=0}^{n} \alpha\text{^}(-(1/3)^k) \qquad n=\{0;1;2;..\} \tag{85}$$

● ratio of elementary charge to electron energy:

$$\frac{e_c}{W_{e,calc}} = \frac{\alpha^2}{\pi^{2/3}} \frac{k_s \Gamma_-}{\Gamma_+} \tag{86}$$

● magnetic moment

$$M_n = \frac{k_s}{k_a} \frac{\Gamma_-^2 \alpha^2}{2\pi^{2/3}} \frac{c_0\, e_c^2}{\varepsilon_c} \{\text{structure factor}_n\} \quad [\text{Jm}^2] \tag{87}$$

● terms for particle interaction
r –> 0

$$W_n(r) = W_n - \frac{e_c^2}{\varepsilon_c r}\left[1 + \frac{2}{3^3} \frac{k_s^3 \Gamma_-^3 \alpha^6}{(4\pi)^2} \left(\frac{e_c}{\varepsilon_c r}\right)^3 \Pi_{\tau,n}\right] \tag{88}$$

r –> ∞

$$W_{pot,nn} = b_0 r^{-1}\left(1 + \varepsilon_c^2 \gamma_1 \left[e_c^2 \int_0^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr\right]^2\right) = \frac{1}{4\pi\varepsilon_c r}\left[e_c^2 + \left(\frac{8\ \alpha^{11}}{3\pi\ k_a k_s^3 |\Gamma_{-1/3}|^3}\right)^2 \left(\frac{\pi^{2/3}\Gamma_+}{k_s \Gamma_-} \frac{e_c}{\alpha^2} \frac{1}{\Pi_{\tau,n}^{1/3}}\right)^2\right] \tag{89}$$

53 Existing in an excited state.

Conclusion

Using the exponential function $\Psi(e_c, \varepsilon_c)$ as probability amplitude for the electric field E(r) gives the following results:

- a numerical approximation for the value of the fine-structure constant α,
- a quantization of energy levels given by a partial product of terms α^(-1/3^n),
- magnetic moments, calculated directly from the electromagnetic fields,
- qualitative explanations for particle properties such as the lepton character of the tauon or the decay of kaons,
- a possibility to quantitatively express gravitational force entirely in electromagnetic terms,
- an indication of a common base for strong force, electromagnetism and mass/gravitation, based on a common set of -electromagnetic- coefficients, the expansion of the incomplete gamma function and a possible quadratic relationship between characteristic terms of potential energy.

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