

# Parking Space Detection and Trajectory Tracking Control for Vehicle Auto-Parking

Shiuh-Jer Huang, Yu-Sheng Hsu

**Abstract**—On-board available parking space detecting system, parking trajectory planning and tracking control mechanism are the key components of vehicle backward auto-parking system. Firstly, pair of ultrasonic sensors is installed on each side of vehicle body surface to detect the relative distance between ego-car and surrounding obstacle. The dimension of a found empty space can be calculated based on vehicle speed and the time history of ultrasonic sensor detecting information. This result can be used for constructing the 2D vehicle environmental map and available parking type judgment. Finally, the auto-parking controller executes the on-line optimal parking trajectory planning based on this 2D environmental map, and monitors the real-time vehicle parking trajectory tracking control. This low cost auto-parking system was tested on a model car.

**Keywords**—vehicle auto-parking, parking space detection, parking path tracking, intelligent fuzzy controller.

## I. INTRODUCTION

CURRENTLY, the vehicle parking electronic systems can be classified into parking assistant and auto-parking two types. The parking assistant system can provide driver more vehicle surrounding information and help the driver to complete the parking action by using additional sensors, i.e. ultrasonic sensor, CCD camera and radar system. Auto-parking system employs camera or radar system to detect the appropriate parking space and relative location with ego-car first, then it can automatically plan a parking trajectory for auto-parking system to track when driver starts the parking action. Driver still can control the brake and vehicle backward speed or the vehicle automatically completed the parking process. Three main steps of auto-parking process are the 2D relative position constructing between ego-car and parking space by using appropriate sensors; optimal parking path analysis and planning based on available space; and on-line vehicle motion path estimation and parking trajectory tracking control. This idea was firstly proposed by Volkswagen Company [1] in 1992. Volvo [2], Audi [3] and BMW [4] developed their auto-parking system, too. It includes electric power steering system (EPS), ultrasonic or radar environment detecting system and auto-parking trajectory planning and tracking control system. This system can automatically search the appropriate parking space based on four ultrasonic sensors and then announce a signal for notifying driver to switch on the auto-parking system for executing the parking operation.

There have been some researches on the parking path

tracking control of car-like mobile robot based on visual based environmental information [5], [6]. For the parking trajectory planning, Lyon [7] proposed a 5<sup>th</sup> order polynomial as the roadside parallel parking path with curvature and nonholonomic constraints. Laumond et al. [8] proposed an automatic parking path planning strategy for a mobile robot with incomplete constraints. Zhu and Rajamani [9] proposed a nonlinear state feedback controller for roadside auto-parking with triangular function parking path planning. Neff et al. [10] designed a minimum roadside parallel parking path with two continuous arc segments by using Pontryagin's principle. Here, a total solution for both the roadside parallel and garage auto-parking process was proposed. An onboard ultrasonic sensors system can detect the 2D relative position between ego-car and parking space and judge the type of available roadside parking space. A model-free intelligent self-organizing fuzzy control strategy is proposed to design an auto-parking path tracking controller based on system learning mechanism without expertise knowledge or trial-and-error process. This control system is implemented on a 1/6 model vehicle for experimental study.

## II. AUTONOMOUS MODEL VEHICLE SYSTEM STRUCTURE

In order to investigate the auto-parking system dynamic performance, a DC motor actuated 1/6 remote control model car was retrofitted with PC based control system for the auto-parking process monitoring. An aluminum structure is constructed for installing the control circuit and power supplier. SRF10 ultrasonic sensors are installed on the aluminum pillars of vehicle side surface for detecting the parking space. The motion speed of this model vehicle is controlled by rear wheel axle motor and the motion orientation is regulated by front wheel steering motor. The overall system structure is shown in Fig. 1. A PC is chosen to handle all the I/O data for the whole system and to calculate the control parameters. The digital counters of PCI-1784 are used to decode the servo motors rotational angle or motion distance. The resolution of these servo motor encoders is 1092 pulse/rev. The control software is written with Borland C++.

## III. VEHICLE MOTION KINEMATICS

Since, this model car only uses two servo motors to control the front wheels steering angle change and rear wheels running velocity, respectively, two degrees of freedom motion kinematics is established to represent the vehicle forward and backward motions, acceleration/deceleration and turning motion. The simplified 2D bicycle model can be represented as

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Fig. 2 where  $\psi$  is the angle between X axis and the longitudinal direction of vehicle chassis. It is called vehicle orientation angle.  $\delta$  is the front wheel steering angle.  $V_f$  and  $V_r$  are the velocities of front wheel axle and rear wheel axle, respectively. L, wheelbase, is the distance between front wheel axle and rear wheel axle. Then the vehicle reverse motion

kinematical correlation can be represented as:

$$\begin{aligned} \dot{x}_r &= -V_r \cos \psi \\ \dot{y}_r &= -V_r \sin \psi \\ \dot{\psi} &= -\frac{V_r}{L} \tan \delta \end{aligned} \quad (1)$$

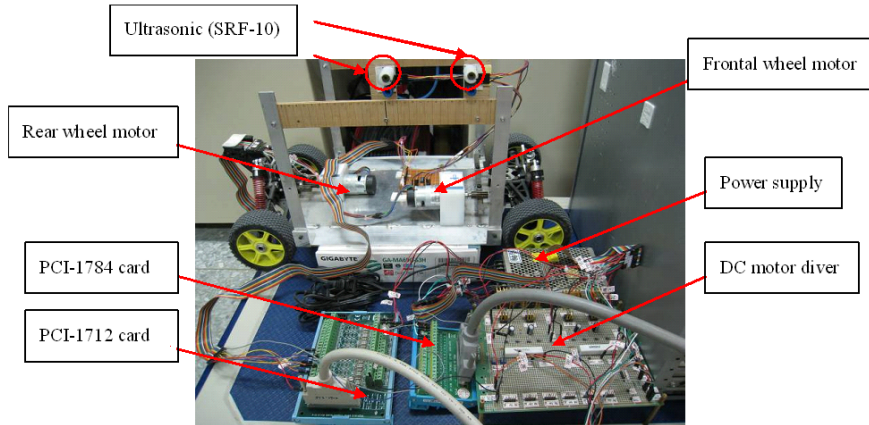


Fig. 1 Model vehicle structure and control circuit

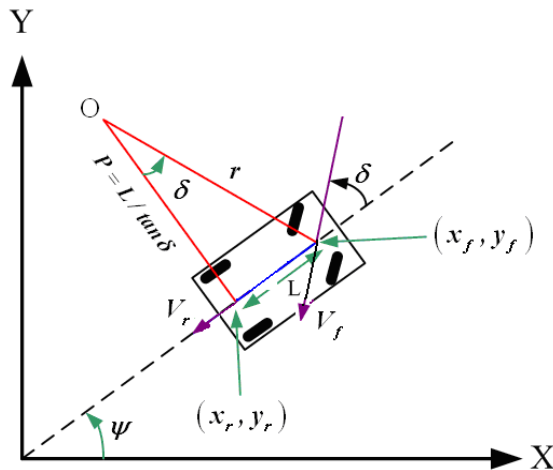


Fig. 2 Simplified 2D bicycle model geometry relationship, where  $(x_r, y_r)$  is the desired tracking trajectory  $(x, y)$  for parking process

#### IV. PARKING TRAJECTORY PLANNING

The optimal parking path analysis and planning based on detected available space is another preceding step for auto-parking operation. An on-board path planning algorithm should be designed based on 2D environmental map and the vehicle's dynamics and constraints to determine a reference parking trajectory. The vehicle backward auto-parking paths include road side parallel parking and garage parking two kinds of reference trajectories. For the parallel parking, several feasible and smooth curves have been used to present the reference trajectory such as two circular arcs, sine curve, cosine curve, and a fifth-order polynomial curve etc. These curves are determined by the initial position of the vehicle and final

position of parking zone. Lyon [7] had shown that a fifth-order polynomial needs the least extra time penalty in straightening out the wheel at the maneuver end-points of parallel parking. The reference trajectory for backward parallel parking is represented as a function  $y_r = f(x_x)$ . The general form of a fifth-order polynomial and the triangular function are given by [5], [9]

$$y_r(x_x) = y_0 \left[ 6 \left( \frac{x_r}{x_o} \right)^5 - 15 \left( \frac{x_r}{x_o} \right)^4 + 10 \left( \frac{x_r}{x_o} \right)^3 \right] \quad (2)$$

$$y_r(x_r) = \frac{y_f}{2} \cdot \cos\left(\frac{\pi x_r}{x_f} + \pi\right) + \frac{y_f}{2} \quad (3)$$

where  $y_0$  and  $x_0$  are the vehicle initial position components before parking process. And  $y_f$  and  $x_f$  are the vehicle final position components after parking process. Then the specified vehicle orientation angle  $\psi_d$  and the corresponding vehicle steering angle  $\delta$  can be presented as:

$$\psi_d = \tan^{-1}(y') = \tan^{-1} \left[ -\frac{\pi y_f}{2x_f} \cdot \sin\left(\frac{\pi x_r}{x_f} + \pi\right) \right] \quad (4)$$

$$\delta = \tan^{-1} \left[ \frac{L \cdot y''}{(1 + y'^2)^{3/2}} \right] \quad (5)$$

According to our driving experiences of backward garage parking, the steering wheel is turned to the right limit and driving back the car. Then the car will result in an arc motion trajectory for entering the garage. Finally, the vehicle is

preferred to keep backing straight in the garage. Hence, the complete desired parking trajectory for garage parking is the connection of a quarter circle and a short straight line at the end of the quarter circle. The planning trajectories for roadside parallel parking and garage parking are shown in Figs. 3 (a) and (b), respectively. Fig. 3 (b) shows the proposed garage parking trajectory, where  $(x_e, y_e)$  is the virtual center of the quarter circle,  $(x_o, y_o)$  is the initial location of the reference trajectory, and  $(x_f, y_f)$  is the final vehicle parking location for  $(x_r, y_r)$ . The reference rear wheel center trajectory during backward garage parking is represented as a function  $y_r = f(x_r)$ . The general

form for circular motion is given by

$$(x_r - x_e)^2 + (y_r - y_e)^2 = (y_e - y_{in})^2 \quad (6)$$

And the straight line motion path is

$$x_r = x_f \text{ and } y_e \leq y_r \leq y_f \quad (7)$$

Similarly, the specified vehicle orientation angle  $\psi_d$  and the corresponding vehicle steering angle  $\delta$  can be derived from (4) and (5).

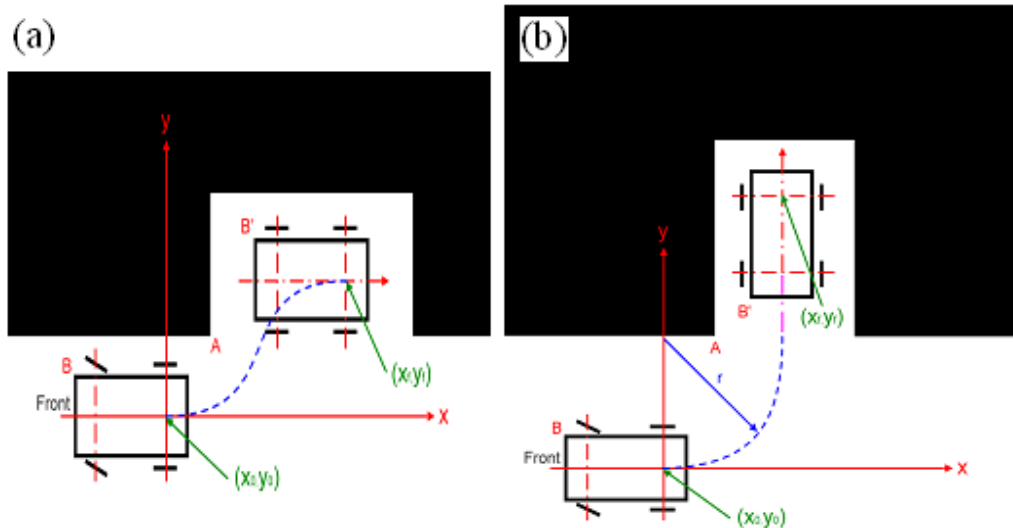


Fig. 3 (a) Roadside parallel parking and (b) garage parking planning trajectories

#### V. VEHICLE BACKWARD AUTO-PARKING CONTROLLERS DESIGN

Since the vehicle auto-parking control process has non-autonomous property and uncertainty behaviour, each parking trajectory and initial/target positions are different. Usually, the 2D parking path tracking control is executed by monitoring the front wheel steering angle only. The X and Y axes' motion path components are not measurable and directly controlled state variables. The controller only generates the steering angle command for indirectly regulating the motion trajectory. It is an indirect and in-complete control problem. In addition, the complicate vehicle dynamic model is not well known for state estimation. Here, a model-free self-organizing fuzzy control scheme is proposed to design an intelligent controller for a 1/6 model vehicle auto-parking monitoring process. The 2D bicycle kinematics model is used to estimate the vehicle real time position only. The control strategy generates the front wheel steering angle command only. Then the vehicle needs an appropriate controller to quickly regulate the steering angle for tracking the desired command. Here, a PI controller is designed for this purpose. The self-organization fuzzy controller is designed and explained shortly in following paragraphs.

The main difference between a SOFC and a traditional fuzzy controller is the properties of their database and fuzzy rules.

The database and fuzzy rules of a traditional fuzzy controller are fixed after the design step. However, the database and fuzzy rules of a SOFC are accumulated or modified continuously based upon a learning strategy during the control processes to improve the system output performance. Then the output error and error change were employed directly to modify the linguistic fuzzy rules table. The fuzzy rules table of this SOFC can be started with zero initial fuzzy rules. Here a real-time linguistic SOFC control strategy shown in Fig. 4 (b) is proposed by using two parameters to take care of the function of performance measure instead of the performance decision table. The state output error, E, and error change, CE, are divided into 11 fuzzy subsets from -1 to +1 with interval 0.2. For each control step, the fuzzy control input variables, i.e. output error and error change of the selected system state variable will stimulate two fuzzy subsets of the E and CE universe of discourse, respectively. Since the steering angle control input command  $\delta$  is derived from the fuzzy rules inference, the rules modification will influence four fuzzy rules for each control step. The correction value of each fuzzy rule is proportional to its excitation strength  $w$ . The excitation strength is designed as a triangular membership function and calculated with a linear interpolation algorithm. Then the steering angle control input command of the  $i^{th}$  rule is

$$\begin{aligned} \delta_i(nT+T) &= \delta_i(nT) + \Delta\delta_i \\ &= \delta_i(nT) + w_{ce} w_{cei} \frac{\gamma}{M} [(1-\xi) \cdot \Delta x(nT) + \xi \cdot \Delta \dot{x}(nT)] \end{aligned} \quad (8)$$

The term  $\frac{\gamma}{M}$  can be considered as the correction weighting. In this study,  $M$  is chosen as 1 in order to eliminate the identification procedure and reduce computing time during implementation. The correction weighting is regulated by the parameter  $\gamma$  only. The general form of a self-organizing fuzzy control rule can be expressed as

$$R_i: \text{ If } \Delta x \text{ is } A_1 \text{ and } \Delta \dot{x} \text{ is } A_2, \text{ Then } \delta \text{ is } C_1 \quad (9)$$

where  $R_i$  is the  $i^{\text{th}}$  rule,  $\Delta x$  and  $\Delta \dot{x}$  are the system output state variables control errors and  $\delta$  is the steering angle

control input command.  $A_1$ ,  $A_2$  and  $C_1$  are the corresponding fuzzy subsets of the input and output universe of discourse. Since the SOFC has learning capability, it does not need to find the appropriate shape of membership functions. The equal span triangular membership function is employed in this paper. The range of the fuzzy variables also can be adjusted according to the system variation by using the parameters  $ge$ ,  $gce$  and  $gu$ . The membership function used for the fuzzification is a triangular type. The function can be expressed as

$$\mu(x) = \frac{1}{W} (-|x-a|+W) \quad (10)$$

where  $W$  is the distribution span of the membership function and  $a$  is the parameter corresponding to value 1 of the membership function.

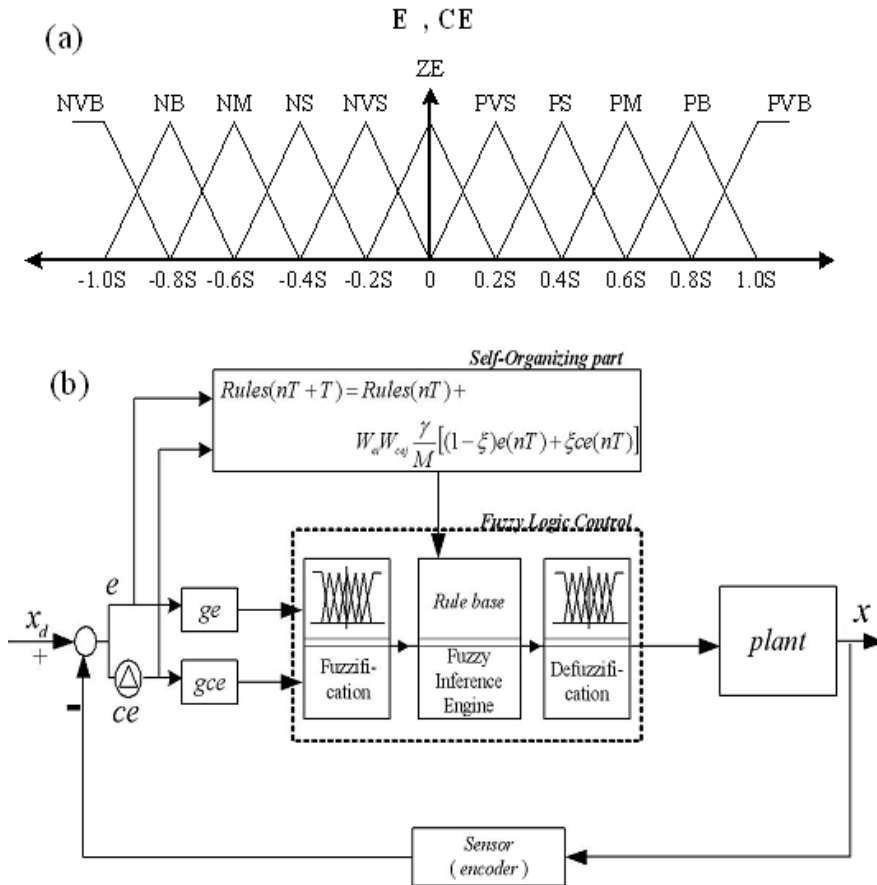


Fig. 4 (a) Fuzzy inputs membership function and (b) SOFC control block diagram

Since the output response error and error change stimulate two fuzzy subsets of the universe of discourse  $E$  and  $CE$ , respectively in each sampling interval, four fuzzy control rules in fuzzy rules table are fired only in spite of the whole rules bank. A linear interpolation fuzzy operation scheme is employed to defuzzy the fuzzy control law from these four fuzzy control rules for obtaining the steering angle control command of the front wheel steering motor in each control step.

The equation can be described as

$$\begin{aligned} U_1 &= \underline{U}_{i,j} + (\underline{U}_{i+1,j} - \underline{U}_{i,j}) \mu_{a_{i+1}}(a) \\ U_2 &= \underline{U}_{i,j+1} + (\underline{U}_{i+1,j+1} - \underline{U}_{i,j+1}) \mu_{a_{i+1}}(a) \\ \delta &= U_1 + (U_2 - U_1) \mu_{b_{j+1}}(b) \end{aligned} \quad (11)$$

where  $\mu_{a_j}(a)$  is the linguistic value of the fuzzy set variable and  $\delta$  is the resulting fuzzy steering angle control command

value of the  $i^{th}$  fuzzy rule of each control step.

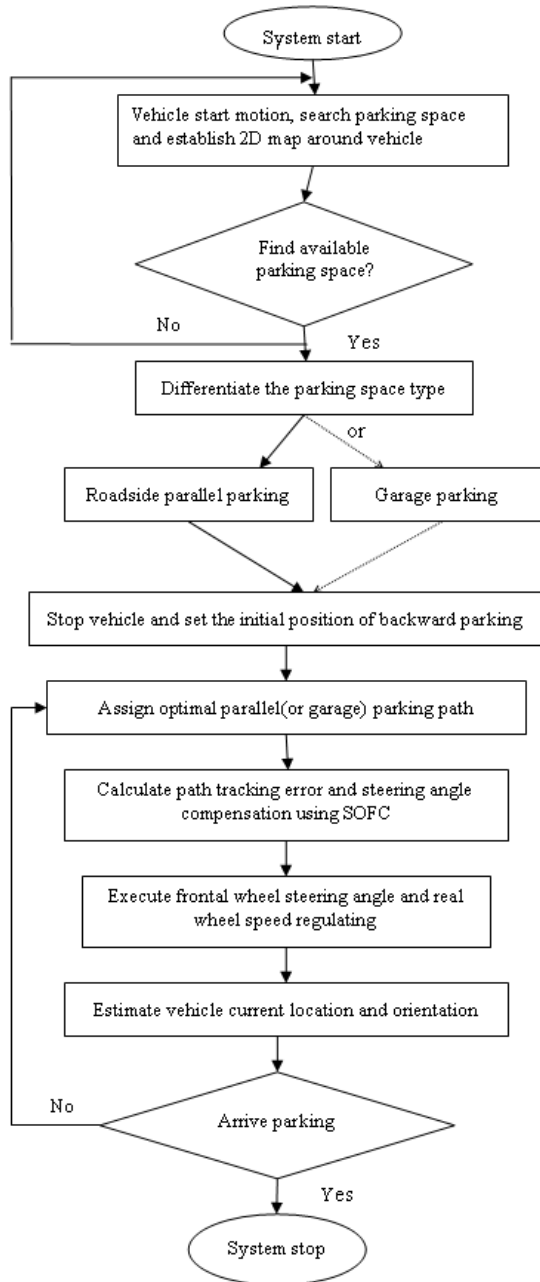


Fig. 5 Overall operation flow chart of this auto-parking control system

## VI. EXPERIMENTAL RESULTS

In order to evaluate the overall performance of this auto-parking system, both the proposed ultrasonic sensor parking space detecting system and SOFC intelligent controllers are implemented on this 1/6 model vehicle constructed in our Lab. and described in Section II. The parking space detecting system can establish the ego-car 2D environmental map and identify the available parking space type. The SOFC control law is used to generate the real-time steering angle control commands during reversed parking process. A PI position controller is designed to quickly regulate

the steering angle step change command come from auto-parking controller in each control cycle. The control gains are selected as  $K_p = 0.04$  and  $K_i = 0.001$ . A PI velocity controller is designed to control the rear wheel servo motor for monitoring the vehicle velocity. The control gains are selected as  $K_p = 3$  and  $K_i = 1$  based on experimental test. If driver starts the auto-parking system, this smart environment detecting system will begin to search an available parking space first, and then execute the complete auto-parking process. BCB user interface can be designed to display the 2D environmental map of ego-car with respect to the surrounding obstacles in screen with an icon to show parking space available information for reminding driver to hand over the control right to reversed vehicle auto-parking operation. Then the vehicle will stop at a suitable position which will be set as the auto-parking initial position  $(x_o, y_o)$  and an appropriate location in the available parking space is selected as the target parking position  $(x_f, y_f)$  based on vehicle dimension and the size of parking space. Secondly, the triangular function or a fifth-order polynomial can be chosen as the desired parking trajectory for connecting the vehicle initial position  $(0, 0)$  and target parking position  $(x_f, y_f)$  for parallel parking, (3), (4) as described in Section IV. Then the corresponding vehicle orientation angle and front wheel steering angle trajectory can be derived from (4) and (5).

$$\psi_d = \tan^{-1}(y') = \tan^{-1}\left[-\frac{\pi y_f}{2x_f} \cdot \sin\left(\frac{\pi x_r}{x_f} + \pi\right)\right] \quad (12)$$

For a specified parking path, the motion trajectories of  $x_{rd}$ ,  $y_{rd}$  and  $\psi_d$  are correlated as (4), (12). Hence, one of them can be chosen as the parking path tracking control variable. The overall system flow chart of this auto-parking process is shown in Fig. 5.

(Case 1) Available Roadside Parallel Parking Space with Constant Model Car Speed:

The model car is moving forward to detect available parking space and execute backward roadside parallel parking operation with constant speed 0.1 m/sec. The test site, vehicle motion trajectory, tracking error history of Y axis component and the steering angle tracking error history by using SOFC controller are shown in Fig. 6. Nine instantaneous motion pictures are shown in Fig. 7. It can be observed that the model car follows the specified auto-parking path backward to the target position with small tracking error. The parking path trajectory error is less than 0.14 cm and the steering angle tracking error is less than  $0.14^\circ$ , respectively.



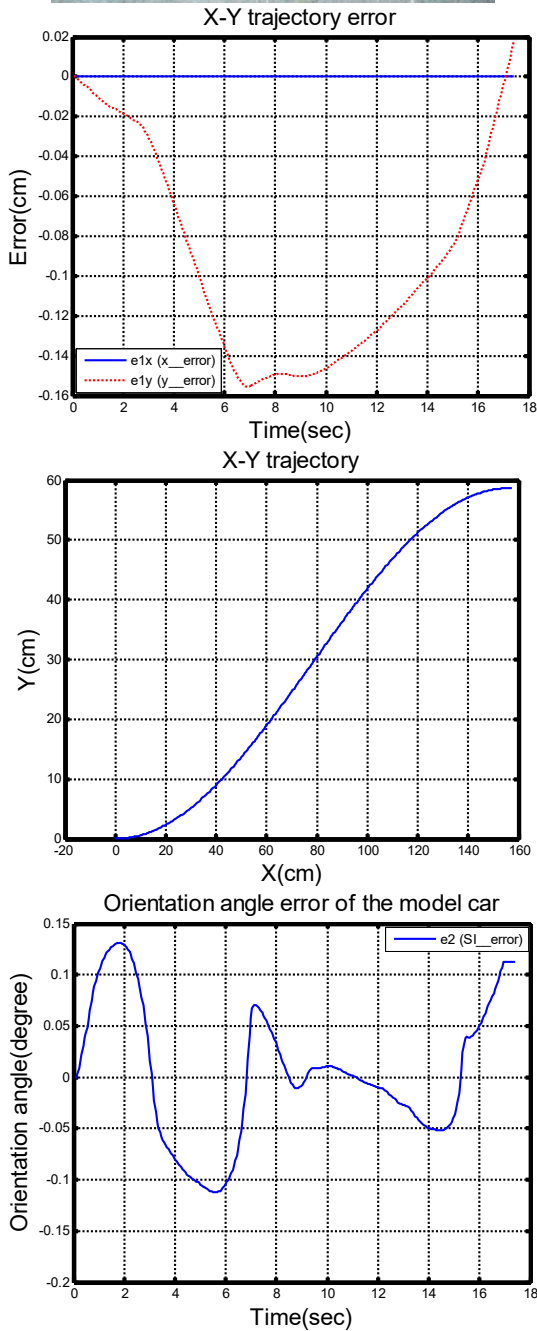


Fig. 6 Test site, vehicle parallel parking motion trajectory, tracking error histories of X and Y axes components and the steering angle tracking error history

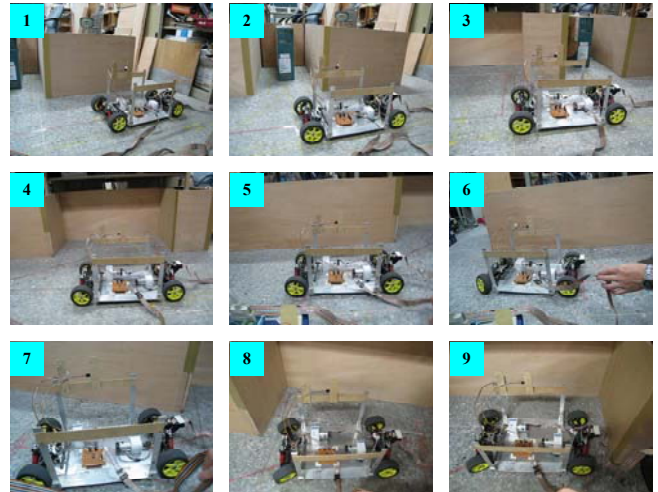


Fig. 7 Nine instantaneous motion pictures of vehicle roadside parallel auto-parking operation with SOFC control

### (Case 2) Available Garage Parking Space with Constant Model Car Speed

The model car is driven forward to detect available garage parking space and execute backward garage parking operation with constant moving speed 0.1 m/sec. The test site, vehicle motion trajectory, tracking error histories of X and Y axes components and the steering angle tracking error history by using SOFC controller are shown in Fig. 8. Nine instantaneous motion pictures are shown in Fig. 9. It can be observed that the model car follows the specified auto-parking path backward to the target position with small tracking error. The parking path trajectory error is less than 0.12 cm and the steering angle tracking error is less than  $0.25^\circ$ , respectively.

It can be observed from these experimental results that this smart auto-parking system can automatically detect the available parking space type and accurately monitor the vehicle model to the target position following the planning trajectory with small tracking error. This smart parking space detecting system is low cost and the SOFC control structure is easy to implement.

## VII. CONCLUSION

An ultrasonic sensor parking space detecting system and self-organizing fuzzy trajectory tracking controller are proposed for backward auto-parking operation. The ultrasonic parking space structure can establish the 2D environmental map around ego-car. On-board parking monitoring system can identify the type of available parallel or garage parking space and plan the corresponding optimal parking path. The intelligent SOFC controller can manipulate the steering angle to drive the vehicle backward to the parking target. The experimental results show that this intelligent auto-parking system can achieve completely automatic parking process for different parking situations. The trajectory tracking error of this auto-parking process is kept within 0.2 cm under non-holonomic, un-measurable state variables and internal dynamics behaviours.

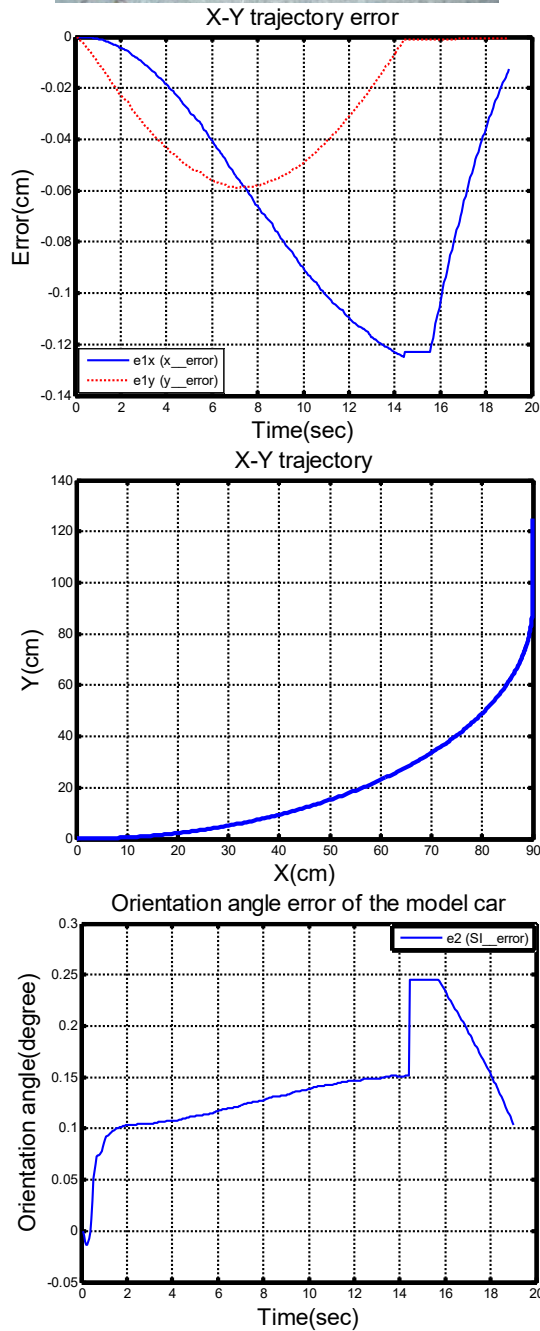


Fig. 8 Test site, vehicle garage parking motion trajectory, tracking error histories of X and Y axes components and the steering angle tracking error history

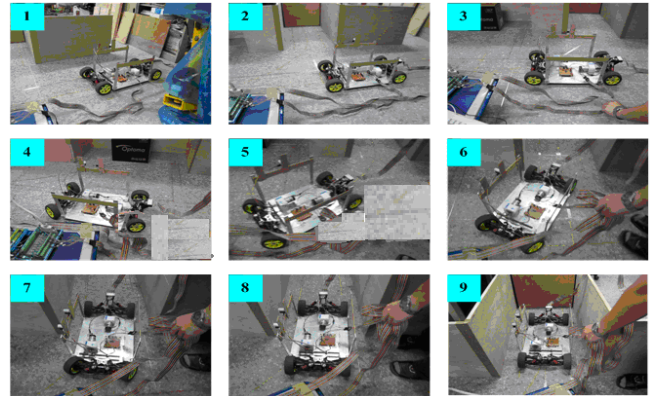


Fig. 9 Nine instantaneous motion pictures of vehicle garage-parking operation with SOFC control

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