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Practical Problems as Tools for the Development of Secondary School Students' Motivation to Learn Mathematics

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***Abstract*—**This article discusses plausible reasoning use for solution to practical problems. Such reasoning is the major driver of motivation and implementation of mathematical, scientific and educational research activity. A general, practical problem solving algorithm is presented which includes an analysis of specific problem content to build, solve and interpret the underlying mathematical model. The author explores the role of practical problems such as the stimulation of students' interest, the development of their world outlook and their orientation in the modern world at the different stages of learning mathematics in secondary school. Particular attention is paid to the characteristics of those problems which were systematized and presented in the conclusions.

***Keywords*—**Mathematics, motivation, secondary school, student, practical problem.

I. Introduction

THEORETICAL understanding of mathematical concepts requires the generalized application of empirical experience students derive from their practical activities. Conversely, abstract concepts learnt in theory may be effectively applied in practice. A series of stages for the teaching of mathematics to students has been identified to incorporate both elements. These stages are based on the results of our experimental work with secondary school students.

The first teaching step was the acquisition of mathematical concepts and applications through an empirical approach. The second stage was to enable an understanding of the system of empirical concepts and the relationships between them. During this stage those concepts were supplemented by theoretical content. This created the conditions for students to progress to the theoretical level. At the final stage, students actively applied mathematical concepts and their theoretical content in practice. This methodology can be implemented throughout the whole teaching-learning process of mathematics at secondary level, albeit that, different motivational factors can predominate depending on the age range of pupils.

As we know, mathematical activity highlights the quantitative relations and spatial forms which are inherent in all objects and phenomena. In other words, mathematics is a universal scientific method for research, descriptions and cognition of nature [2], [4], [7], [12]-[14]. Thus, the problems arising from real-life practice tend to be the major drive for the subsequent mathematical activity [3], [10], [11], [15]. The connection of mathematical activities and real-life practice is reflected in Fig. 1.

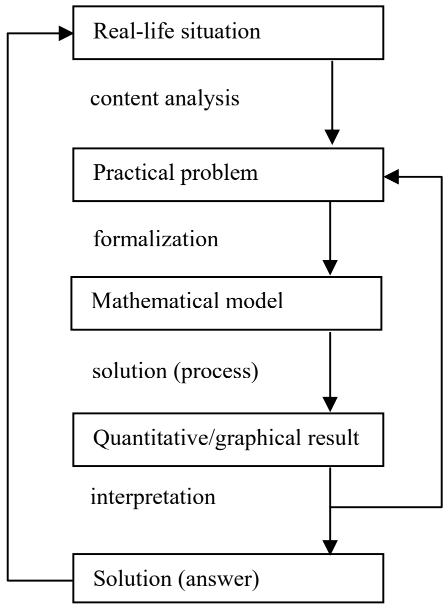

Fig. 1 Mathematical activities and real-life practice

Fig. 1 shows that research activity begins with a content analysis of the real-life situation leading to the appearance of specific practical tasks. This analysis includes clarifying the meaningful parameters, checking the initial information, the addition of missing data when necessary, the choice of a range of a numerical data corresponding to the characteristics of the given problem and the preliminary evaluation of the result [2], [8], [11], [13]-[15]. The next step is then the creation of a mathematical model of the problem. This reproduces the features of the mathematical structure using mathematical terms and symbols. This model is then transformed to obtain a quantitative or a graphical result. Then, students make conclusions about solutions by estimating the viability of those values in the initial real-life situation. They also make decisions about the possibility of using the method for solving similar kinds of problems [5, p. 22-23]. An urgent need to find

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a solution of this practical problem is the dominant factor for students.

Practical problems allow the transformation of real-life concepts, ideas and approaches inherent in natural science and the humanities into the area of pure mathematics. Practice can be a source of plausible reasoning, based on intuition, experiment, analogy and constructive induction. This reasoning differs from methods based on proof and does not ensure the reliability of the mathematical theory, but is by its nature inherent in any mathematical discoveries for various level of abstraction [10, p. 21]. Polya [9] gives many examples of how plausible reasoning may lead to the discovery of certain mathematical relations. He directly links the application of such reasoning to motivation that may be a guide for formulation of the relevant hypotheses. The relationship between mathematics and real-life practice inherent in school mathematics is very clear [1], [2], [6], [8], [11], [13], [15].

II. Stimulating Students' Interest in a Topic through Practical Problems

The initial purpose of practice is to encourage students to learn a new topic. Students have a tendency to "cling" to the familiar, concrete and physical facts or situations. Therefore, the teaching of mathematics is a process of empirical cognition, where observation and practice (calculation, measurement, construction, etc.) play an important role. At this stage, the main motivator is the intention to establish a connection between the studied mathematical concepts and everyday life.

The role of practice ceases to be dominant at the next stage of teaching. However, it continues to maintain students' interest in the concepts of facts being studied. Here, mathematical fact emerges as a result of empirical generalization of the solution of mathematical problems based on real life situations. At this stage, the possibilities to explore mathematical concepts can significantly increase through the integration of concepts of related disciplines. For example, in physics lessons, students learn about the shortest distance light can be reflected from a mirror. Before they start to find a solution to this problem, students must convert it from "physics" to "mathematics" language, producing a geometric problem.

Problem 1. The points A and B lie in the same half-plane determined by the line t. At the given line t, find the point X such that the sum of the distances from A and B to X is minimum. To solve this problem, students must use symmetry. At this point, a physics problem is a motivational factor for an application of mathematical methods to then solve geometric problems.

The use of abstraction and generalization methods to teach pupils new concepts or real-world phenomena does not exclude the stimulation role of practice in the teaching of a new topic. To solve a problem or to prove a theorem it is important to show the need to know specific facts related to the studied content. While this problem or theorem has a mathematical content, it is also necessarily connected with the empirical experience of students. For example, at the primary level students meet challenges to find the sum of certain numbers in a number sequence: 1+2+3+…+99+100.

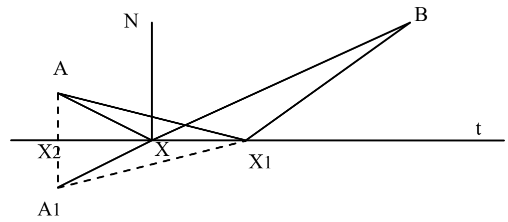

Fig. 2 Visualization of physic problem

To find the sum, students change the order of numbers in the sequence and group them accordingly:

$$\frac{(1+100)+(2+99)+...+(100+1)}{2} = \frac{101\times 100}{2} = 5050$$

At the senior level, students can apply this fact using geometric material. Here students must find out the quantity of lines connecting a certain number of points. After some practical actions with two, three, or four points, students determine how many lines can be drawn through 100 points. As a result of discussions, students establish that the 100th point can be connected with other points by 99 lines, the 99th point by 98 lines, the 98th point by 97 lines, and so on. This allows a transition from a geometrical problem to an algebraic problem. Further analysis helps to find the quantity of lines passing through n points. To solve this problem, students need to find the sum of the first n natural numbers, i.e. finding the sum of arithmetic progression corresponding to a number sequence.

In the given example, students were stimulated to learn the progression from manipulating geometric objects to formulating a geometrical problems and then converting it into an algebraic problem. On the one hand, the use of geometric material gives a subjective feeling of novelty to pupils. On the other hand, it allows them to engage in practical activities.

III. Developing Students' World Outlook through Practical Problems

At the next stage of teaching mathematics, the world outlook role of practice becomes dominant. This implementation is possible through the integration of mathematical content from other school subjects, consideration of the history and the evolution of scientific concepts and methods and the presence of elements of mathematical modeling in real-world processes. Students are aware of the role of mathematical knowledge as an essential component of human culture and they demonstrate their intention to apply mathematical knowledge and skills in related school disciplines and real life.

A traditional way to show the importance of mathematics is through real-world problems. Students are acquainted with the basic mathematical method of understanding reality by solving real-world problems. It is a method of modeling, and its application involves the construction of mathematical models corresponding to a real situation, the selection of a method of investigation of the model, its implementation and the analysis and interpretation of quantitative and qualitative data.

Real world problems are beginning to be used in primary schools. Implicitly they prepare students to grasp modeling ideas in the future. At this stage, we can call them pseudo problems. A problem contains as much data as students need to solve it. An answer to a posed problem question is unambiguous and accurate. A process of solving these problems is "monological"; it does not require special definitions and additional refinements for application. Accordingly, the motivational effect of these problems is determined by an artificial problem situation - often having an entertaining character. The situation relates to the real experience of the students to a certain extent and does not relate to the mathematical content. In this case there are a minimum set of requirements:

1. The given data of the problem should correspond to the real world, not to mathematical objects.
2. The content of the problem should be understood by students and reflect significant aspects of their experience.
3. The level of the students' mathematical competence should be sufficient to solve the problems.

With the growth of students' knowledge and skills in mathematics, there is a noticeable increase in the world outlook role of practice. Firstly, essential mathematical tools (e.g.: types of equations, functional dependencies, algebraic expressions, etc.) are significantly enriched. Each of them represents a mathematical model of a real phenomenon or process. Secondly, the study of related school disciplines (physics, chemistry, geography, etc.) provides for wide practical applications. These connections enhance the development process of the motivational components of students' activity. In particular, students master real processes from related fields of knowledge through this mathematical modeling method. They demonstrate the ability to apply mathematical knowledge and skills widely. It is desirable to make some provision for the effective implementation of the world outlook role of practice in the teaching of mathematics:

1. By expanding mathematical content through using the widest possible range of issues from related school subjects and real life. The possibility of making such connections depends on the time allocated to teaching concepts in different school subjects, a unity of approaches to the development of these concepts and their application, and consistency of terminology, symbols and units of measurement systems.
2. By targeting the development of mathematical intuition through using the specific characteristics of an activity related to applied mathematics. On the one hand, it may be the awareness of the basic methods and skills used in practical problems (e.g.: data selection, preliminary evaluation of results, methods of approximation, etc.). On the other hand it may be conducting the next steps: analysis of a real situation, a formulation of the problem, choice of a method to construct a mathematical model, and an interpretation of the real meaning of the result.
3. By using the historical data to demonstrate the features of mathematical concepts and methods to meet real needs.

These conditions become most effective when dialogue-teaching forms dominate in a classroom. This allows the actualization of additional stimuli. All of this is significantly influenced by the characteristics of the participants in the dialogue: the level of emotional stress, the cognitive abilities of the interlocutor, mutual support and respect, tolerance, and elements of competition, etc. The practical content of a problem has some degree of uncertainty and greater opportunities for dialogue. In particular, some of the conditions of such problems may be excessive or insufficient. Accordingly, at the stage of problem analysis, conditions should be specified through different sources of information. Involving students in dialogue creates a favorable motivational environment for them and encourages cognitive activity and initiative. As an example, let us consider a practical problem described in a real situation associated with the construction of a cottage.

Problem 2. The cottage has width a = 5 m and length m = 8. What is the size of the mansard roof a, if the distance from the attic flooring to the top of the mansard is h = 3 m?

After discussing the meaning of the terms related to the given situation (mansard roof, attic flooring, and top of mansard) students find out that the length of the mansard is the length of the cottage. To find other sizes, the students analyze the situation and answer the following four questions.

1) What geometric shapes correspond to the images of the façade of the cottage and the cross section of the mansard? The answer is a rectangle inscribed in a triangle.
2) Draw some variants of the specified geometrical configuration. When does the mansard become the most spacious? Check your answer by calculation. The answer is the rectangle sides are slightly different from each other.
3) Mathematically characterize the most rational form of the cross section of a mansard. Its area is maximum.
4) What problem can you formulate using this fact? You need to find the maximum cross-section of the mansard.

The problem requirements are not imposed on students by the teacher. They are requirements which develop as an intellectual acquisition as a result of discussion.

Next, some preliminary quantitative estimation of an expected result should be carried out. The estimated value of this result should be determined. Rough approximation shows that the result is equal to the area of the cross section of the mansard which has a triangular shape ($15 m^2$). More precisely, this value is determined through a drawing using students' visual and intuitive reasoning. This value equals half the cross-sectional area of the mansard approximately ($7.5\ m^2$). This process allows students to control the progress of the search.

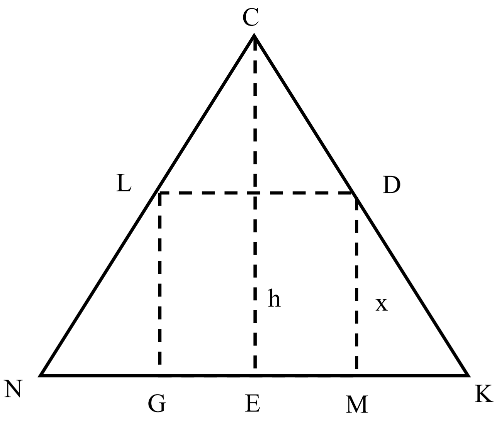

Fig. 3 The mansard roof

At the next stage of solving the problem, the students' constructive team work continues, where they build a mathematical model. Using the geometrical drawing, students formulate a mathematical analog of the practical problem:

Find the sides of a rectangle inscribed in the biggest triangle. The triangle has base a and height h, (Fig. 3). Introduce the notation: LD=y; CE= h; DM=LG=x; NK=a. Then, students construct an analytical model of this practical problem using similarity of the triangles NCK and LCD:

$$S = \frac{a}{h}(h - x) \times x,$$

where S is the area of the rectangle LDMG.

An investigation of the relationship between the area of the rectangle and the second and the third factors (a/h constant) leads to the mathematical question: At what value of x is the product of (h–x)x a maximum (x<h)? This product can be presented in the form $-x^2+hx$ and corresponds to the quadratic function $f(x) = -x^2 + hx$. In this case, only a graphical method is possible. Students construct a new graphical model of this practical problem. This model is a part of the parabola which opens downward (Fig. 4). It leads to the next mathematical question. At what argument value is the function $f(x) = -x^2+hx$ a maximum?

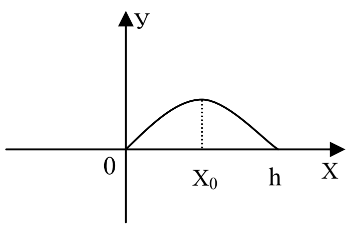

Fig. 4 Graphical model

Clearly, the argument value corresponds to the abscissa of the parabola vertex. From the concept of symmetry or the formula, this value is $x_0=h/2=1.5$.

We come to different interpretations of the results.

1. An initial situation: The maximum cross-section of the mansard is when the height of the mansard is equal to half the distance from the attic flooring to the top, i.e. 1.5 m. The area of the cross-section is $S = a/h\times(h-x)\times x$=3,75 m^2 ($S < 7.5$ m^2). Maximum mansard capacity is $V= S\times b$=30 m^3.
2. A geometrical model: The side of the rectangle inscribed in the biggest triangle is the middle line of this triangle.
3. An analytical model: The product of two positive factors has maximum value if they are equal (the sum of these factors is constant).
4. A consequence: Among all rectangles with the same perimeter, a square has the maximum area.
5. A graphical model: The relationship between an area of a rectangle inscribed in a triangle and its side is a part of a parabola. The abscissa at the top of the parabola corresponds to the side when an area is maximum. The abscissa of the points of intersection of the parabolas and the coordinate axes corresponds to the side when an area is minimal, etc.

As a result of this work, teachers inform the students that the practical problem considered is one from a wide range of problems where one needs to find a maximum and minimum. These problems are of great practical importance. It is useful to give students homework: choose similar problems from their life experience. This example illustrates all the major stages of solving practical problems. Among these stages, the key element is the choice of the basic model of the studied situation. Practice shows that students have challenges in choosing this model without the training activity organized by a teacher. For this to be effective, the teacher must help students develop particular skills. For example, students must establish the similarity between various explanations in order to estimate the outcome of each of them in a specific situation, whilst evaluating different approaches. This kind of work can be organized through specially chosen sets of practical problems.

Problem 3. What kind of measuring instruments do you need to determine the area of the steel plate in the form of an equilateral triangle measuring in cm?

Most students propose to use the formula $S = \frac{a^2\sqrt{3}}{2}$ for the area of the triangle. It is sufficient to measure the side of a triangle with a ruler. The content of the school physics course allows them to determine the area through the volume and density of the material of the steel plate: $S = \frac{V}{n} = \frac{m}{p \times h}$. Students should weigh this steel plate and find the appropriate value of density in a table. After this activity, the students can obtain identical results using different formulas and different measuring instruments. The relationship between mathematical and physical reasoning then becomes clear to them.

Students can see that those two approaches produce an approximate measurement. Then they have to explain the equality $\frac{m}{p \times h} = \frac{a^2\sqrt{3}}{2}$. The obtained "motivational impulse" can be used by the teacher to clarify the range of possibilities of the methods used. A teacher gives students steel plates of different forms (a circle, a rectangle, ellipse etc.) and discusses the benefits of different approaches to solve practical problems. The practical approach is optimal for any given activity. Students may see plates of various shapes. Also, they can use the weighing method to determine the characteristics of geometric shapes. This method was successfully applied by Archimedes for the volume of a sphere. During this activity, students recognize how a practical method relates to a mathematical method. This method allows students to consider from a given problem how to find areas and volumes of geometrical shapes in general. As a result of this activity students will develop a long-term motivation in learning future mathematical analysis. It is important to note that in the practical problems considered, mathematical activity is not a closed structure; it is a natural component of the universal system of knowledge about the world.

IV. Students' Orientation in the Modern World through Solving Practical Problems

At the last stage of teaching, the role of practice in providing mathematical orientation becomes dominant. Practice gives students a new motivation and understanding of how mathematical skills are needed to fully participate in the modern world and for the successful implementation of future professional activity. However, this is not always easy. One challenge is that many modern scientific fields operate in such a way that their own various types of models do not smoothly integrate with the traditional mathematics curriculum. Indeed, the practical application of mathematical knowledge can be perhaps better realized in a less direct and more 'dotted' ways- to students' future professional development and in relation to environmental and other extracurricular activities.

Let us consider an example from the field of medicine. In studying the exponential function in the school, students' attention is drawn to the traditional formula expressing the laws of growth (y=ekx) and decay (y=e–kx). At the beginning of the 20th century, American scientists revealed the law related to the latter formula, which reflects the approximate dependence of the area of protracting wounds from the time when the wound becomes sterile. This dependence can be traced with a special device, a planimeter. A planimeter is used for the approximate measurement of the surface area bounded by lines. The perfect curve of wound healing is described by the formula: S=S1e–kt, where S1 is the wound area at the initial time. The perfect curve is also called the prediction curve. A prediction curve is compared with an actual curve (Fig. 5). The wound is infected if the observed wound area is larger than the area defined by the perfect curve. If a wound heals faster than the perfect curve shows, this indicates the appearance of secondary ulcers. A wound is healing well if the prediction curve is the same as the actual curve. Such examples, as shown in our monitoring of the teaching-learning process, significantly enhance the subjective attractiveness of mathematics for most students.

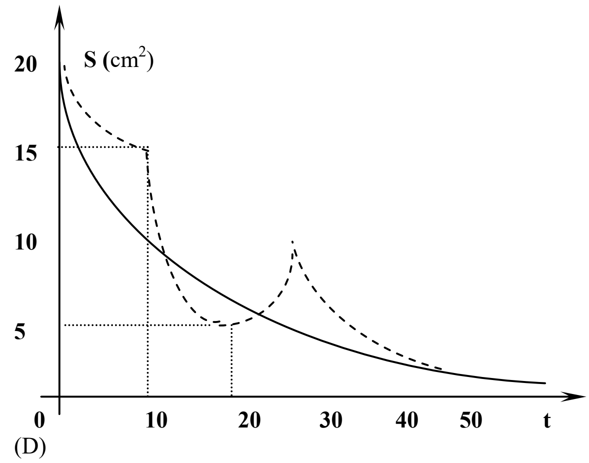

Fig. 5 Graph of curves

TABLE I
SYSTEMATIC FORM OF PRACTICAL PROBLEMS

Stage	Role of practical problem	Motivating factor	Objective	Instructional techniques	Students' activity
1	Stimulate students' interest	Intention to link learning material with their own life experiences	Deriving of mathematical relations; understanding concepts, theorems, algorithms and their application	Organization of empirical support for students' activity	Using students' life experience, applying it to solve non-standard word problems
2	Develop students' world outlook	Intention to apply mathematical material which was studied in other related school disciplines and real-life practice	Demonstration of practical application of mathematics in various branches of human knowledge	Forming of modeling method	Using plausible reasoning to solve practical problems related to other school disciplines
3	Students' orientation in the modern world	Intention to apply mathematical tools for associated to their environment	Awareness of importance of mathematical knowledge and skills for further education and profession	Understanding of the features of mathematical models and their use for practical activity	The development of research with the involvement of the appropriate mathematical tools

V. Conclusion

Based on the previous discussion, the motivational characteristics of practical problems can be presented in a systematic form in Table I.

This table demonstrates the implementation of the motivational role of practice and instructional techniques related to students' activity. The majority of the characteristics reflected in Table I are not tied to a particular stage of the teaching process. Mathematical knowledge and skills are enriched when students move to the next stage, which ensures the stability and depth of their motivation.

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