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Selection of Rayleigh Damping Coefficients for Seismic Response Analysis of Soil Layers

Huai-Feng Wang, Meng-Lin Lou, Ru-Lin Zhang

***Abstract*—**One good analysis method in seismic response analysis is direct time integration, which widely adopts Rayleigh damping. An approach is presented for selection of Rayleigh damping coefficients to be used in seismic analyses to produce a response that is consistent with Modal damping response. In the presented approach, the expression of the error of peak response, acquired through complete quadratic combination method, and Rayleigh damping coefficients was set up and then the coefficients were produced by minimizing the error. Two finite element modes of soil layers, excited by 28 seismic waves, were used to demonstrate the feasibility and validity.

***Keywords*—**Rayleigh damping, modal damping, damping coefficients, seismic response analysis.

I. Introduction

DIRECT time integration is an important analysis method in seismic response analysis, especially when nonlinearity was be involved. A number of procedures are available for the modeling of damping in time-domain analysis. Viscous damping is routinely assumed, and Rayleigh damping is a popular choice (the main reasons being that Rayleigh damping preserves the undamped natural modes of the system as discussed below and, sometimes, that most finite element codes offer few other models to choose from). It is generally acknowledged that there is little physical evidence to support Rayleigh damping. Many real systems encountered in civil engineering practice display hysteretic damping which is largely independent of frequency [1]. Modal damping, which is constant for all frequencies, is the damping typically specified in seismic analysis Codes and Standards. On a more fundamental level, it can be shown that the mass-proportional damping matrix αMdoes not remain invariant under a Galilean transformation as it must do to comply with the classical principle of relativity [2]. Despite these limitations, Rayleigh damping can and has been used as a heuristic, as opposed to strictly physical, attenuation mechanism.

When Rayleigh damping is specified, the damping matrix C is linearly dependent on the mass and stiffness matrices, M and K, such that:

$$C = \alpha M + \beta K \tag{1}$$

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where α and β are real scalars, called Rayleigh damping coefficients. Rayleigh damping belongs to the group of classical damping models: this implies that the damping matrix satisfies an orthogonality condition:

$$\varphi_i^T C \varphi_j = \begin{cases} 2\omega_i \xi_i, i = j \\ 0, \quad\quad i \neq j \end{cases} \tag{2}$$

whereω_i and φ_i are the undamped natural frequency and mode shape of mode i and ξ_i is the modal damping ratio of mode i, which for Rayleigh damping is given by the familiar formula:

$$\xi_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} \tag{3}$$

This produces a curve that can be modified to match a modal damping value at one or two natural frequency points. Consequently, if a structure has one or two very dominant frequencies, Rayleigh damping can closely approximate the behavior of a prescribed modal damping. However, for more complicated systems with many modes over a large range of natural frequencies, Rayleigh damping can cause significant variation in response as compared to modal damping. Considering that, some researchers [3]-[5] have proposed methods to reconstruct the damping matrix on the base of extensional Rayleigh damping matrix (Caughey damping matrix). However, this modification led to the abandonment of the proportional property of classical damping. What demanding urgent solution is that Rayleigh damping matrix is still adopted by many general finite element programs and widely used in analysis of engineering. Thus, the selection of Rayleigh damping coefficients, which is worthy of a deeper study, is the purpose of this paper.

A conservative approach would be to enforce a Rayleigh damping curve that matches a prescribed modal damping for the highest and lowest modes of the system. This however can result in an unreasonably conservative response for intermediate modes.

More generally, analysts conceptually enforce a Rayleigh damping curve that matches a prescribed modal damping at two special frequency points, such as the natural frequency of the first mode or other major mode, the frequency corresponding to the peak of response spectrum or Fourier frequency spectrum of seismic excitation, the frequency corresponding to centre of gravity of response spectrum or Fourier frequency spectrum of seismic excitation, and so on [6]-[11].

The standard least-squares method is a widely used mathematical method and it is also used to determine Rayleigh damping coefficients [12]. The sum-of-the-squares error of modal damping ratio is defined as:

$$E = \sum_{i=1}^{n}\left(\frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} - \xi_i^*\right) \tag{4}$$

where ξ_i^* is the prescribed modal damping ratio of mode.

Minimizing E gives:

$$\begin{cases} \frac{\partial E}{\partial \alpha} = \sum_{i=1}^{n}\frac{\alpha}{2\omega_i^2\xi_i^{*2}} + \sum_{i=1}^{n}\frac{\beta}{2\xi_i^{*2}} - \sum_{i=1}^{n}\frac{1}{\omega_i\xi_i^*} = 0 \\ \frac{\partial E}{\partial \beta} = \sum_{i=1}^{n}\frac{\alpha}{2\xi_i^{*2}} + \sum_{i=1}^{n}\frac{\beta\omega_i^2}{2\xi_i^{*2}} - \sum_{i=1}^{n}\frac{\omega_i}{\xi_i^*} = 0 \end{cases} \tag{5}$$

then,

$$\begin{cases} \alpha = \frac{2\left(\sum_{i=1}^{n}\frac{1}{\omega_i\xi_i^*}\sum_{i=1}^{n}\frac{\omega_i^2}{\xi_i^{*2}} - \sum_{i=1}^{n}\frac{\omega_i}{\xi_i^*}\sum_{i=1}^{n}\frac{1}{\xi_i^{*2}}\right)}{\sum_{i=1}^{n}\frac{\omega_i^2}{\xi_i^{*2}}\sum_{i=1}^{n}\frac{1}{\omega_i^2\xi_i^{*2}} - \left(\sum_{i=1}^{n}\frac{1}{\xi_i^{*2}}\right)^2} \\ \beta = \frac{2\left(\sum_{i=1}^{n}\frac{\omega_i}{\xi_i^*}\sum_{i=1}^{n}\frac{1}{\omega_i^2\xi_i^{*2}} - \sum_{i=1}^{n}\frac{1}{\omega_i\xi_i^*}\sum_{i=1}^{n}\frac{1}{\xi_i^{*2}}\right)}{\sum_{i=1}^{n}\frac{\omega_i^2}{\xi_i^{*2}}\sum_{i=1}^{n}\frac{1}{\omega_i^2\xi_i^{*2}} - \left(\sum_{i=1}^{n}\frac{1}{\xi_i^{*2}}\right)^2} \end{cases} \tag{6}$$

Considering the difference of contribution of different modes to dynamic response, [13] recommended the maximum strain energy ratio of mode as weight and used the weighted least-squares method to determine Rayleigh damping coefficients for spatial structure. Although the numerical result is not bad, there is a lack of supporting argument for choosing maximum strain energy ratio as a weight.

Considering the product of the effective mass and acceleration response of a single-degree-of-freedom system to be shear force at the base of structure, which is a major controlled factor, [14] proposed an iteration method to select Rayleigh damping coefficients by controlling the sum of the product of the cumulative effective mass and acceleration response of a single-degree-of-freedom system with prescribed modal damping corresponding to each mode to match the sum of product of cumulative effective mass and acceleration response of a single-degree-of-freedom system with Rayleigh damping corresponding to each mode.

On the basis of the square root of sum of square method (SRSS), [15] suggested an optimized solution for Rayleigh damping coefficients and analyzed the numerical errors of a high-rise building subjected to 10 earthquake waves.

This paper aims to find an improved approach for selection of Rayleigh damping coefficients to limit the numerical error by Rayleigh damping matrix. The approach proposed in this paper is intended to define a Rayleigh damping curve that minimizes the variation in the response as compared to modal damping. In the next sections, we first introduced the method and formula. Then, the effect is assessed and the feasibility and validity is demonstrated by comparing from soil model runs using modal damping and Rayleigh damping with coefficients selected using the proposed method. Meanwhile, the error of some common methods proposed by aforementioned studies is offered for contrast and discussion.

II. Procedure of Selecting Rayleigh Damping Coefficients

The dynamic equilibrium for the multi-degree of freedom system under synchronous seismic excitation can be given as:

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = -M\{1\}\ddot{u}_g(t) \tag{7}$$

where $\{1\}$ represents a column of ones; $\ddot{u}_g(t)$ represents the free-field input acceleration applied at the base of the system; $u(t)$, $\dot{u}(t)$, $\ddot{u}(t)$ represents, respectively, the relative displacement, velocity, acceleration.

Acceleration spectral response of mode *I* is defined by:

$$S_a(\xi_i,\omega_i) = \left|\frac{\omega_i^2}{\omega_{iD}}\int_0^t \ddot{u}_g(\tau)exp[-\xi_i\omega_i(t-\tau)]sin[\omega_{iD}(t-\tau)]d\tau\right|_{max} \tag{8}$$

where $\omega_{iD} = \omega_i\sqrt{1-\xi_i^2}$ is the damped natural frequency of mode i.

On the basis of complete quadratic combination method (CQC), the maximum acceleration of the k^{th} degree of freedom of system with modal damping and Rayleigh damping are respectively:

$$a_k^* = \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\rho\left(\xi_i^*,\xi_j^*,\omega_i,\omega_j\right)\gamma_i\gamma_j\phi_{ik}\phi_{jk}S_a(\xi_i^*,\omega_i)S_a\left(\xi_j^*,\omega_j\right)\right] \tag{9}$$

$$a_k = \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\rho\left(\xi_i,\xi_j,\omega_i,\omega_j\right)\gamma_i\gamma_j\phi_{ik}\phi_{jk}S_a(\xi_i,\omega_i)S_a\left(\xi_j,\omega_j\right)\right] \tag{10}$$

where $\gamma_i = \frac{\varphi_i^T M\{1\}}{\varphi_i^T M\varphi_i}$ is the modal participation factor of mode i; φ_i is the i^{th} mode-shape vector; ϕ_{ik} is the response of the k^{th} degree of freedom of the i^{th} mode-shape vector; ρ is the correlative coefficient, defined as:

$$\rho\left(\xi_i,\xi_i,\omega_i,\omega_j\right) = \frac{8\sqrt{\xi_i\xi_j}(\xi_i+\lambda\xi_j)\lambda^{3/2}}{(1-\lambda^2)^2+4\xi_i\xi_j(1+\lambda^2)\lambda+\left(\xi_i^2+\xi_j^2\right)\lambda^2} \tag{11}$$

here $\lambda = \omega_j/\omega_i$.

The error of the maximum acceleration of the k^{th} degree of freedom is:

$$e_k = a_k - a_k^* \tag{12}$$

thus,

$$e_k^2 = (a_k - a_k^*)^2 = \begin{pmatrix} \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\rho\left(\xi_i,\xi_j,\omega_i,\omega_j\right)\gamma_i\gamma_j\phi_{ik}\phi_{jk}S_a(\xi_i,\omega_i)S_a\left(\xi_j,\omega_j\right)\right] \\ -\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\rho\left(\xi_i^*,\xi_j^*,\omega_i,\omega_j\right)\gamma_i\gamma_j\phi_{ik}\phi_{jk}S_a(\xi_i^*,\omega_i)S_a\left(\xi_j^*,\omega_j\right)\right] \end{pmatrix}^2 \tag{13}$$

Expanding the acceleration spectral response of mode i based on first order Taylor series gives:

$$S_a(\xi_i,\omega_i) \approx S_a(\xi_i^*,\omega_i) + S_a'(\xi_i^*,\omega_i)(\xi_i - \xi_i^*) \tag{14}$$

where $S_a'(\xi_i^*,\omega_i) = \frac{\partial S_a(\xi_i^*,\omega_i)}{\partial \xi_i^*}$ is the partial derivative of the acceleration spectral response with respect to prescribed modal damping ratio.

On the basis of many engineering practices and studies, the first mode is one of the most important modes for dynamic response of the system. Thus, the first undamped natural frequency was chosen as one of the two frequency points with prescribed damping ratio, namely, $\omega_i = \omega_1$. Meanwhile, assuming that the other frequency point is ω_x, namely, $\omega_j = \omega_x$. It usually is assumed that the same damping ratio applies to both control frequencies and this damping ratio is equal to the prescribed modal damping ratio; i.e., $\xi_i = \xi_j = \xi_i^* = \xi_j^* = \xi^*$. Thus,

$$\xi_i^* = \frac{\alpha}{2\omega_1} + \frac{\beta\omega_1}{2} = \frac{\alpha}{2\omega_x} + \frac{\beta\omega_x}{2} \tag{15}$$

then, rearranging (15) gives:

$$\alpha = \beta\omega_1\omega_x = 2\omega_1\xi^* - \omega_1^2\beta \tag{16}$$

Substituting (16) into (3) gives:

$$\xi_i = \frac{\omega_i-\omega_1}{\omega_i}\frac{\omega_i+\omega_1}{2}\beta + \frac{\omega_1}{\omega_i}\xi^* \tag{17}$$

In seismic analysis Codes and Standards, it usually is assumed that the same damping ratio applies to all frequencies, namely, $\xi_i^* = \xi^*$. Thus, substituting (17) into and (14) gives:

$$S_a(\xi_i, \omega_i) \approx S_a(\xi^*, \omega_i) + S_a{}'(\xi^*, \omega_i)\left(\frac{\omega_i-\omega_1}{\omega_i}\frac{\omega_i+\omega_1}{2}\beta - \frac{\omega_i-\omega_1}{\omega_i}\xi^*\right) \tag{18}$$

Substituting (18) into (13) gives:

$$e_k^2 = \left(\begin{matrix}\sum_{i=1}^n\sum_{j=1}^n\left\{\begin{matrix}\rho(\xi_i,\xi_j,\omega_i,\omega_j)\gamma_i\gamma_j\phi_{ik}\phi_{jk} \\ \left[S_a(\xi^*,\omega_i) + S_a{}'(\xi^*,\omega_i)\left(\frac{\omega_i-\omega_1}{\omega_i}\frac{\omega_i+\omega_1}{2}\beta - \frac{\omega_i-\omega_1}{\omega_i}\xi^*\right)\right] \\ \left[S_a(\xi^*,\omega_j) + S_a{}'(\xi^*,\omega_j)\left(\frac{\omega_j-\omega_1}{\omega_j}\frac{\omega_j+\omega_1}{2}\beta - \frac{\omega_j-\omega_1}{\omega_j}\xi^*\right)\right]\end{matrix}\right\} - \\ \sum_{i=1}^n\sum_{j=1}^n\left[\rho(\xi_i^*,\xi_j^*,\omega_i,\omega_j)\gamma_i\gamma_j\phi_{ik}\phi_{jk}S_a(\xi^*,\omega_i)S_a(\xi^*,\omega_j)\right]\end{matrix}\right)^2 \tag{19}$$

For minimizing e_k^2 (namely minimizing $|e_k|$), the value of β should meet the following condition:

$$\frac{\partial e_k^2}{\partial\beta} = 0 \tag{20}$$

thus, substituting (19) and into (20), and meanwhile ignoring the difference of $\rho(\xi_i,\xi_j,\omega_i,\omega_j)$ and $\rho(\xi_i^*,\xi_j^*,\omega_i,\omega_j)$, namely:

$$\rho(\xi_i,\xi_j,\omega_i,\omega_j) = \rho(\xi_i^*,\xi_j^*,\omega_i,\omega_j) = \rho^*(\omega_i,\omega_j) \tag{21}$$

gives:

$$(a\beta^2 + b\beta + c)(2a\beta + b) = 0 \tag{22}$$

and

$$\beta_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \tag{23}$$

$$\beta_3 = -\frac{b}{2a} \tag{24}$$

in which:

$$a = \sum_{i=1}^n\sum_{j=1}^n\left[\begin{matrix}\rho^*(\omega_i,\omega_j)\gamma_i\gamma_j\phi_{ik}\phi_{jk}S_a'(\xi^*,\omega_i) \\ S_a{}'(\xi^*,\omega_j)\left(\frac{\omega_i-\omega_1}{\omega_i}\right)\left(\frac{\omega_j-\omega_1}{\omega_j}\right)\left(\frac{\omega_i+\omega_1}{\omega_i}\right)\left(\frac{\omega_j+\omega_1}{\omega_j}\right)\end{matrix}\right] \tag{25}$$

$$b = \sum_{i=1}^n\sum_{j=1}^n\left\{\begin{matrix}\rho^*(\omega_i,\omega_j)\gamma_i\gamma_j\phi_{ik}\phi_{jk}\times \\ \left[\begin{matrix}S_a(\xi^*,\omega_i)S_a'(\xi^*,\omega_j)\left(\frac{\omega_j-\omega_1}{\omega_j}\right)\left(\frac{\omega_j+\omega_1}{2}\right) \\ + \\ S_a(\xi^*,\omega_j)S_a'(\xi^*,\omega_i)\left(\frac{\omega_i-\omega_1}{\omega_i}\right)\left(\frac{\omega_i+\omega_1}{2}\right) \\ - \\ \xi^*S_a'(\xi^*,\omega_i)S_a'(\xi^*,\omega_j)\left(\frac{\omega_i-\omega_1}{\omega_i}\right)\left(\frac{\omega_j-\omega_1}{\omega_j}\right)\left(\frac{\omega_i+\omega_1}{2}\right) \\ - \\ \xi^*S_a'(\xi^*,\omega_i)S_a'(\xi^*,\omega_j)\left(\frac{\omega_i-\omega_1}{\omega_i}\right)\left(\frac{\omega_j-\omega_1}{\omega_j}\right)\left(\frac{\omega_j+\omega_1}{2}\right)\end{matrix}\right]\end{matrix}\right\} \tag{26}$$

$$c = \sum_{i=1}^n\sum_{j=1}^n\left\{\begin{matrix}\rho^*(\omega_i,\omega_j)\gamma_i\gamma_j\phi_{ik}\phi_{jk}\times \\ \left[\begin{matrix}\xi^{*2}S_a'(\xi^*,\omega_i)S_a'(\xi^*,\omega_j)\left(\frac{\omega_i-\omega_1}{\omega_i}\right)\left(\frac{\omega_j-\omega_1}{\omega_j}\right) \\ -\xi^*S_a(\xi^*,\omega_i)S_a'(\xi^*,\omega_j)\left(\frac{\omega_j-\omega_1}{\omega_j}\right) \\ -\xi^*S_a(\xi^*,\omega_j)S_a'(\xi^*,\omega_i)\left(\frac{\omega_i-\omega_1}{\omega_i}\right)\end{matrix}\right]\end{matrix}\right\} \tag{27}$$

then, the value of α can be solved through (16). In some cases, the value of α is negative, which is illogical, and the corresponding β should be excluded.

Substituting (21), (23) and (24) into (19) gives:

$$e_{k3}^2 > e_{k1}^2 = e_{k2}^2 = 0 \tag{28}$$

thus, β_3 is not what we wanted, and there are two solutions remaining: $\beta_1$and β_2. To choose the better one for Rayleigh damping matrix, the standard least-squares method was used to calculate the sum-of-the-squares error of the modal damping ratio (E_1 and E_2) by (4). The β corresponding to the smaller one of E_1 and E_2 is the final choice.

III. Calculation of the Partial Derivative of Acceleration Spectral Response with Respect to Prescribed Modal Damping Ratio

The aforementioned procedure of Rayleigh damping coefficients involves calculation of the partial derivative acceleration spectral response with respect to prescribed modal damping ratio$\left(\frac{\partial S_a(\xi_i^*,\omega_i)}{\partial\xi_i^*}\right)$.

For almost all seismic waves, the curve of the acceleration spectral response is highly irregular, so it is difficult to obtain the expression of that and the partial derivative acceleration spectral response with respect to the prescribed modal damping ratio. Thus, here the numerical differentiation method [16] is introduced:

$$\frac{\partial S_a(\xi_i^*,\omega_i)}{\partial\xi_i^*} \approx \frac{S_a(\xi_i^*+\varepsilon,\omega_i) - S_a(\xi_i^*-\varepsilon,\omega_i)}{2\varepsilon} \tag{29}$$

whereε is a real number closed to zero.

IV. Test Model

To show the difference in the results and demonstrate the effectiveness of the aforementioned method, five sets of test model runs are evaluated. One of the five sets is using modal dynamic analysis with modal damping (MPA), as standard results, and the other four sets are using direct integration with Rayleigh damping, including coefficients defined with the approach proposed by Pan [15] (PAN), coefficients defined with (6) (SLS), coefficients defined with the approach proposed by [11] (LOU) and coefficients defined with the aforementioned method (CQC). A total of 28 seismic acceleration time histories are used to perform the five sets of model runs. The seismic event for these models is run in the horizontal direction. There are two different test finite element models. One is representative of soil layers with 250 meter depth, the parameters of which are shown in Table I, in Shanghai, China (model 1). And the other one is representative of soil layers with 60 meter depth, the parameters of which are showed in Table II, in Tianjin, China (model 2). The prescribed modal damping ratio is 5%.

Displacement and acceleration results at the surface of soil layers are then established and compared. Relative error is introduced to evaluate those selecting methods of Rayleigh damping coefficients:

$$e_u = \frac{u_R - u_M}{u_M} \times 100\% \tag{30}$$

$$e_a = \frac{a_R - a_M}{a_M} \times 100\% \tag{31}$$

where u_M and a_M are respectively the dynamic peak response of displacement and acceleration at the surface of soil layers corresponding to MPA; u_R and a_R are respectively the dynamic response of displacement and acceleration at the surface of soil layers corresponding to PAN, SLS, LOU or CQC. Consequently, the positive values indicate that the Rayleigh damping model produces a higher peak response.

Figs. 1 and 2 are the relative errors of displacement and acceleration of model 1. Figs. 3 and 4 are the relative errors of displacement and acceleration of model 2. Table III is the statistical parameter of the relative errors of model 1. Table IV is the statistical parameter of relative errors of model 2.

The relative errors of the dynamic response of model 1 are more conspicuous than that of model 2, which is due to the lower natural frequency of model 1. Meanwhile, the relative errors of displacement are less significant than those of acceleration, which is due to the high sensitivity of calculation error of acceleration at high frequency points. Generally, the errors of damping ratio, owing to the mathematical properties of Rayleigh damping matrix, are often limited within a relatively smaller range at low frequency points. Thus, the aforementioned phenomena come out and, predictably, the calculation error of acceleration of the system with a relatively longer natural period would be hard to control.

Obviously, the SLS method brings about the most conspicuous error: the absolute values of relative errors of displacement and acceleration of model 1 can respectively be up to 60% and 159% and that of model 2 can respectively be up to 27% and 21%. The absolute values of relative errors of the CQC method are the smallest: for model 1, the absolute values of relative errors of displacement and acceleration are respectively, 27% and 36%; for model 2, the absolute values of relative errors of displacement and acceleration are respectively, 2% and 15%.

TABLE I
Parameters of Soil Layers (Model 1)

No.	Property	Thickness (m)	Shear wave velocity (m/s)	Density (kg/m^3)
1	Soil	2.1	100	1900
2	Soil	1.2	110	1860
3	Sand	6.2	120	1840
4	Soil	5.5	150	1760
5	Soil	9	200	1820
6	Soil	3.5	260	2010
7	Soil	10.5	280	1900
8	Sand	22	330	1940
9	Soil	12.5	350	1900
10	Sand	6	360	1930
11	Sand	21.5	380	1940
12	Sand	10	420	1970
13	Sand	10	480	1970
14	Sand	10	520	1970
15	Sand	10	540	1970
16	Sand	10	520	1970
17	Soil	15	480	2000
18	Sand	12	500	1950
19	Sand	12	530	1950
20	Sand	11	530	1950
21	Sand	10	530	1950
22	Sand	10	530	2000
23	Sand	10	530	2000
24	Sand	10	530	2000
25	Sand	10	530	2000
26	Rock	-	800	2400

TABLE II
Parameters of Soil Layers (Model 2)

No.	Property	Thickness (m)	Shear wave velocity (m/s)	Density (kg/m^3)
1	Soil	6	190	1880
2	Sand	4	233	1930
3	Soil	2	278	1920
4	Sand	4	310	1950
5	Cobble	6	388	2010
6	Soil	2	272	1960
7	Soil	2	333	1970
8	Sand	2	384	2010
9	Soil	4	330	1990
10	Cobble	6	381	2040
11	Sand	2	400	2030
12	Cobble	4	418	2040
13	Sand	2	460	2030
14	Soil	6	448	2010
15	Sand	2	488	2030
16	Soil	4	442	1990
17	Sand	2	498	2010
18	Rock	-	800	2400

TABLE III
STATISTICAL PARAMETER OF RELATIVE ERRORS (MODEL 1)

statistical parameter	PAN		SLS		LOU		CQC	
	u	a	u	a	u	a	u	a
$\frac{\sum_{i=1}^{n}\lvert e_i\rvert}{n}$ (%)	12	12	20	55	12	22	14	9
$\sqrt{\frac{\sum_{i=1}^{n}(e_i)^2}{n}}$ (%)	16	15	24	65	17	26	14	11

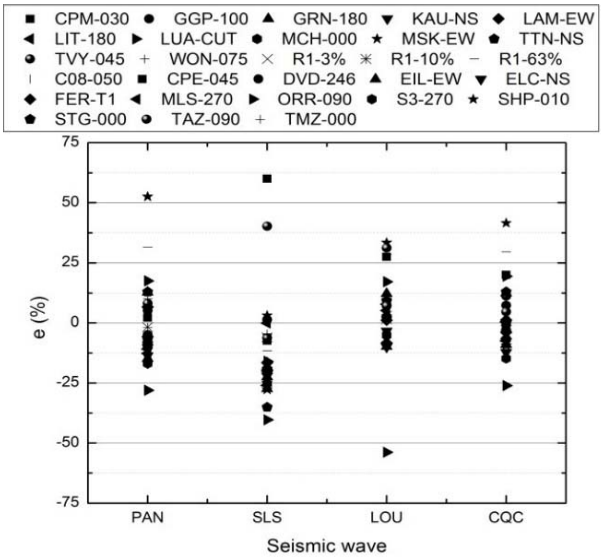

Fig. 1 Relative errors of displacement (model 1)

TABLE IV
STATISTICAL PARAMETER OF RELATIVE ERRORS (MODEL 2)

statistical parameter	PAN		SLS		LOU		CQC	
	u	a	u	a	u	a	u	a
$\frac{\sum_{i=1}^{n}\lvert e_i\rvert}{n}$ (%)	1	6	16	9	1	5	1	4
$\sqrt{\frac{\sum_{i=1}^{n}(e_i)^2}{n}}$ (%)	1	8	17	11	2	6	1	5

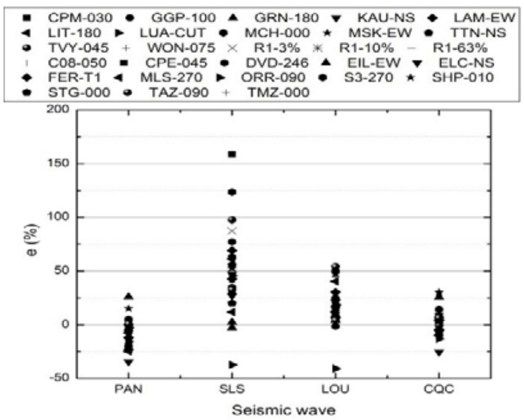

Fig. 2 Relative errors of acceleration (model 1)

The relative error is much dependent on the seismic acceleration time history. For the same method of selecting Rayleigh damping coefficients, different seismic waves lead to different calculation errors. Thus, the dispersion is a significant consideration in measuring the selection method. No matter whether for displacement or acceleration, the mean of the absolute value and standard deviation of relative errors of the CQC method is the minimum. It indicates that the CQC method has great adaptation for different seismic wave, and should be reasonable for a wide range of problems.

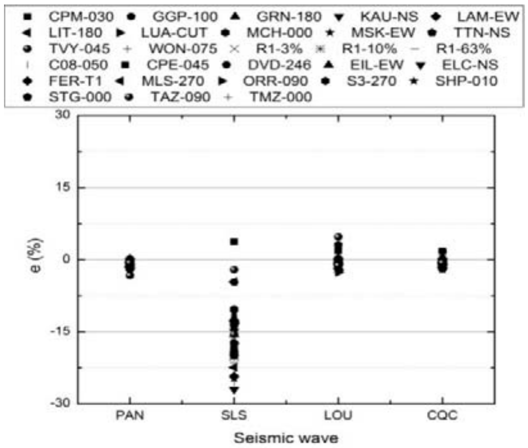

Fig. 3 Relative errors of displacement (model 2)

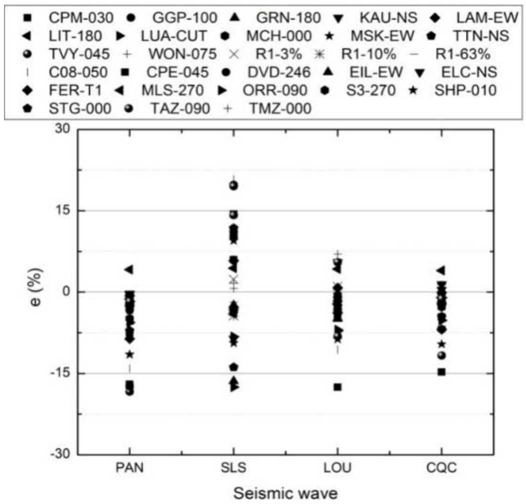

Fig. 4 Relative errors of acceleration (model 2)

V. CONCLUSION

Nonlinearities, whether geometric or material, need to be addressed in seismic analysis. This may motivate an analyst to evaluate the seismic response through time-domain analysis, where Rayleigh damping was widely used. A process has been set up to solve Rayleigh damping coefficients which are used in seismic analyses. Firstly, through the complete quadratic combination method, the functional relation of the error of peak response and Rayleigh damping coefficients was established. Secondly, those coefficients were produced by minimizing the error of peak response. At last, two example problems were included to clarify the feasibility and validity of the proposed process.

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