

Application Reliability Method for Concrete Dams

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Abstract—Probabilistic risk analysis models are used to provide a better understanding of the reliability and structural failure of works, including when calculating the stability of large structures to a major risk in the event of an accident or breakdown. This work is interested in the study of the probability of failure of concrete dams through the application of reliability analysis methods including the methods used in engineering. It is in our case, the use of level 2 methods via the study limit state. Hence, the probability of product failures is estimated by analytical methods of the type first order risk method (FORM) and the second order risk method (SORM). By way of comparison, a level three method was used which generates a full analysis of the problem and involves an integration of the probability density function of random variables extended to the field of security using the Monte Carlo simulation method. Taking into account the change in stress following load combinations: normal, exceptional and extreme acting on the dam, calculation of the results obtained have provided acceptable failure probability values which largely corroborate the theory, in fact, the probability of failure tends to increase with increasing load intensities, thus causing a significant decrease in strength, shear forces then induce a shift that threatens the reliability of the structure by intolerable values of the probability of product failures. Especially, in case the increase of uplift in a hypothetical default of the drainage system.

Keywords—Dam, failure, limit-state, Monte Carlo simulation, reliability, probability, simulation, sliding, Taylor.

I. INTRODUCTION

MAINTEINING dam safety requires the importance of developing methods and support tools for taking decisions that assess their performance with assurance of security from both technical and economical sides to the works during the period of exploitation or construction. Without forgetting that reliability analysis plays a major role in addressing the uncertainties that affect the design of concrete gravity dams or embankment dams [1].

Reliability problems are often based on modelling the mechanisms of degradation and constraints of the environment structure leading to the definition of a failure function called limit state function, which involves different geometrical and physical parameters of the studied system [2].

The structural reliability is formulated in terms of a vector of random variables of a structural system that can describe the loads, the structural dimensions of the system, materials and their characteristics. The geometry and material of the structure are typical strength variables.

When no damage or excess is allowed, the condition $r=s$ is applied. This is known as the ultimate limit state condition.

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The probability of failure is the probability that the loading exceeds the strength, that is, that $s > r$, and the reliability is then defined as the probability that $s = r$ [3].

For resistance (r) material constitutes the structure and the (load) stresses (s) acting on this structure. Typically, a probability of failure will be quoted as a percentage, with the implicit time unit understood (Fig. 1).

Under the condition that r and s are independent, we define the probability distributions for the strength and loading are $f_r(r)$ and $f_s(s)$, respectively, the probability of failure is given by [4]:

$$P_f = \int_{-\infty}^{+\infty} f_R(x) f_S(x) dx \quad (1)$$

The probability of this occurrence is given by $f_s(x) dx$ (i.e., the probability that s lies close to x , within an interval of length dx). Failure will occur if the strength r is less than x . (x is the probability that the strength r is less than x , so the integrand is the probability that for a given load $s = x$, failure will occur [5].

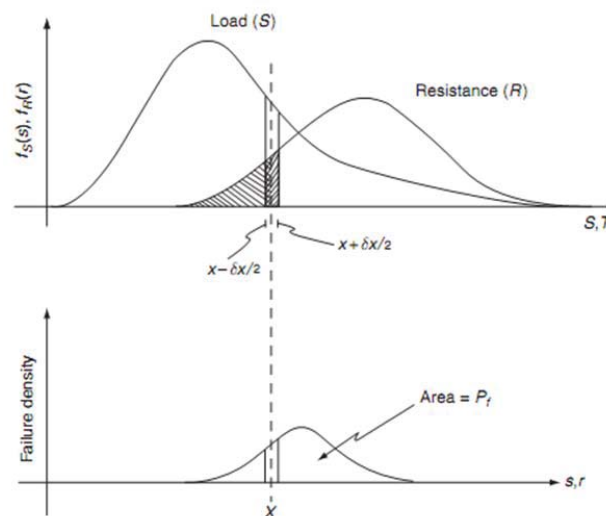


Fig. 1 Diagram explaining the probability of failure [3]

In general, a reliability function g is defined as:

$$\int_G f_G(\underline{g}) d\underline{g} \quad (2)$$

where $\underline{g} = (x_1, x_2, \dots, x_m, x_{m+1}, \dots, x_n)$, $f_g(g)$ is the joint probability function of g and the integral is over a volume defined in n dimensions.

$$G = R - S = R(x_1, x_2, \dots, x_m) - S(x_{m+1}, x_{m+2}, \dots, x_n) \quad (3)$$

where x_1, x_2, \dots, x_m are strength variables, $x_{m+1}, x_{m+2}, \dots, x_n$ are load variables and probability of failure corresponding to the probability that $g < 0$ is evaluated by :

$$\iint \int_{G < 0} f_{x_1}(x_1) \cdot f_{x_2}(x_2) \dots f_{x_n}(x_n) dx_1 dx_2 \dots dx_n \quad (4)$$

where $f_{x_1}(x_1) \cdot f_{x_2}(x_2) \dots f_{x_n}(x_n)$ are the marginal probability density functions of the loading and strength variables. However, the resolution of (3) poses difficulties to evaluate the function of reliability.

Thus, prompting the development of various approximation methods that can be divided into four categories or levels of resolution, as shown in Table I.

Resolution level	Principle of the method
Level 0	Traditional methods that use characteristic values of strength and loading. Based on the traditional design approach, characteristic values of strength, r , and load, s , are used to ensure that r is sufficiently greater than s to meet the design requirements.
Level 1	Quasi-probabilistic methods, which assign safety factors to each of the variables to account for uncertainty in their value.
Level 2	Probabilistic methods, which approximate the distribution functions of the strength and load variables to estimate (2). This method has been further subcategorized as first-order risk methods (FORMs), and second-order risk methods (SORMs), depending on the order of the approximation to the reliability function.
Level 3	The most complex probabilistic methods, which estimate (2) either directly or through numerical simulation technique

For a structure, reliability may be defined as the relationship between the probability of failure and security of an event by Bomel [6].

$$P_s = 1.0 - P_f \quad (5)$$

where P_f and P_s are respectively the probability of failure and the security of an event, the probability of failure is noted on a normalized scale from 1 to 5. The ratings are converted to theoretical probability values of failure, as it is indicated in Table II.

TABLE II
RELATIONSHIP BETWEEN LIKELIHOOD RATING AND PROBABILITY OF FAILURE (AGS, 2007)

Likelihood Classes	Indicative value of annual probability of Failure P_f
Likely (5)	10^{-2}
Quite Common (4)	10^{-3}
Unlikely (3)	10^{-4}
Unusual (2)	10^{-5}
Rare (1)	10^{-6}

The derived probability values can be used to give an indication of the potential need for remedial works to be undertaken by reference to the Health and Safety Executive guidelines given in Reducing Risk Protecting People [7], as shown in Fig. 2. In fact, for the probability of failure of greater than 10^{-4} or 1 in 10,000 for an identified hazard potentially leading to loss of life or societal risk [8], a lower limit value of 1 to 1,000 times indicates low risk situations where no loss of potential life is identified and consequences may be minor.

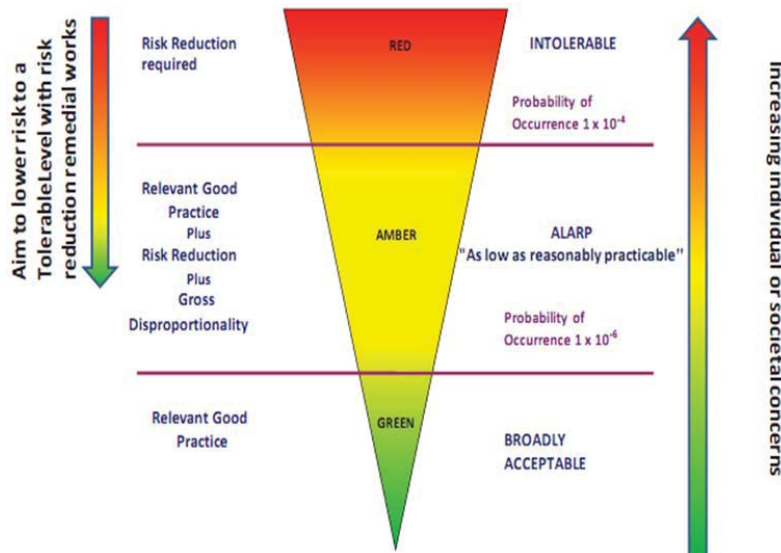


Fig. 2 Health and safety executive likelihood of failure guidelines [7]

II. METHODOLOGICAL APPROACH

In the present study, solving the problem of reliability basically comprises two approaches. A first approximation based on an analytical approach to reliability for level 2,

where the limit state is then essential. This involves having an explicit writing of this limit state, which is by default an approximation. A second approach is based on the application

of Monte Carlo simulations. Such methods constitute a family of complex methods named level 3.

A. Series Approach of Taylor Method.

Level 2 methods introduce the concept of probability distributions to the calculations. At the beginning of the development of these methods, the load and strength variables were considered to be independent, normally distributed variables. These methods are therefore intrinsically linked to that famous point of conception; specifically, the distance from this point to the failure surface in the original space [9]. This distance is called the reliability index or safety, whose decomposition is performed in three stages as:

- a) First phase: to transform the original space of the basic variables in a standard Gaussian area, called the u-space.
- b) Second phase: it determines the point of famous design into the new space.
- c) Third phase: finally, we must appreciate the failure surface at that point to approximate the searched probability.

For the FORMs, the problem associated with this approach is that the reliability index will depend on the choice of linearization point that is not invariant. Hasofer and Lind [10] suggested the linearization of the limit state function in the so-called "design point" in the standard normal space. In the latter, each variable to zero mean admits deviation.

The reliability index is defined by:

$$\beta = \min \left[\sum_{i=1}^n y_i^2 \right]^{1/2} \quad (6)$$

where, y_i represents the coordinates of any point up to the limit state function in the normal space and β reliability index. While there are several methods to estimate the probability of failure, one method is commonly used by Taylor series approximation according to method forms. The basic steps consist of:

- Determine the values of the parameters involved and calculate the factor of safety or the function of performance M for the particular case.
- Estimate the standard deviation parameter which contains uncertainty.
- Calculate the performance of each function parameter m by an increase then decrease. It will generate different values of m , allowing saying that M_1, M_2, M_3, M_4 where M_1 and M_2 belong to the first parameter M_3 and M_4 belong to the second parameter.

Computing the variance of performance function M using equation by:

$$Var(M) = \left[\frac{M_1 - M_2}{2} \right]^2 + \left[\frac{M_3 - M_4}{2} \right]^2 \quad (7)$$

The reliability index is determined by:

$$\beta = \frac{M}{\sqrt{Var(M)}} \quad (8)$$

The method of confidence (SORM) was developed in a series of works that deal with asymptotic analysis. Unlike the FORM method, which aims to replace the limit state by an order 1 hyperplane, the SORM approach replaces it with an order surface hyperplane 2 [11]. The principle is to achieve an approximation of the function state boundary to the point of designing a surface of second order (Taylor development of order II), using the principal curvatures of the limit state function at the design point.

The probability of failure is then approximated by:

$$P_f = \Phi(-\beta) \prod_{i=1}^{n-1} (1 + \beta k_i)^{-1/2} \quad (9)$$

where k_i indicates the curvature of the objective function at the design point, β is the reliability index estimated by the FORMs method; Φ is the standard normal distribution.

The meaning of the curvature has an influence on the probability of failure. A positive bending (convexity turned towards the origin) tends to decrease the probability of failure with respect to the approximation FORMs.

It should be clarified that the possible difference between the failure probabilities obtained by SORMs. FORMs and can be linked to the presence of high nonlinearity or strong curvatures.

A. Numerical Approach Monte-Carlo Simulation

The numerical approach, Monte-Carlo simulation level 3, the most general of the reliability techniques, is to obtain an estimation of the integral in equation (1) through numerical means [12].

The complexity of the integral (in general) means that numerical, rather than analytical, methods are used. There are two widely used techniques [13]:

- a) Monte Carlo integration.
- b) Monte Carlo simulation.

The Monte Carlo simulation methods are general methods for multidimensional integral estimation and mathematical expectation. They can thus be used to estimate the probability of failure in structural reliability.

The idea of the method is to reproduce the operation of the real system by means of an analytical approach and analyze the effects of changes in inputs on the outputs for a system. It includes six key elements:

- 1- Define the problem in terms of random variables design;
- 2- Identify the probabilistic characteristics of all random variables in terms of probability density function and associated parameters (mean and standard deviation);
- 3- Generate values for these random variables;
- 4- Evaluate the deterministic problem for each data set;
- 5- Conclude on the likelihood of product failure, i.e., assess the probability of failure and determine the mean and standard deviation of the output variables of the problem;
- 6- Determine the accuracy of the simulation [14].

Such methods are mainly the processes that are used to estimate sampling of the probability of failure of a structure.

This method is useful for obtaining numerical solutions to complicated problems to be solved analytically.

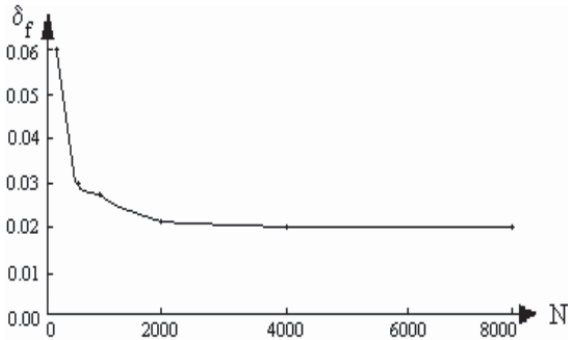


Fig. 3 Variation coefficient of the probability of failure as a function of number of simulations for the MCS [17]

Latin Hypercube Sampling can be considered as an alternative to Monte Carlo simulation defined by: Such methods are mainly the processes that are used to estimate sampling of the probability of failure of a structure. This method is useful for obtaining numerical solutions to complicated problems to be solved analytically.

Latin Hypercube Sampling can be considered as an alternative to Monte Carlo simulation defined by:

$$P_f = \frac{N_f}{N} \quad (10)$$

with N_f : number of simulation runs which corresponds to the failure structure. N : total number of simulation cycles, the probability of failure P_f is expressed by:

The variance of the probability of failure:

$$\text{Var}(\bar{P}_f) = \frac{(1 - \bar{P}_f) \bar{P}_f}{N} \quad (11)$$

The variation coefficient:

$$\delta(\bar{P}_f) = \frac{1}{\bar{P}_f} \sqrt{\frac{(1 - \bar{P}_f) \bar{P}_f}{N}} \quad (12)$$

There are many formulations to estimate the necessary number of simulations [15].

An estimation of the probability of failure of a monitored system is primordially ensured with proper convergence. The simplest formula is proposed [16]:

$$N > \frac{-\ln_{P_f}(1-c)}{P_f} \quad (13)$$

where, N : number of simulations for a confidence level, c : confidence level and P_f : probability of default.

The probability failure related to sliding safety factor (SSF) is given by:

$$P_f = \frac{n(SSF < 1)}{N} \quad (14)$$

with, n (SSF); number of simulation which there will be product the failures.

B. Calculating Failure Sliding of Gravity Dam

This is an application of the previous two methods for reliability assessment of the probability of failure P_f of different types of concrete gravity dam sliding, located in northern Algeria. These dams have different characteristics from the point of view of geometry, including the value of the slope of the downstream face and upstream and in geotechnical characteristics of the foundations. The latter represent the random variables in the calculation of the mechanical parameters, in this case the cohesion and angle of friction of the sliding section.

Initially, a review of the reliability of structures has been conducted according to the method recommended by Taylor approximation, based on the calculation of the objective function M and β reliability index.

Secondly, it was the probabilistic method according to the MCS method based on the principle of generating random variables depending on the number of cycles, which is pronounced as a failure (Figs. 4 and 5). Thereafter, a calculation of validation by CADAM calculation software [18] has been recommended in order to test the validity of the results associated with the pair safety failure of the work based on different combinations of charges and circumstances of the dam and the drainage device that each dam of this study is submitted to.

The performance function M can be evaluated by varying the average value of each variable based on its standard deviation:

$$M = \frac{R}{H} - 1 = \frac{\sum F_v \cdot \text{Tang } \varphi + C \cdot L}{\sum F_H} - 1 \quad (15)$$

The variation of the performance function for the sliding is provided by the precedents by:

$$M_1(\mu_f + \delta_f, \mu_c) = \sum F_v \cdot \tan(\mu_f + \delta_f) + L \cdot \mu_c \quad (16)$$

$$M_2(\mu_f - \delta_f, \mu_c) = \sum F_v \cdot \tan(\mu_f - \delta_f) + L \cdot \mu_c \quad (17)$$

$$M_3(\mu_f, \delta_f + \mu_c) = \sum F_v \cdot \tan(\mu_f) + L(\mu_c + \delta_f) \quad (18)$$

$$M_4(\mu_f, \delta_f - \mu_c) = \sum F_v \cdot \tan(\mu_f) + L(\mu_c - \delta_f) \quad (19)$$

The overall results of failure calculations and safety according to the condition of the uplift expressed according to the functioning of the drainage system are summarized in Tables III and IV.

TABLE III

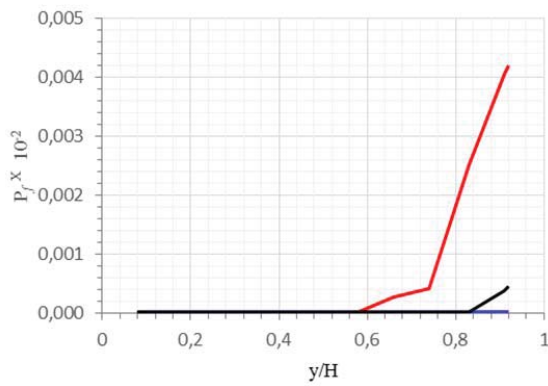
COMPILATION OF THE RESULTS OF THE PROBABILITY OF FAILURE FOR SAFETY SLIDING DAMS FOR COMBINATIONS OF EXTREME LOADS AND ACCORDING TO THE CONDITION OF EFFECTIVE DRAINAGE

(1) Wadi Fodda Dam: H= 101.0 m; Values facings m = 0.675 n= 0.1							
Water level	Filling rate (y/H)	Foundation $P_f \times 10^{-2}$		Interface $P_f \times 10^{-2}$		Dam body $P_f \times 10^{-2}$	
NNR	0.95	0.0911	99.9089	0.0912	99.9089	0.8207	99.1793
(2) Beni Haroun Dam H= 118.0 m; Values facings m= 0.85 n= 0							
NNR	0.86	0.00003	99.9999	0.81106	99.1889	0.49742	99.5025
(3) Hamiz Dam H= 50.00 m; Values facings m= 0.50 n= 0.25							
NNR	0.70	0.00281	99.99719	0.16014	99.8398	0.00003	99,9999

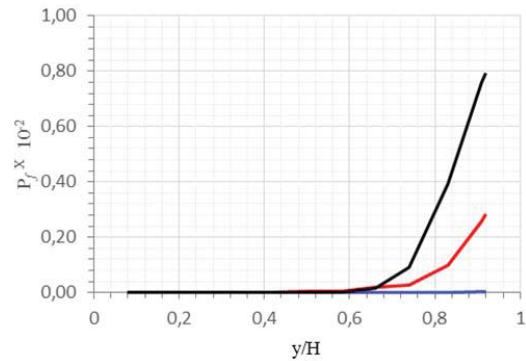
TABLE IV

COMPILATION OF THE RESULTS OF THE PROBABILITY OF FAILURE AND SAFETY SLIDING DAMS FOR COMBINATIONS OF EXTREME LOADS AND ACCORDING TO THE CONDITION OF FAULTY DRAINAGE SYSTEM

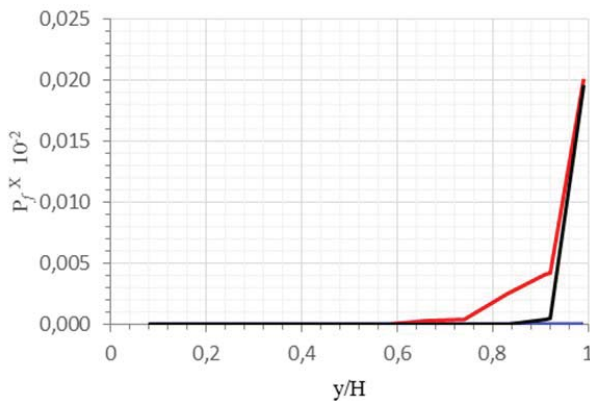
(1) Wadi Fodda Dam: H= 101.0 m; Values facings m = 0.675 n= 0.1							
Water level	Filling rate (y/H)	Foundation $P_f \times 10^{-3}$		Interface $P_f \times 10^{-3}$		Dam body $P_f \times 10^{-3}$	
NNR	0.95	100.0	0.00001	100.0	0.00001	100.0	0.00001
(2) Beni Haroun Dam H= 118.0 m; Values facings m= 0.85 n= 0							
NNR	0.86	0.02576	99.9746	32.0246	67.9766	20.1477	79.853
(3) Hamiz Dam H= 50.00 m; Values facings m= 0.50 n= 0.25							
NNR	0.70	0.1221	99.8780	1.5113	98.489	0.00003	99,999



(a) Probability of failure normal case

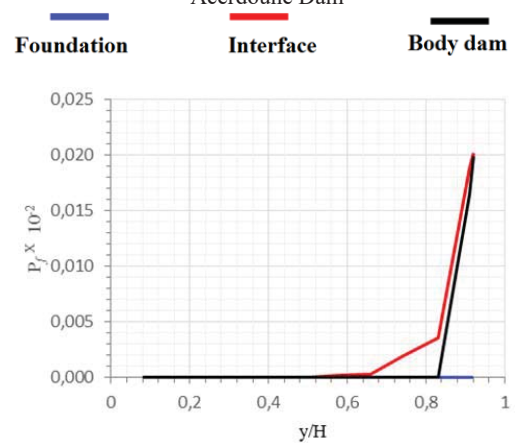


(c) Failure probability extreme case

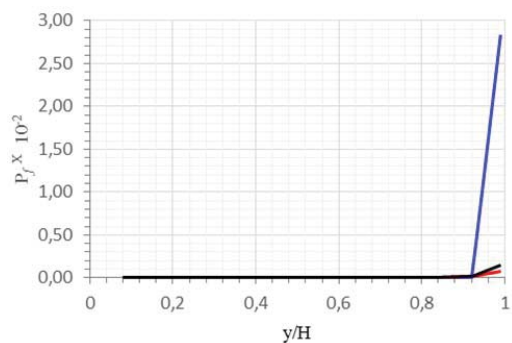


(b) Probability of failure exceptional case

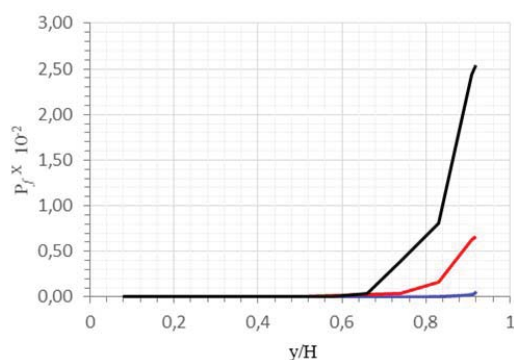
Fig. 4 Evolution of the probability of failure p_f according to the reservoir filling ratio (y/h) for effective drainage the Koudiat Acerdoune Dam



(a) Normal cases Probability of failure



(b) Probability of failure exceptional case



(c) Failure probability extreme case

Fig. 5 Evolution of the probability of failure P_f , according to the reservoir filling ratio (y/H) for faulty drainage the Koudiat Acerdoune Dam

— Foundation — Interface — Body dam

III. RESULTS AND DISCUSSION

The results presented in Fig. 6, illustrate the consistency of the results of the calculation of Monte Carlo Simulations established by the r code corresponding to level 3 and accrediting the validation of the latter. It is the same for the

Taylor approximation method of order two corresponding to level 2. The findings are valid for both modes of operation of the drainage system of the studied dams.

The results obtained by the two methods of failure probability calculation support the validity of the results obtained because of the good correlation. While one considers solely the results for the case of the effective functioning of the drainage system, the correlation coefficient has a tendency to converge to a value close to one.

On the results of Fig. 6, the study of the failure probability P_f according to the relationship based on the ratio (y/H), by application of the Taylor approximation method based on changing uplift pressure, represented by the state operation of the dam drainage system.

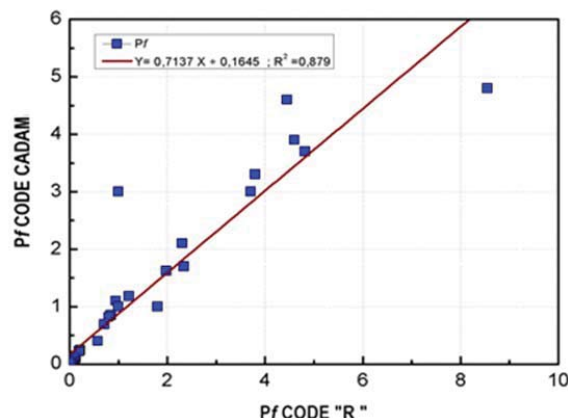


Fig. 6 Comparison of the probability of failure of Taylor r model code according to the simulation code by CADAM software

The analysis of the probability of default was carried out according to the different load combinations of situations namely: usual, unusual and extreme situation. To do this, the analysis of the results was done on a comparative study using the methods of advocated calculations and mechanical and geometric characteristics listed above from the dams discussed in this study. It follows, that in the case of an effective drainage system, and according to the various filling situations (y/H) and for all the studied dams, the probability of failure at the foundation of sliding sections is almost zero, i.e. $P_f \approx 0$ (very near zero), which explains the default risk almost unlikely and it can be said that the behavior of the studied dams is acceptable. However, as regards the concrete-rock section (interface) and concrete-concrete section (on raising joints) and for the same operating condition of the drainage system, we find that the probability of failure $P_f \leq 3 \cdot 10^{-4}$ has a tendency to grow from the ratio (y/H) > 0.80 for a combination of exceptional loads (if raw), the risk is unlikely to be where it can be assumed that behaviour is acceptable.

For the combination of extreme loads (NR and earthquake), can be noted that for all the dams a failure probability P_f increases from the value of the ratio (y/H) > 0.8 and the maximum value reached is equal to $P_f \leq 8.4 \cdot 10^{-2}$ for: Beni-Haroun and Wadi Fodda dams. However, the value of the probability of failure tends to decrease in case of Hamiz and Koudiat Acerdoune dams the probability failure is less $P_f < 0.61 \cdot 10^{-2}$.

We can say that in the first category of the mentioned dams the risk is greater compared to the second category Hamiz and Koudiat Acerdoune dams. Therefore, it is necessary that the risk required a decrease in order to save the development of tensile stresses that could cause sliding sections in the body of the dam, including the Wadi Fodda Dam which is located in an area of higher risk of seismic activity.

IV. CONCLUSION

The present study aims to undertake an evaluation of methodology of probabilistic failure to sliding and computing security for gravity dams. The results provided information in the light of a study based on sample of various dams in

operation by following structural reliability analysis of various load combinations including:

- a) For usual of combination loads that correspond to the nominal level (NNR) with the level of pressure and the associated buoyancy, also taking into account thrust sediments and ice eventuality.
- b) For an exceptional of combination loads corresponding to the maximum filling level envisaged in case of floods (PHE) [19].
- c) For an extreme of combination loads corresponding to the usual loads and seismic effect.

The study results in a physical space formed by the resistors R and the solicitations S, and it is divided into three areas: security, failure domain and a borderline separating the two previous areas.

The geometrical distance between the origin of the normalized space and the limit state curve is called reliability index, giving also an outline on the safety of the structure, as well as the higher is the reliability index, the more the likelihood that failure is low and therefore the structure becomes reliable. Comparing the methods based on approximation of Taylor (FORMs) and (SORMs), the approach to calculation of reliability by MCS integrates an adequate number of random variables simulations offering results with a minimum margin of error.

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