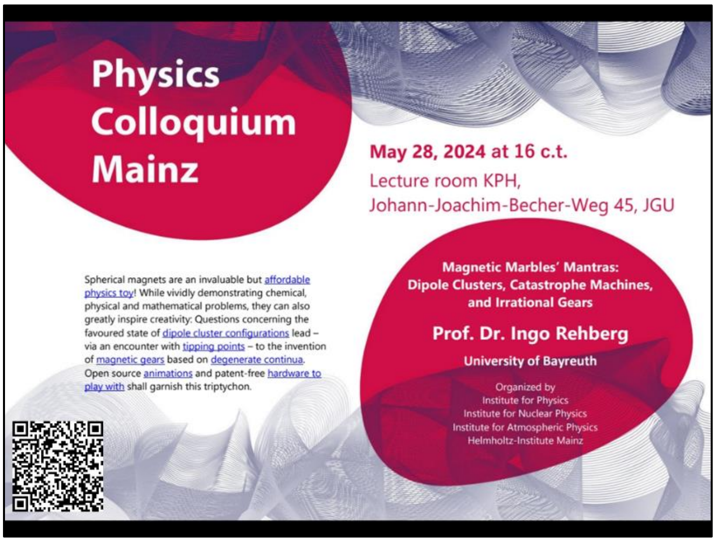


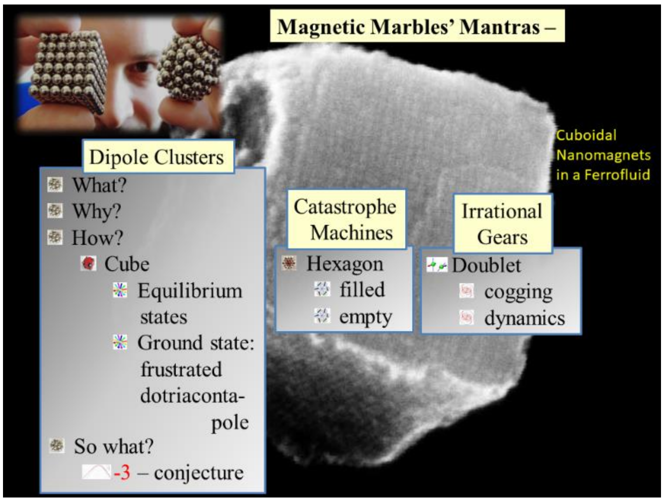
Technical remarks:

○The QR-code leads to the location of this poster from JGU Mainz. In that pdf, blue links in the abstract provide background information.

The slides of the talk are shown below, together with some notes, where hints to
○ links are blue,
○ animations green,
○ experimental demos red.

The abstract with links:

Spherical magnets are an invaluable but affordable physics toy! While vividly demonstrating chemical, physical and mathematical problems, they can also greatly inspire creativity: Questions concerning the favoured state of dipole cluster configurations lead – via an encounter with tipping points – to the invention of magnetic gears based on degenerate continua. Open source animations and patent-free hardware to play with shall garnish this triptychon.



- The background image shows a magnetic cluster formed by about 30000 nanomagnets floating in a ferrofluid.

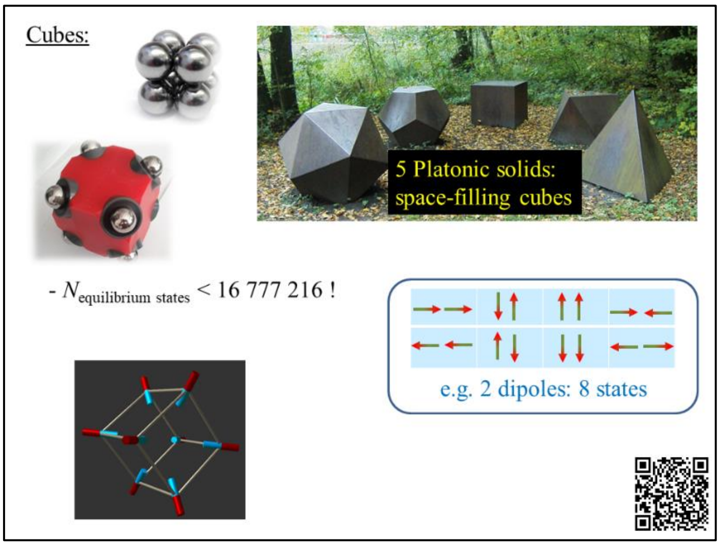
- Magnetic marbles (demo) can form mesoscopic clusters.

- On a macroscopic scale, magnetic interaction of planetesimals might be worth considering – for understanding the formation of the earth (figure).

- A personal motivation to study dipole clusters stems from the wish to understand the dynamical behaviour of magnetic fluids (movie).

- All these clusters form due to dipole-dipole attraction, which can be summarized in a single formula (figure).

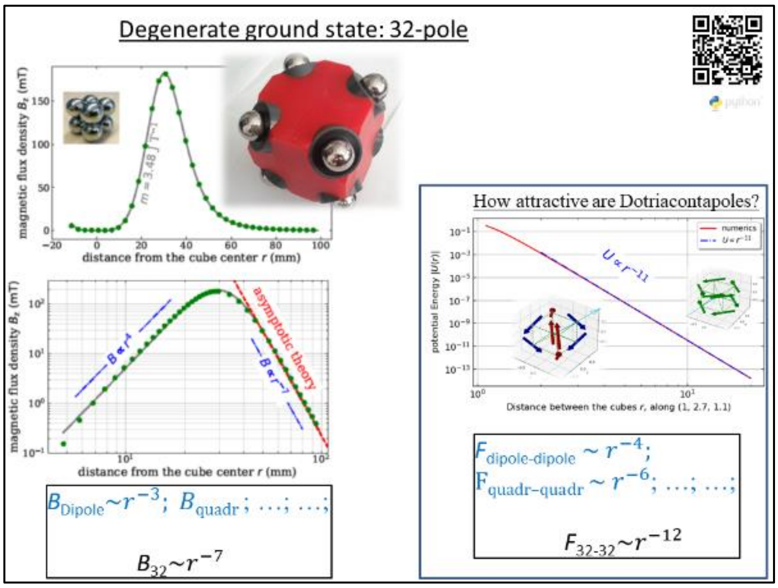
- Three cluster geometries will be addressed here: a *cube*, a *hexagon* and a *doublet*.



- The simplest cluster is formed by two dipoles, which like to align North Pole to South Pole. That is the ground state, but six other orientations are also equilibrium solutions, not necessarily stable ones.

- 3-d clusters are more important. Out of the five Platonic bodies, the cube might be the most significant, because it is the only space-filling one. The number of equilibrium states in a dipole cube (demo) was proofed to be smaller than 2^{24}, nine years ago by J. Schönke. In his interactive animation (QR) you can watch them all.

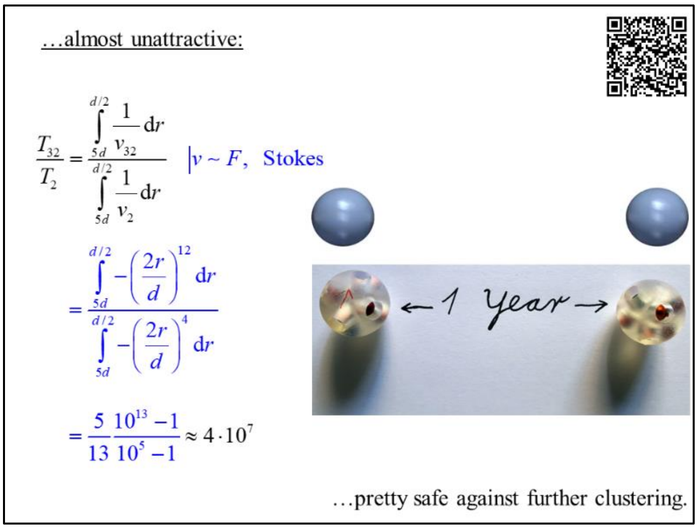
- The ground state deserves attention. It can be experimentally demonstrated on stage, with the help of eight hands from volunteers (Cube demo).



In the lab (QR next page), we drilled a hole into the red plastic body and measured the magnetic field along a straight line. The result is shown also on logarithmic plot bringing it out that the field increases from zero in the centre with the 4th power, and decreases with the -7th in the far field. To illustrate this fact, an interactive animation should help (QR).

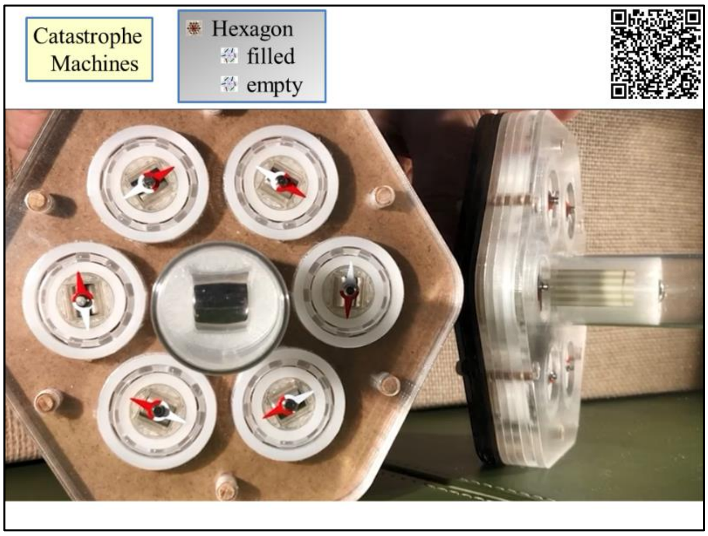
- Keeping in mind that the field of a monopole decays with the second power, this cube formally corresponds to a dotriacontapole.

- The (Coulomb)-force between two monopoles decays with the second power, two dipoles with the 4th …, making it plausible that dotriacontapoles interact with the -12th power, as illustrated in the animation.

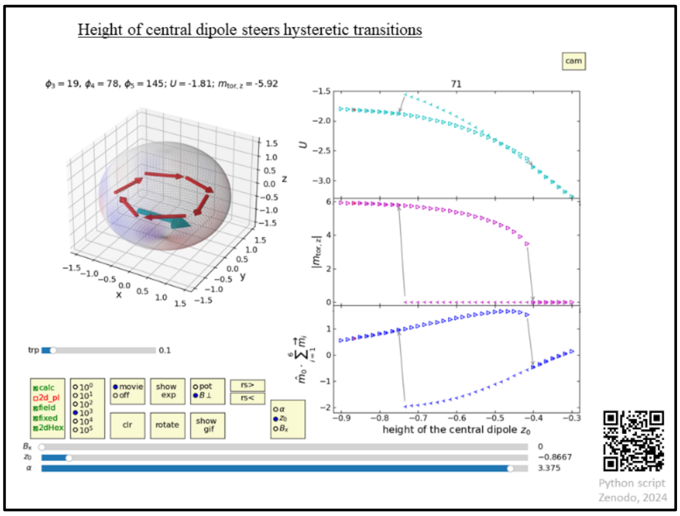


To illustrate this experimentally, let us try to watch these two spheres attracting each other (demo). They collide in less than a second. These two dotriacontapoles – formed by eight spheres embedded in plastic – are not willing to perform (demo). This can be understood by this “back of an envelope”- calculation. If the dipoles swimming in a magnetic fluid decide to fall into this configuration, the suspension would be safe against further clustering.

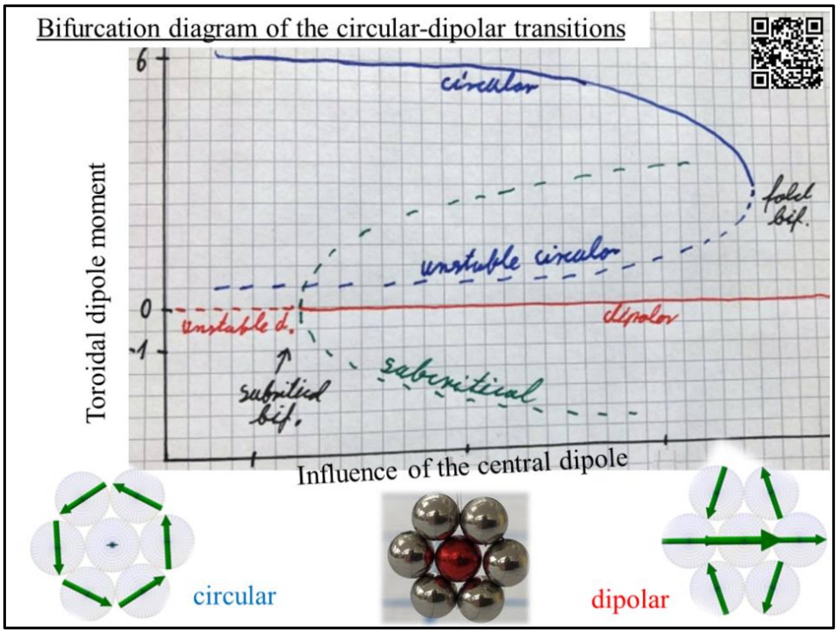
Having realized that there are many equilibrium solutions for dipole clusters lets us now focus on transitions between such states:



I would like to demonstrate the maybe most elementary transitions between two stable cluster configurations with a special catastrophe machine (demo) – seven dipoles forming a hexagon. An illustration of that mathematical term is provided via an animation (QR). Showing the experiment on stage (and passing the machine around) is possible. The movie provided here makes use of an additional helpful mirror.



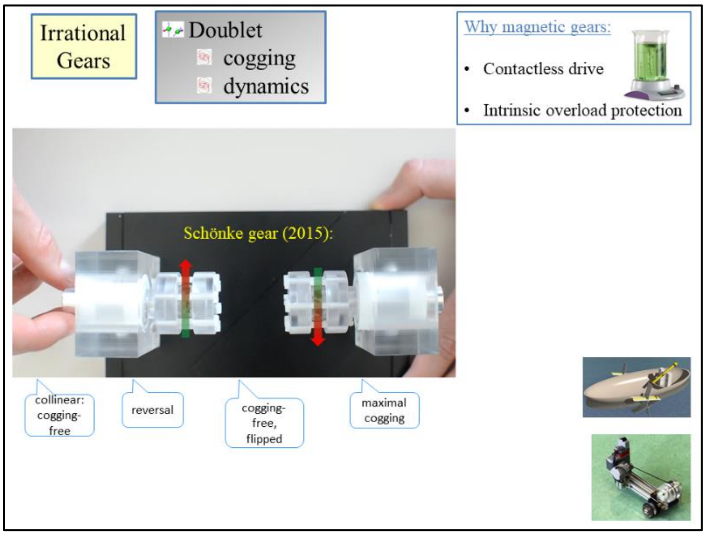
You can interactively play with that configuration in an animation (QR). The corresponding movie brings out the two discontinues jumps between two stable states and the corresponding hysteresis.



A bifurcation diagram clarifies the nature of this two transitions and the connection between two states. If the central dipole is weak – far away – the dipoles favour the circular configuration. By a strong central dipole, they are forced into a dipolar configuration, like compass needles on earth.

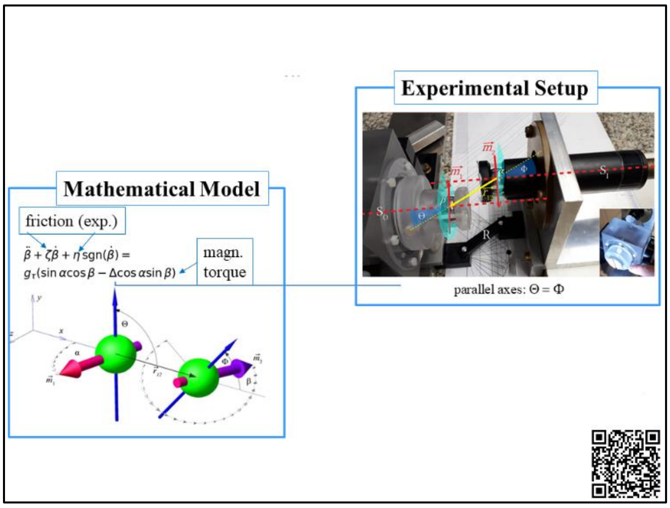
Decreasing that force leads to an instability, where the system jumps up to the stable branch of the circular solution. The subcritical symmetry-breaking branch is neither stable nor connected with the circular solution. The circular state terminates in a fold, that is the smaller jump observed in the experiment. Such a fold bifurcation is currently more prominent under the name “tipping point” in connection with the climate catastrophe. (The QR-code is a link to an open-access version of the corresponding numerical calculations).

Besides multi-stability, the other interesting aspect of the cube was the degenerate ground state, a sevenfold magnetic clutch. Shouldn’t that be useful for something?



A classical application of magnetic gears is the magnetic stirrer, as used in chemistry labs – but also in your milk frother at home, if you happen to have a magnetic version. The term cogging is best illustrated if you play with this machine yourself (demo), a tricky kind of gear, proposed by Johannes Schönke in 2015: The design is based on two spherical magnets, magnetic dipoles as indicated by the arrows, which can turn around a mechanical axis. The motor is the hand of the experimentalist; the driven output is on the right hand side. Depending on the angle between the shafts and the line connecting the 2 dipoles, this gear changes the rotation of the output. At two specific angles the driving is smooth.

- The two "machines" shown here make use of the counterintuitive *cogging free* coupling.



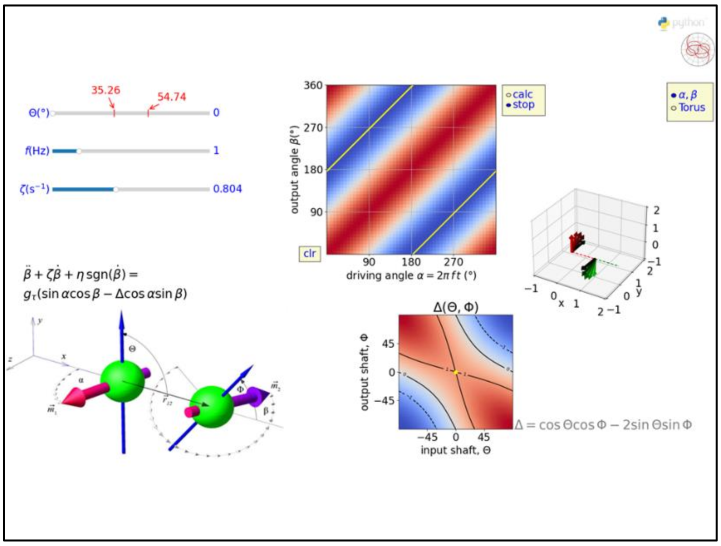
In the lab (the QR-code hints to a more detailed description), a motor replaces the driving hand. Under the given circumstances, an animation of the mathematical model must do.

- The torque stemming from the dipole-dipole interaction provides the angular acceleration of the output.

- Its strength is modified by a purely geometrical pre-factor.

- The acceleration is reduced by friction, its strength we determined experimentally.

Animation:



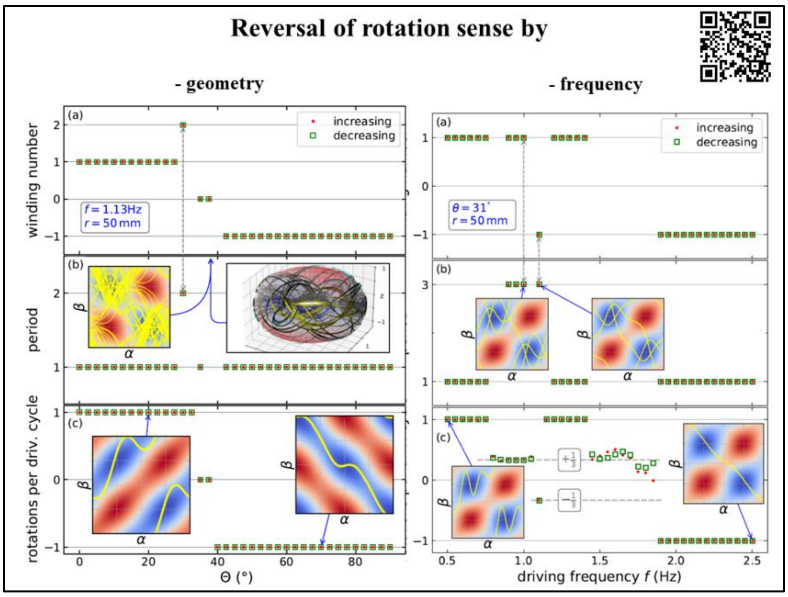
- Let us first watch the reversal of the rotation sense by changing the geometry: The shaft angles Θ and Φ are the same in our parallel setup. Try Θ = 0, 20, 35.26, 40 (Period 3, better shown on a torus), 50, 54.75, 90.

While the change of rotation sense can be understood by looking at the static interaction, a change of rotation with the driving frequency is less intuitive:

With Θ =31° fixed, set f= 0.5, (0.9, 1.7, 1.8) 1.9 (robust up to 3 Hz,…, 4 Hz).

The hysteretic reaction to overloads is illustrated by the reaction at a driving frequency of 4 Hz: With Θ =0°, set f to 0.5, 4, 6 Hz (show with α, β). However, jumping suddenly to 6 Hz overloads the gear (step from 0.5 Hz to 6 Hz). It does not break, but rotates slowly when stepping back to 4 Hz.

- To illustrate the influence of locking, switch to Θ =10°, which leads to a periodic state with a long period.

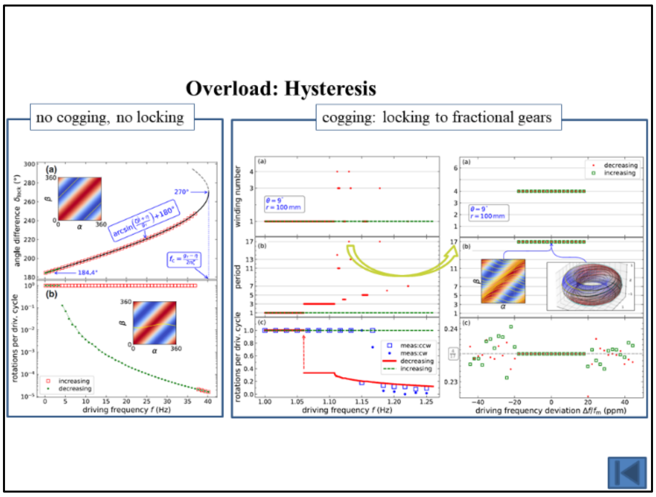


Here is the summary of the animation (the [QR] denotes the python code):

- Increasing Θ lead to a flip of the rotation.

- So does the frequency, which is less intuitive.

In both cases, the crossover regime is not simply periodic with the driving frequency.



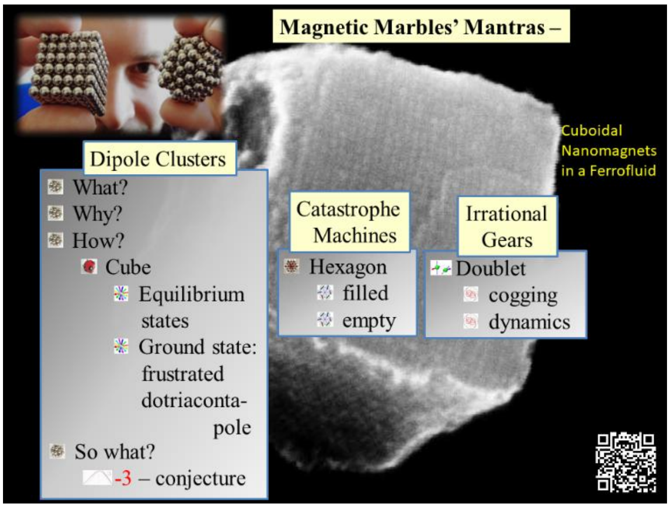
The angle difference between input and output growth with friction. The gear gets out of synchronisation when the magnetic torque is smaller than torque provided by friction. The output does not really stop, but flips through with a very small frequency: One rotation of the output requires about 10000 rotations of the input!

- In the flip through situation without cogging, the relation between driving and driven speed is a smooth function.

- This changes with cogging: Here the structured energy landscape favours rational relations between the driving and reaction frequency.

- The silliest locking seen here is the frequency 4/17: A periodic state with a period of 17 rotations of the drive, in which the driven output performs 4 rotations. Well, that finding is clearly fairly special, and its range of existence tiny, on the ppm-level.

Is there not any bigger take-home message concerning dipole clusters?



Yes, I think there is. I have not yet illustrated the -3 – conjecture.

- Recall the cuboid (animation, linked via the QR-code): The sum of the two powers for the increase (4) and decrease (-7) of the field is 4-7 = -3.

- For the tetrahedron, we get: 2-5 = -3, the same.

- The soccer ball in
  - star configuration: 5-8 = -3,
  - a relaxed state: 2-5 = -3.

The conjecture is that this "minus three" is universal for *all* dipole clusters, not just counting the magnetic ones. I am not able to present a mathematical proof (Maxwell equations with symmetry considerations?), though. If you have a clue, that would be great!