Conduction Accompanied With Transient Radiative Heat Transfer Using Finite Volume Method

A. Ashok, K.Satapathy, B. Prerana Nashine

Abstract—The objective of this research work is to investigate for one dimensional transient radiative transfer equations with conduction using finite volume method. Within the infrastructure of finite-volume, we obtain the conservative discretization of the terms in order to preserve the overall conservative property of finite-volume schemes. Coupling of conductive and radiative equation resulting in fluxes is governed by the magnitude of emissivity, extinction coefficient, and temperature of the medium as well as geometry of the problem.

The problem under consideration has been solved, for a slab dominating radiation coupled with transient conduction based on finite volume method. The boundary conditions are also chosen so as to give a good model of the discretized form of radiation transfer equation. The important feature of the present method is flexibility in specifying the control angles in the FVM, while keeping the simplicity in the solution procedure.

Effects of various model parameters are examined on the distributions of temperature, radiative and conductive heat fluxes and incident radiation energy etc. The finite volume method is considered to effectively evaluate the propagation of radiation intensity through a participating medium.

Keywords—Radiative transfer equation, finite volume method, conduction, transient radiation.

I. INTRODUCTION

NOWADAYS, thermal radiation accompanied with conduction, has extensive practical applications in the field of heat exchangers, combustions chambers, spacecraft, furnaces for the boilers, electronic cooling systems, industrial furnaces, porous volumetric solar receivers, surface and porous IR-radiant burners, etc. To enhance the range of applications for different applications, over the years, various types of angular quadrature schemes do exists. In any radiative transfer method, angular quadratures are required to evaluate the source term and the radiative heat flux. For an accurate and computationally efficient solution in any method, a simple and an accurate quadrature scheme is an important component. Radiation in the 3-D space needs to be examined because of angular consideration even if the solution domain is 1-D one, which makes evaluation of thermal radiation more difficult than that of conduction or convection. Persistence of radiative information for the energy equation is the most time consuming component in conjugate mode problems. Therefore, numerous efforts have been made to develop numerical methods to deal with various types of radiative transport problems and also to make the present methods computationally more efficient. FVM is one of the efficient methods which are gaining momentum for radiative transport problem. The Finite volume method (FVM) was extensively used to solve the energy equation of a transient conduction–radiation heat transfer problem. This method is widely used for the analysis of heat transfer and flow in complex industrial geometries.

Radiative heat transfer in the participating medium has a significant role in various engineering applications. The problem of absorbing and emitting rectangular medium has been studied by Modest [1], Razzaque et al. [2], Fiveland [3]. In the case of an isotropic scattering medium, several different techniques have been used. As to coupled problem with radiation transfer, the spatial distributions of radiative information as source terms are first solved, and then energy equation can be solved by finite difference method (FDM) [4], finite volume method (FVM) and finite element method FEM [5]. Combination of the methods mentioned above can be used to deal with the coupled radiation and conduction in multi-dimensional complicated geometry. Due to their high accuracy, wide applicability and relatively low computational cost and computer memory the FVM and the DOM have emerged as the most efficient methods for modeling radiative transfer these years. In the FVM, the radiative transfer equation is integrated over both the control angle and the control volume, which is different from the DOM, where the RTE is integrated only over the control volume [6] and the conduction equation is solved using two dimensional equation.

The FVM has been prominent during past few years due to very favorable characteristics [7] especially that the FVM allows for conserving radiant energy and can be easily incorporated in computational fluid dynamics (CFD) simulations [8]. Raithby and co-workers [9]-[11] investigated new angular and spatial discretization practices. In this approach, radiant energy is conserved with in a control angle, control volume, and globally for any number of control angles and control volumes arranged in any manner.

In radiative heat transfer, the finite volume method (FVM) [9], [12] is extensively used to compute the radiative information. This method is a variant of the DOM [14]. It does not suffer from the false-scattering [9], [10] as in DOM and the ray-effect is also less pronounced as compared to other methods. Since the FVM for the radiative heat transfer utilizes the same theory as that of the FVM of the CFD its computational grids are compatible with the FVM grids that

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are utilized in the solution of the momentum and energy equations [15], [16].

The objective of the present work is to establish the compatibility of the FVM for the solution of the radiative transfer equation and for the determination of radiative information. The radiative transfer equation is solved using the FVM of the CFD and the conduction term is computed using error function. Here a Transient conduction and radiation heat transfer problem in 1-D slab is considered. For various parameters like heat flux, incident radiation energy, the conduction–radiation parameter and the boundary emissivity, results of the FVM have been computed. The number of iterations and CPU times for the converged solutions has also been reported.

II. MATHEMATICAL FORMULATION

The radiative transfer equation at any direction s and at any location for an absorbing, emitting and scattering gray medium is governed by:

$$\int_{\Delta\Omega}\int_{\Delta x}\int_{\Delta t}\frac{1}{c}\frac{\partial I}{\partial t}dtdxd\Omega + \int_{\Delta t}\int_{\Delta\Omega}\int_{\Delta x}\frac{\partial I}{\partial s}dxd\Omega dt$$
$$= \int_{\Delta t}\int_{\Delta\Omega}\int_{\Delta x}(-\beta_m I + S_m)dxd\Omega dt \quad (1)$$

The first term on the RHS in eq. (1a) represents an attenuation of radiation intensity due to absorption and out-scattering, while the last term account for an augmentation of intensity due to gas emission as well as in-scattering.

$$\frac{1}{c}\frac{\partial I^l}{\partial t} + \frac{\partial I^l}{\partial s} = -\beta_m^l I^l + S_m^l \quad (2)$$

where β_m^l is the modified extinction coefficient and S_m^l is modified source term and can be written as in (3a) and (3b) respectively.

$$\beta_m^l = k + \sigma_s - \frac{\sigma_s}{4\pi}\Phi^{l'l}\Delta\Omega^l \quad (3a)$$

$$S_m^l = kI_b + \frac{\sigma_s}{4\pi}\sum_{l'=1, l'\neq 1}^{M}I^{l'}\Phi^{l'l}\Delta\Omega^{l'} \quad (3b)$$

The order of integration is chosen according to the nature of terms. The first term of left hand side can be discretized as:

$$\int_{\Delta\Omega}\int_{\Delta x}\int_{\Delta t}\frac{1}{c}\frac{\partial I}{\partial t}dtdxd\Omega = \frac{1}{c}(I_P - I_P^0)\Delta x\Delta\Omega \quad (4)$$

where I_p^0 shows the intensity at previous time step and I_p at present time step. Pursueing the discretization practice of Chai.et.al. [13], [17], the second term in the left hand side and the term of right hand side can be written as:

$$\int_{\Delta t}\int_{\Delta\Omega}\int_{\Delta x}\frac{\partial I}{\partial x}dxd\Omega dt = D_{cx}\int_t^{t+\Delta t}(I_e - I_w)dt \quad (5)$$

Using the fully implicit scheme (5) can be written as:

$$D_{cx}\int_t^{t+\Delta t}(I_e - I_w)dt = D_{cx}(I_e - I_w)\Delta t \quad (6)$$

I_e and I_w are the intensities at east and west control volume faces.

$$\int_{\Delta t}\int_{\Delta\Omega}\int_{\Delta x}(-\beta_m I + S_m)dxd\Omega dt = \int_t^{t+\Delta t}[(-\beta_m I_P + S_m)\Delta x\Delta\Omega]dt \quad (7)$$

Integrating (7) over the time interval result in following equation.

$$\int_t^{t+\Delta t}[(-\beta_m I_P + S_m)\Delta x\Delta\Omega]dt$$
$$= (-\beta_m I_P + S_m)\Delta x\Delta\Omega\Delta t \quad (8)$$

Combining (4), (6) and (8) gives:

$$\frac{1}{c}(I_P - I_P^0)\frac{\Delta x}{\Delta t}\Delta\Omega + D_{cx}(I_e - I_w)$$
$$= (-\beta_m I_P + S_m)\Delta x\Delta\Omega \quad (9)$$

The scattering phase function is described as:

$$\Phi_w^+ = \frac{I_W - I_{WW}}{I_P - I_{WW}} \quad (10a)$$

$$\Phi_e^+ = \frac{I_P - I_W}{I_E - I_W} \quad (10b)$$

By solving (9) a final discretization equation (for $D_{cx} > 0$) can be reduced in a standard form of finite volume method as:

$$a_P I_P = a_W I_W + b \quad (11)$$

For the solution of (11) the initial intensity is prescribed according to initial boundary condition and the terms a_p the coefficient of discretization equation and b the source term in the discretization equation can be written in a discretized form as follows:

$$a_w = D_{cx} \quad (12a)$$

$$a_P = D_{cx}(1 + S_{pe+} - S_{pw+}) + \beta_{m,p}\Delta x\Delta\Omega + \frac{1}{c}\frac{\Delta x}{\Delta t}\Delta\Omega \quad (12b)$$

$$b = D_{cx}(S_{cw+} - S_{ce+}) + S_{m,P}\Delta x\Delta\Omega + \frac{1}{c}I_p^0\frac{\Delta x}{\Delta t}\Delta\Omega \quad (12c)$$

$$D_{cx} = \int_{\Delta\Omega}(\hat{s}.\hat{n}_x)d\Omega \quad (12d)$$

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The problem of TRTE coupled with conduction in a one dimensional slab investigated. The one dimensional transient conduction equation which will be coupled by RTE is given by (13):

$$\frac{\partial T}{\partial t} = \alpha_t \frac{\partial^2 T}{\partial x^2} \qquad (13)$$

and the solution of conduction equation is expressed in terms of the error function given below in (16):

$$T(x,t) = T_0(x)\left[1 - erf\left(\frac{x}{2\sqrt{(\alpha_t t)}}\right)\right] \qquad (14)$$

where erf is the error function, α_t is the thermal conductivity of slab, t is time in seconds. $T(x,t)$ is temperature in Kelvin, $T_0(x)$ is temperature at previous time step.

III. Problem Description

The problem is being computed for one-dimensional radiative heat transfer by employing finite volume method in the existence of participating media. The space is divided into 300 control volumes of unit depth in x-direction and is maintained at cold temperature. The scattering coefficient and absorption coefficient of medium is 0.5 respectively. To discretize the angular space is divided in to 40 polar and 1 azimuthal control angles. The left boundary temperature is promptly raised to an emissive power of value equal to *Phi* at time *t=0*.

IV. Result

A formulation of the finite volume method was proposed. After collapsing the radiative information to the 1-D solution plane, formulation procedure of the conduction equation was incorporated. The FVM is developed to study transient radiative transfer in one-dimensional (1D) anisotropic scattering, emitting and absorbing medium. The formulation is feasible for both transient and steady-state radiative transfer. It is found that the present method is accurate and efficient. It is found that the ballistic component of the laser propagates with the speed of light in the medium, and its value is reduced dramatically with the advance of propagation.

A. Variation of Heat Flux

Heat flux is highest at the hot surface and reduces with time because a thick medium absorbs nearly all the radiation. Fig. 1 shows the variation of heat flux without conduction along *x*-direction. Due to back scattering the radiative heat flux near the hot surface reduces with time. Transient radiative heat transfer is accompanied with transient conduction and the variation of heat flux along *x* coordinate is shown in Fig. 2, and the heat flux is maximum at the left boundary and reduces while radiation reaches the opposite wall (*x=L*) of the slab. The effect in increase of heat flux can be seen when conduction is incorporated with radiative properties. The effect of radiative energy is at its peak at the left boundary

because it has been provided with an emissive power and the radiative energy decreases while reaching the extreme end of the slab.
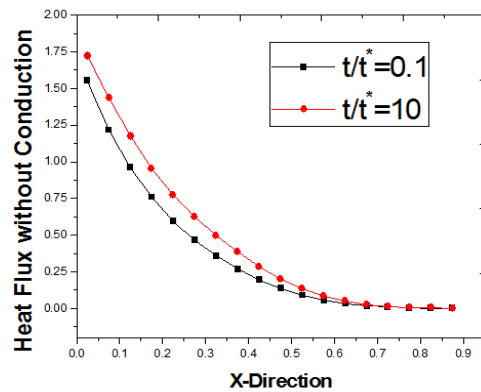

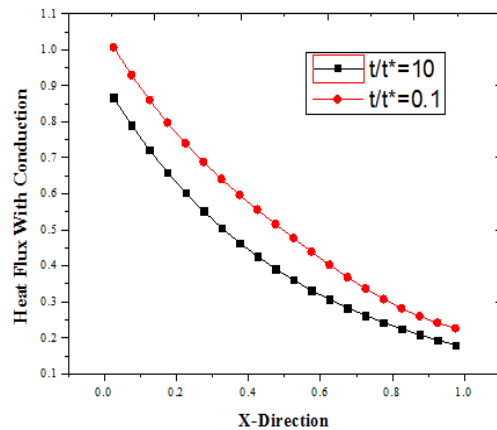Fig. 1 Variation of heat flux without conduction with X Direction

Fig. 2 Heat Flux with conduction along X-Direction

B. Variation of Incident Radiation Energy

Incident radiation energy is accompanied with and without transient conduction, and the results plotted in Figs. 3 and 4. The incident radiation energy without conduction near the hot surface decrease with time due to back scattering effect before reaching steady state values. The variation of incident radiation energy without conduction along *x* coordinate is shown in Fig. 3. The incident radiation with conduction is maximum in the left boundary and decreases while reaching the extreme right of the slab at (*x=L*). Radiation coupled with conduction has been plotted in Fig. 4. The value of irradiation is more when radiation is coupled with conduction and incident radiation energy is at its peak at the left boundary and decreases while reaching the extreme right boundary.

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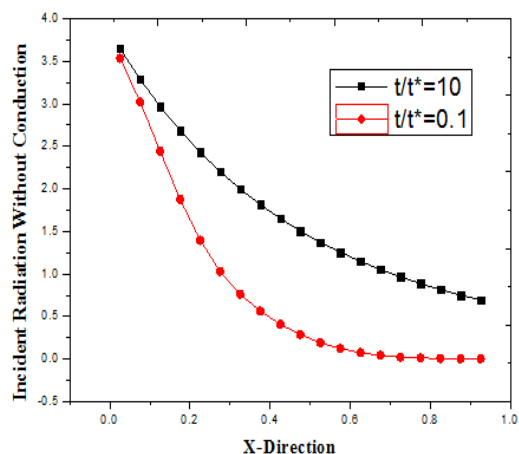Vol:8, No:9, 2014

Fig. 3 Variation of incident radiation without conduction along x-axis
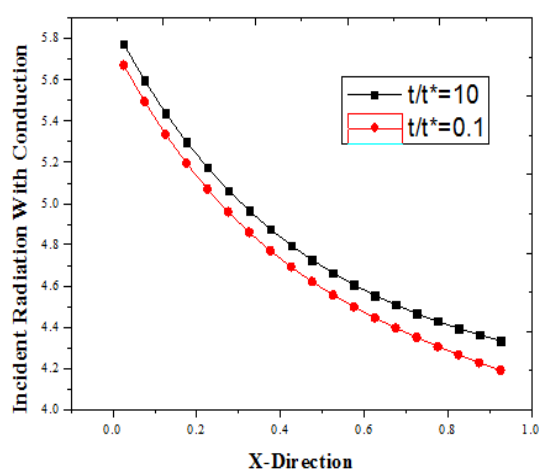

Fig. 4 Variation of incident radiation energy with conduction along x-direction

V. CONCLUSION

The accuracy of the method depends on the discretization of the radiative transfer equation and calculation of the source function. A finite volume method has been proposed for the prediction of radiative heat transfer that can be employed to predict radiative properties. The theory is advanced for an absorbing, emitting and scattering medium in an enclosure, but extensions to a broad bandwidth model and to include walls with other reflective properties are straight forward. The method has been demonstrated for a one-dimensional problem, for which benchmark solutions and other approximate solution are available. Boundary conditions are more strongly a function of the incident energy and not the emissive power therefore incident energy at boundary propagate in to boundary conditions and finally into field of radiant intensities. We get the increased value of heat flux and incident radiation energy when radiation is coupled with conduction as compared to radiative properties without conduction

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