The following derivation is based on this paper. https://doi.org/10.1002/nme.1620151205

$$\begin{bmatrix}\beta_x^4\\ \beta_x^5\\ \beta_x^6\end{bmatrix} = \begin{bmatrix}c_4 & & \\ & c_5 & \\ & & c_6\end{bmatrix}\begin{bmatrix}\beta_n^4\\ \beta_n^5\\ \beta_n^6\end{bmatrix} - \begin{bmatrix}s_4 & & \\ & s_5 & \\ & & s_6\end{bmatrix}\begin{bmatrix}\beta_s^4\\ \beta_s^5\\ \beta_s^6\end{bmatrix}, \quad \begin{bmatrix}\beta_y^4\\ \beta_y^5\\ \beta_y^6\end{bmatrix} = \begin{bmatrix}s_4 & & \\ & s_5 & \\ & & s_6\end{bmatrix}\begin{bmatrix}\beta_n^4\\ \beta_n^5\\ \beta_n^6\end{bmatrix} + \begin{bmatrix}c_4 & & \\ & c_5 & \\ & & c_6\end{bmatrix}\begin{bmatrix}\beta_s^4\\ \beta_s^5\\ \beta_s^6\end{bmatrix}, \tag{1}$$

$$\begin{bmatrix}\beta_n^4\\ \beta_n^5\\ \beta_n^6\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & 1 & & & & \\ & & 1 & 1 & & \\ & & & & 1 & 1\end{bmatrix}\begin{bmatrix}\beta_{n,4}^1\\ \beta_{n,4}^2\\ \beta_{n,5}^2\\ \beta_{n,5}^3\\ \beta_{n,6}^3\\ \beta_{n,6}^1\end{bmatrix}, \quad \begin{bmatrix}\beta_{n,4}^1\\ \beta_{n,4}^2\\ \beta_{n,5}^2\\ \beta_{n,5}^3\\ \beta_{n,6}^3\\ \beta_{n,6}^1\end{bmatrix} = \begin{bmatrix}c_4 & & \\ & c_4 & \\ & c_5 & \\ & & c_5\\ & & c_6\\ c_6 & & \end{bmatrix}\begin{bmatrix}\beta_x^1\\ \beta_x^2\\ \beta_x^3\end{bmatrix} + \begin{bmatrix}s_4 & & \\ & s_4 & \\ & s_5 & \\ & & s_5\\ & & s_6\\ s_6 & & \end{bmatrix}\begin{bmatrix}\beta_y^1\\ \beta_y^2\\ \beta_y^3\end{bmatrix}, \tag{2}$$

$$\begin{bmatrix}\beta_x^1\\ \beta_x^2\\ \beta_x^3\end{bmatrix} = \begin{bmatrix}\theta_y^1\\ \theta_y^2\\ \theta_y^3\end{bmatrix} = \begin{bmatrix}\cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1\end{bmatrix}\begin{bmatrix}w^1\\ \theta_x^1\\ \theta_y^1\\ w^2\\ \theta_x^2\\ \theta_y^2\\ w^3\\ \theta_x^3\\ \theta_y^3\end{bmatrix}, \quad \begin{bmatrix}\beta_y^1\\ \beta_y^2\\ \beta_y^3\end{bmatrix} = \begin{bmatrix}-\theta_x^1\\ -\theta_x^2\\ -\theta_x^3\end{bmatrix} = -\begin{bmatrix}\cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot\end{bmatrix}\begin{bmatrix}w^1\\ \theta_x^1\\ \theta_y^1\\ w^2\\ \theta_x^2\\ \theta_y^2\\ w^3\\ \theta_x^3\\ \theta_y^3\end{bmatrix}, \tag{3}$$

$$\begin{bmatrix}\beta_s^4\\ \beta_s^5\\ \beta_s^6\end{bmatrix} = -\begin{bmatrix}w_s^4\\ w_s^5\\ w_s^6\end{bmatrix} = \frac{3}{2}\begin{bmatrix}\frac{1}{l_4} & -\frac{1}{l_4} & \\ & \frac{1}{l_5} & -\frac{1}{l_5}\\ -\frac{1}{l_6} & & \frac{1}{l_6}\end{bmatrix}\begin{bmatrix}w^1\\ w^2\\ w^3\end{bmatrix} + \frac{1}{4}\begin{bmatrix}1 & 1 & & & & \\ & & 1 & 1 & & \\ & & & & 1 & 1\end{bmatrix}\begin{bmatrix}w_{s,4}^1\\ w_{s,4}^2\\ w_{s,5}^2\\ w_{s,5}^3\\ w_{s,6}^3\\ w_{s,6}^1\end{bmatrix}, \tag{4}$$

$$\begin{bmatrix}w_{s,4}^1\\ w_{s,4}^2\\ w_{s,5}^2\\ w_{s,5}^3\\ w_{s,6}^3\\ w_{s,6}^1\end{bmatrix} = \begin{bmatrix}\cdot & c_4 & s_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & c_4 & s_4 & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & c_5 & s_5 & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & c_5 & s_5\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & c_6 & s_6\\ \cdot & c_6 & s_6 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\end{bmatrix}\begin{bmatrix}w^1\\ \theta_x^1\\ \theta_y^1\\ w^2\\ \theta_x^2\\ \theta_y^2\\ w^3\\ \theta_x^3\\ \theta_y^3\end{bmatrix}, \tag{5}$$

$$H_x = \begin{bmatrix}0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ -\frac{3s_4}{2l_4} & -\frac{3c_4s_4}{4} & \frac{c_4{}^2}{2} - \frac{s_4{}^2}{4} & \frac{3s_4}{2l_4} & -\frac{3c_4s_4}{4} & \frac{c_4{}^2}{2} - \frac{s_4{}^2}{4} & 0 & 0 & 0\\ 0 & 0 & 0 & -\frac{3s_5}{2l_5} & -\frac{3c_5s_5}{4} & \frac{c_5{}^2}{2} - \frac{s_5{}^2}{4} & \frac{3s_5}{2l_5} & -\frac{3c_5s_5}{4} & \frac{c_5{}^2}{2} - \frac{s_5{}^2}{4}\\ \frac{3s_6}{2l_6} & -\frac{3c_6s_6}{4} & \frac{c_6{}^2}{2} - \frac{s_6{}^2}{4} & 0 & 0 & 0 & -\frac{3s_6}{2l_6} & -\frac{3c_6s_6}{4} & \frac{c_6{}^2}{2} - \frac{s_6{}^2}{4}\end{bmatrix} \tag{6}$$

$$H_y = \begin{bmatrix}0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0\\ \frac{3c_4}{2l_4} & \frac{c_4{}^2}{4} - \frac{s_4{}^2}{2} & \frac{3c_4s_4}{4} & -\frac{3c_4}{2l_4} & \frac{c_4{}^2}{4} - \frac{s_4{}^2}{2} & \frac{3c_4s_4}{4} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{3c_5}{2l_5} & \frac{c_5{}^2}{4} - \frac{s_5{}^2}{2} & \frac{3c_5s_5}{4} & -\frac{3c_5}{2l_5} & \frac{c_5{}^2}{4} - \frac{s_5{}^2}{2} & \frac{3c_5s_5}{4}\\ -\frac{3c_6}{2l_6} & \frac{c_6{}^2}{4} - \frac{s_6{}^2}{2} & \frac{3c_6s_6}{4} & 0 & 0 & 0 & \frac{3c_6}{2l_6} & \frac{c_6{}^2}{4} - \frac{s_6{}^2}{2} & \frac{3c_6s_6}{4}\end{bmatrix} \tag{7}$$