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Self-organization versus hierarchical organization

a mathematical investigation of the anarchist philosophy of social organization

Dissertation submitted for the degree of Doctor in the Interdisciplinary Studies

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This is life
What a fucked up thing we do
What a nightmare come true
Or a playground if we choose
And I choose

The Offspring - I Choose

Abstract

In combining anarchist theory with mathematics, this thesis wishes to better understand what power and hierarchy are in order to explore how we can live without coercion. My motivation to study these concepts stems from observing a lack of freedom in contemporary society despite a lack of obvious coercion or clear hierarchical structure.

I divide this issue into three main research questions. What are, on the one hand, authority and hierarchy, and, on the other hand, what are freedom and autonomy? How does hierarchy evolve in social systems? And how can we shift from hierarchical control to a more free social organization? To answer these questions, I make use of social theory, anarchist theory, complex systems theory, mathematics and computer simulations.

I distinguish several aspects of power: control, coercion, constraint, determination and hierarchy. Defining these aspects leads to different understandings of freedom. Internal control refers to control over your own situation, while external control is directed towards the (whole) environment. Coercion forces a person to do something he does not want to do, while constraint limits a person's possibilities. External determination, wherein one is completely influenced by an external force, makes one vulnerable to coercion.

Determination and coercion are associated with a hierarchical structure. In a hierarchy, each element has no more than one influence and this influence works in only one direction. These concepts are described using mathematical tools such as graphs and entropy in cybernetic models.

Self-organization can lead to the development of a controller. Working together to reach your goals can lead to a higher-level system. This system can acquire goals of its own, which can become disconnected from the goals of the entities that created the system. The rise of such a controller can be avoided by constantly opposing any seed of hierarchy or coercion. In this manner, no power can grow too big. This mechanism of constant opposition is illustrated in a simulation.

Overall, this thesis illustrates how to think in a less hierarchical way by focusing on local coherence. In this way, there can be jointly related ideas rather than a single, primary concept with several sub-concepts. The tension between hierarchy and local coherence recurs throughout the thesis—in the difference between Marxism and anarchism, in internal versus external control, in the structural component of hierarchy, in hierarchical models versus their non-hierarchical variants, and in human agency versus determination.

Acknowledgements

As a life trajectory is always shaped by many influences, so is mine. Some of these influences have specifically enabled me to write this PhD, and supported me in the years it took to complete this effort.

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Contents

Chapter 1

Introduction

1.1 Aims and methods

The motivation behind this thesis is to better understand what power and authority are in order to explore how we can live without coercion. The method used will be mainly abstract reasoning to generate a coherence between two of my main interests: mathematics and anarchism. This thesis aims to answer three main questions:

- What are, on the one hand, authority and hierarchy, and, on the other hand, what are freedom and autonomy?
- How does hierarchy evolve in social systems?
- How can we shift from hierarchical control to a more free social organization?

I wish to develop a clear understanding of concepts like freedom that many people find appealing, but that are often ill defined. My intuition is that there is a differentiating factor defining power, beyond visible coercion, but which is difficult to pinpoint. In particular in today's changing world, where, on the one hand, organization becomes more and more distributed without a central leader, and on the other hand, a lot of people seem to feel like they cannot fit in with the pace of this change, the problem of alienation seems to be widespread. People experience confusion, a void. This may result in a lot of mental problems. The social system itself is perceived as limiting one's freedom.

In my understanding, hierarchy is when one's freedom is limited, though not necessarily through the existence of any specific structure. Hierarchy often evolves from previously free systems. Through this evolution, organizations start to serve their own goal, revolutions end up in dictatorships, passions evolve into *idées fixes*, and so on.

In this manner, I want to find out how we can stop this phenomenon—how we can live and pursue our dreams without being dominated, nor having to be authoritative ourselves.

I will not provide one clear answer to all these questions. On the one hand I have sometimes been frustrated myself because I sensed I could not adequately define freedom or authority. How could I be so attracted to a concept like freedom, if I did not even know what it meant? On the other hand, freedom is so attractive because it is more than a mere formula, which is perhaps even one of its core properties.

Reality is complex, and it can not be fitted into a deterministic model. Since a model is always simpler than what it is describing, no model can be a total, exhaustive description. *The map is not the territory.* In every model, there will be aspects lacking and things that do not correspond perfectly with reality. Still, models can be useful. I think it can be valuable to search for an abstract core that differentiates freedom from authority, and to construct models of how hierarchy emerged and how to avoid it. Every theory, every sentence, is a model. In this case, I simply use a more mathematical toolbox. For me, freedom is not a hollow or meaningless word, and I want to find its core.

For some, the ideas in this thesis might seem too reductionistic or simplistic, while others might miss clarity, single, concise answers or an obvious conclusion.

There is no main thesis in this PhD, but it consists of a lot of interconnected ideas. This is not a coincidence, but one of the messages this thesis wants to convey. One does not need one main idea with some subordinated ideas. Coherence can emerge because ideas are related. Having no hierarchy does not equate with structurelessness—lack of one simple explanation does not equate with meaninglessness.

I am not interested in trying to “prove” that self-organization is better than hierarchical organization, I rather want to investigate the specific nature of hierarchy and freedom and find methods to develop self-organization. Whether you think a non-authoritarian structure is better, depends on your values. I personally value freedom, and I know I am not the only one. I want to think with these people on how we can move towards this aspiration, and I hope this thesis can be a step and an inspiration in that direction. I don’t want to be a messiah who wants to convince others that my ideas are the best, I simply want to develop knowledge to navigate this world, for myself and other people with similar aims.

To answer these questions, I make use of social theory, anarchist theory, systems theory, complexity science, cybernetics, mathematics and computer simulations.

I started my PhD by doing computer simulations, but I have moved away from these methods. While simulations have the advantage that they can have unpredictable outcomes and can model more complex phenomena than deterministic formuli, I also stumbled upon some drawbacks. The results of a simulation are often difficult to map to the actual phenomena, and *tout court* challenging to interpret. Usually there are a lot of parameters, and when changing a parameter gives different results, then what can be said about reality, which is different than the model, and for which the parameters are unknown? I furthermore realized I did not yet have a sufficiently adequate understanding of what power actually is, and so I could not model its functioning.

↑ 2

Once I became aware of the deficiencies in my understanding of these concepts and the limitations of computational modeling for understanding broader themes within social structure, I dove into some literature to distill the abstract core behind the ideas in question, to see whether I could formulate these ideas more mathematically—the goal being to portray these ideas sharper and with more clarity.

That is why my thesis begins with a literature study, and becomes more and more mathematical as it progresses.

1.2 Overview of the thesis

Chapter 2 reviews how social theory understands concepts like authority and power, and the difference between *anarchist and Marxist thoughts* regarding how hierarchy evolved and how to change this form of social organization.

↑ 9

In chapter 3, I explain some of the concepts of *complex systems theory* by applying them to anarchist ideas. I define self-organization and show how even in these systems hierarchy can emerge, as the organizational structure develops to form a higher-order control and begins to serve nothing beyond its own perpetuation. But I also propose some principles of coordination without hierarchy, and discuss what freedom and autonomy could mean in abstract complex systems terms.

↑ 38

↑ 76 Chapter 4 presents a more structural view of what can be understood as hierarchy in the mathematical sense. For this, I use *order theory* to investigate how sets can be ordered, and I discuss graphs (directed and undirected) and hypergraphs. In directed networks, I examine three dimensions that can measure how hierarchical a graph is, and review the *theory of controllability.*

↑ 125 Chapter 5 focuses on process and change. First, I recapitulate how a hierarchical structure can be established from minimal conditions. A network can be made more or less hierarchical by creating or destroying specific edges. I furthermore present a simple simulation of how a mechanism of constant opposition can counteract an unequal distribution and give rise to a more dynamic distribution. Finally, I introduce *chemical organization theory*, a useful tool to model processes.

A simulation of how the network structure affects its function is ↑ 139 presented in chapter 6. Agents adapt to their neighbors, becoming influenced by them. This increases their fitness. I investigate which kinds of networks produce most fitness overall, which nodes have the highest fitness, and which have undergone most adaptation or influence.

↑ 154 In chapter 7, I explore autonomy and control. I differentiate internal from external control as the control of an agent over its own situation or control over its environment. We want to measure how much an agent is determined by an external force. I argue why determination coincides with a hierarchical structure.

I present two models of cybernetic control: the quantitative model of *perceptual control theory*, and the qualitative model in the *law of requisite variety.* These models are extended to a *perceptual control hierarchy* and a *law of requisite hierarchy*, both claiming a hierarchical structure. But I argue that a hierarchical structure is nowhere implied, only an agent that changes the functioning of other agents.

Diverse manifestations of control can be classified among two dimensions: to what extent the control is global, and whether the links, methods or goals ↑ 206 of other agents are changed. In chapter 8, I situate different theories into this classification: *controllability theory*, *self-organized control* and a model where the *methods of other agents* are changed. I do so by translating them into a shared language.

↑ 8 At the end of this chapter I show in a map how some of the important concepts of the thesis connect, transcending the chapter borders.

1.3 How to read the thesis

A text is always linear, which makes its structure hierarchical. A thesis has multiple chapters, each of which has multiple sections, and multiple subsections. Usually there is one main hypothesis which is further developed.

While writing my thesis, I noticed problems putting my ideas into a hierarchical structure and creating a clear outline. Ideas are usually related to multiple other ideas, so in what order should I write them? There are several dimensions by which I could order them. For example, I could structure them based on the different questions (but the answers are related), or based on the different domains that answer these questions. Furthermore, clarity and an overall structure often only comes when ideas are written down.

I see this as a general problem with hierarchy. Hierarchy creates friction because elements have to fit into one category, which they rarely do. Furthermore, a structure often emerges from the details, bottom-up, and cannot be known beforehand, so that details cannot be filled top-down.

Because of these arguments, I have tried to make as many links as possible between different ideas. Therefore, certain aspects might become clearer only further on. Because of the link structure, you can read the thesis in the order you want, and only the parts that interest you. While I tried to put a logic to how I have structured my thesis, and some parts will build further on previous parts, whenever this is the case, there will normally be a link to the necessary information.

The ideas developed in this thesis are partly original and partly borrowed from other people. But ideas are often difficult to attribute to a specific person since they stem from a collective knowledge, where people are often building further upon the reasoning of other people. While I have tried to cite as much as possible and some ideas clearly come from a specific paper, other ideas are extracted from a combination of things I have read, discussions I have had, and general life experiences, and then the source is less clear. A lot of my views are formed by being in anarchist circles: reading books and pamphlets, having discussions, but also in trying to put anarchism in action. In contrast to academics, anarchists usually care more about the ideas themselves than about the source from which they originate. Thus, anarchist texts are often anonymous and there is no habit of citing whenever an idea is expressed that is not the author's own. Moreover, there is neither a strong habit of citing in mathematics, the discipline I come from.

To clarify from where parts of the text stem, I have used two symbols which are visible in the margin:

🔍 means that part of the text is borrowed from other work. Thus, if the text summarizes different ideas, this categorization is not mine (but there might be some observations which are mine). I can still speak of ideas from other authors outside of this environment, but the general structure is then decided by me.

📝 means that part of the text comes from one of my papers. This thesis is partly based on papers I have written before. Some of them are almost completely taken over in a chapter, for others, parts are used in multiple places. The reference year of the paper cited will be next to the symbol.

For both symbols, there will be a line in the margin when the specific environment has ended. For example, the next paragraph is borrowed from other work, while the paragraph thereafter comes from my paper (Busseniers, 2016) (according to the symbols, not really).

🔍

When a word or idea links to another part of the thesis, this is visible by a hyperlink. The word will be in gray, and in the digital version clicking on it directs you to the part of the thesis it refers to. Furthermore, there is an upward arrow with the page number of the target, so that also in a printed version one can look up the link. For example, this link directs you to my previous discussion about links in my thesis. ↑ 5

Every linked concept is summarized in a term, and all these terms are put into the index. ↑ 255 In the index, the target page is in bold font, since here the main idea is usually developed, while the pages that link to this target are also mentioned. Sometimes the concept gains form out of the links, where it is developed in all of these pages.

The index should not be seen as a complete overview of the most important terms of the thesis, but more as an overview of the links. It also does not necessarily sum up all the mentionings of a certain word. Use the search function of your digital reader if you want to find whether and where a certain word is used.

Finally, definitions are put in bold font, so that it is visible when a concept is important.

A folder with additional information can be found on the web[1]. Here, I created a supplementary information document, partly because I had difficulties completely deleting parts of my thesis, but it can also be of use to

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[1] http://alturl.com/rm887, http://student.vub.ac.be/~ebusseni/PhDEvoAnnex.zip or https://zenodo.org/record/1098302#.Wi6KizeDOUl (last one without the thesis itself)

the reader. It contains longer versions of sections of the thesis, so that you will be able to see all the proofs and understand the content better. It also contains more details on the results of simulations, and on which specific tests are used. The document moreover discusses appealing theories that did not make the thesis because they did not really fit. Please note that this supplementary information should not be seen as part of the thesis, for it is rather drafty, often containing simple copy-pastes of old sections without much editing.

Besides the supplementary information, this folder contains the computer code I used for the simulations and for extracting a network from the index. It moreover contains the original network files that represent the thesis structure, and the latex-files of the thesis. Finally, there is a little movie of the evolution of the distribution in the constant opposition simulation.

↑ 255

I wish you a nice exploration, hope you enjoy it!

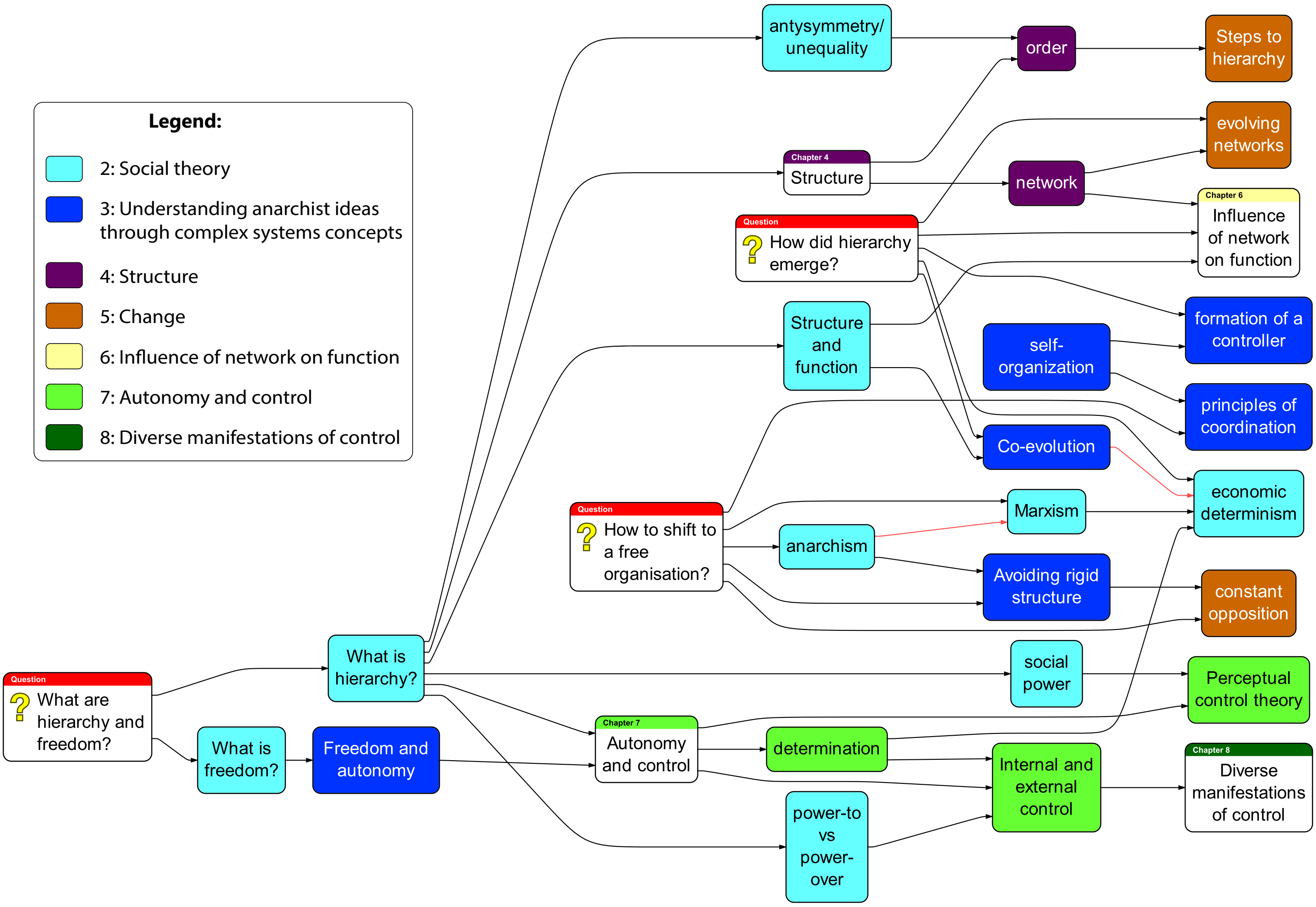

Figure 1.1: A map of some important concepts of the thesis. Red lines means concepts are opposites.

Chapter 2

Social theory

The first set of questions for my PhD thesis relate to the definition of concepts. ↑ 10
That is: “What is hierarchy, authority, power?” And in contrast: “What is freedom?”

Intuitively, these concepts seem clear, yet they are difficult to grasp with concise definitions. Further, none of these words seem to fully comprehend an obstruction of one’s freedom since each one relates to different aspects of obstruction. Hierarchy focuses on the structural aspect. Power can also be used as simply the ability to act. An authority can also mean someone who has more knowledge on a certain topic. Domination and oppression puts the focus on the use of force, while control can both be used as wanting to have control over your own life, and someone controlling you.

For now, I will use these concepts interchangeably and as they are used by the corresponding authors until they are better clarified in the context of this thesis.

The second and third questions my thesis addresses are: “How did hierarchy ↑ 25
evolve?”, and, as an extension: “How can we evolve to a free society?”. These ↑ 30
three questions are, of course, interrelated. The answer to how hierarchy evolves depends on what hierarchy is understood to be. If someone has a different understanding of freedom, his view of how a free society will look, and thus the way to reach this society, will be different. To have a free society, there should be a mechanism to prevent the emergence of hierarchy. Thus different views on how hierarchy evolved will bring different outlooks on the evolution to a free society.

Because these questions are so intertwined, it was sometimes difficult to split them into different sections. For this reason, you will find that individual theories are discussed in more than one section, and that an answer to one question is actually given under a separate question. Actually, we could also split up based on different currents, where we divide based on the difference in the answers these currents provide.

In this chapter, I will give an overview of some of the answers the social science literature provide. In the next chapters, these questions will be answered from different perspectives: from complex systems science, mathematically and cybernetically. But the answers given are often quite similar.

2.1 What is freedom and hierarchy?

2.1.1 What is freedom?

Freedom is a notion that is generally attractive to most people. However, there is a substantial variation in what different people understand it to mean. I see three ways of defining this concept.

The first is to define it negatively, as the **absence** of constraints, for example, as the absence of domination and oppression. What freedom is, then, depends on what its opposite is. (I will discuss definitions of domination
↑ 12 in the next section.) Malatesta's definition of freedom as when "no one could constrain" fits in here (Malatesta, 1965, p47-48). One is free when she is not constrained in her acts. Freedom is then an opposition to certain forces, it
↑ 40 is a decision one makes. This conception of freedom will be discussed later in this section.

A more positive approach sees freedom as **self-actualization**, as the ability to develop oneself to the fullest, or "the tendency to actualize, as much as possible, [the organism's] individual capacities" (Goldstein, 1939). This is quite in line with how Bakunin defines freedom (Bakunin and Kenafick, 1950, p17):

> No, I mean the only liberty which is truly worthy of the name, the liberty which consists in the full development of all the material, intellectual and moral powers which are to be found as faculties latent in everybody, the liberty which recognizes no other restrictions than those which are traced for us by the laws of our own nature; so that properly speaking there are no restrictions, since these laws are not imposed on us by some outside legislator, beside us or above us; they are immanent in us, inherent, constituting the very basis of our being, material as well as intellectual and moral; instead, therefore, of finding them a limit, we must consider them as the real conditions and effective reason for our liberty.

> I mean that liberty of each individual which, far from halting as at a boundary before the liberty of others, finds there its confirmation and its extension to infinity; the illimitable liberty of each through the liberty of all, liberty by solidarity, liberty in equality; liberty triumphing over brute force and the principle of authority which was never anything but the idealized expression of that force, liberty which, after having overthrown all heavenly and earthly idols, will found and organize a new world, that of human solidarity, on the ruins of all Churches and all States.

Bakunin contrasts his approach of freedom to a notion of freedom considered as “individual liberty”. This is a freedom licensed and regulated by the state, a selfish freedom where one limits the freedom of someone else by exercising his own. This view considers freedom **as a right**. A right is always given by some central authority, and always implies duty. Additionally, this freedom is understood as the ability to choose, where the possibilities from which one can choose are given and often limited (for example by a state). This is how freedom is understood under capitalism, where, for example, a person can choose the color of her coffee machine, but can’t really decide how she wants to live her life.
There are some overlaps and differences between these three notions of freedom—freedom as an opposition, as self-actualization or as a right to choose. The fewer constraints there are, the more space there is for an individual to develop himself, and the more choice he has. To be able to develop oneself means having more possibilities, having more choice. Freedom as a right to choose considers these possibilities as well-defined and set beforehand from the outside, while freedom as self-actualization considers these possibilities as open-ended, where an individual can choose her own path. Freedom as a right to choose doesn’t consider how these choices are often set or manipulated from the outside, while the notion of freedom as an opposition highlights the potential for manipulation. The danger in considering freedom as self-actualization (not really in the notion of Bakunin, but in general), is that this can result in the isolation of the individual. Consider an artist who creates a happy little bubble where she develops herself with her art without being aware anymore of very real oppression in the world. On the other hand, when someone purely defines freedom as an opposition to oppression, there is a danger that he may become an empty shell. He only knows what he does not want, but does not know what he wants. His whole life stands in relation to this oppression.

Malatesta (Malatesta, 1965) and Bakunin (Bakunin and Kenafick, 1950) also differ on how they see nature and its laws. Bakunin considers the laws of nature as immanent in us, and not the limit of, but the reason for our liberty, an enabler of our freedom. Malatesta, on the other hand, considers human life a struggle against nature (which he considers ruthless), an overcoming of natural law, an adaptation. That made me wonder whether life might, in general, be defined as the overcoming of a natural law. Biological life is an overcoming of the second law of thermodynamics that without selection, a system will become more and more disordered. Life succeeds in becoming more and more complex. Human life might then be an overcoming of a natural law on a new level, where biological life evolved to become a natural law. If there is some natural law that states that there will always ↑ 18 be hierarchy in the system (like the social contract of Hobbes), then freedom ↑ 25 ↑ 63 is the overcoming of this natural law (see section 2.2.1 and 3.6.2).

If a restriction is only proper when it is imposed by some outside legislator (as stated by Bakunin), what happens when power becomes less and less central? The mechanisms of power become more and more automated, incorporated into our heads. Do these mechanisms of power simply become immanent in us, part of human nature and thus no longer problematic? ↑ 19 What about restrictions that do not come from some central body, like social norms?

This dissimilarity between Bakunin and Malatesta also highlights the distinction between considering freedom as a spontaneously developed order (where nature is the enabler of our freedom), and as a decision (where laws ↑ 40 of nature should be overcome), as discussed in the next chapter (Passamani, 2010). Freedom as a decision treats it as opposing a dominant current. It thus falls under the first notion of defining freedom as a negation.

↑ 59 Section 3.6, offers a more elaborate discussion of the concept of freedom.

In particular, if we are to adopt the view of freedom as opposition, we must first try to understand its opposites: power, authority, and hierarchy.

2.1.2 What is hierarchy?

This section provides an overview of how social theory currently understands the concepts of power, authority, and hierarchy. Different perspectives on the properties, behaviors, and definitions of each concept will be discussed.

The three terms—power, authority, and hierarchy—are related but differ slightly in meaning. Arendt (Arendt, 1970) provides the following five definitions:

- Power: the human ability to act in concert. Belongs to a group. Extreme: All against One

- Strength: the property inherent in an object or person; independent of relation to other things/persons; individual
- Force: the energy released by physical or social movements
- Authority: unquestioning recognition by those who are asked to obey; neither coercion nor persuasion is needed. In persons or offices
- Violence: instrumental character. Used for the purpose of multiplying natural strength until they can substitute for it. Extreme: One Against All

For Arendt, power comes from a group, and is something social. This relates to the concept of social power. This and other understandings of power are discussed further on.

↑ 19

In *Frameworks of power*, Clegg discusses the evolution of the understanding of power (Clegg, 1989, p11). Bachrach and Baratz consider **two faces** of power (Bachrach and Baratz, 1962). The first one is covered by behaviorism, which studies behaviors in their environment by the same models that explain non-social phenomena. Here a formal model of power is made by measuring responses in social experiments. This view thus looks purely individualistic to power. The second face considers the structural component of power, which is always "lurking in the dark". There are two manifestations of this face: through non-decision-making and through mobilization of bias. Non-decision-making is about what should be decided upon, and which things do not get done. Mobilization of bias is about the structure that prefigured the exercise of power.

Lukes extends these two faces to three dimensions: decision-making power, non-decision-making power, and ideological power (Lukes, 1974). **Decision-making power** is generally exercised through political decisions, while **non-decision-making power** (also in one of the two faces), is what sets the agenda. **Ideological power** is about influencing people's desires and thoughts, even mold them to go against their own self-interest. This relates to the discussion about intentionality.

↑ 20

2.1.2.1 Power to versus power over

Power is used in two distinct meanings: either as the ability to do certain things, or more negatively, when there is coercion, as the enforcement of one's will. Following list gives definitions of power given by different authors (Wrong, 2005):

- Wrong: "the capacity to produce effects on the world".
- Hobbes: "man's present means to any future apparent good".
- Russell: "the production of intended effects".
- Weber: "power as the ability to enforce one's will even in the face of conflict or resistance".
- John E. Stewart: "the ability to influence or constrain without being influenced in return".

The last definition comes from (Stewart, 2014). Thus in the first three definitions, power is defined positively, as the ability to do something, while Weber and Stewart define it as a form of coercion or antisymmetry. This is the difference between *power to* and *power over*. There are some more differences between the definitions. Hobbes only considers goods, while Russell looks at any intended effect. Russell still considers there should be an intention, while in the first definition this is not necessary.

This difference between *power-to* and *power-over* is explained as follows. **Power over** assumes asymmetry: one produces more and greater effects on others than the reverse. There is still some reciprocity though: it defines a social rather than a physical relation. There is one who has "power over" someone else, which are both roles restricted by certain norms. **Power to** on the other hand is any means to any desired end, and is universally wanted.

Of course there is a similarity between the concepts: if you produce effects in the world for your desired ends, you will shape and sometimes constrain what is possible for others. How to have influence without exercising power, becoming a new controller? How to have more influence without this resulting in less influence for others, and thus more asymmetry? Of course, being able to reach its desires, doesn't necessarily constrain someone else in any way.

↑ 154

I will formalize this difference further by the concepts of internal and external control. Internal control relates to *power-to*, to have control over your own life. While external control relates to *power-over*, to determine one's environment.

2.1.2.2 Antisymmetry

This antisymmetry present in the definition of power as *power over*, can give rise to a ranking. This is when everything can be compared, and thus the one is put over or under the other. This means there is one-dimensionality. Everything can be put on one line, and is in this sense linear. Antisymmetry however does not imply that everything can be compared, and thus that there is a linear structure: some elements might be incomparable. In chapter 4 this is worked out more. ↑ 76

One can differentiate between two sorts of hierarchy: a structural or a functional. A **structural hierarchy** can be thought of as different non-overlapping sets, that are part of a bigger set, which is part of an even bigger set, and so on. The general representation of a hierarchy is a tree structure, where there are different levels. We do not differentiate however what the represented relation means. When we understand this relation as that one higher can influence or command something lower, but not the other way around, we speak of a **functional hierarchy**. Here the relation has a functional meaning. Simon makes a similar distinction, who calls the latter a formal hierarchy. ↑ 90

Here a hierarchy includes a chain of command. The positions on this chain are certain roles, both top and bottom have to behave in a certain way.

Graeber (Graeber, 2007) investigates these roles in what is called joking and avoidance relations. **Joking relations** are mutual, while **avoidance relations** are generally hierarchical, where one party is considered inferior to the other. Joking relations are between people who are excepted to tease and make fun of each other. In avoidance relations, there is often shame expected from the inferior party. The inferior party should avoid the superior party, who can decide whether to start contact. In joking relations, bodily fluids flow freely, the body is not closed off, and sex and excretion is not taboo. In avoidance relations, the self and the body of the superior party is separated from the other and the external world. Nothing can get in or out the body, thus excretion is taboo.

This brings us to the concept of **property**. Property can be used both in the sense of something I own, and as what defines something, what gives something its identity. Graeber argues these two are similar, in that a property set something apart from the world, as an extension to the self. Owning something means it gets separated from the world, is not anymore for anyone to use. Abstract properties of a person distinguishes this person from the rest of the world. It is when relationships started to become focused on exchange, that avoidance relations became generalized.

The difference can also be made on the level of universalization and particularization. In avoidance relationships, the superior party is made more universal, more abstract, for example by using formal titles or family names, while joking relations refer to a particular individual. This can be related to Graeber's discussion of hierarchy.

Two sorts of hierarchy can be distinguished: a **linear hierarchy**, where things are ranked, and there is only one criterion of ranking, and **hierarchies of inclusion**. The latter is about subsets being part of bigger and bigger sets, which are more and more general and abstract. This can be represented as a tree structure, while in a linear hierarchy everything can be put on one line. There is a link between the two: usually in a hierarchy of inclusion, every set has a representative which claims to represent the whole below them. For example, a king is said to represent the country, with under him town rulers who are said to represent the town, under them heads of family said to represent the family. This is the universalization spoken about above: a superior is equated with the whole. We can thus represent this in different classes, and these classes form a linear hierarchy.

But actually, this is a system of exclusion, not inclusion. At every level, the ones at that level set themselves apart from the "undifferentiated mass" below them. This is thus an avoidance relation. The king excludes himself from the general population below them, who cannot get his privileges. The town ruler does the same, undifferentiating between the town dwellers.

↑ 87

In a next chapter, the mathematical difference between the two will be further explored.

One way to see hierarchy is thus when there is linearity, one-dimensionality, when there is only one criterion of ranking. This can give rise to having only one goal at play (since there is only one criterion on which to improve). The hypothesis is that a hierarchical organization works better when there is only one main goal to achieve.

One way one-dimensionality is achieved is when everything is aggregated into one utility measure. Here there is a linear hierarchy in that everything is ranked, but being higher in the ranking is not necessarily associated with having more power or control.

2.1.2.3 One-dimensional utility

In (Clegg et al., 2006, p26), Clegg argues that **utilitarianism** gave rise to a focus on efficiency in work. Utilitarianism is a theory that claims that the best moral action is the one that maximizes overall utility, which is usually related to the well-being of sentient beings (noa, 2017). Frederick William

Taylor brought utilitarianism into work organizations by emphasizing efficiency through the political economy of the body. People did exactly what they were supposed to do, there was no room for uncertainty nor innovation. The worker can here be seen as just an extension to the machine, doing a precise task, while it does not matter what the worker is thinking.

Later, the soul started to be disciplined, rather than just the body. The consent of those that are ruled is given importance. Thus the brain, rather than just the body, is being controlled. This is the knowledge management of contemporary times. This relates to social power, as those that are ruled sustain the power.

↑ 19

Critical Theory questions the usefulness of efficiency as a measure of value. Not everything can be quantified, and they thus focus on qualitative aspects. They will also often stress emancipation, where they want a more just society, rather then just a more efficient one.

There are some drawbacks for reducing all value to a one-dimensional utility measure. In the book *Evolution's Rainbow* (Roughgarden, 2013), Roughgarden argues that biological fitness cannot be measured as one value. Different strategies work in different environments, there is not one solution. There are different kinds of societies possible, and there is **social evolution**: individuals adapt to social rules. Evolution's Rainbow provides a large number of examples of the huge gender and sexual diversity that exist in nature. Roughgarden gives the example of the difference between bonobos and chimpanzees, which are biologically very similar. Bonobos are quite peaceful, they have frequently sex as a social bonding mechanism, females have a lot of influence, and conflicts are resolved through sex.

Chimpanzees, on the other hand, are male-dominated. Aggression and domination play a big role, and there is usually one alpha-male. In some primates, the other men are cast out, and sometimes form homosexual bonds that overthrow the alpha-male. It is thus not that (for example) male dominance is an unavoidable consequence of natural selection, but it evolved in some societies and not in others, while it is impossible to say which of these societies is the "fittest". These traits evolved due to cultural pressure. This relates to the concept of co-evolution: a specie evolves to a certain environment, but it also shapes that environment.

↑ 49

The main criticisms of utility focuses on an explicit, measurable utility, exposing that a lot of utility is not measurable and implicit. The complex, multi-valued world around us cannot be adequately reduced to a one-dimensional utility measure.

2.1.2.4 Authority: a social relationship or a personality trait?

There are two different views on **authority**: either to see it as a characteristic of an individual, or to look at it as a social relationship.

Weber falls under this first view (Ryan, 2005). He considers authority as legitimate domination, and calls this **Herrshaft**. Both authority and domination happen through coercion. Authority is the "probability that certain specific commands (or all commands) will be obeyed by a given group of persons". Domination can become authority through legitimacy, this is when there is consent based on rules and the like shared by ruler and followers.

For Dahrendorf, authority is derived from social positions, rather than the characteristics of individuals. There are two distinct sets of positions: subjection and domination. These are social roles. Authority can thus be seen as a relational form of power, it is exercised by actors in position of leadership. The source of compliance is legitimacy or another form of consent. Authority is distinct from forms of domination based solely on coercion, and influence, where the compliance is obtained by persuasion or argument.

The concept of **legitimacy** can be elaborated (Haugaard, 2012). There are four sources of legitimacy: purposely rational, value rational, affectual and traditional action. Purposely rational entails bureaucratic legal authority, the legitimacy is obtained through giving importance to formal procedures. Value rational is about ideological authority, there is an attraction to certain ideas, while affectual works on the emotions, and is based on charisma: the perceived skills and characteristics of a leader. In traditional authority, the legitimacy is obtained because of historically grown norms, for example when ruling is inherited.

For Hobbes, it is about a **social contract**: in order to overcome our "human nature" (which is considered detrimental), people agree to submit to some authority.

Parsons sees a link between power and money: they can both influence the environment, and are both based on trust. The legitimacy of authoritative power is obtained through trust. However, money can be used universally, while power can only be used for specific purposes. Power can increase through use. If it is used in a way that it is not spent, but increases, it is "legitimate". How power is used thus matters.

On the other hand, Marxists do not directly accept authority as legitimate. They consider the legitimacy of authority as a false consciousness, as hegemony, where there is consensus only as a manifestation of bourgeois control over knowledge.

Historically, in classical Greece, the right to command was based on true knowledge. This was later transformed to the church, where legitimacy was obtained through some claim on God's law. With the Enlightenment, a reversal took place: authority now became associated with the absence of truth and reason. Beck suggests that there is now a return to the classical view, with the concept of expert authority, to which people turn in a risk society.

(Legitimate) authority is only one of **five** possible **sorts of power** discussed in (Wrong, 2005). This power is achieved through a sense of the obligation to obey. Another sort is naked power, the power gained through coercion: imposition of negative sanctions, or the threat with it. Obedience can also be reached through rewards, which is nothing more than positive sanctions. Persuasion is the power relation when one party possesses much greater persuasive abilities than another. The fifth sort of power is manipulation: compliance is than achieved through the concealment of the power holder's intention from the power subject.

Most anarchists also consider authority to be a social relationship. One argument is that the master is as enslaved as the slave, in the sense that both are social roles that have to conform to expectations and obey protocol (Serge, 2009, 1911). Thus, the social structure itself forms a constraint. In the next chapter, it is further developed how a social structure can become a controller that acquires autonomy and strives for its own interests, rather than those of the agent(s) who created it.

↑ 43

This relates to the concept of social power, which considers power as alignment. It is when everyone around is pressuring towards the same that it is difficult to do something else (McClelland, 1994). In section 7.2, a model of this phenomenon will be presented. Social power can, for instance, be social norms you cannot avoid, for example, gender norms. This phenomenon can also be observed in economics, in that one cannot opt out of capitalism—this is what Gelderloos calls the myth conflating consumption with agency (Gelderloos, 2011). Seeing power as social, and not just as a central trait, also points to seeing control as present everywhere and working in different interrelating facets. This ever-present control is not just in the state or capitalism, but also in the family, in school, at work. There is no hierarchy in these different oppressions, but they influence each other mutually. This is an alternate view to marxism, which claims economic oppression is the most important type, and considers the other oppressions only as a consequence of these.

↑ 162

↑ 52

2.1.2.5 Determinism

Marxists thus believe in economical determinism—where the economical ↑ 34 structure determines the social fabric. Determinism relates to a *power-over* ↑ 13 relation, where an objective law can influence society, without being influenced. Here it is assumed that societal evolution could be modeled by objective laws. A contrasting perspective highlights the subjective and the intentionality of people, as opposed to the determinism of the system in which they exist.

'Objective versus subjective' is one of the two dimensions by which different views in social theory are categorized in (Burrell and Morgan, 1979). The other dimension is **radical change versus regulation**.

A preference for either objective or subjective explanations is what separates the viewpoints on intentionality (Clegg, 1989). This division ↑ 13 also relates to the two faces of power (and the three dimensions). The first face is described by behaviorism, which treats social explanations as no different in principle than the explanations of non-social phenomena. There is event causation here: the intentionality of people is left out, and the world is seen as objective. Post-structuralism on the other hand argued that there is no originating source of action, only an endless series of contingencies. It is acknowledged that the intention of people plays a role. Subjectivity is emphasized, as in the structural face of power.

The dimension of regulation versus radical change relates to the elitist versus pluralist debate (Clegg, 1989), which discusses whether there should be elites at all. The emergence of corporate elites, and the separation of the powers of ownership and control (by the emergence of a manager), is observed. Some argue that there are different forms of power, and thus not one center.

Hobbes and Machiavelli are in opposing positions in both dimensions. ↑ 18 Hobbes legislated what power is (focusing on legitimation and the "social contract") (Clegg, 1989). Machiavelli interpreted what power does. The two also differ on whether or not power can be described mechanically. Hobbes investigates the causal, atomistic and mechanical nature of the relations of power, while the Machiavelli has a disinclination to believe in any single, originating and decisive center of power.

2.1.2.6 Structure and function

This section discusses the dualism of function and structure.

According to the **theory of structuration**, social structure is "produced by and [acts] back on the knowledgeable agents who are the subjects of that structure which they 'instantiate' through their constitution of it" (Clegg, 1989, p15). This is about the interplay of how the components form a structure, but are also determined by it. This theory investigates the connections between power and organizational forms, and the underlying structures of power. Power exerts the role of both an enabling and a constraining force.

↑ 49

Giddens (Clegg et al., 2006) considers control over two types of resources: **allocative resources** (control over material things) and **authoritative resources** (control over people). History is the mutual evolution of these two kinds of resources. Control over people thus influences the control over material things, and vice versa.

Marxists, on the other hand, assume that economic circumstances completely determine social structure. Thus, they thus do not consider any acting back of the social structure on the economy.

↑ 34

Clegg (Clegg, 1989, p197) considers three **circuits of power**—episodic, dispositional, and facilitative. The episodic circuit works on the micro level and considers the "irregular exercise of power as agents address feelings, communication, conflict, and resistance in day-to-day interrelations". The dispositional circuit deals with the macro level and is more about "rules of practice and socially constructed meanings that inform member relations and legitimate authority". The facilitative circuit also acts on the macro level, but it is about "technology, environmental contingencies, job design, and networks". This circuit empowers or disempowers, punishes or rewards, but agency is in the episodic circuit.

Chapter 6 presents a simulation that investigates how structure influences function.

↑ 139

2.1.2.7 Example: gender

In this section, I will use gender as an example of how the previous distinctions and definitions can be applied to a social context. I will link discussions in feminist, LGBT and queer studies to some of the concepts presented above. These movements have in common that they defy the existing gender norms, but they differ in the facet they emphasize and the category of people they work with. This also leads to different views on how to act and what the goal is. Although the difference between these movements are at least as big as the similarities.

Feminism focuses on women and criticizes the norms women are socially expected to fulfill, and demands equal opportunity, rights, and treatment for men and women. **LGBT** means lesbian, bi, gay and transgender. These struggles thus focus on so called heteronormativity, which refers to the collection of norms around sexual orientation and gender identity. **Sexual orientation** is about to who you are sexually attracted, while **gender identity** is about which gender you identify as. Anyone who identifies as different from the gender assigned at birth is called **transgender**: this can be as male, female, in between, neither or both genders, or something else. The term trans* is used to include everyone who fits under this broad umbrella. Transman, female-to-male or FTM are terms used for a person who is assigned female at birth, but identifies as male, while people assigned male at birth but with a female identify, are called transwomen, male-to-female or MTF.

Queer is a bit a label that does not want to be a label, thus is difficult to define. A simple definition of queerness could be the defiance of standard sexual orientation and gender identity norms. It tends to emphasize that there is a broader set of possible sexual orientations and gender identities beyond the typical hetero-homo and man-woman duality (Shannon et al., 2013, p13). Queerness, therefore, depends on what these norms are since it is an opposition to them and may evolve as sex and gender norms evolve. For example, in contemporary western culture, homosexuality is quite acceptable. Two gay people who marry, buy a house, and raise children together still fulfill heteronormative social expectations. They are thus less queer than, for example, a woman who has a polyamorous relationship with two men. Rather than an identity, queer can be viewed as a point of view, as wanting to struggle against current sexual and gender norms. In this view, queerness is about destroying current norms, not changing them to accommodate queer identity, or adjusting queer identity to meet current
↑ 10 norms. This relates to seeing freedom as opposition, as the absence of constraints.

This discussion of **identity** is one of the main points of divergence between the three different movements. The feminist and LGBT movement organize based on a shared identity. In contrast, the queer movement rejects the concept of shared identity. But these divisions are also at play within each of these movements.

Critiques on focusing your struggle around identity, is that usually they focus on getting a better position on the social ladder, rather than destroying the ladder. It is about being accepted by this society, rather than destroying the norms. Usually, some people are left behind, because they are so unacceptable for society that their struggle will be for 'later'. For example, in the struggle for gay marriage, transgenders and less traditional gays where often considered disadvantageous for their image. This relates to reformism. ↑ 33

This division can be viewed as an example of whether authority is seen ↑ 18 as a social trait or as central, part of one's personality. Struggles based on identity consider categories of people—one group is the 'oppressed' (usually considered the 'good'/ the in-group), another are the 'oppressors' (usually considered the 'bad'/the out-group). The alternative view considers that there are norms for all groups, which are collectively maintained by all groups (although there might be a group which is more privileged). This is the difference between feminism that emphasizes 'male dominance' or considers men oppressors of women and feminism that fights social expectations for both men and women (although the focus may still be placed on the expectations that impact women more than men).

Another criticism of identity is that it makes abstraction of the complex reality. A person is more than a member of some category, for example, more than just a woman, or a transgender. For instance, a black transman living in the suburbs of some city has little in common with a rich transwoman who is the boss of a big company in a major international city. This helps explain the emergence of the term **'intersectionality'**. Intersectionality attempts to account for how different oppressions are linked and reinforce each other. Intersectionality encourages observers to see different struggles versus one determinative struggle. ↑ 52

Gender can likewise serve as an example for other concepts introduced in previous sections.

Power-to can describe emancipation—people who are no longer bound by ↑ 13 sexual or gender norms, while the patriarchy or strong gender norms exemplify *power-over*. But *power-to* can evolve to *power-over*, in that *power-to* can give rise to privilege (for example male privilege), where some have more opportunities than others. These others feel *power- over*, and can depict the resulting oppression or lack of agency as coming from the privileged group. A natural question then follows regarding how to ensure that an emancipatory struggle does not simply create a new set of norms, where the oppressed become the oppressors, and start to have *power-over*.

Creating a strict binary categorization of gender, versus seeing gender as more diverse, relates to one-dimensionality. In reality, sex and gender can be ↑ 16 categorized in a multitude of ways: by chromosomes, genitalia, or hormone levels (if one looks at biological sex), by some combination of gender norms one fulfills, or by how one defines himself. None of these ways of categorization are completely correlated, and there exists a variety of combinations of them. Yet, common usage collapses all these dimensions into just two categories: ‘male’ and ‘female’—even though these terms are seldom defined consistently.

2.1.3 Conclusion

In this section several definitions related to freedom and hierarchy are given.

↑ 10 I gave three notions of freedom: as an opposition, as self-actualization or as a right to choose. The concept of freedom will be further elaborated ↑ 59 ↑ 154 in section 3.6 and chapter 7.

↑ 15 Further on, I will use the term hierarchy for all concepts related to domination and authority. I will imply the functional conception when using the term hierarchy, unless it is clear from the context that structural aspects ↑ 200 take precedence. Further in this thesis, I will explain how a structural hierarchy, where every element has only one direct influence and influence is one-directional, gives rise to a functional hierarchy.

↑ 15 One of the properties of hierarchy is antisymmetry, which can be understood functionally as a power-over relation. Influence only happens in one direction, and there often is coercion—resulting in an agent doing something he does not want to do. Power-over is different from power-to, which expresses the capacities of an agent. This difference will be used ↑ 154 further on, where I will use the concepts internal and external control. When speaking about *power*, I usually mean the power-over variant, although this can depend on the context.

↑ 18 Authority is seen as legitimate domination, where there are several sources of this legitimacy. Authority is only one of five possible sorts of power. It can also be seen as a social relationship, where power is a social force and not just in an individual.

This difference recurs in the comparison between determinism, where the structure determines the function, and theories focusing on subjectivity and intentionality, where there is an interplay between structure and function. This difference reappears in the different views on how hierarchy has evolved and how we could evolve away from it. How this difference manifests itself throughout the chapter, is illustrated in table 2.1.

2.2 History of hierarchy

In this section, I will present several author's views on how hierarchy and oppression emerged in human evolution. In the next chapter, I will offer, in a more systematic manner, an account of how coordination can evolve into a controller.

↑ 43

This is a very subjective selection of some theories on how hierarchy arose, and by no means an extensive overview of the full history of hierarchy. I present three explanations of how a new configuration emerged, in which there is a certain ranking. This does not mean, however, that there was no prior hierarchy present.

Gelderloos argues that hierarchy arose when there was no opposition to prevent it, and not because of certain economic circumstances. I contrast this with Rifkins' view of how capitalism emerged, which underlines the economic and technological circumstances. This clarifies how certain new economic classes appeared, though other economic classes already existed before. The last subsection focuses on gender oppression, describing how certain genders started to be ranked higher than others.

2.2.1 Gelderloos: rise of hierarchy

In the essay "Rise of Hierarchy" (Gelderloos, 2005), Gelderloos argues that hierarchy arose where there was no organization to prevent it. Technology and agriculture provide a positive enforcement of this hierarchy, but it did not emerge explicitly because of either of them.

There were two kinds of hierarchy in hunter-gatherer communities: patriarchy and gerontocracy. In patriarchal societies, there were monogamous households. Gelderloos argues that patriarchy did not arise because men gained influence as warriors or providers—there is no correlation between the two. Societies that are more war-like, or where the men provide most of the food are not more patriarchal. He gives the example of the Mbuti, a comparatively gender-free society, that has a ritual-game to restore gender tensions. The men and women start at opposite sides in a classical tug-of-war, but as soon as one side is winning, someone from the winning game moves to the other side. At the end, everyone has changed its gender multiple times. This is an example of how a society builds a mechanism to prevent hierarchy from emerging. Further on, I will discuss more on how patriarchy and a gender binary emerged.

↑ 27

Gerontocracy is a hierarchy based on age, existing when there is a segmentary lineage. This is a pecking order of leaders consisting of the fathers. (In that sense it also relates to patriarchy.) For every younger age group, there is the promise of possible inclusion—as one grows older, he can become part of the more privileged group. This leads to the keeping of the status quo because the youth enforces the will of the age group above through policing. In contrast, in non-gerontocratical societies, the youth often plays the role of the rebel, of autonomous defenders of justice, thus changing the status quo.

On the other hand, there were also egalitarian agricultural societies, which is why scholars tend to agree that agriculture did not automatically lead to the emergence of hierarchy. However, agriculture did lead to a positive feedback, making the hierarchy of already non-egalitarian societies more profound and complex. In patriarchal societies there were monogamous households, and this led to the emergence of private property (since there were separate families). With gerontocracy, the status quo was defended, and this led to economic disparities, where the decision-making elite got the fruit of the labor. These older people were not as physically fit anymore to provide food, and thus wanted to ensure their survival by getting more power.

The military advantages of these hierarchical agricultural societies made it spread, with more egalitarian communities more likely to be conquered. However, the advantage does not hold with respect to internal tensions. Gelderloos concludes that domination and accumulation-based civilizations spread not because of any freely chosen assurances of material improvement (in contrast to Hobbes 'social contract'), but because of the military advantages, and the imperative to dominate (egalitarian societies did not want to conquer other societies). Oppressive dynamics thus are not inherent to any material mode humans would choose (as opposed to those forced from the top, like Western-style industrialism). Oppressive hierarchies allow technologies to become oppressive, and technologies define the range of complexity in which these hierarchies can develop.

↑ 18

This relates to the concept of constant opposition as mentioned before, and is opposed to the Marxist idea of historical materialism.

↑ 63 ↑ 12
↑ 34

2.2.2 Rifkin: enclosures

Another view than that of Gelderloos is to see societal changes as being

↑ 25

caused by economic and technological forces, rather than focusing on human agency. This is the idea behind Marx' "base and superstructure" concept. ↑ 34

Rifkin (Rifkin, 2014) follows this logic in his overview of how the enclosures gave rise to the birth of a market economy and capitalism. Before the enclosure movement, in the feudal era, the land was common, and was leased by landlords. Property was a series of trust administered pyramidally and was never exclusively owned. Production was for immediate use and was carried out in communities that were isolated from each other, and most people were illiterate.

With the enclosure movement, communally held land was enclosed, transformed into private property and exchanged in the marketplace. There are two phenomena that undermined the feudal order and triggered this movement. The first was the rising demand of food following the rise of an urban population. The second was the emergence of the textile industry, which increased the price of wool. Raising sheep thus became more lucrative, and land previously used to feed families was now enclosed to raise sheep.

The second wave of enclosures came with the First Industrial Revolution. An expanding urban population encouraged the emergence of a legal system—there now was an anonymous market where strangers exchanged goods and services.

The soft proto-industrial market is where capitalism arose from—it did not directly emerged from feudalism. A combination of print revolution and water and wind power altered the existing power relations. Water and wind mills replaced the labor of many people and were easy to install. They were used in the textile industry and caused the power of urban craftsmen and merchants to expand, matching that of feudal lords. Printing caused a standardization and removed the subjective element. Information could now be transmitted over distances and stored in time. This lead to the development of commercial contracts (demanded due to the anonymous market) and maps for travel.

This provides an explanation for how capitalism emerged. In section 2.3.2, I present how Rifkin believes the economy will develop in the future. I explain the underlying theory and discuss why such technological determinism does not always agree with reality. ↑ 33

2.2.3 Transgender warriors: gender more in depth

In Transgender Warriors (Feinberg, 1997), Leslie Feinberg gives an overview of the history of transgenders, how bigotry and more rigid gender categories emerged, and how they have always been people resisting and transgressing gender boundaries. I will specifically summarize the author's view on how gender distinctions evolved. This as an example of how a specific sort of hierarchy emerged, and shows how different types of hierarchy can evolve together and reinforce each other.

According to Feinberg, there are two simultaneous evolutions that broadened the gap between 'male' and 'female': economically the concentration of wealth, and religiously the evolution to monotheism.

With agriculture came the accumulation of wealth, where it was in general men who were in charge of the stock (while before this was commonly owned). The notion of inheritance sprouted from this evolution, where property was passed along from father to son. This lead to the enrichment of some men over women and the whole tribe, where a few forced the rest to work for them. This describes the origin of slaves, yet there are some mechanisms needed in order for the few to be able to rule over the many. That is why some people were labeled as different, to create an excuse for why these people were not worth a free status. The majority of people were divided so that a minority were still able to exert control. Patriarchy and the devaluation of transgender identities have the same origin: men gained power, women were thus considered inferior and anyone who crossed the created boundary was invalidated and rejected on the basis of being a danger to the created order.

Communal societies in general were matrilineal. Matrilineal means women were the head of so-called gentes, which differ from the nuclear family. When a man married, he moved to the gens of his wife, a living unit, where the wife lived together with other relatives. Thus, the family line was traversed through the female side. However, it is different from patriarchy because society was not really based on rule or dominance (by either gender).

Feinberg borrowed the concept of gens from Engels (Engels, 2004), who discussed the work of Morgan (Morgan, 1877). A gens is understood as a group related through a common female ancestor. Since it was only possible to be sure about who the mother was, lineage happened through the mothers side.

Marriage is understood more broadly than the current meaning, including, for example, group marriage and class marriage (where an entire class is married to an entire other class, or everyone of one class is married to each other). Marriage seems to be sometimes used as simply the social norms existing in a certain culture concerning who is allowed to have sex with who. These norms became more and more restricted to prevent inbreeding, where first entire classes were married, then there was group marriage where groups of sisters were married to groups of (unrelated) brothers. Even remotely related people became excluded until only the pair was practically possible (in which the man was still allowed polygamy, but the woman wasn't, to ensure the knowledge of the father). Both in group marriage and smaller pairings, lineage was still maternal. This evolved to a monogamous family, which was based on male rule, and inheritance happened through the paternal line (property already became important in the pairing family).

Simultaneously, there was a religious evolution. A communal (matrilineal) society in general worshiped many deities. In these societies some sort of "Mother Goddess" was worshiped, and there were often some cross-dressing priests and gods. With patriarchal rule arose the worshiping of only one god. Reactions against cross-dressing were a measure against the worshipping of "Mother Goddess" since the ancient polytheistic religions were a danger for male rulers.

This evolution happened Greece and Rome among other places. From origin, Greece was a communal society, but with the rise of city-states, divisions based on class and gender sharpened, and slaves became common. The attitudes towards trans* people also changed, which can be observed in some of the Greek myths, like the one of Kaineus: a female-to-male hero who was considered 'rival of the Gods' and is buried by the Centaurs. The worshiping of Dionysus is another interesting case. While he emerged as a replacement of the old goddesses, Dionysus was still portrayed as a transgendered, cross-dressing God. Worshipers often cross-dressed, for example, women wore men's clothes and carried large phalluses (the 'ithyphalloi'). While Dionysus was one of the main gods, he was still considered a god of the oppressed, of the marginalized. He symbolizes how the ruling class was not able to completely wipe out the old beliefs.

But in the Roman Empire, repression increased and male and female became legal categories. The worshiping of Dionysus was banned, as was any male effeminacy or same-sex love. There are some reasons for this evolution. One of them was war, wherein they realized that these effeminate men might not want to become soldiers. Another reason was, again, inheritance—there were property-owning males in a heterosexual family. In order to protect male heritage, male and female had to become legal categories.

But there was some resistance against these tendencies. An example is about a famous circus performer in Thessalonica, Greece, who was quite feminine. He was arrested by Butheric, the head of the militia, but the people rose up in protest, and killed Butheric (the authorities then killed 3000 people as a collective punishment).

Despite all the effort of the ruling elite to crush gender traversing, transgender identities and protest continued to exist throughout history. Examples are cross-dressing in festivals that mocked the authorities in the 16th-17th century, "General Ludd's wives"—cross-dressing workers who led a crowd to burn down a factory in 1812—and "Rebecca and her Daughthers"—farmers dressed in women's clothes to destroy toll gates in 1839-1843.

2.3 How to evolve to a less hierarchical system?

This section is again a subjective selection of approaches on how we can evolve to a less hierarchical system. I first discuss anarchist theory, and then contrast this with avenues like Marxist theory that try to change the system from within. But we do not only need grand theories, as change and the difficulties to obtain it, are often very personal. That is why in the last subsection I focus on the more psychological obstacles preventing people from being anti-authoritarian and how these can be overcome.

2.3.1 Anarchist theory

Anarchists hold the ideal as a world without any domination. Though there have always been people resisting authority, the present anarchist movement originated in the 19th century, from an anti-authoritarian current in the early socialist movement. It stems from a split in the First International between Marx and Bakunin: Marx and his followers wanted to conquer the state to get rid of it, while Bakunin and other anarchists thought this would simply lead to a new ruling order and wanted to abolish all states. Eventually, Bakunin and others where expelled from the First International (Marshall, 2009).

Anarchism does not want to be an ideology, and is a resistance against all dogmas. Thus, every anarchist will define anarchism differently, and there are many currents in the anarchist movement. In what follows, I will present some of the general ideas most anarchists agree upon and discuss some of the points where there is disagreement.

Most anarchists agree that aims and methods should be aligned. Thus, one cannot abolish authority by becoming an authority. This leads to a criticism of the Marxist thought of conquering the state to abolish it. Anarchists believe that corruption is an inherent property of power regardless who holds it. The communist states formed in the 20th century more or less confirm this hypothesis.

But the alignment of aims and methods goes further than simply no involvement with the state apparatus. It requires putting ideas into practice, being consistent between words and deeds. It is the opposite of "the end justifies the means." For anarchists, the idea is that an end can only be reached if it is embodied in the means. This is the idea behind prefigurative politics, which seeks to already embody the desired future in the practices of today. Direct action is also about aligning aims and methods. It means directly acting against a certain oppressive practice, instead of, for example asking the government to do something about it.

↑ 39

One of the distinctions which often surfaces is between building alternatives and merely attacking or critiquing existing structure (capitalism, authority,...). Building alternatives is often criticized because domination is so wide-spread in this world that it is difficult to avoid these alternatives being incorporated into the present status-quo. However, others criticize a focus on attacking for lacking constructivism and neglecting important questions about what to do once the present structure is destroyed.

But this sharp distinction is actually a false one (Gelderloos, 2007). Building an alternative means there is already a criticism of present affairs, and when the practice is effective, it will endanger the powers that be. Only when there is an explicit rejection of something, can it be avoided to get back into this (seen as a reason why hierarchy could rise). The organization of our attacks envision how to organize differently, and a revolt can create the space for realizing an alternative way of living. The strength of a certain practice is in the combination of building alternatives and attacking.

↑ 25

These two points imply that everything is intertwined, which causes some overlap between the sections of this chapter. "What is wrong", "how the wrong emerged", "what we want" and "how we want it", are inseparable questions. Because aims and methods should be aligned, "what we want" and "how we want it", are the same thing. Because the distinction between building alternatives and attacking is a false one, "what we want" is simply the opposition of "what is wrong" ("what we do not want"). "How the wrong emerged" is embedded in "what is wrong".

↑ 59 One of the primary values anarchists embrace, is autonomy, meaning that an individual can accomplish his goals himself, and is not dependent ↑ 33 on an outside structure. That is why paternalism or reformism is often seen as just the other side of the same coin as prison or repression. Paternalistic actions, such as the church giving free meals to the homeless, make these people more dependent on these organizations, and does nothing to increase their autonomy. That is why anarchists believe reforms, for example, robust social programs will not solve the problems of society at its roots. Social programs make recipients dependent on the state, and a small reform can incapacitate a protest movement—"do not bite the hand that feeds you". ↑ 19 This relates to how obedience can be achieved through positive sanctions.

Most anarchists do not consider conflict as necessarily bad—diversity and fragmentiveness can be values (Graeber, 2007). There is, however, a split between anarchists on the merits of collectivism. There are two main currents in the anarchist movement: the more collective anarcho-syndicalists, anarcho-communists and platformists, and the more individualistic insurrectionalists and anarcho-individualists.

The collective current in general wants to implement an often formal organization—a platform, a syndicate. They usually focus on the economic realm, which sometimes leads to strange contradictions: one of the first female ministers, for example, was a self-proclaimed anarchist (Marshall, 2009).

Anarcho-individualists not only act against external authorities (like the state or capital), but also warn about authority in our own structures. They are cautious about the possibility of a (formal) organization leading society and undermining individual agency. A modern (related) current is that of ↑ 58 insurrectionalism. Here the idea is to try to create an insurrectionary climate, which could eventually lead to a revolution that abolishes domination. People ↑ 46 organize based on **affinity**: a shared view on how to organize, what to do, and what is important to act against.

One of the differences between the two main currents is that the collec- ↑ 70 tivists still want to make a collective decision, which then everybody acts according to (the decision is, however, not made anymore by some authority or democratic majority, but by consensus). In anarcho-inidvidualism, every individual acts by herself, which can coincide with other acts.

But Gelderloos (Gelderloos, 2007) argues that in both currents, there is still an inclination to what he calls "Risk Board mentality". This is "the assumption that contact between people who are different must result in a missionary relationship, with one converting the other", yet there can be a mutual influence. Anarcho-syndicalists thus tend to take this missionary position, while anarcho-insurrectionalists tend to avoid all contact and simply act by themselves. (As a generalization, most insurrectionalists for example, do search lots of social contact by, for instance, spreading texts in the streets to inspire people for rebellious acts.) I will develop this difference between mutual influence and converting more in my model of internal and external control. ↑ 154

Anarchy Works (Gelderloos, 2010) gives several examples and methods of how people organize in an anti-authoritative way in all aspects of life, without being ignorant to the imperfections.
The next chapter further explains anarchistic principles of organization, in particular, through concepts of complex systems theory. ↑ 38

2.3.2 Within the system

Marxist theory has another view on which methods to use to move away from domination and exploitation. These can be summarized in two related concepts: the “dictatorship of the proletariat” and “historical materialism”. I will elaborate these concepts here and discuss some of the shortcomings of changing the system from within—a method also used by non-Marxists.

The **dictatorship of the proletariat** (Marx and Engels, 2002) describes the notion that there should be an intermediate phase, where the working class takes control over political power, to eventually abolish the state. In the past section, I discussed the anarchist criticism that the state structure an sich is the problem, and that a state would never abolish itself. ↑ 31

A less abrupt version of this principle is **reformism**. This encourages pressure for reforms of laws to make them less unjust. This pressure can happen by gaining political power or by pressuring politicians with, for example, petitions or lobbies. This is the preferred method of social-democrats. The criticism of this approach is that it only treats the symptom and not the structural cause. It does not change the system. The danger here is that the bigger picture is lost. When people put so much effort into changing a specific practice, they may not see how things become worse in other domains, or how different oppressions are linked. Moreover, it usually requires a lot of energy to make even small changes in this manner.

Historical materialism (Marx and Engels, 1970) is the theory that the material conditions, the mode of production (the “base”), determines the social relations, political structures and culture (the “superstructure”) of a society. When this influence is considered uni-directional, this leads to a state of economic determinism. The concept was first developed by Marx, but the idea behind it is still used today to predict social changes based on recent technological evolutions. I've made reference to this theory before, and will do so further in this thesis. ↑ 20 20 25 48 52

↑ 26 Rifkin argues that the decentralized nature of the internet (and other technologies), would lead to a re-emergence of the commons, and fundamentally change the economic and social structures (Rifkin, 2014). The idea is that costs of products will decrease until they become nearly free, and thus become common goods.

This argument sounds surprisingly similar to the argument of the anarchist thinker Kropotkin (Kropotkin, 1906) in the 19th century. He argued that the decentralized nature of electricity would lead to more local production, where before there had to be one central steam engine, which worked better the bigger the factory was. In the present era of multinationals, this prediction sounds overly optimistic, which causes skepticism for similar arguments today.

Another view is that society thoroughly influences the technological changes, and economic evolution, thus, does not act as simply a physical force. As Bookchin (Bookchin, 1982) states: "it is neither technical change nor Marx's "production relations" that changed society, but rather an immanent dialectic within given societies themselves, where organized coercion was not directly involved". ↑ 25 Gelderloos also argues that hierarchy did not emerge solely because of economic or technological factors. An example that affirms this view is that the steam engine was actually already invented in Greece in the first century, but it was at that time just a nice gadget to impress guests at a party by automatically opening a door (Bookchin, 1994). Nobody thought about using it to replace labor, since there were, at that time anyway, enough slaves to do the work.

What both concepts have in common, is that the principle is to change the system *from within*: either by trying to change the government, or by changing technology. The rapper Immortal Technique summarizes nicely the pitfall of this approach: "The problem [...] when you try to change the system from within, it's not you who changes the system. It's the system that will eventually change you." (from the song The Poverty of Philosophy).

2.3.3 Mental struggle

All of these theories presented so far neglect psychological aspects, which play a big role. In this section, I will discuss some of the psychological barriers causing people not to act, even if they agree in principle with the goal. This will be exemplified by straight edge, an approach that helped me personally in dealing with psychological obstacles.

I think a big part of the problem is **addiction**, but I consider this more broadly than drug addiction. Emotions serve as a sort of compass: they tell us when we want a certain direction (giving positive emotions), and when we don't (resulting in negative emotions). Often, however, we start to consume emotions, searching for these emotions without them being linked anymore to benefits. The emotional compass starts to drift, and is no longer pointing you in the right direction.

The emotions can moreover be manipulated by an external force, who provokes certain emotions knowing that it leads to certain acts that are wanted by this external agent, but which is not necessarily good for the agent performing the acts.

Straight edge is a subculture within hardcore punk of people who refrain from recreational drugs, including alcohol and tobacco (Kuhn, 2010). I understand it more broadly as trying to avoid addiction, understood as doing behavior which you cannot stop doing, but which is actually not good for you, and not really what you want.

With this definition, we could say that a lot of people are addicted to the existing social system, in the sense that they do not really like it, but cannot get out and end up maintaining it. In a sense, everybody maintains the status-quo, in that people often want a better position on the ladder, and those on top want to maintain their position, thus no one tries to destroy the ladder.

There are some benefits from the status-quo and living a 'stable life'. It requires a lot of energy to fight the status-quo, and it often comes with negative consequences. For example, with state repression. An *idée fixe* is attractable, as one of its properties: once you have it, it is difficult to quit. But it is still often unwanted. ↑ 43

No one wants to completely give up control over his own life, but there is also no one who can be completely autonomous and never make compromises. Thus, everybody tries to balance between autonomy and comfort, but with different thresholds, which also depend on the situation.

But there is a difference between falling for an ideology and being coherent in certain values throughout life. With the former, an idea is followed without questioning, even if it no longer agrees with some of ones values. On the other extreme one could become nihilist, having no values since they could all rule you. Here one simply goes with the flow, without any chosen direction or goal, thus serving the dominant idea.

A balance between the two thus seems to be required for autonomy,
although another dimension might be at play. This search for balance
↑ 32 is present in the conflict between the more individualist and collectivist
currents in anarchism. As such, anarchism might seem paradoxical, being an anti-dogmatic dogma. But I think most anarchists are constantly critical to their ideas, without neglecting their values. This tension will probably always be present.

Within anarchist movements, there is also the constant tension of staying anti-authoritarian. Often people disappear from the movement due to lack of energy because of state repression or internal dynamics. Personal conflicts can also disrupt the movement. For example, some people can be criticized for taking too much power while they may simply feel like they are doing a lot because no one else is doing it. There will always be people doing more than others. The trick is to empower people to do the things they want to, and let no-one feel obliged to do more than they want to.

2.4 Conclusion

↑ 10 ↑ 25 In this chapter, I discussed the different views on how to see hierarchy, how
↑ 30 ↑ 18 it evolved and how we can move away from it. Authority can be seen as
↑ 20 deterministic, as a natural law. In this view hierarchy evolved because of
↑ 26 technological or economical circumstances, and we will rid ourselves of it
↑ 33 ↑ 20 when these circumstances change. The other view emphasizes intentionality,
how the choices of people can have an influence. Instead of considering the
↑ 34 economy as determining everything, there are different related struggles and
↑ 13 oppressions that reinforce each other. Power is seen as social, and not just a
characteristic of some individuals.

Table 2.1 shows how this difference is manifested throughout this chapter. The first column shows the domain, while the second column is contrasted with the third. The second and third column are not complete opposites though, there is for example also asymmetry in the structural face of power or when authority is a social relationship.

↑ 30 This difference is exemplified by the difference between anarchist and
↑ 33 ↑ 38 Marxist thought (although the correspondence is not perfect). In the next
chapter, I will further discuss some anarchist methods, and put these into a
complex systems framework.

2.1.2.4 Authority ↪ legitimacy	personality trait	social relationship
a sort of power	objective 2.1.2.5 determinism event causation	subjective intention no single originating center
2.1.2 Two faces of power Lukes' 3 dimensions 2.1.2.1 Power	(covered by) behaviorism decision-making power Power over	structural face non-decision making ideological power Power to
Hierarchy	2.1.2.2 antisymmetry avoidance ranking 2.1.2.3 one-dimensionality (e.g. in utilitarianism)	joking
↪ 2.1.2.6 structural and functional	structure determines function	interplay between two
2.2 History of hierarchy	2.2.2 by economic and technological forces	2.2.1 when no opposition
2.3 How to evolve	Marxism 2.3.2 within the system	2.3.1 anarchism
2.1.1 Freedom	a spontaneously developed order	a decision

Table 2.1: Difference between a deterministic and two-directional view throughout this chapter.

Chapter 3

Understanding anarchist ideas through complex systems concepts

In this chapter, I apply concepts from complex systems theory to anarchist ideas. The purpose is two-directional. On the one hand, the intent is to explain some general principles in systems and offer anarchist practices as merely an explanatory example. On the other hand, I want to provide a better view of how anarchists organize themselves and which methods they use to create a free world, and I will use concepts from complex systems theory to explain them.

Although there are some differences between how complexity science and systems theory is practiced, the general principles are very similar. Sometimes the one is considered a part of the other, in both directions. I will use both terms or "complex systems theory" without distinction, focusing on the ideas rather than getting into an etymological discussion.

Systems theory tries to distill some general mechanisms that happen in systems regardless of whether the manifestation is biological, ecological, social, or something else. These are always simplifications of reality, but I hope to extract some useful perspectives from it.

I consider as a **system** a "whole", a collection of components connected through relations, where the interactions within the system are more dense than with the rest. This is often an arbitrary distinction, where we draw some boundary between what we consider the "system" and the "**environment**" (the unspecified everything else). It is we who give a collection some identity.

An **agent** is a system that has agency, i.e. it has its own goals and acts to reach these goals. Examples are human beings, bacteria, social systems and autonomous robots.

A system is seldom completely isolated. It interacts with the environment. These interactions can be split into inputs and outputs. The system receives certain information and resources from the environment, and also disseminates certain information and material. When an output is generated so as to bring the input closer to a goal state, we say there is a **cybernetic loop**.

3.1 Self-organization

In (Heylighen, 2011a), what is meant by self-organization and coordination is investigated. The concepts used in this section are borrowed from this paper. 2014b

An **organization** is defined as a structure with a function. This means the structure as a whole aims at a global goal or a global pattern. It is not necessary that the goal is clear from the beginning, but there should emerge some general direction. Now, the difference between self-organization and hierarchical organization can be defined on the level of the functionality.

Self-organization is when local elements interact in such a way that they exhibit some global behavior—they coordinate. This global activity, the function, thus arises spontaneously through local interactions (Heylighen, 2014a). The common goal is thereby set by the collective. In a **hierarchical organization**, in contrast, the structure and function is decided from above, by one or a few agents who determine the common goal.

When anarchists speak about self-organization, they speak specifically about organization between people, about social organization. Organization of this type occurs when people organize directly between themselves to meet their needs, without any mediation of the state or another outside agent. It is related to the prefigurative method of direct action—this is when people ↑ 52 directly act to change something, instead of asking the government, the media or other people to do something about it. For example, when people squat a building to organize community activities, they directly give a solution to empty houses and homelessness, instead of asking the government to house people. They organize between themselves to meet their needs, for example, by organizing popular kitchens, give-away shops or bike workshops.

3.1.1 Guided self-organization?

Does self-organization mean the same in anarchism and systems theory? In anarchism, it's about an organization emerging purely out of local interactions. But systems science, like science in general, often wants to predict and control a system, so terms like '**guided self-organization**' and 'guided evolution' emerged (Banathy, 2013). This is when someone does assign an overall goal to a system, and experiments to see which local rules give the desired behavior, so that the most lucrative rules can then be assigned to the system. Thus, guided self-organization is not compatible anymore to the anarchist idea of self-organization, where there is no mediation from the outside.

However, anarchists are usually neither for some "laissez-faire" strategy, where one simply lets the world spontaneously organize in whatever direction, without having any control over its environment or life (since one's environment confines much about one's life, as further argued in section 3.4.1). Passamani (Passamani, 2010), in this respect, argues that hierarchy and the state might as well have arisen spontaneously, and that freedom is thus not a spontaneously developed order, but a decision. And that's exactly the charm of freedom, that it's not a determined condition, but something an individual chooses (see also the discussion about freedom in section 2.1.1). This is also what Gelderloos says in "Rise of Hierarchy" (Gelderloos, 2005) (see section 2.2.1): that it is only in those societies where people consciously chose to oppose hierarchy, where hierarchy did not emerge (I will explain this mechanism of constant opposition more later on).

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↑ 25
↑ 63

Anarchists do want to intervene in the world. Insurrectionalists, for example try to intervene in existing struggles to give some more oxygen to them so that they might possibly evolve into insurrections. Gelderloos also explores this tension in (Gelderloos, 2007), in what he calls the risk board mentality: the belief that contact between people who are different must result in a missionary relationship, with one converting the other (while there could actually also be a mutual influence). The theory I developed about internal and external control also explores how one can have control over one's life without controlling others.

↑ 32
↑ 32
↑ 154

The idea is to create the environment that helps people to develop and enables them. But there are two different perspectives on how to do this (Busseniers, 2016). The first is to start from yourself, constructing the world you would like to live in. The second is to start from the other, constructing a world where an assumed better behavior is more easily achieved. Libertarian paternalism (Thaler and Sunstein, 2003) fits the latter category. The idea here is to ‘nudge’ people into ‘good’ behavior. This distinction is similar to what is described in section 3.7.2. One could try to reach one global view, one global decision, or one could see the world as a diverse amalgam, where many ways of living are possible.

↑ 70

There can be self-guidance, where an agent guides itself, and is not just passively drifted by external forces. This is one of the essences of autonomy and of living systems. But this self-guidance is something else than the urge to guide others.

↑ 59

3.1.2 Coordination

2014b

To be able to reach a goal, some coordination between the different agents involved can help. **Coordination** is defined as the structuring of actions to minimize friction and maximize synergy (Heylighen, 2011a). **Friction** is when actions hinder or oppose each other, while there is synergy if these actions reinforce each other. There are four processes which can help with that: alignment, division of labor, workflow and aggregation. In the following paragraph, I will define these concepts and explain how they can also work in self-organization. Further on in this thesis, I will do some simulations based on these concepts.

↑ 139

If different agents aim at the same target to avoid friction, we speak about **alignment**. The direction of an agent accords with the target it is aiming for. In a hierarchical organization, this is achieved by all agents adapting towards one leader or preset goal. In self-organization, agents adapt towards their neighbors, so that a common direction is achieved. This adaptation usually happens by variation and selection. That is, an agent varies a bit in its direction, and the best direction—the one with the least friction with its neighbors—is selected. An example is magnetization: molecules will locally align the direction of their magnetic field to that of their neighbors until all molecules point in the same direction, thus creating a global magnetic field that has a power that none of the individual molecules, nor the molecules all pointing in a different direction, could have.

↑ 54

But it is not always necessary to align to the same target to avoid friction. In fact, sometimes friction is even avoided by aligning to a different target. For example, in traffic or a crowd, it is better to spread over different lanes or roads so that everyone is less hindered. If there are two animals together at an equal distance to two equal food resources, it is better for both if they go to a different food resource.

Synergy is defined as the surplus gained by working together. A task which couldn't be fulfilled by one individual, can be completed by the work of different individuals together. To maximize synergy, first, the initial task is divided into different sub-tasks. Different agents perform different tasks, which is called **division of labor**. An end product of one work is used for another work, which is called **workflow**. Finally, everything needs to be put together. We call this **aggregation**. This isn't as linear as it looks. At every step in the process it can happen that a task is divided into sub tasks or aggregated with other tasks.

At first sight, it may seem that hierarchical organization is needed for this: there needs to be one agent who divides the tasks, who has an overview. But these things can also work by self-organization with simple rules. For division of labor and workflow, any time an agent doesn't have anything to do, he picks up a task he is most skilled at. An example of this is the evolution of different species. A species chooses a niche in which it further develops. For aggregation, there are two possibilities: a shared medium or the interaction of the products of different activities. An example of a shared medium is the earth with ant pheromones. Some ants put pheromones on the earth, other ants can pick up this trace and follow it. An example of the products of different activities that interact, is the ecosystem. The output of several species is used as a resource or a service for another species.

The principles behind this are, again, variation and selection. For division of labor, it works as follows: In the beginning, there is already some variation among the agents. This means that some are more skilled for a certain task, so they will select it. By doing the task, they will become even better in it. This is how certain species become extremely good in a certain niche. In the case in which aggregation happens by different activities that interact, in the beginning, there will be some random interactions. Then, the best of these interactions are selected.

In human society, we see this in the difference in functioning between a classic firm and the open source community. In a classic firm, there is a boss deciding who does what, dividing the labor between workers, typically encouraging each one to specialize. Open source communities often work with an issue queue where developers can post tasks that still have to be done, which other people take up based on their skills and what they think is important. This way projects can be effectively organized without any need for central control.

In general, this works via stigmergy, which will be explained in section 3.4.2.

↑ 53

3.2 The formation of a controller

2017

Coordination can, however, evolve to a state where it does not benefit the individual agents anymore. A system could start to live "its own life" so to speak, wherein it strives for its own survival, instead of that of the agent(s) who created it. Stirner (Stirner, 1907) describes this process on several levels. In the individual mind, first you have the creative process where ideas originate. Then this transforms into an ***idée fixe***, a dogma where the person starts to live to serve the dogma, instead of the idea serving the person. This *idée fixe* can be religion, money, humanism,... The same mechanism happens on the societal level: first people start to cooperate because doing so makes them all better off. A society is created from this cooperation. But then rigidity comes into play. This social mode (for example, a state) becomes a higher value, to which the people constituting it are subordinate. The goal of the system thus stops being aligned with that of the agent(s). This can be seen in an organization for the sake of it, that only exists to preserve itself, and doesn't fulfill any of the goals of the people constituting it anymore.

Victor Serge (Serge, 1911, 2009) also describes this process, speaking about a society or a crowd that "has a mentality, a life, a destiny distinct from the individuals that compose it", and that tends to maintain itself.

One of the main points of Stirner, is that an individual should follow its own desires, and not be lead by a goal outside of herself.

3.2.1 Meta-system transition

Heylighen (Heylighen, 2006) explains the process by which a controller is formed in several steps. First, a collective forms a medium, a support for carrying interactions. These interactions begin to get coordinated, the medium becomes a mediator. This is when agents commence to coordinate to minimize friction and maximize synergy, as described in the previous section. Finally, this mediator evolves into a manager. Instead of passively mediating actions of the agents, it starts to actively initiate and control such actions. This is when the system begins to have its own goals since a control function arises. Thus, there emerges a higher-order control mechanism, the meta-system, that starts to have goals on its own. This mechanism is called a **meta-system transition**. First these goals might be aligned with those of the constituting agents, but usually they separate from each other, as the main goal of a system is generally to preserve itself. So then it must be asked whether the goals of the system can be in the best interest of the agents.

↑ 41

3.2.2 From exploiter to cultivator

Heylighen (Heylighen, 2006) and Stewart (Stewart, 2014) address the above question by noting and describing the possible evolution from exploiter to cultivator. An **exploiter** is an agent that gets most of the benefits, being in an asymmetric relationship with others who mostly get the losses. An exploiter that is too successful will weaken, and eventually kill, the exploited, and thus endanger its own survival. This is why exploiters tend to evolve into **cultivators**—they become more benign, thus being able to harvest an ongoing stream of benefits from those they control. However, there is still an asymmetrical relationship between the cultivator and the cultivated. While the cultivator will let the cultivated survive as long as they act according to its interest, it won't enable them to grow and develop, to live. It is only interested in the aspects of the agents that give the cultivator benefits, and does not care about the rest. Both an exploiter and a cultivator thus have influence without being influenced. This influence could be only negative (the case of an exploiter), or positive (as with a cultivator).

An example of the emergence of a cultivator is the rise of the welfare state. First, there were factories that exploited workers by providing horrible working conditions. The workers could not accept this and started to protest against this in various ways, such as strikes, sabotage and demonstrations. Upon beginning to see this protest as a legitimate threat to its survival, the state initiated reforms meant to mitigate that threat, such as voting rights and social programs. In doing so, the state transformed into a cultivator, becoming more benign in order to maintain stability of the system it controlled. However, the fundamentals of the system were not really changed— people still were not able to form the society they wanted to live in, and they still had to work in factories for little (although a bit more) money, while others made fortunes simply because they owned the factories.

This often gives rise to behavior that is actually not in the best interest ↑ 35 of the individual. This could be one definition of addiction, wherein a person continues to perform a certain behavior because it was beneficial in the past or in the short term, but it is not anymore, or not in the long term. This relates to the concept of **supernormal stimuli** (Barrett, 2010). These are stimuli that are present in an exaggerated amount, so that they often elicit an exaggerated or inappropriate reaction. An example is sugar, which was rare in prehistoric times and highly valuable, but is now available in such gargantuan amounts that it has become detrimental to human health. Supernormal stimuli can be viewed more largely than as simple abundant resources. One could, for example, say that people are addicted to contemporary society, in the sense that it alienates them and is not really the best for them, but still they maintain it because they do not know how to do differently. Society could be seen as a golden cage—it gives us wealth, but we are actually imprisoned by it.

One of the main characteristics of the situation described is that there is some **dependency**. The agents usually need this bigger structure to survive. Examples are the cells in a human body, or humans in society (most humans will not survive anymore in the jungle). But it can also be another goal than survival that cannot be reached anymore without this structure.

This dependency also manifests itself in an asymmetry of influence—the bigger structure can influence the agents, but the agents that are constituted in it cannot influence the bigger structure. This is why this structure can be rigid and maladapted to the agents on its own. The agent loses its autonomy, since it can no longer accomplish its goals itself, but depends on the bigger structure to provide its needs. ↑ 59

Whether one considers this dependency problematic is dependent of one's value system. In some cases and for some agents, a loss of autonomy might increase survival. I personally value autonomy, and thus consider dependency to be problematic. I can give some arguments for this (as done before), but in the end there is no accounting for taste. That is why I am mainly focusing on how this rigid structure can emerge, and how it can be avoided, rather than trying to prove why this rigid structure is indeed problematic.

3.2.3 Autopoiesis

Luhmann's theory (MOELLER, 2012) also states that humans are not really part of the social system in the sense that they are interchangeable. The social system will maintain itself, it is an autopoietic system. **Autopoiesis** is when a system can reproduce and maintain itself—it can produce the resources it needs (Maturana and Varela, 1991). Chemical organization theory(COT) defines an **organization** (which can be viewed as an autopoietic system) mathematically as a system that is self-maintaining and closed (Dittrich and Fenizio, 2007). Self-maintaining is here understood as when everything that gets consumed is at least as much produced, and closed means that everything that is produced is part of the system. But the things that get consumed or produced do not need to be material objects. Chemical organization theory in general considers reactions, where some instances get ↑ 136 transformed into other instances. The input of the reactions is said to be consumed, while the result is called produced. These instances can, for example, also be ideas or social norms.

A social system can thus be considered autopoietic in the sense that it reproduces itself, there are, for example, some social norms or expectations that create other expectations, which in the end produces the expectations from the start. There is thus a social system which one simply feels subject to, without having the ability to influence it. Humans are simply means of transport for communications of the social system, but they do not play a vital role in it. Lenartowicz (Lenartowicz, 2016) applies Luhmann to interpret social systems as intelligent, evolving 'semio-creatures'.

According to Luhmann (MOELLER, 2012), society has changed from stratified differentiation to functional differentiation, with function systems that are autonomous. This means that in the past differentiation happened through physical components, while today's society is split into functional systems. Later, I will explain the idea of functional systems more using the terminology of aspect systems.

↑ 47

3.2.4 Avoiding a rigid structure

How anarchists try to avoid this system rigidity, is to avoid too permanent of organizations. The focus then is on informal organizations, which form to achieve a certain goal, and dissolve once this goal is reached or is no longer desired by the people forming the organization. Thus, organizations are constantly evolving, and the dissolution of an organization isn't necessarily considered bad. Organizations of this type are based on **affinity**, certain shared desires and views on how to struggle (Bonanno, 1998). This is similar to Stirner's concept of **"Union of egoists"** (Stirner, 1907). Egoists work together with other individuals because together they can achieve more than alone, but they are never being led by this union (then it would no longer be a union of egoists). It is a dynamic constellation, where the relations between the egoists is constantly renewed. If the union starts to cause suffering for one of its members, it has dissolved into something else.

Thus we are searching for ways of coordination without evolution toward control. In section 3.4, I will discuss several principles of coordination, but first I want to discuss more about centrality and decentrality.

↑ 52

3.3 (De)centrality

We could, in fact, differentiate three configurations of influence in a system: 2017

- the ‘dictator’: one (or few) agents can influence the bigger structure, the other agents have no influence.
- ‘not-my-meta-system’: none of the individual agents have any influence on the bigger structure. Though the structure emerges out of these individual agents, they are components of the system, but they are interchangeable.
- ‘shared world’: every agent can partly shape the world around him, where and how he wants to live, everyone has influence.

A lot of systems, most democratic countries, for example, are in the second configuration. This is pretty difficult for a lot of people to grasp because there is not a clear structure ruling over another structure.

3.3.1 Aspects

The concept of aspect system (Heylighen, 2006) can bring some clarity to the matter. An **aspect system** is a subset of the set of relations, interactions and properties that characterize the structural components of a system. The idea is thus to distinguish on the basis of function instead of structure. It is therefore important to note that the system that emerges out of local interactions is often not some external agent or well-defined body, but more an aspect system of the whole system (although it has distinguishable attributes). Often people will search for a small group of people responsible for a particular situation in the world (the first configuration). They do not see that the problem lies in how society is configured, in that the individual agents are interchangeable. Probably, even if these people were to get into power, the situation would remain the same. ↑ 47

A lot of conspiracy theories fall into this trap. For example, claiming that the world is ruled by reptilian-like aliens that steal our gold because they live from it on their home planet. It is easy to just laugh such people away, but often they do have the correct observation that the world is beyond their control and seems to have its own goal(s). Yet they clearly have a wrong understanding of how power works, and always try to explain things with a single, central cause. Thus, they assume there needs to be one central power and do not understand that everyone, in a sense, contributes to the

↑ 19
↑ 47

functioning of the system, that power is often social (see also the previous chapter). They mistake the second configuration (‘not-my-meta-system’) for the first (‘the dictator’).

↑ 43

In the past, power was associated with some central control, a hierarchy, like a king ruling the population. Nowadays, power isn’t that easy to locate in a specific person or group. We have all incorporated and contribute to the power mechanisms,—“the cop has entered into our heads”. This is also related to the discussion in the previous section, that society as a whole somehow rules us.

Traditionally, we tend to look at the world as different objects and persons, all having different properties. I.e. we make distinctions on the structural level, looking at subsystems. We can, however, also make the distinction on a functional level. Doing so results in what we call aspect systems. For example, we can look at two points as being two subsystems with respective coordinates (x_1, y_1) and (x_2, y_2). Or we could look at these points as existing out of two aspect systems: the x-coordinates (x_1, x_2) and the y-coordinates (y_1, y_2). If now these two points are bounded so that the distance between them remains the same, we can also represent them by the aspects of the middle between the two points, (x_m, y_m), and the angle, α, formed between the x-axes and the line through the two points. Because of this extra condition (the bounding between the two points), the system can thus be best represented (as in needing the least parameters) through aspect systems.

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In society, different aspect systems are the cultural, political and economic systems. Because of all the complex conditions and interactions in this world, it sometimes makes more sense to look at those systems, than only at the direct micro interactions between people (which doesn’t help us to understand the larger world). It thus depends on how we represent things, whether we will see centrality or not. There might not be one person dominating us, but there might be one aspect system dominating us. For example, having only one technological medium that forms how we function. Although we might work decentralized on the platform, there is still a centralized platform in place. One aspect system can also dominate another, which happens, for example, with economic determination. This also sheds some new light on the previous section, where society, an *idée fixe*, or an organization is an aspect system that dominates or controls us.

3.3.2 Co-evolution

Whether it is between aspect systems or structural systems, usually there is a mutual influence (in its simplest form) between two systems, though sometimes this influence only goes one direction.

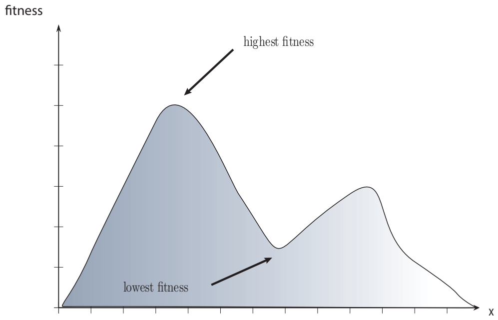

Figure 3.1: An example of a fitness landscape.

The standard model of evolution assumes an agent adapts to a fixed environment. The concept of **co-evolution** encompasses that usually an agent also shapes its environment (as described by niche construction (Laland et al., 2001)).
This can be described by the concept of **fitness landscape** (see figure 3.1). The idea is to have a fitness measure for every state that can be visited, and visualize this as a graph to see which state has the highest fitness (these states can be physical states, as the place in space, but it is in general just all the possible parameter values). Of course, in reality, fitness cannot be put simply in one clear measure, and all the possible states are unknown. But in computer simulations fitness can be used to find the optimal solution for a given problem.

The traditional view assumes this fitness landscape is fixed and an agent simply moves through it to find the highest peak, where the fitness is the highest (or the lowest point in some representations). With co-evolution, the fitness landscape changes as an agent moves through it and acts in it. You can view this, for example, as a swamp-like fitness landscape, where whenever you go to a certain place, you sink deeper, and the fitness function decreases (this is a negative example, often the fitness will increase). Figure 3.2 illustrates this idea. Thus, the fitness landscape depends on the path the agent has walked.

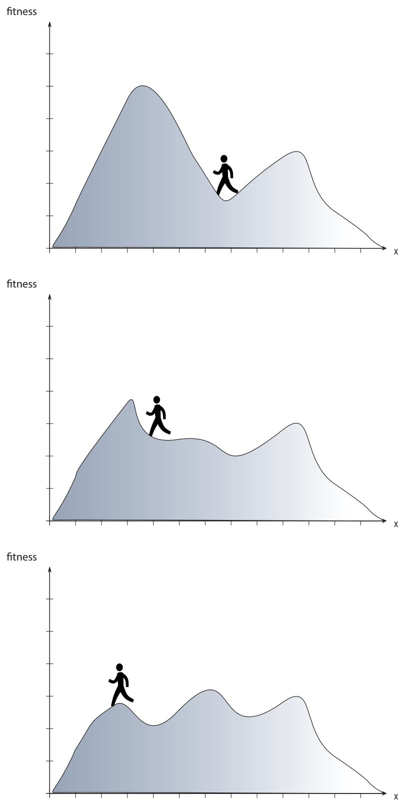

Figure 3.2: Co-evolution in a swamp-like fitness landscape.

Such interplay between agent and environment can also take place between different aspects: between ‘nature and culture’, ‘social and infrastructure’, ‘function and structure’, ‘society and technology’, ‘decisions and actions’, ‘theory and practice’, ‘micro and macro’, and so on. These specific ↑ 69 ↑ 52 cases will be further explained in section 3.7, section 3.4.1, and were already ↑ 20 touched on in the first chapter.

Influence between two such aspects could happen in either only one direction, or be two-directional. Influence without being influenced was one ↑ 13 definition of power (over).

But usually there are more than two systems that influence each other. This of course depends on where the boundaries are drawn, and thus which collections are considered ‘systems’. Often one only considers an agent and its environment. But in this environment, a lot is going on and it also includes other agents. It could make more sense to treat these agents as separate systems. An environment with different agents means the fitness landscape will change, even when the agent remains static.

3.3.3 Networks

Another way of investigating centrality involves observing a lot of agents that influence each other to see which agent has the most influence. This can be done by looking at a network. Networks are composed of points (called nodes) and links in between them. Investigating this structure provides insight into how and where power and influence might be concentrated in webs of connected agents. In the next chapter, I will define these concepts more formally. ↑ 107

Of course, what you represent with these points and links, will strongly influence what you observe (see the previous discussion on centrality's dependence on how we represent things). ↑ 48 We can look at the number of connections a node has with other nodes, and call this the **degree**. Furthermore the **cluster coefficient** is a measure of how well connected neighboring nodes are (where a node is a neighbor if there is a direct link). We can then consider a network hierarchical if nodes with a high degree have a low cluster coefficient, while nodes with a low degree have a high cluster coefficient. This means that for a node with a lot of links (lets call this a leader), its neighbors aren't well connected in between each other, and often will have to pass by this leader to reach each other. On the other hand, for a node with few connections, these neighbors will be well-linked, often being in the same cluster, and that node is often dispensable.

Some behavior we see in hierarchical networks, can be explained from this perspective. In general, a hierarchical network gives rise to several clusters being only connected through a leader. This allows for the divide and conquer strategy of setting different groups against each other. This explains why taking away the top, for example, through a revolution, often gives rise to a civil war. The tops of these different groups will fight so as to become the new top, because that's the only way the network can get reconnected. But this doesn't show a general problem with non-hierarchical organization. On the contrary, this stems from an inherent problem of hierarchical networks. A network can become less hierarchical if less connected nodes connect with nodes outside their cluster. ↑ 130 Thus, not having clear groups, and being part of different groups that don't contain the same persons, can tackle hierarchy.

3.4 Principles of coordination

In this section, I discuss some of the principles anarchists use to organize themselves, and some of the methods of coordination distilled from systems theory. This is meant to give an impression of how we could coordinate without it leading to the formation of a controller. I will show how anarchists practices can be explained by system science's concepts.

3.4.1 Coherence

Most anarchists agree that there should be a coherence between what one thinks and how one acts. That is, what you say shouldn't contradict what you do, aims and methods should be aligned. That's why direct action is the anarchists' preferred method. It isn't coherent to want the end of the state and then make a petition to ask the government to kindly stop existing or stop a certain practice. The idea is to be prefigurative, to already act in a way we want our ideal world to look like.

↑ 31 ↑ 39

One activity shouldn't be subordinated to another. Thinking without acting and acting without thinking both require the subordination of one activity to the other. Thinking without acting is the constant spreading of an opinion, or the spreading of information about a certain oppressive practice without actually trying to do something about it. Acting without thinking might leave one vulnerable to being used by all kinds of people with their own agenda.

Passamani (Passamani, 2010) argues that you can't really differentiate between the two, that the body and mind are inseparable. Thinking and spreading information is also an act. It is because neither is subordinated to the other that a separation is impossible. But the body and the mind is neither a homogenous whole. There are lots of processes going on in it, which sometimes even oppose each other.

This is analogous to how different struggles can relate to one another. The classical Marxist view is that the economic struggle is the most important, and the one to which all other struggles are subordinate. The suggestion is that once the economic oppression is wiped out, all other oppressions will automatically disappear (see also here). The other view is that there should be no hierarchy of struggles, but different struggles are linked. There is no homogenization of struggles, but different struggles can reinforce each other.

↑ 34

Moreover, an individual cannot be separated from the world. This is what

Stirner (Stirner, 1907) argues, that relationships are part of an individual (“the ego and its property”). It might seem offensive to call relationships property, but Stirner uses the term property for everything an individual uses or benefits from, this can also include mutual use. In this use of the term, property is not necessarily something material, and doesn’t mean the ego has a dominance over its property.

This is how Stirner’s individualism distinguishes itself from capitalist individualism. Stirner considers relationships as a central desire of the individual. He considers someone who does everything for money, as being ruled by money, and thus not really an egoist, because she neglects her desire for relationships and thus part of her ego. Stirner describes his idea of social relationships in his concept of “union of egoists”. ↑ 46

Some systems theorists don’t consider a clear distinction between the body and the mind, and don’t consider an individual as completely separate from its environment (Heylighen and Beigi, 2016). Such theorists criticize the “brain in a vat” idea—that’s the idea that you could simply have a brain in a vat, separated from the world, that is intelligently processing information (Heylighen, 2009). This doesn’t work. On the contrary, our intelligence is highly contextual and depends on our environment. We constantly learn by the input we receive from this environment, and we are adapted to this environment. We won’t be intelligent in another.

Because ideas are so intertwined, they can’t really be put in a hierarchy or a linear order. That’s why it is sometimes so difficult to structure a text, and that’s why you will have noticed a lot of links between parts of this thesis.

In section 4.1.2.1, I will discuss in a more mathematical sense this difference between local coherence, where all elements interact, and having an overall, encompassing object (for example, a theme or goal), with all elements being in a hierarchical structure. ↑ 88

3.4.2 Stigmergy - propaganda of the deed

Stigmergy is a way of coordinating without any need for a central commander or even meeting place or direct interaction. It occurs when agents leave traces in the environment, which other agents pick up and build on further. A classic example is ant pheromones. When an ant finds a food source, it leaves pheromones so that the other ants can also find it. The shorter the path ants take, the more often a particular spot will be passed, thus the more pheromone will be on the path, and the bigger the chance an ant will take this path. Another example is Wikipedia. Here, someone starts an article, which someone else picks up to elaborate further, and so on.

The anarchist practice of **propaganda of the deed** can be understood from this perspective. Propaganda of the deed is the idea that deeds are used to show that resistance is possible, to inspire for certain practices, to give an example of how one can act (rather than just say how you want things to be). Thus, an action is a catalyst for more actions, and a certain struggle can emerge without the actors even knowing each other. For example, people can do solidarity actions for an anarchist arrested in some country, which can spread across the globe. This also relates to not separating your acts and your thoughts ↑ 52, and using prefigurative action ↑ 52.

3.4.3 Variation and selection - diversity of tactics

One of the general mechanisms of evolution is **variation and selection**. A variety of things are tried out and the ones that work are selected. If there is not enough variation, better solutions won't be found, and the system can become rigid. If you don't select for what you want, you won't get there. Of course, how and with which criteria ↑ 69 a selection process works can make a huge difference. For example, in capitalism there is a selection for egoistic behavior, only profit-making counts. In general, what kind of society you have will largely influence the behavior selected.

The origin of this description comes from natural selection, where the genes that survive get selected. But this mechanism is far more applicable than genetic evolution, like in our example with the magnets ↑ 41.

The notion that variation can be useful is expressed in the principle of "diversity of tactics". This is the idea that everyone should perform the actions she thinks works the best, instead of trying to convince all others to all perform the same kind of action. And it is precisely this variation of different tactics that often works better than any one tactic alone. On the other hand, there should still be room for enough critique and discussion to evaluate which actions worked better or worse, and to see whether we want to abort, improve or simply continue with a given action.

3.4.4 Antifragility - growing due to repression?

This variety helps systems to be **antifragile**. This is when a system actually becomes stronger after a shock (Taleb, 2012). One of the principles through which this can be achieved, is called the **order from noise** principle

(Heylighen, 2014a). The idea is that because of more variation (the noise), the system finds a more orderly configuration. For example, consider having a bin with some stones or other heterogeneous object in it. Shaking up the bin will cause the stones to arrange with less space in between because if there is less space in between, this arrangement will stick and get selected. Thus, it will be possible to put more stones in the bin, and the configuration is more orderly.

A system can thus be more antifragile by having enough variation to deal with the variety of disturbances. This is what the law of requisite variety ↑ 166 states: that the variety of actions should be at least as big as the variety of disturbances.

Often trying to get more control leads to having less control, which we could similarly call the **noise from order** principle. When controlling everything, there will not be a lot of variety in these controlled disturbances, and the system will not have a lot of capacity to deal with disturbances. It should only have more variety of actions than the little variety of its input. When there then is an unexpected disturbance (because it is never possible to control everything), the system will not be able to deal with it. This relates to the difference between trying to control everything versus leaving room for self-organization as discussed in my model of internal and external ↑ 154 control.

An example is how trying to shield you completely from microbes and viruses, often causes more illnesses. When your body regularly comes into contact with germs, it will develop responses for dealing with them. When a body is kept as sterile as possible, it will become weak, and any germ will be able to spread and grow without obstruction.

Of course, whether a system will get stronger, weaker or completely fall apart, depends on the strength and character of the shock. In the end, every system will fail if a shock is too strong. But some systems can withstand bigger shocks than others. For example, the principle of hormesis in the human body works because of antifragility. The idea is that if one does a harmful or unexpected activity (for example, eating a poisonous food) in a small amount, the body will learn how to deal with this, and actually get stronger from it (Heylighen, 2014b). Vaccinations work according to this principle. By injecting a certain disease in a small amount into the blood, the body will make anti-bodies, and will also be able to deal with the disease the next time it encounters it. Of course, the disturbance shouldn't be too big, we don't want someone to die or get sick from a vaccination or poisonous food. On the other hand, it should also not be too small for the organism to notice (which is often the case with hormesis proposed in alternative medicine).

↑ 57

In general, it is best if the distribution of the disturbances follows a power-law. This is when there are only a small amount of intensive disturbances, while most disturbances are small. An intensive disturbance will cause a system (for example, the body) to go into an alarm state to take a serious action against it, while the encountered small disturbances ensure that the system already has some repertoire of possible actions against disturbances. A continuous flow of average disturbances, for example, following a normal distribution, will simply slowly weaken the system, while it will not go into this alarm state to adequately deal with the challenge. A metaphor to explain this concept is that if you put a frog into hot water, it will immediately jump out of it, but if you put it in cold water and slowly increase the temperature of the water, it will not realize it gets boiled until it is too late. (It is debatable whether this is what would actually happen. While there are some 19th century experiments that confirm the premise, some contemporary scientists reject this idea, with one of the arguments that frogs will not simply stay into water for you, whatever the temperature.)

But this metaphor can also be used to explain that when repressive laws are imposed gradually, with each new law confining the possibilities a bit more, this will often just be accepted and will not lead to any protest. While if the resulting repressive law would be enforced immediately, nobody would accept it, and it would not be possible to maintain it.

We see antifragility in revolutionary movements when they grow even in the face of repression. The goal of repression is to isolate and demotivate individuals or tendencies. But we see that it is often exactly this repression that causes people to be shocked by the present way of doing things, and start engaging in subversive activities. For example, in Exarcheia (Athens), the killing of one kid, Alexis Grigoropoulos, caused riots to spread across Greece, with also a lot of solidarity demonstrations across the globe. The Arab Spring (2010) started from just one street vendor burning himself in protest of police brutality and confiscation of his wares.

In both cases, we see that there was already a strong movement. In Athens, the anarchists where already strong with a lot of social centers, and in the Middle East there were already lots of protests going on. The respective systems had already experimented with a repertoire of different responses to (perceived) smaller disturbances, so that the movement could deal with a (perceived) bigger disturbance. But there was one event that triggered a cascade and let it burst out.

3.5 Cascading effects

3.5.1 Positive and negative feedback

Such cascading effects usually happen through **positive feedback** (Heylighen, 2014a). This is when a disturbance gets amplified. The simplest form is when A creates B, and B creates A. Having a bit of A, will create B, which will create more A. Thus the creation of A (and B) will explode.

Positive feedback is one of the mechanisms that causes power-law probability distributions, a distribution that is widely observed. This is a distribution where most occurrences have a low value, while there are only a couple of cases with a high value. This is also explained in the Pareto principle. An example of the Pareto principle states that most of the capital is in the hands of only a few people. This results from the "rich getting richer effect" wherein the more money a person has, the more money he can earn. This is a positive feedback, which leads to a power-law distribution (Mitzenmacher, 2003). A positive feedback amplifies differences—the more you have, the more you'll get. If you don't have that much, you wont be able to get that much. You can avoid this power-law by constantly opposing those who have the most. For example, if instead of taking your money equally from all other people, you take it from the one that has the most of it, the power-law will disappear and we will have a more dynamic behavior. (In section 3.6.2, I will explain how this opposition is analogous to life). Further in this thesis, I will do a little simulation to explain this mechanism.

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A vicious circle is a manifestation of a positive feedback, where A remains because it is produced by B, which itself is produced by A. That is why it is so difficult to get out of such a circle. When A is simply taken away, it will come back through B. This is how addiction often works. For example, consider the classic statement "I drink because I feel bad, and I feel bad because I drink". Drugs are a special case of supernormal stimuli.

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The opposite mechanism is **negative feedback**. This is when disturbances gets reduced (Heylighen, 2014a). An example is the legal system, which wants to suppress all disturbances it considers illegal (whether it succeeds in this or not is another discussion). But the "negative" or "positive" in a feedback isn't about a value judgment, it is simply in whether a signal gets amplified or decreased. There is a negative feedback when more A creates more B, while more B creates less A. Thus, the amount of A will stabilize. If the amount of A increases, this will let the amount of B increase, which will cause the amount of A to decrease again. A positive feedback can have detrimental consequences. For example, the spreading of a virus or poverty caused by an unequal income distribution.

3.5.2 Butterfly effect - revolution

Positive feedback causes what is called the **butterfly effect**. The butterfly effect occurs when a small event causes a big effect, or when a small change in the initial conditions results in large differences in a later state (Lorenz, 2000). The classic example is that the flapping of the wings of a butterfly can cause a hurricane at the other end of the planet.

When a failure of one part of the system triggers failures in other parts, this is called a **cascading failure**. This is an example of the butterfly effect, where one small failure causes bigger failures, until even the whole system fails. This is often investigated in the context of failures in, for example, power grids, where a failure in one element might cause a nationwide blackout. But of course determining whether an event is a failure depends on what the goal is and whether maintaining a certain system is desirable or not.

There can also be cascading effects that have nothing to do with a failure of any system. For example, an information cascade, where information spreads further and further.

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A goal of anarcho-insurrectionalists is also to spread subversion by intervening in existing struggles to create an insurrectional climate from which a revolution could sprout. This is the purpose of propaganda of the deed, as explained in section 3.4.2. Reclus (Reclus et al., 2004) also states that "every event, even if it seems to be of minimal importance, will be capable of creating shock waves of change". He sees revolution as happening after evolution, lagging behind it because of resistance from the environment. He gives the example of a dam in a river: First there are slow changes, evolution, the water forms a pool before the dam, until all of a sudden, the water breaks the dam, starting with a small opening that then makes the water flood suddenly, breaking the resistance of the dam and river banks. This is revolution, which can only happen by an effort more violent, more forceful than whatever the forces are that maintain the status-quo

But how could such a revolution now be sustainable? How can it be avoided that the system evolves back into the old state or a similar one? For this I take inspiration from life and how it can maintain itself while being in constant flow. I think the more general principles underlying this are freedom and autonomy.

3.6 Freedom and autonomy

In this section, I will discuss the diverse notions that can be assigned to the concepts of freedom and autonomy. This will be partly based on two papers, *Freedom as a Natural Phenomenon* (Zwick, 2017) and *Founding autonomy: The dialectics between (social) environment and agent's architecture and powers* (Castelfranchi and Falcone, 2003), although other concepts will also be introduced. Rather than giving an overview of these papers, I will incorporate them in the framework I developed here to explore freedom and autonomy.

Zwick discusses several manifestations of freedom, distinguishing between simple and complex living systems. Castelfranchi comes from a computer science background, and discusses several notions of autonomous agents (with rather artificial agents in mind, although the ideas can be applied more generally). He distinguishes between non-social autonomy, which is autonomy from the environment, and social autonomy. Social autonomy can be further split into autonomy as independence, and autonomy in collaboration.

These distinctions will be further elaborated and explained in the coming passages, but first I will start from a basic understanding of freedom.

3.6.1 Degrees of freedom and constraints

One of the characteristics of a system is its **degrees of freedom**. This measures by how many independent parameters the system can be represented. This can be viewed as a measure of the freedom of a system (Maldonado and Mezza-Garcia, 2016). When there are more degrees of freedom, there are more independent dimensions by which the system can vary. Note that degrees of freedom denotes the number of independent parameters—it is thus the minimal number of parameters by which the system can be represented. This is independent of the specific representation of the system. Consider, for example, a circle, which can be represented in two dimensions (by the x- and y-coordinate), but these are not independent. Given an x-coordinate, the y-coordinate can, at most, have two values (since $x^2 + y^2 = r^2$, with r the radius of the circle). But it is enough to know the angle to know which point in the circle we are speaking about. Thus, a circle can be represented by one parameter, it is one-dimensional and its degree of freedom is 1.

The reason for this is that there is a constraint working on the two-dimensional points. We consider only the points that are at an equal distance (the radius) from a certain center (here the origin). In general, a **constraint** is a relation between the parameters so that only certain parameter values are possible. Constraints limit the degrees of freedom. Thus constraints can be seen as confining the freedom of a system.

↑ 47 In the section about aspect systems, we also discussed that when there are constraints in a system, it needs less coordinates to represent it by its aspects, rather than its structural dimensions. A problem with aspect systems is that it makes the constraints invisible and more absolute. For example, when representing a circle by one parameter, it is less clear that actually originally there were two dimensions that got constrained. And since this constraint is made invisible, it is more difficult to break out of it. The constraint is considered as absolute, unchangeable, given, and it is made impossible to imagine what would happen if the constraint would disappear or change.

3.6.2 (Moving out of) attractors

How a constraint arises can be envisioned by a dynamical system that gets into an attractor. A **dynamical system** is a system that changes its parameters as it moves through its state space. A state space is all the parameter values the system can possibly adopt. For example, if the system has two coordinates, the state space is the two-dimensional plane. The rules of the dynamical system can be formed by a function, which determines where the system will go when it is at a certain point. An **attractor** is a part of the state space which can be entered, but cannot be exited. This means that when the system is at a point in the attractor, it can move to other points within this attractor, but it cannot move to other points. The **basin** of an attractor is all the points that will end up in the attractor. That is, when the system is within the basin, it will move to a point in the attractor where it cannot get out. There are several attractors possible. The attractor in which the system will end up depends on in which basin it started. Attractors constrain the places that the system can visit. The most extreme case of a system without constraints is when the attractor comprises the whole state space, i.e. any state can be visited from any starting position. The most constrained case is when there is one attractor consisting of one point, and the basin of this attractor comprises the whole state space except for this point. This means that all points will end up in this one point. Figure 3.3 shows these cases. While the previous section analyzed the stage when the constraints had already arisen, dynamical systems are about how these constraints appear.

In reality, systems are more complex than being governed by simple rules. It is thus possible to move out of an attractor by changing the dynamics of the system or through external influences. Take, for instance, a ball that has rolled to the bottom of a valley as a simple example of a

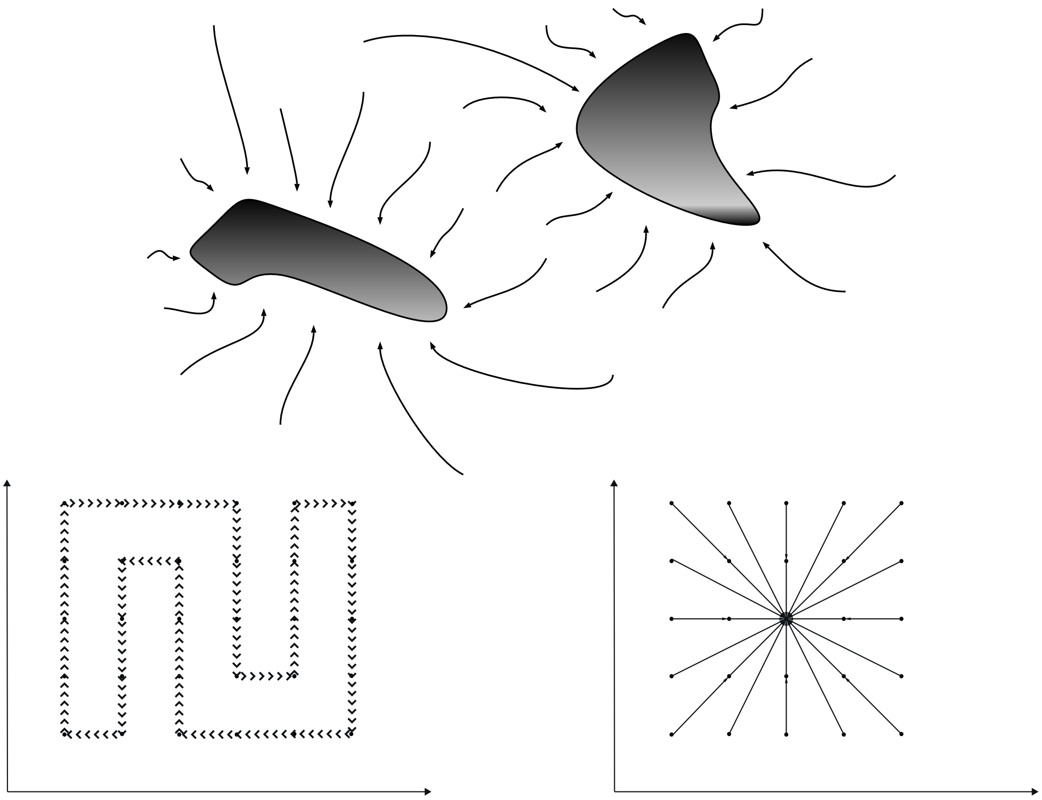

Figure 3.3: Examples of attractors. On top a system with two attractors is shown. The least constrained case is that where the attractor comprises the whole state space, which consists of discrete points (lower left). The arrows, which follow the dynamics of the system, thus visit all points. The most constrained case is that of a system with one point attractor to which the whole system is attracted (lower right).

system in an attractor. If there is an external force working on the ball, like a kick, it may move out of that attractor.

Freedom and autonomy can be understood as breaking out of constraints and moving out of an attractor. A living system can be seen as an autonomous system. There is usually a cybernetic loop here, where the system acts on the environment so as to influence its input. The system is no longer passively drifting by according to the complex processes going on in the world, but actively guides itself in a certain direction.

Life seems to surpass the second law of thermodynamics, which states that in a closed system, a system will evolve to a state of maximal entropy. Entropy can be seen as a measure of disorder. In life, complexity increases.

Complexity lies in between order and disorder. Thus living systems do not evolve to a state of maximal entropy. This state of maximal entropy is an example of an attractor. Because the world is an open system, there are constant challenges and a constant influx of energy, making it more dynamical, which is why a complex phenomenon like life is possible. We can further explain this by using Heylighen's (Heylighen, 2014a) interpretation of the second law of thermodynamics, which is that without selection, the entropy of a system will always increase. Shannon **entropy** (Shannon, 2001) is used here, which describes how the probability distribution of a system is scattered, and how unpredictable a system thus is. It is defined by the following formula: ↑ 168

$$H(X) = -\sum_{i=1}^{n} P(x_i) \log P(x_i)$$

where X is a discrete random variable with possible states $\{x_1, ..., x_n\}$, and each state x_i has a probability $P(x_i)$. The log is usually at base 2, thus $\log 2^n = n$. The rationale behind this is that a string with length n where each element is either 0 or 1, can take 2^n possible values. While $P(x_i)$ denotes the probability of a state x_i, $-\log P(x_i)$ expresses the minimum string length by which this state can be represented. For example, when a probability is 1/2, it is like that of a coin flip, and can be represented by one bit.

A uniform probability distribution gives maximal entropy. Consider that there are 2^n states (thus binary strings of length n), where each state is equally probable, thus with a probability of $1/2^n$. The entropy is thus $H(X) = -\sum_{i=1}^{2^n} \frac{1}{2^n} \log \frac{1}{2^n} = 2^n \frac{1}{2^n} n = n$. The more possible states there are in the system, the bigger the entropy. But if we consider the number of states to be fixed, the maximal entropy is reached. An example is a gas where all molecules are randomly moving. It is impossible to predict at which position a molecule would be.

When there is zero entropy on the other hand, there is complete order, as there is only one certain state. There is thus complete predictability. If there is only one state, x_1, the probability can be represented by $P(x_1) = 1$, while for all the other states it is $P(x_i) = 0$. Thus, the entropy is $H(X) = -1 \log 1 - \sum_{i=2}^{n} 0 \log 0 = -1 \cdot 0 = 0$.

It would seem to follow from the second law of thermodynamics that life is not possible: how can there be such complex structures, and why don't they just fall apart? This is because there is a mechanism called selection that makes certain states more probable, since they have a higher chance of survival (the principle of natural selection). In the second law of thermodynamics there is a uniform distribution where all states are equally probable - there is maximum entropy. However, because of selection, this changes, where certain states are more probable. Thus, despite constant outside pressure to fall apart, life manages to sustain itself.

Analogously, there is the common belief that there will always be power imbalances and people dominating other people. This is expressed in statements like: “Those with power will get more” or “The rich get richer, the poor get poorer”. These are specific manifestations of a positive feedback mechanism causing a power-law distribution. An attractor is thus reached, but this is because there is no mechanism to prevent this. If there is a constant opposition, so that as soon as someone gets more than the others, he gets a headwind that restrains him from accumulating it, the hypothesis is that the distribution would get flattened.

↑ 57

To sum up, in both of these cases there is a certain law that seems difficult to avoid (either the second law of thermodynamics or a positive feedback mechanism giving rise to a power-law). In general, the system moves into an attractor, either a uniform distribution when there is maximal entropy, or a power-law distribution. In both cases, there can be a mechanism (selection or constant opposition) to overcome this.

I could formulate the above observation in a law similar to the second law of thermodynamics, as follows: without a constant opposition, domination will always increase. However, we see that whenever there is domination, there will be people opposing it (Foucault, 1998). Thus, I believe that just as we have seen that life is possible and can flourish in the world, a world without domination is possible. This is also the lesson that can be drawn from Gelderloos' *Rise Of Hierarchy*. A world without domination will require a constant effort of opposition to keep it that way, and this autonomous action is exactly what makes us living beings. Further on in this thesis, I will present a small simulation of how constant opposition can counter-act a power-law. I already mentioned the basic mechanism in the section about positive feedback.

↑ 25
↑ 131
↑ 57

Zwick (Zwick, 2017) concords with this idea in his conceptualization of freedom as when there is ‘available energy that is unblocked and holistic’. The idea is that there is a potentiality, but there are still potential constraints that need to be broken out of. This is explained in the language of triads in Bennett's "systematics" (Bennett, 1966), where freedom is a 321 scheme: there is a neutralizing force (3) that neutralizes an opposing force (the passive force, 2) before it occurs, leaving the active force (1) to act freely.

3.6.3 Dependency

One way autonomy is understood is when there is no **dependency**, as when ↑ 45 an agent does not need help to reach its goals. I already discussed dependency ↑ 44 in section 3.2.2 about the formation of a controller. Here, an agent needed a bigger structure to survive, and there is an asymmetry in influence—the bigger structure can influence the agents, but the agents that are constituted in it cannot influence the bigger structure.

Castelfranchi (Castelfranchi and Falcone, 2003) mentions this notion by the facet of autonomy as Independence, which he defines as "an Agent is completely autonomous (relative to a given goal) when it does not need the help or the resources of other Agents neither to achieve its goal nor to achieve that goal in a better way". But this simply states that the agent is able to reach its goal without help. Its goal can still be externally determined. On the other hand, when there is dependency, an agent is vulnerable to external determination. Since an external agent will only help this agent when it is also in its interest, that agent will determine how it will help and what will be the outcome.

Zwick (Zwick, 2017) coheres by regarding freedom from a fixed materiality. ↑ 45 When there is autopoiesis, there is self-production, and thus there is a constant flow of materials, that do not have to be fixed. The system is thus less dependent on its environment.

3.6.4 Internal determination

Autopoiesis also makes a system closed under causal implication—it is thus "causa sui", **self-caused** (Zwick, 2017). In an autopoietic system understood as a chemical organization, everything that is consumed, thus that is a cause, is produced by the system itself. Zwick differentiates between freedom from a fixed materiality in simple living systems, and what he considers one of the highest forms of freedom: self-reference, self-knowledge. Surprisingly, the mechanisms he uses to describe this 'simplest' and 'highest' form of freedom are pretty similar, i.e. autopoiesis, self-cause.

Self-cause leads to **internal determination**, the agent chooses his goals and values himself. This relates to agency, which Zwick defines as being able to choose or alter one's environment.

Castelfranchi (Castelfranchi and Falcone, 2003) also regards being goal-oriented as a key value of autonomy. This can be summarized into the following points:

- To be self-interested and goal-driven;
- To actively perceive and select stimuli;
- To have internal states, with own internal dynamics and evolution. The reaction to stimulus depends on this state.
- To choose which goals to adopt. When there are conflicting goals, it can make decisions about these. It can adopt goals from the outside (other agents), but only when it enables the achievement of some of its own goals. It is not possible to directly change the goals of an agent, this can only happen by modifying the 'beliefs' of an agent. These beliefs cannot be automatically altered.
- To be the source of own norms, laws and obligations. This is the etymological origin of the word autonomy, "auto-nomos". The agent is thus not subject to other authorities and norms.

But this last point is also about not being externally determined. It shows how having internal determination is so intertwined with not being externally determined.

3.6.5 No external determination

There are other points Castelfranchi (Castelfranchi and Falcone, 2003) mentions as forms of autonomy that are more related to not being externally determined. He also considers being autonomous although being delegated by another agent. **Delegation** is understood as when an agent likes or needs an action from another agent, and has the goal that that agent performs that action. He mentions two forms of autonomy in this case. The first being that the agent is still able to choose which plan to follow to reach a given goal, and the second being that there is no monitoring or intervention. This is, of course, a very limited form of autonomy, where one simply has more freedom to do as he's told. But it is in general easier to escape external determination when one is not constantly monitored and can choose plans by himself.

Castelfranchi suggests two aspects of non-social autonomy related to not being externally determined. Namely, when one is not being pushed by external forces, and when behavior cannot be completely determined and predictable on the basis of the current input.

But is being goal-oriented the same as not being predictable? When something cannot be predicted, it cannot be determined externally, but is there internal determination? There could simply be a more complicated mechanism at play, which we did not figure out how to predict yet. The outcome from flipping a coin cannot be determined, but we would not consider the coin as having an internal determination.

Zwick (Zwick, 2017) calls one of the properties of freedom that it is partly deterministic and partly random. This is also the stage of complexity, in between zero entropy and maximal entropy. The randomness makes it impossible to externally determine, while the deterministic part implies that there should be some determination, thus this should be internal. This is not completely true, however. There can be external determination that makes certain states more probable, although there is some variation in them. How can we then differentiate between internal and external determination?

A possibility is that in an autonomous system, the internal state is determined, while the output is not, since output depends on the input. An autopoietic system wants to maintain itself for different inputs, but it might still generate different outputs. It is also important to differentiate between output as waste and output that has evolved to change the environment as wanted, that functions in a cybernetic loop.

There is always some influence from the environment. A system is not closed off (a complete chemical organization that does not need any external input nor generates output, is an idealization. Usually one incorporates an influx and outflux inside the system, as given). How can we then decide whether this environment determines the system? A possibility is to differentiate between an environment as a field of realization, which increases the possibilities of the system, and an environment as a colonizer, which restricts the possibilities. Increasing or decreasing the possibilities can be described by a change in the system's degrees of freedom.

3.6.6 Rapport between parts

One of the problems of differentiating between external and internal determination is that it is not always clear what is internal and what external. Being self-caused can also feel like there is (a part of) your state that determines yourself. I introduced autopoiesis in the context of an imposing structure that is self-maintaining, and thus more difficult to break out of. Freedom could be seen as a process, where a mechanism to break out of an attractor is found, which then becomes an attractor in itself, from which a new mechanism to escape it is found. This relates to the *idée fixe* discussed by Stirner. ↑ 43

Three of the types of freedom of Zwick (Zwick, 2017) are about how a part of the system could determine the rest: 'wholeness', 'hierarchy' and the 'modeling subsystem'.

Wholeness means there is some congruity between the parts, there is no part that causes negative consequences for other parts, who are just subject to this part. Zwick mentions three mechanisms by which this can happen: 1) a unitary utility, where there is some aggregation from the parts; 2) an order of priority of the parts; or 3) one utility that got selected. Thus, he does propose one 'decider', assuming there should be one overall utility, while I argued in section 3.4.1 that coherence can also be achieved by having local consistency. Zwick does acknowledges that it is incorrect to conclude that multiplicity is adverse to freedom, referring to the Law Of Requisite Variety. ↑ 52 ↑ 166

This relates to the second aspect Zwick associates with freedom, namely **hierarchy**. The idea is that complex living systems have a hierarchy of needs, for example, Maslow's hierarchy (Maslow, 1943) for human beings. Higher level needs can only be pursued after lower level needs are (more or less) met, and higher level needs usually address something else than mere viability. This mainly emphasizes that there is some development, that additional possibilities can be enabled. But this merely means there is directionality, which does not necessarily induce a hierarchical structure. There can be different possible paths for reaching a certain state (thus it does not necessitate the fulfillment of an A, if it can also be reached via a B), and one situation can also cause multiple effects. Moreover, there can be cycles, where an effect also enables a starting cause. This is necessary for autopoiesis, where there should be self-production. For example, in Maslow's pyramid, intellectual development is higher than survival, since it is only when some basic needs are met that there is room for intellectual exploration. But intellectual efforts can also create tools to make survival easier, for example by developing more sophisticated agriculture methods to aid food production. There is not one 'final goal'—an idea that is further developed in the next chapter. ↑ 78 ↑ 89

The third aspect mentioned by Zwick is that complex living systems have a **modeling subsystem** that regulates the system, its interactions with the environment, and even itself, and models its environment. This can increase the freedom of the system by increasing its possibilities.

Automatism, being on auto-pilot, could be seen as an absence of freedom, which is outrun by sensitivity and subjective experience. One could both see reason as the slave of passion, where passion sets the goals while reason is just a method, or find that desire should be governed by reason, where otherwise one is simply led by impulse. Consciousness enables deliberation, while mobility of attention prevents being captured by internal or external impressions. These are thus mechanisms to prevent being led by a part of oneself.

These mechanisms can also prevent indirect environmental control. Different levels of freedom could be differentiated, depending on how much external control there is. The most immediate is freedom from direct external control. Already more difficult to see is freedom from **external colonization**. Where there is alienation, the system acts on behalf of the environment ↑ 35 rather than itself. This can be seen in the case of addiction, and I discussed ↑ 43 this case further in section 3.2. One might also wants to free itself from compromise with environmental codetermination. The environment is the least freedom-constraining when it is a field of realization. This links to the ↑ 66 previous subsection, where the environment could either increase or decrease the possibilities.

3.6.7 Conclusion

Freedom and autonomy could be seen as choosing your own goals, not just ↑ 64 doing as you're told. This means being self-guiding, having internal determi- ↑ 65 ↑ 64 nation and not being externally determined. Being self-caused means there is autopoiesis, there are self-maintaining, connected processes. There is not one main goal (or one utility function) with several sub-goals, but many ↑ 89 ↑ 66 interacting goals that create some coherence (becoming autopoietic). Still, all these goals can evolve and change.

↑ 66 These processes are thus not completely determined, and an external agent cannot just predict and influence the system. It is, however, also not completely random, since the system is goal-directed and prefers certain ↑ 64 states over others. When the system is dependent on the environment for reaching its goals, it is also more susceptible to external determination. The ↑ 66 environment, but also internal influences, can increase or decrease possibili- ties—it could thus be a field of realization, or act as a colonizer. A process ↑ 59 could create more constraints or more degrees of freedom. Freedom can be ↑ 60 viewed as a process of always getting out of a new attractor. Thus, it is a constant opposition.

3.7 Applied on domains

In this section, I will explore some aspect systems in more depth to give some examples to previous concepts, like co-evolution and meta-system transition, but also to discuss these domains in themselves.

3.7.1 Technology - creating the environment

Technology is in interaction with a certain kind of society and ideas. Technology strengthens a certain type of society, while it is also out of current ideas that a technology is created. Technology creates the circumstances, the environment, in which one can act. Even if in the beginning or in its roots a technology is not configured for the current social mode, a technology can easily be recuperated for a certain dominant idea. Thus, technology often reinforces the status quo, the current tendency.

This is a basic manifestation of co-evolution. The classic view of evolution is that species adapt to an assumed fixed environment. With the rise of technology, humans more and more created their environment themselves. By this I mean we created the selection criteria for our species ourselves. This is the flaw in using the survival-of-the-fittest argument in present human society. The argument is that it is only natural that only the strongest individuals, firms, ... survive. But we artificially created the selection criteria of what defines ‘strongest’ (in capitalism, this is basically what can generate the most profit). These selection criteria could be changed so that a wholly different kind of social organization would rise.

↑ 49

But new ideas from society can create new technology, which could change society. So technology could help liberation, either because it is constructed to do so or because technology does not always follow the path its creator had in mind. By liberation I mean moving away from a dependency relation and becoming autonomous. This relates to the concept of self-actualization, as explained in section 2.1.1 about freedom from the previous chapter.

↑ 45
↑ 59
↑ 10

But no one can self-actualize someone else, one can only self-actualize himself. One of the core attributes of how I conceive freedom, is that it is a decision (Passamani, 2010). That is why technology can never liberate in itself. Expecting from technology to create a more free society is in contradiction with this notion of freedom. This is similar to the economic determinism in Marxism, that assumes that the economic mode determines the society completely, and it is through changing this economic mode that a free society can be created. In these scenarios, technology cannot save us

↑ 40
↑ 34

because we are still not the drivers, the players, of our own future.

Technology can reinforce certain liberating tendencies, but if these tendencies are not present, even the most liberating technology will evolve to serve the current system rather than change it. We can find support and tools in technology to liberate ourselves but it may just as easily create circumstances that make it more difficult to do so.

We can see an example of these mechanisms of influence in the technology of agriculture. Private property and patriarchy could flourish by this technology , which drastically transformed the way society was organized (Feinberg, 1997). This is a clear example of how technology shapes society.

Today, we see how every technology gets recuperated by capitalism. The main drive is to make profit, thus every technology will serve this goal. A classic example is the story of bio-fuel. First, some people found out they could repurpose their used vegetable oil to fuel their cars. Recycling things, and being less dependent on fossil fuels, is better for the environment, right? But then big businesses saw profit in it, and now there are lots of big fields that grow plants just for fuel. This means there is less land available for food production, and brings forth a series of problems with mass monoculture: deforestation, soil erosion, loss of biodiversity, ... This example shows how a certain ideology (for example, capitalism) can create and perpetuate a certain kind of technology.

3.7.2 Democracy - separating thinking and acting

Today's democracy creates a sharp separation between decision making and acting. Only politicians make the decisions, which other people put into practice. This makes it structurally possible for both the decision makers and the executors to avoid responsibility, which can lead to alienation. Dreams cannot evolve into acts.

↑ 52

By **alienation** I mean when there is an incongruity, a discrepancy. This can, for instance, be between self and environment; between thoughts and acts; or between one part of self and another. Often the cause is a maladaptation, where one part has undergone sudden changes, and the other part cannot follow. This is the case with supernormal stimuli.

↑ 44

Distributed governance is a step in the right direction to prevent alienation. But often there is an assumption that global decisions are the ideal and should be acted on by everyone (for example in (Banathy, 2013)). Although these decisions and acts have come about in a distributed way, there is still a separation between them. A global decision is made out of local decisions, which lead to local acts bringing forth a global act. Another practice is where local decisions lead to local acts, out of which a global behavior, a global direction, emerges (see figure 3.4).

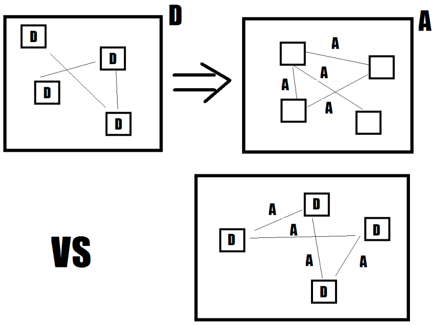

Figure 3.4: In the upper figure, local decisions (D) lead to a global decision, which determine local acts (A) that brings forth a global act. Decisions and acts thus get separated. In the lower figure on the other hand, local decisions directly lead to local acts. From all of this there pops up an emergent behavior.

Consider, for example, the shaping of the neighborhood you live in. One way this may happen is through people from the neighborhood coming together to share ideas on how they want their neighborhood to look and forming a consensus plan on what should change, then ultimately acting on that plan. Though the plan is formed by consensus, this often does not feel very empowering. Imagine the case that on the day of the planning you did not really know yet what you wanted. This only becomes clear to you once you see the plan being put into action. You may feel pretty alienated because it does not really feel like your plan. In contrast, a completely different way to shape a neighborhood involves people just acting on what they think should happen, sometimes discussing with others to see whether there is support, but not necessarily obtaining a full consensus. Then others build further on this when they see something they like. This way people are not restrained in acting because they do not go anymore through a whole (bureaucratic) process before being able to act.

The scientific process also sometimes creates a separation between thinking and acting (acting is usually by communicating thoughts to the world). Right now, a researcher develops a plan for an experiment, performs an experiment and writes down the results in an article, and only then are his ideas peer-reviewed. At that stage, it may be realized that actually there are some problems with the experimental setup. A more continuous peer-review instead could be interesting, where every step gets peer-reviewed. Something comparable is already happening with crowd-sourced research (Silberzahn and Uhlmann, 2015).

3.7.3 Global Brain

The discussion of technology and democracy can be applied to global brain research. The **global brain** can be defined as the distributed intelligence emerging from the coordination of humans and technology through the internet (Heylighen, 2014c).

↑ 69

Since the global brain is highly technological, it is relevant to see how the considerations in the technology section affect the global brain. The global brain could enable people to build the world they want to live in, where the technology, structure and coordination will be formed to aid with this liberation. The internet could enable people to put their ideas into practice by providing tools, resources and people. But as I argued, people will still have to put effort forth to actually do these things, and decide to liberate themselves.

It is also possible, and this partly depends on whether this decision will be made, that the global brain becomes another technology from which people are totally dependent, that influences their life, but which they have no influence over. This can be seen in how contemporary technology is often used for surveillance and repression by the state apparatus (for example, with internet surveillance that often uses big data mining, or with tracking people through smartphones and an increased number of CCTV-cameras).

↑ 70

The examination of alienation in the democracy section can also be applied to the global brain. We already see nowadays how increased connectivity can actually create isolation, where people are constantly behind their computer or smartphone and no longer have much deep human contact; or when they are constantly in a computer game and no longer spend much time outside interacting with the social or physical environment. This thus creates a separation and incongruity between the self and the environment. Theoretically, the internet could further alienate our decisions from our acts. Imagine people stuck in a virtual world where they can raise all kinds of opinions, but without these being connected to their acts and everyday lives.

But this could also evolve more positively, where the global brain enables people to ease the transformation of ideas and thoughts into actions, by making it easier to coordinate and find other people to accomplish shared wants.

The global brain is a meta-system transition: a transition to a higher ↑ 43 level of complexity. It is thus not that difficult to imagine that this structure will develop its own goals, which might become more and more independent of individual goals (although these individuals constitute and sustain the global brain). I discussed how such a process is possible even with the evolution to a cultivator. This is already more or less ↑ 44 happening today (where we for example see that a state is not really fulfilling individual needs), but the danger with the global brain is that it would be more intelligent than the hierarchical system of today. It would be a self-organizing, emergent system, and thus it could not simply get dismantled by taking away the top. The stronger this structure will be, the more difficult it will be to break it down. Thus, if it would be omnipotent and omnipresent (as argued in (Heylighen, 2014c)), will it not be also impossible to resist?

But the global brain can also be viewed in a more positive light. The global brain could be a constantly evolving structure, a dynamical play full of differentiation and experimentation. To make this possible, I think there needs to be a constant opposition to avoid being stuck in a stable attractor ↑ 63 state. We should avoid that the new vision becomes a dogma, where it becomes a restricting structure that owns us instead of we owning it. There ↑ 43 should be a diversity of methods, and we should as much as possible avoid to make one global decision. ↑ 41

I do not think the path the global brain will take is already predetermined, it is up to us to build the type of global brain we would like to have.

3.8 Conclusion

In this chapter, I developed a set of theories describing how self-organization ↑ 39 can emerge, and how this can evolve into a rigid structure, a controller, a colonizer. Often co-evolution occurs, where agents create a system, but this ↑ 49 system in turn influences the agents. This system could then become more rigid, where it escapes out from under the control of the agents and starts ↑ 43 to follow its own goals. But there does not necessary have to be systems and subsystems on the structural level, there can also be different aspect ↑ 47 systems, where the division happens on the functional level.

A solution to this structure that imposes itself could be a constantly

↑ 54 evolving structure, a variation and selection of different ways of organizing
(Veitas and Weinbaum, 2014). There is not one utility measure that imposes
a hierarchical ordering (Roughgarden, 2013). Instead of trying to reach a
global, united decision or view, there would be local groups or individuals
who develop themselves and work together to do so. It would be diverse
and even contradictory. This conflict will boost a dynamic play.

A method to coordinate in a self-organizing fashion, without even the need
↑ 53 ↑ 54 to be at the same place at the same time, is stigmergy. Variation and selection
↑ 57 can cause certain new arrangements to emerge, where positive feedback can
↑ 54 make it significant. Antifragility means a system can grow through shocks,
which makes this constellation sustainable. We can draw some lessons from
↑ 59 living systems on how to be sustainable, and what autonomy and freedom
↑ 60 means. A constant opposition can results in an attractor being repeatedly
escaped. Freedom can thus be viewed as a process, where there are different,
ever-changing goals and processes, without one being overarching. There
↑ 52 is however a coherence reached by being locally consistent. Autopoiesis is
achieved, which makes the system self-maintaining. This makes the system
self-caused, giving it internal determination.

But this process could evolve into being led by an *idée fixe*, or an external colonizer could use the mechanism to let an agent act on behalf of this colonizer, and not in the best interest of itself, thus determining it externally. The question is then when external as well as internal influences determine a system. An answer might be to differentiate between processes that create more degrees of freedom, and processes which decrease the degrees of freedom, inflicting constraints.

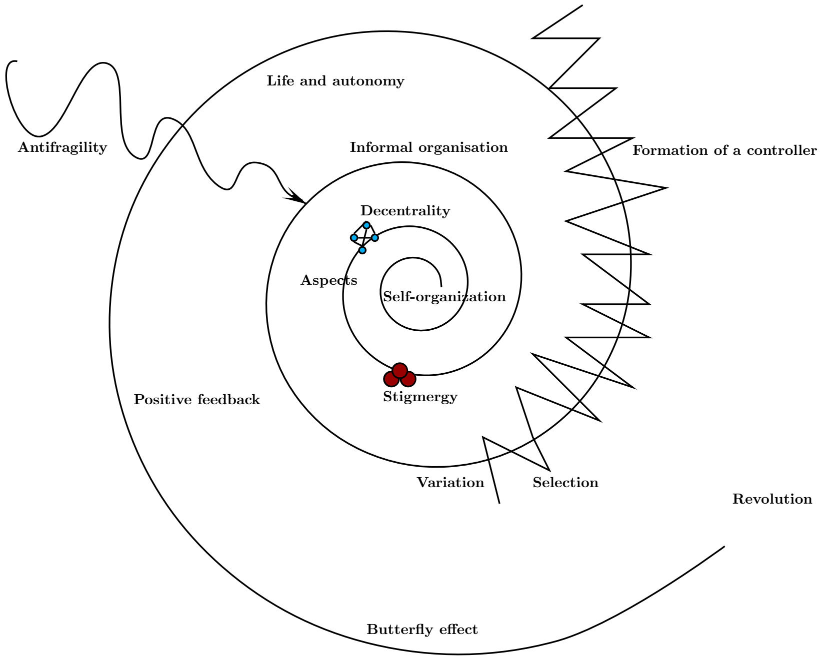

Figure 3.5: A visualization of some of the concepts of this chapter.

Chapter 4

Structure

In this chapter, I investigate the structural component of hierarchy. I first introduce the mathematical notion of order. This can be understood as properties of network structures, where there are multiple ways the connections between elements can be represented. This mathematical notion of order is an idealized notion that is seldom completely present. Thus, I will discuss a measurement that can classify structures as more or less hierarchical. Other approaches of categorizing networks as more or less hierarchical are also discussed.

This discussion will provide more clarity on the question of the definition of hierarchy, as already touched upon in chapter 2 on social theory. These definitions will be used in further chapters. I will show how a hierarchical structure relates to the functional aspect of determination, and check whether hierarchy is necessary in certain models. Undirected networks will be made more or less hierarchical and will be used in the simulation of chapter 6.

↑ 9
↑ 200
↑ 182
↑ 139

4.1 Order

A common understanding of hierarchy is an ordering in which one element is considered ‘higher’ than another. I will provide an overview here of how ordering is understood in mathematics.

First, we will review some general definitions. A **set** is a collection of elements where all elements are distinct (thus, no element is twice in a set), and the order of elements is not important. A set S of elements a, b, c is denoted as $\{a, b, c\}$, and we then say $a \in S$. A set can contain a finite or an infinite number of elements, noted as $|A|$.

$A \subseteq B$ denotes that A is a subset of B, and is defined as:

$$A \subseteq B \Leftrightarrow \forall a \in A : a \in B$$

When we want to specify that B is a proper subset, different from A, we say $A \subset B$, which can be defined as $A \subseteq B \wedge B \nsubseteq A$. The symbol $\wedge$ denotes *'and'*, while $\vee$ denotes *'or'*. Two sets are equal, $A = B$, when they contain the same elements. This can be formalized as $A \subseteq B \wedge B \subseteq A$.

↑ 82

Given a set S, we can define the **power set** $P(S)$ of S as the set of all the subsets of S, $P(S) := \{A | A \subseteq S\}$. We can then consider a certain $F \subseteq P(S)$, which we call a **family** of subsets (this is a set where the elements are themselves sets).

An **ordered pair** (also called a couple) is a pair (a, b) of elements. Here the order is important, $(a, b) \neq (b, a)$. This can be generalized to an **n-tuple**—an ordered list of n elements, denoted as $(a_1, ..., a_n)$.

The **cartesian product** $A \times B$ of two sets A, B is the set of all ordered pairs where the first is an element of A, and the second of B, ie:

$$A \times B := \{(a, b) | a \in A, b \in B\}$$

This can be generalized with n-tuples to the cartesian product $A_1 \times ... \times A_n$ of n sets.

An n-tuple is noted in bold, i.e. $\boldsymbol{x} = (x_1, ..., x_n)$. Sets are noted with capital letters, i.e. A, while specific instances (known or unknown) are noted with small letters, i.e. a.

A **binary relation** is an $R \subset A \times B$. In the following we consider binary relations with twice the same set. Thus, $R \subset A \times A$. This is sometimes also noted as (A, R), and we may note aRb for $(a, b) \in R$. In this notation, there is often a symbol used for R, for example $\leq$. In the following properties the set A over which the relation is considered, is also important (it might be that the same relation can work over different sets, and some properties hold for the one set, but not the other). Examples are given further on.

↑ 79

***Definition* 4.1.1** A **pre-order** is a relation for which following properties are met:

- **reflexive**: $\forall a_i \in A : (a_i, a_i) \in R$
- **transitive**: $(a_i, a_j) \in R$ and $(a_j, a_k) \in R \Rightarrow (a_i, a_k) \in R$

When this pre-order relation is **symmetric**, this means that if $(a_i, a_j) \in R$ then $(a_j, a_i) \in R$, we speak of an **equivalence relation**.

The **transitive closure** $T(R)$ of R is often considered. This is the smallest relation that is transitive and for which $R \subseteq T(R)$. It can be constructed by first taking $T(R) = R$, and then adding all (a_i, a_k) to $T(R)$ for which $\exists j : (a_i, a_j) \in T(R)$ and $(a_j, a_k) \in T(R)$, and continue until nothing can be added. It can be proven that if R is reflexive and symmetric, so is $T(R)$, and thus $T(R)$ is an equivalence relation.

Given an equivalence relation R over A, and a certain $a \in A$, we can consider all b which are equivalent with it:

$$E(a) = \{b \in A : (a, b) \in R\}$$

This is called the **equivalence class** of a. We have $E(a) = E(b) \, \forall b \in E(a)$. The equivalence classes form a **partition** of X: a selection of non-empty subsets (called parts), such that every element belongs to one and only one of the subsets. Conversely, given a certain partition of X, we can construct an equivalence relation R by setting $(a, b) \in R$ when they are in the same part (it is easy to prove that such a relation is an equivalence).

When symmetry never hold, we speak of a partial order:

***Definition* 4.1.2** A relation is a **(non-strict) partial order** when following properties are met:

- **reflexive**: $\forall a_i \in A : (a_i, a_i) \in R$
- **antisymmetric**: $(a_i, a_j) \in R \land a_i \neq a_j \Rightarrow (a_j, a_i) \notin R$
- **transitive**: $(a_i, a_j) \in R$ and $(a_j, a_k) \in R \Rightarrow (a_i, a_k) \in R$

***Definition* 4.1.3** A **strict partial order** never meets the first property, but does meet the other properties:

- **non-reflexive**: $\forall a_i \in A : (a_i, a_i) \notin R$
- **antisymmetric**: $(a_i, a_j) \in R \land a_i \neq a_j \Rightarrow (a_j, a_i) \notin R$
- **transitive**: $(a_i, a_j) \in R$ and $(a_j, a_k) \in R \Rightarrow (a_i, a_k) \in R$

I speak about **directionality** when the order of a pair is important. In this case the relation is not symmetric—there are a, b for which (a, b) holds, while (b, a) doesn't. When a relation is symmetric, (b, a) holds whenever

(a, b) holds, and we can work with unordered sets of two elements (e.g. $\{a, b\}$) instead of ordered pairs. Antisymmetry is when there is always only one direction, i.e. when (a, b) holds, (b, a) doesn't.

Definition **4.1.4** A **total order** is a partial order for which any two elements can be compared:

$$\forall a_i, a_j \in A : (a_i, a_j) \in R \vee (a_j, a_i) \in R \vee a_i = a_j$$

A total order is also called a **linear** order because all the elements can be represented on one line. A **chain** is a totally ordered subset of a partially ordered set (although it is sometimes simply used as a synonym for totally ordered set).

An example of a (non-strict) total order is $(\mathbb{R}, \leq)$, the standard $\leq$-relationship on the set of real numbers. When we demand strictness (thus we want the relation to be anti-reflexive), the $<$ relation satisfies.

An example of a partial order that is not a total order is $(P(S),\subseteq)$ for the power set of a set S. It is easy to prove that this relation is transitive. It follows from how we defined $A = B$ (as $A \subseteq B \wedge B \subseteq A$) that this relation is anti-symmetric and reflexive. Usually there are $A, B \in P(S)$: $A \nsubseteq B$ and $B \nsubseteq A$, thus the order is not total. The relation $\subset$ is anti-reflexive, and thus $(P(S), \subset)$ is a strict partial ordering. We can also easily prove that $(P(S), =)$ is an equivalence relation. ↑ 77

Since we have an ordering, now we can consider which element of a set is the biggest. The emerging definitions are exemplified in figure 4.1. I'll work with the partially ordered set $(A, \leq)$, where $a \leq b$ denotes $(a, b) \in R$ (with R thus a partial order). Now we define maximal as:

Definition **4.1.5** $m \in S$ is a **maximal element** of $S \subseteq A$ if:

$$\forall s \in S : m \leq s \Rightarrow m = s$$

A maximal element of a subset is thus an element of that subset for which no other element is larger. Since in a partial order it is not necessary that all elements can be compared, there can be more than one maximal element (these maximal elements are then incomparable). An element that is larger than all other elements is defined as an upper bound:

***Definition* 4.1.6** $u \in A$ is an **upper bound** of a set $S \subseteq A$ if:

$$\forall s \in S : s \leq u$$

A maximal element is not necessarily an upper bound—it is possible that it is incomparable with some elements. An upper bound does not have to be part of the set it is the upper bound of. However, if we demand that the upper bound be part of the set, we call this a greatest element:

***Definition* 4.1.7** $g \in S$ is the **greatest element** of a set $S \subseteq A$ if:

$$\forall s \in S : s \leq g$$

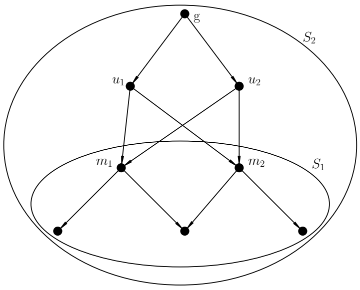

Figure 4.1: m_1 and m_2 are maximal elements of S_1, while u_1, u_2 and g are its upper bounds. g is the greatest element of S_2. A downwards arrow from a to b means $a \geq b$.

In a total order, a maximal element and the greatest element coincide, and we call this the **maximum**. There can be at most one greatest element of a set (since if there were two greatest elements, g_1 and g_2, it follows from the definition that $g_1 \leq g_2$ and $g_2 \leq g_1$. Then, because of antisymmetry, $g_1 = g_2$). There does not have to be a maximal element, upper bound or greatest element in a set (in a finite set there is always a maximal element, but in an infinite set this is not necessary, for example in the order $(\mathbb{R}, \leq)$).

Similarly, if we take $\geq$ instead of $\leq$, we can define for a set a **minimal element** (an element in the set for which no element is smaller), a **lower bound** (an element which is smaller than all elements in the set) and the **least element** (a lower bound that is in the set).

A set can have more than one upper bound, but the smallest upper bound is the most interesting. That is why we define a supremum and infimum:

***Definition* 4.1.8** The **supremum** of a set $S \subseteq A$ is the least upper bound, i.e. an upper bound $a \in A$:

$$\text{For all upper bounds } u \text{ of } S : a \leq u$$

Similarly, the **infimum** of a set $S \subseteq A$ is the greatest lower bound, i.e. a lower bound $b \in A$:

$$\text{For all lower bounds } l \text{ of } S : l \leq b$$

A supremum or infimum does not have to exist, like for set S_1 of figure 4.1. If it exist, it is unique (since the greatest and least element of a set is unique). We denote the supremum of a set S as $\mathbf{sup}(\boldsymbol{S})$, and the infimum as $\mathbf{inf}(\boldsymbol{S})$. If any two elements have a supremum and infimum, we speak of a lattice:

***Definition* 4.1.9** A **lattice** is a partially ordered set $(L, \leq)$ for which any two elements have a supremum and an infimum in L.

If we only demand that any two elements have a supremum (and thus not necessarily an infimum), we speak of an **upper semi-lattice**, while if we only demand that any two elements have an infimum, we speak of a **lower semi-lattice**.

A total order is a lattice: any two elements a, b are comparable, take $a \leq b$, and then b is the supremum of $\{a, b\}$, and a is its infimum.

A lattice can have no more than one maximal and minimal element. Since if there were two distinct maximal elements m_1, m_2, these elements would have a supremum, $\sup(m_1, m_2)$ distinct from at least one of the elements, which would be greater than that element, implying that that element is not maximal.

***Definition* 4.1.10** A **bounded lattice** is a lattice that has a greatest element and a least element.

A finite lattice is necessarily bounded: it has (only one) maximal element, which is the greatest element of the set: the supremum of the maximal element with another element is always the maximal element itself (since the supremum cannot be bigger than the maximal element, by definition of maximal element), and thus the maximal element is bigger or equal than all other elements, since it is a supremum of it.

The greatest element is often noted as 1, while the least element is noted as 0.

When there are only two elements, we denote

$$a \wedge b := \inf(a, b)$$
$$a \vee b := \sup(a, b)$$

If we consider the relation $\subseteq$ over sets, $\wedge$ is the intersection $\cap$, and $\vee$ is the union $\cup$, since we can prove that $\inf(A, B) = A \cap B$, and $\sup(A, B) = A \cup B$. This can be linked to how in logical operations $\wedge$ denotes **AND**, while $\vee$ denotes **OR**. I.e. "Property $A \wedge$ Property B" means both property A and property B holds, while "Property $A \vee$ Property B" means property A or property B should hold (they can also both hold). Define a property set as all the states for which a property is fulfilled. Then "property A and property B" holds in the intersection of the property sets of A and B,while "property A or property B" holds in the union of both property sets.

A lattice can also be directly defined by a set L and two binary relations $\wedge, \vee$:

***Definition* 4.1.11** $(L, \wedge, \vee)$ is a lattice if following properties hold

- commutativity: $a \wedge b = b \wedge a$ and $a \vee b = b \vee a$
- associativity: $a \wedge (b \wedge c) = (a \wedge b) \wedge c$ and $a \vee (b \vee c) = (a \vee b) \vee c$
- absorption: $a \wedge (a \vee b) = a$ and $a \vee (a \wedge b) = a$

We can now go back to a partial order by defining a relation as: $a \leq b \Leftrightarrow a = a \wedge b$ (or equivalent: $b = a \vee b$). We can prove that both definitions of a lattice are equivalent.

4.1.1 Emphasizing comparable or incomparable elements

↑ 79 Some of the sets of $(P(S), \subseteq)$ can be ordered, where we could say that one is "less" than another, but others cannot be compared (since it is a partial order, but not a total order). We could then look at certain families of subsets $F \subseteq P(S)$, where in some F, there will be more elements that can be compared than in other families.

First, consider $F_1 \subseteq P(S)$ for which following property P_1 holds:

$$\forall A, B \in F_1 : A \subseteq B \vee B \subseteq A$$

(this is a constraint on all the possible families of subsets, not all F fulfill, but there are still different F_1 possible). The relation $\subseteq$ restricted to a subset F_1

for which P_1 holds is a total order, thus $(F_1, \subseteq)$ is a chain. F_1 is a collection of a set, a bigger set containing this set, an even bigger set containing this bigger set, and so on (see figure 4.2). This can be made broader by defining another property P_2 as:

$$\forall A, B \in F_2 : A \cap B = \emptyset \vee A \subseteq B \vee B \subseteq A \tag{4.1}$$

An F_2 fulfilling P_2 thus consists of a set containing different subsets, who themselves contain different subsets, and so on - see figure 4.2. $(F_2, \subseteq)$ is not a total order, but any two sets can be contained in a bigger set, and thus have a supremum, or at least F_2 can be split into smaller families that are completely distinct, within which this property holds. What I mean by 'completely distinct' families is that F_2 can be partitioned in what I call a **set-disjoint partition**: a partition of F_2 for which if two sets are in different parts (subfamilies), they are disjoint. i.e., $F_2 = \bigcup_i G_i : \forall A \in G_i, B \in G_j, i \neq j : A \cap B = \emptyset$ (in a standard partition, it is only demanded that the elements in different parts are distinct, ie $A \neq B$). There is no relation whatsoever between these subfamilies—they are completely distinct—and we can limit ourselves to one subfamily. I prove the above statement and more in the following theorem:

Theorem 4.1.12 *A family F_2 for which property P_2 (4.1) holds either is an upper semi-lattice, or can be partitioned in a set-disjoint partition in which each subfamily is an upper semi-lattice.*

If $\emptyset \in F_2$, F_2 is either a lattice, or has a set-disjoint partition in which each subfamily is a lattice.

Proof For an upper semi-lattice, we have to prove that any two sets have a supremum. Thus consider two sets $A, B \in F_2$. If $A \subseteq B$ or $B \subseteq A$, B respectively A is the supremum of $\{A, B\}$. Thus consider $A \cap B = \emptyset$ (because P_2 holds this is the only other option). If $\exists C \supseteq A, D \supseteq B : C \cap D \neq \emptyset$, then either $C \subseteq D$ or $D \subseteq C$, and thus both A and B are contained in D (respectively C). $\{A, B\}$ thus has an upper bound. Since any two upper bounds have an intersection (since they both contain A and B), one is contained in another, thus all upper bounds are comparable, and there is a least upper bound. $\{A, B\}$ thus has a supremum.

If $\forall C \supseteq A, D \supseteq B : C \cap D = \emptyset$, there is no connection whatsoever between A and B, and we can consider the subfamily G_1 consisting of A, all the sets (of F_2) containing A, and all the sets (of F_2) contained in one of these sets. Similarly, we can consider a subfamily G_2 around B. These subfamilies are semi-lattices (since for any two sets within such a family, the above properties are met). Sets from different subfamilies do not intersect.

If there are still sets not in G_1 and G_2, we work further on $F_2 \setminus \{G_1, G_2\}$ like before, splitting it further into subfamilies if necessary. We can thus split F_2 in a set-disjoint partition of upper semi-lattices.

If $\emptyset \in F_2$, any two sets of F_2 also have an infimum: if the two sets are distinct, this is $\emptyset$, and otherwise one set is contained in the other, and then the smallest set is the infimum. Thus $(F_2, \subseteq)$ is a lower semi-lattice, and F_2, is either a lattice, or has a set-disjoint partition in which each subfamily is a lattice.
□

The previous properties tried to find sets which were comparable. We can also work the other way: to define properties in order to avoid comparability. For example, we could restrict possible $F_3 \subset P(S)$ by following property P_3 (see figure 4.2):

$$\forall A, B \in F_3 : A \not\subset B \wedge B \not\subset A \tag{4.2}$$

↑ 79 Possible maximal families fulfilling this property are all the sets with an equal number of elements (e.g., the family containing all the sets with two elements is a maximal family, and the same for another number of elements). Because if we consider F_3 as all the sets with i elements, then none of them can be contained in or contain another set of F_3 since they have the same number of elements (thus property P_3 (4.2) holds). Consider any other set. This set either has fewer elements than i, thus is contained in a set with i elements, or it has more elements than i, and thus contains a set with i elements. Adding any other set to F_3 results in property P_3 (4.2) no longer holding. Thus, F_3 is a maximal set fulfilling the property.

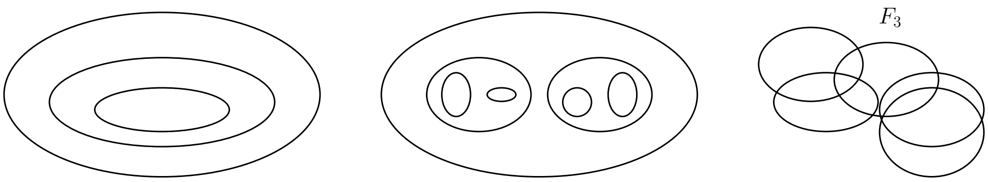

Figure 4.2: Examples of a family F_1 (left), F_2 (middle) and F_3 (right).

We can now use this principle to define another relation R_1, where we want to relate two sets when they are incomparably linked. We first define the **non-inclusively intersecting** sets of a set A as:

$$P'(A) := \{B \in P(S) : A \cap B \neq \emptyset \wedge A \not\subset B \wedge B \not\subset A\}$$

We consider all the sets which intersect with A, but which are not simply contained in A, and neither contain A.

A set in $P'(A)$ is the union of a proper subset of A, and a proper subset of the complement of A:

$$P'(A) = \{B \cup C : \emptyset \subset B \subset A; \emptyset \subset C \subset S \setminus A\} \cup A$$

(A is a special case that is still in $P'(A)$).

We can now define R_1 as:

$$(A, B) \in R_1 \Leftrightarrow B \in P'(A)$$

This relation is symmetric: if $B \in P'(A)$, then $A \in P'(B)$: A and B are interchangeable in the condition in $P'(A)$.

It is also reflexive: $A \in P'(A)$.

This transitive closure $T(R_1)$ is an equivalence relation (since we can prove that the other properties of R_1 still hold).

↑ 78 ↑ 78

We can use this mechanism to in general construct an equivalence relation R_2 starting from any strict partial order R as:

$$(a, b) \in R_2 \Leftrightarrow (a, b) \notin R \wedge (b, a) \notin R \tag{4.3}$$

(note that this is less strict than the definition in sets, where we also demanded the sets were intersecting). This relation is symmetric, as a and b are interchangeable in (4.3), and it is reflexive, since $(a, a) \notin R$ because R is a strict order. The transitive closure $T(R_2)$ is thus an equivalence relation.

However, when $R_2 \neq T(R_2)$ (thus when R_2 is not transitive), there are necessarily elements $(a, b) \in T(R_2)$ which are also in R (when they are not in R_2). They are elements which can be compared, and for which one is smaller than the other according to R, but which are considered equivalent according to $T(R_2)$. Thus, we can connect two comparable elements by a path of incomparable elements.

For sets, neither R_1 nor R_2 is transitive in $P(S)$ (in the latter non-intersecting sets are allowed). We can imagine $A \subset C$ where both A and C are non-inclusively intersected by a B. Thus $(A, B), (B, C) \in R_1$, hence $(A, C) \in T(R_1)$. We can however restrict R_1 to a certain family, for example, when we restrict R_2 to a family F_3, R_2 is transitive with respect to this family.

In general, it is not difficult to define an equivalence relation: for a set X, the relation $R = X \times X$ is an equivalence relation where all elements are equal.

↑ 78 An equivalence relation defines a partition into equivalence classes. We can now prove that in $P(S)$ all the sets except for the empty set, the set S itself, and the sets of one element, are equivalent according to $T(R_1)$:

Theorem 4.1.13 *The power set $P(S)$ of a set S is partitioned into equivalence classes by $T(R_1)$ as:*

$$\{F, \emptyset, S, \{s_1\}, ..., \{s_n\}\}$$

with $F = P(S) \setminus \{S, \emptyset,$ all one-element sets$\}$, $S = \{s_1, ..., s_n\}$.

Proof Consider $A, B \in F$, we will prove that $(A, B) \in T(R_1)$, then we have proven that F is (part of) an equivalence class. First consider that there exist $b \in B \setminus A$, and $a \in A \setminus B$. Since A and B have more than one element, $\{a, b\} \in P'(A) \cap P'(B)$, and thus $\{a, b\}$ is in R_1-relation with both A and B, thus $(A, B) \in T(R_1)$ (if $A \cap B \neq \emptyset$, we already have directly that $(A, B) \in R_1$). Now consider that this is not the case, for example that $B \setminus A = \emptyset$ (the case $A \setminus B = \emptyset$ is similar). This means that $B \subseteq A$. Then $\exists x \notin A, b \in B$, and $\{b, x\} \in P'(A) \cap P'(B)$ (since B and A have more than one element, and $A \neq S$). Thus $(A, B) \in T(R_1)$.

Now we prove that all other sets (not in F) form their proper equivalence class, thus they are equivalent with no other set. The empty set has an empty intersection with all sets, thus $\forall A \in P(S) : A \notin P'(\emptyset)$, thus $\emptyset$ is its proper equivalence class. Any set is contained in S, thus does not belong to $P'(S)$, thus S is its proper equivalence class. For a set $\{s_i\}$ with only one element, another set with one element is necessary completely distinct from it, and a set with more than one element that is not distinct from it, necessary encloses $\{s_i\}$. Thus $\{s_i\}$ is its proper equivalence class. □

F has no greatest element, and this holds in general for an equivalence class (that exist of more than one element) from an equivalence $T(R_2)$ constructed out of a strict partial order (4.3). Assume such an equivalence class $E(a)$ would have a greatest element $s \in E(a)$, thus $\forall b \in E(a) : b \leq s$. This means $(b, s) \in R$, thus $s \notin P'(b)$, hence $(b, s) \notin R_2$, $\forall b \in E(a)$. Since we assumed $s \in E(a)$, $(a, s) \in T(R_2)$. This means there exist $b \in E(a)$: $(b, s) \in R_2$, a contradiction. Thus $E(a)$ cannot have a greatest element.

4.1.2 Connection with previous chapters

Some of the ideas from the previous chapters can be explained with the above concepts. This will hopefully clarify these concepts further.

A relation is in general directional, i.e. (a, b) is not the same as (b, a) (we work with ordered pairs). We can understand this as a influences b. *Power over* ↑ 13 was understood as when a has influence over b, but b has no influence over a. This is an antisymmetric ↑ 78 relation, where whenever (a, b) exists, (b, a) doesn't (we can understand this more in general as when a can do something to b that this b could not do in return).

But another interpretation of power-over is when there is a difference in the sort of influence between the directions, where there is thus still two-directional influence. A power-over relation (a, b) can then be understood as that a has influence over b of sort 1, while b has influence over a of sort 2. Such a relation will still be anti-symmetrical, when b cannot have influence over a of sort 1.

↑ 77

Transitivity is when this *power-over*-relationship is transmitted, i.e. when if a has power over b, and b has power over c, it follows that a has power over c. We can imagine a situation where there is a cycle, with a having influence over b, b over c, and c over a, while there is no influence in the other direction. When "power over" is only understood directly (b has no direct influence on a), it is thus not transitive, but since b can then still influence a by influencing c, it is questionable whether we can still speak about "power over" (and thus whether we allow such cycles or demand transitivity). When the relation is also transitive, it is a partial order ↑ 78 (if we assume it is either reflexive or non-reflexive, thus either non-strict or strict, but we assume this is not that important since we usually compare different elements).

A **hierarchy** can be understood as an upper semi-lattice ↑ 81, i.e. a partial order where any two elements have a joint 'commander'. When it is finite and connected (which we usually assume), there is one top 'commander'. Further on, a (perfect) hierarchy ↑ 101 is understood in a more limited way, with some extra conditions added.

When it is moreover a total order ↑ 79, all elements can be but on one line. This is what was meant with the 'one-dimensionality' ↑ 16 in the previous chapter.

There is, however, a difference between a partial order and a total order. In a partial order, there can be elements that cannot be compared. A set containing different sets, which in turn contain different sets—F_2 in figure 4.2—is a partial order (it is, moreover, an upper semi-lattice). This is the

↑ 16

hierarchy of inclusion mentioned by Graeber (Graeber, 2007). This is different from what Graeber calls a linear hierarchy, which is the same as a total order (F_1 in figure 4.2).

In a social system (for example, a family inside a clan inside a tribe), at each level there is someone said to represent the whole (the head of the family, clan or tribe). It is thus possible to divide into social classes, depending on which level someone represents. These social classes then do form a total order (the heads of tribes form a class above the clan-leader-class, which is above the class constituted of the heads of family).

In general, we can create a total order from a partial order by first considering all the maximal elements as one class, bigger than everything else, and then leave all these maximal elements out, look which elements are now maximal, and put them together into one class, smaller than the one(s) already created, but bigger than anything else. Then we continue this procedure until all elements are put into a class. When the number of steps to go from a maximal to a minimal element is the same for all maximal and minimal elements, the resulting classes are the same when one starts at the bottom, putting all minimal elements in one class, and then moves up.

In a partial order, one order dimension can be distilled by projecting it to a total order.

A representative of a certain whole will set himself apart from the whole it represents and is, in this sense, a hierarchy of exclusion. From the perspective of a (representative of a) whole at a certain level, it is included in ever-increasing levels, while it simply includes an undifferentiated mass. In this sense it is a total order (namely F_1 from figure 4.2, sets contained in each other).

In a social system, usually both the partial order view and the total order view are at play. Consider that probably someone who has a higher rank, but is not a representative of your group, has to be treated differently than someone who is equal to you. Yet you will still not have to follow his commands like you will have to for your own representative.

4.1.2.1 Universality vs (local) coherence

↑ 84
↑ 84 ↑ 84

But we can also consider other structures where there is no ordering. For example, we could emphasize the sets in F_3 (which cannot be compared) or, on the contrary, the sets in F_1 or F_2. In general in a partial order, we could emphasize the elements that can be compared (if we only consider those, we have a total order), or those that cannot be compared (thus leaving the partial order empty).

We can relate this to the previous chapter. As humans, we usually do not have one main goal, which we then split up into sub-goals, which we further split up into more specific goals (F_2). We have several goals which are related to each other, sometimes benefit each other, but can also contradict each other. For example, I might both want to be physically fit and develop myself intellectually. Being fit may help me better concentrate, and my intellectual exploration may also lead to knowledge about how to be physically fit more easily. (But even if it doesn't, that does not mean I want to be fit simply to help my intellectual development. I might receive other benefits from physical fitness.) On the other hand, we must take into account the tradeoffs. If I constantly do sports, I cannot do any intellectual work. Thus, both goals also partly contradict each other. In general, our goal is more than mere survival (then we would be completely determined by nature), nor is it completely determined by culture. A human is a unique constellation, influenced by elements of both 'nature' and 'nurture', while both also influence each other. It is impossible to say what will come out of any interaction between them. ↑ 84

We are not determined solely by nature or solely by nurture (the nature vs nurture debate is about what determines us, which is impossible to say), but because of these interplaying influences. We become unique and autonomous, we go beyond what was determined for us. Further in this thesis, I develop how having multiple influences means you are not determined by them. ↑ 200

This relates to either having one main thesis, which is further developed into several sub-theses, and so on, versus having several ideas where some relate to others, and by this create a coherence. When writing a text, someone adopting the first view would start from an outline, where a main idea is taken and further elaborated into different points, which is then written out. In the other perspective, someone would elaborate some ideas, see how these are linked, and then create a broader vision from this. A metaphor for this is the two ways someone could solve a jigsaw puzzle.

Some will look at the depiction of the puzzle, take a piece and see where it fits in the picture, while others will try to fit pieces together (probably by some common property, like similar colors), and see which picture will come out. A global behavior emerges from local interactions, the definition of self-organization. Both strategies have advantages and disadvantages. In reality, we do not necessary know the big picture, and putting one image upfront and then fitting the pieces in it, will cause friction because the pieces might be on the right place in the image, but do not nicely fit together. The disadvantage of the other strategy is that it can be too undirected, the outcome might not be interesting. ↑ 39

↑ 34

Another example is regarding struggles: the view that the economic struggle is the most important, with all the other struggles subordinated or part of this struggle (e.g., gender oppression is simply caused by economic circumstances, thus is only a specific manifestation of economic oppression), versus seeing struggles as interrelated, influencing each other but with none more important than the other. This is the difference between seeing one struggle as contained in another, an seeing struggle as intersecting. This relates to considering the economic sphere to determine all the rest, versus considering an interplay between multiple spheres.

↑ 78

In reality, there is never a perfect partial order—there will always be cycles, things will go in two directions. But there might be an asymmetry, where it is easier to go one direction than the opposite, where certain connections are weaker than others. We always neglect certain connections when modeling reality (since it is too complex to consider all), but which connections we neglect and which we don't influences what kind of structure we see in the model.

4.1.3 Hierarchy according to Simon

↑ 84

For Simon (Simon, 1962), a **hierarchy** means "a system that is composed of interrelated subsystems, each of the latter being, in turn, hierarchic in structure until we reach some lowest level of elementary subsystem." This is a partial order in the form of a family F_2 consisting of a set containing several subsets who themselves contain several subsets, and so on. It is a partition where the parts can in themselves be partitioned into smaller parts.

↑ 15

But this is simply a structural definition, and says nothing about the relation of the subsystems to the bigger system. When the subsystems are subordinated by an authority relationship to the system they belong to, he speaks of a **formal hierarchy**. Before, I called this a functional hierarchy.

One could speak of a flat hierarchy when there are a lot of subsystems under one system, but Simon reserves the word hierarchy to a system that can be divided in only a small or moderate number of subsystems. In that sense a crystal existing of thousands of atoms, is not hierarchical.

Since in reality there will be interactions between the subsystems, the term **"nearly decomposable"** is introduced. This is when the interactions among the subsystems is weak, but not negligible. In the short term, the behavior between subsystems will be independent, while in the long run, the behavior of one subsystem only depends in an aggregated way on the behavior of the other subsystems. This influence will usually happen from the top of the system.

Simon explains why hierarchical systems evolve by the parable of two watchmakers. Both make a watch from 1000 parts, but the first does this as one big system, so that if he is distracted, he has to start from scratch. The second makes small subsystems, which are assembled into bigger subsystems, which are then put together in the whole system. When she is distracted, she only has to start again for the subsystem she was busy with, and the other subsystems remain. Thus, having subsystems that are already partly viable in themselves, makes it easier to build a bigger system.

However, it does not follow from this argument that the subsystems cannot intersect or should only have weak interactions. If a subsystem falls apart as soon as a system it intersects with falls apart (or is unfinished, as in the watchmaker example), a configuration with a lot of intersecting systems would indeed lead to cascading failures. But this assumes a system falls apart, rendering all its elements useless, as soon as one element is missing. But under this assumption, a hierarchical setup would neither work, as all its elements and thus subsystems would be useless until all elements are in place. We can thus assume the interruption of the construction of a system would not disrupt the systems it intersects with.

It is neither investigated whether the constructed system would be resilient. When only one subsystem performs a certain function, the system is quite fragile, as soon as one subsystem fails it stops functioning. **Redundancy** is when two systems perform exactly the same function, while **degeneracy** is when two systems share a function, but also have other functions. Here there is some overlap in functionality.

With the parable it is assumed there is one given overall function (a working watch). This is different from multiple agents who all have a different function and coordinate with others in order to better succeed as is the case in human social organization.

4.2 Mesarovic' notion of hierarchical system

I now give the formalization Mesarovic (Mesarovic et al., 2000) gives of a hierarchical system. This can partly be mapped to the structural notion of order from before, but like Simon, it also touches the functional dimension.

First, Mesarovic states that a hierarchical system should have three characteristics:

1. **Vertical arrangement**: the system consists of subsystems that can

be ordered into higher and lower level units.

2. **Priority of action/ right of intervention of higher level**: the actions and goals of a higher level system are considered more important; the operation of a subsystem is directly influenced by the actions of a higher system. This can be by changing the parameters of the lower system, or because the problem of the lower system depends on the solution of a higher level system—and it is thus only well-defined as soon as the problem at the higher level is solved. An action at a lower level can thus only happen after an intervention from the higher level.

3. **Dependence of higher level upon performance lower level**: success on a higher level, however, depends on the result of the actions on the lower levels. The result of the actions of a lower level can thus be interpreted as a feedback on the intervention of the higher level.

↑ 78

↑ 78

Vertical arrangement is when systems can be ordered, when there is thus an antisymmetry between a 'higher' and 'lower' level system. It is not made completely clear what is meant with 'ordered into higher and lower level units', but I assume this implies a partial order.

The second and third condition is a functional expression of what this direction entails: 'priority of action' can be understood as that the higher system designates the goals of the lower systems, while 'dependence of higher level upon performance lower level' means the lower level only has influence on how good the higher-level-goals are met. In this relation, influence happens in two directions, but there is a qualitative difference between the directions.

4.2.1 Three types of hierarchical systems

Mesarovic then gives three types of hierarchical systems, depending on whether it involves abstraction, decision or organization:

- **Abstraction**: each level concerns another level of abstraction: a higher level system is more abstract, while a lower level system is more detailed. Levels here are called **strata**, and we call this system a **stratified system**. There is an asymmetry: the requirements for proper functioning on one level act as constraints on a lower level.

- **Decision**: there is a family of decision problems, where each solution to a problem determines some parameters in the problem of the lower level. We call each of these levels a decision **layer**, and the system a **multilayer (decision) system**.

- **Organization**: here, there are multiple subsystems at each level. Some of these subsystems act as decision units, and these decision units are structured hierarchically, in the sense that some are influenced by other decision units. An example is a human formal organization. One characteristic is that the higher levels condition, but do not completely determine, the lower levels. A level here is called **echelon**, and the system is called a **multi-echelon system**.

These three types should however not be considered as distinct—a system can belong to more than one type. I will now give the formalization of these three types.

To define a stratified system, as illustrated in figure 4.3, we first define a **system** as a transformation from an input X to an output Y, which is mathematically represented as a mapping $X \to Y$. Imagine that these sets can be represented as Cartesian products, such that ↑ 77

$$S : X_1 \times ... \times X_n \to Y_1 \times ... \times Y_n$$

Assume that we can decompose these into n subsystems, where we map each X_i, together with some coupling B_i from below, and U_i from up, to Y_i. Thus:

$$\begin{aligned} S_n : &\quad X_n \times B_n \to Y_n \\ S_i : &\quad X_i \times B_i \times U_i \to Y_i, \text{ if } 1 < i < n \\ S_1 : &\quad X_1 \times U_1 \to Y_1 \end{aligned}$$

We call a system **stratified** if there exists functions $h_i : Y_i \to B_{i+1}$,$1 \leq i < n$, and $c_i : Y_i \to U_{i-1}$, $1 < i \leq n$, such that for all $\boldsymbol{x} \in X$, and $\boldsymbol{y} = S(\boldsymbol{x})$:

$$\begin{aligned} y_n &= S_n(x_n, h_{n-1}(y_{n-1})) \\ y_i &= S_i(x_i, h_{i-1}(y_{i-1}), c_{i+1}(y_{i+1})), \text{ if } 1 < i < n \\ y_1 &= S_1(x_1, c_2(y_2)) \end{aligned}$$

This means that we can create each sub-output Y_i purely from the sub-input X_i and some mapping of the output of the systems immediately above and below.

Notice that all these definitions are completely symmetrical, and thus it does not matter what you call ‘top’ and ‘bottom’. Mesarovic calls h_i an information function, and c_i a decision function, implying that there is a difference, with a lower system providing information to a higher system that issues its decision to the lower system. But mathematically, there is no difference.

Mesarovic does mention the special case were only the lowest system gets any outside input, which of course leads to a direction (thus where $X_i = \emptyset \, \forall i \neq 1$, or $X = X_1$). This then leads to a reduction in information when going up. One way this reduction can be achieved, is by aggregation: a subfamily of variables of the lower level is represented as one aggregated variable in the higher level. One can see this as the lower level consisting of several subsystems, with interactions in between them. The lower level is mainly concerned with the working of these subsystems, neglecting the interactions, while the higher system looks at the interactions between them, and neglects what is going on inside them.

This reduction of information the more one goes up, also implies that usually, the higher systems consists of less units.

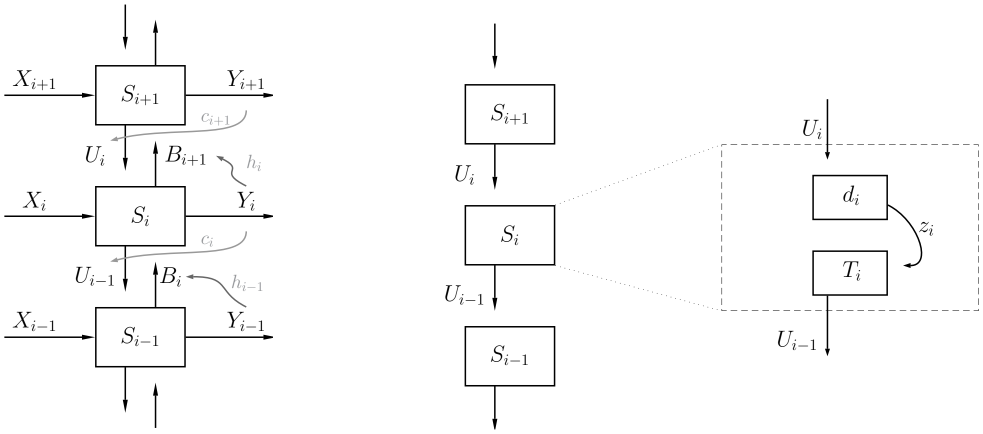

Figure 4.3: Left: stratified system. Right: Multi-layer (decision) system.

For a **layer hierarchy**, each subsystem is a mapping $U_i \to U_{i-1}$. This is illustrated in figure 4.3. These subsystems can be decomposed as decision-making systems, as defined now.

Consider a system $S : X \to Y$. It is a **decision-making system**, if each $x \in X$ defines a decision-problem D_x. Z is the solution set of these problems. Then there is a mapping $T : Z \to Y$. Thus, (x, y) is in the system S (or equivalent, S maps x to y), if and only if there exist a z such that z is a solution of D_x and $T(z) = y$. What this basically does, is splitting a system into a decision unit D_x and an implementer T.

Here, there is a clear antisymmetry, where inputs always come from higher level decision-making systems.

To define a multi-echelon hierarchy, we first define a **decision-making hierarchy** as the pair $(P, >)$, with P a family of decision-making systems S_i, $i \in I$, with I a finite index set. $>$ is a strict partial order, defined as

↑ 78

$$i > j \Leftrightarrow S_i \text{ has priority of action over } S_j$$

A **multi-echelon hierarchy** is now defined as a decision-making hierarchy for which any two systems S_i and S_j have at most one system S_k immediately above it. The condition Mesarovic gives does not behave as claimed. To clearly define it, we need a formal definition of 'immediately above':

***Definition* 4.2.1** k is **immediately above** i, also stated as that k is a **direct commander** of i, is defined as:

$$k > i \ \wedge \nexists l : k > l > i$$

'Direct command' is only a condition of the order relation, and does not presume any meaning (like 'command') of this relation. The condition of a multi-echelon hierarchy that any two systems have at most one system immediately above it, is thus:

$$\forall i, j \in I \ (\nexists \vee \exists!) k : k > i, k > j : \nexists l \in I : k > l > i \wedge l > j \tag{4.4}$$

These classifications can be interpreted in the language of the first section.

↑ 76

The description of a stratified and multi-layer system implies a total order—systems are put on a line, with the highest one on top, and the lowest at the bottom. This is mainly because a level is taken as one whole system, without considering the subsystems.

A multi-echelon hierarchy does consider these subsystems, and is thus a (strict) partial order. We further have following property:

Property 4.2.2 *Condition 4.4 is equivalent with each element having at most one direct commander. Thus any system having at most one system immediately above it, is equivalent with any two systems having at most one system immediately above both.*

Proof When condition 4.4 holds, we can take $i = j$, and this implies one system has at most one direct commander.

When each system has at most one direct commander, if there would be i, j for which condition 4.4 does not hold, there would thus be k_1, k_2 directly above i and j. Both k_1 and k_2 are then directly above i, which has thus two direct commanders, a contradiction. Thus condition 4.4 holds. ☐

This condition means any two systems have at most one minimal upper bound, which would thus be a least upper bound, a supremum. It does not mean, however, that any two elements have a supremum, since there might be no minimal upper bounds. There can be two elements on the highest level that have no element above. But two elements i, j without a common upper bound, are necessarily unconnected. There is no common connection with any higher level, and if there would be a lower element l for which $l < i$ and $l < j$, l would have two direct commanders (not necessary i and j, it might also be a $k_1 < i$ or a $k_2 < j$). We can consider only the elements that are connected, and here, any two elements have a supremum, it is an upper semi-lattice. If it is finite, it has one greatest element, the highest level consists of only one system.

↑ 99

In the following chapter, I will repeat and polish this argumentation.

↑ 126

At the end of this thesis, I will formulate Mesarovic's model of coordination in a hierarchical structure, generalize it to any network, and put three models into this framework—one of them being the model of controllability discussed further in this chapter.

↑ 206 ↑ 118

4.3 Definitions graph

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There is quite some literature that investigates networks, for example, the review paper (Dorogovtsev et al., 2008). This section is based on this previous work in network theory and graph theory. One way to represent network structures is as a graph:

***Definition* 4.3.1** A **graph** G is defined as the set of two sets: $G = \{V, E\}$, with V the set of **nodes**, and E the set of **edges**. An edge $e \in E$ connects two nodes, and can be represented as $e = \{v_i, v_j\}$, with $v_i, v_j \in V$.

I will use the terms edge, link and connection interchangeable. A (normal) graph is thus undirected. There is simply a connection between two nodes—it is not specified from which node the connection starts, and in which node it ends (i.e., $\{v_1, v_2\}$ is a set, without ordering). When we want a connection to have a direction, we use a directed graph:

↑ 76

***Definition* 4.3.2** A **directed graph** G is the set of two sets: $G = \{V, E\}$, with an $e \in E$ having the form $e = (v_i, v_j)$, with $v_i, v_j \in V$.

(v_i, v_j) is thus a couple, $(v_i, v_j) \neq (v_j, v_i)$. E can also be viewed as a relation

↑ 77 ↑ 77

on V: $E \subset V \times V$. We can represent a directed graph by an $n \times n$ matrix A, with $n = |V|$ the number of nodes, and with $a_{ij} = 1$ if there exists an edge from v_i to v_j, and $a_{ij} = 0$ otherwise.

An undirected graph can also be represented by an $n \times n$ matrix A, but this is now a symmetrical matrix, for which $a_{ij} = a_{ji}$, since we do not differentiate in direction. $a_{ij} = 1$ thus means that there is an edge $e = \{v_i, v_j\}$.

In these graphs, there is no difference in strength of the edges: there is either a link or not. To allow for more continuity, so that there can be nodes which are weakly linked, and others stronger, we introduce the concept of a weighted graph:

***Definition* 4.3.3** A **weighted graph** can be represented as an $n \times n$ matrix A, with n the number of nodes, and a_{ij} representing the weight of the link from v_i to v_j. When $a_{ij} = 0$, there is no edge from v_i to v_j.

When A is symmetrical, we have a normal weighted graph, otherwise it is a directed weighted graph.

A **clique** is a subset of nodes that are all connected to each other, i.e. a $C \subseteq V$ for which applies $\forall a, b \in C : \{a, b\} \in E$. Such a clique can be seen as a grouping of nodes. However, groups often behave differently than as a clique.

A graph presumes there is a one-to-one connection between nodes, by edges. But this is sometimes too simple for reality. We meet up with other people in groups, which have other dynamics than each of its members meeting everyone else separately. On the internet, we post messages on forums, which lead to other results than emailing someone personally. The behavior of such a group is different than a collection of one-to-one interactions. A clique can only describe one-to-one interactions. While in most groups, an interaction is between all the members of the group at once.

A hypergraph formalizes this idea (Johnson, 2013). It is a generalization of a graph where an edge can connect more than two nodes. Such a hyperedge can hence be seen as a group, a platform that enables group interaction. A hyperedge leads to different behavior than a clique: a message can only be send to all or none of the members of a hyperedge, while an agent in a clique can decide to send a message to only some of the members of the clique. A hypergraph is defined as follows:

***Definition* 4.3.4** An (undirected) **hypergraph** is a couple of two sets (V, E), with an element $E_j \in E$ a subset of V. We thus have $E \subseteq P(V)$. An element $v_i \in V$ is called **node**, an element $E_j \in E$ is called **(hyper)edge**.

We can represent this by a $|V| \times |E|$-matrix R, where $R_{ij} = 1$ if $v_i \in E_j$, and 0 otherwise. All connections have the same strength this way. If we want the links to have weights, we'll work with a matrix of weights W, with $W_{ij} \in [0, 1]$ the weight of vertex v_i in edge E_j. In an undirected hypergraph, this is the same as the weight from E_j to v_i. This won't be the case in a directed hypergraph, which is defined as follows:

***Definition* 4.3.5** A directed **hypergraph** is a couple of two sets (V, E), with an element $E_j \in E$ a couple (I_j, O_j) of two subsets of V. We thus have $E \subseteq P(V) \times P(V)$. I_j is called the set of **input nodes** and O_j is called the set of **output nodes**.

A weighted directed hypergraph can be represented by two matrices: a $|V| \times |E|$-matrix W, giving the weights from vertices to edges, and an $|E| \times |V|$-matrix Z, which represents the weights from edges to nodes.

A standard measure to investigate in these networks, is the degree. That is, how many other nodes does a node connect? We define this concept in the different structures.

***Definition* 4.3.6** The **degree** of a node v (with $v \in V$) in an (undirected, unweighted) graph is defined as the number of edges that contain v. This is the same as the number of neighbors of v, where a **neighbor** is a node which is connected by an edge with v. This is $|\{\{v, v_i\} \in E, \text{with } v_i \in V\}|$.

In a directed graph, we have to differentiate whether an edge is pointing towards or away from a certain node:

***Definition* 4.3.7** The **in-degree** of a node v ($v \in V$) in an (unweighted) directed graph is the number of edges that point towards v, this is
$|\{(v_i, v) \in E, \text{with } v_i \in V\}|$.

***Definition* 4.3.8** The **out-degree** of a node v ($v \in V$) in an (unweighted) directed graph is the number of edges that start from v, this is
$|\{(v, v_i) \in E, \text{with } v_i \in V\}|$.

We now extend this definition to a weighted graph:

***Definition* 4.3.9** In an (undirected) weighted graph, the **degree** of a node v_i is the sum of all the weights of the edges it is contained in, this is $\sum_j a_{ij}$.

We again differentiate two sorts of degrees in a directed weighted graph:

***Definition* 4.3.10** The **in-degree** of a node v_i in a directed weighted graph is the sum of all the weights of the edges that points towards it, this is $\sum_j a_{ji}$.

The **out-degree** of a node v_i in a directed weighted graph is the sum of all the weights of the edges that starts from it, this is $\sum_j a_{ij}$.

These three formula's for the degree also work when A represents an unweighted graph. They correspond to the previous definitions.

A hypergraph has two sorts of degree:

***Definition* 4.3.11** The **degree of a node** v_i in an (undirected, unweighted) hypergraph, is the number of hyperedges it is contained in, this is $|\{E_j \in E : v_i \in E_j\}|$.

The **degree of a hyperedge** E_j is the number of nodes it contains, this is $|E_j|$.

These definitions can easily be extended to a directed and weighted hypergraph as we did for a graph.

A **hub** is a node with a degree greatly bigger than the average degree.

The following definitions permit how nodes can be connected by more than one direct edge. I will assume a directed graph, but these definitions can easily be extended.

***Definition* 4.3.12** A **path** is a (possibly infinite) sequence of edges of the from: $(a_i, a_j), (a_j, a_k), (a_k, a_l), ..., (a_q, a_r)$. The ending node of the previous edge is thus the beginning node of the next edge.

***Definition* 4.3.13** A **cycle** is a directed path starting and ending in the same node, i.e. $(a_i, a_j), (a_j, a_k), (a_k, a_l), ..., (a_q, a_i)$.

In an undirected graph, we have:

***Definition* 4.3.14** $S \subseteq V$ is a **connected** component of V if it is a set of nodes for which there exist a path between any two nodes. Two components are **unconnected** if there does not exist a path between any two nodes of different components.

A similar definition can be used in directed graphs. However, we often understand a component connected when it is connected in the undirected version of the directed graph. Thus, in the first definition, two nodes are only connected when there exist a path between them, a traverse of edges which keeps on pointing in the same direction. While in the second definition, two nodes are also connected when the direction of the edges change when traversing. For example, if a node points to another node, which is pointed at by yet another node, this last node is connected with the first one.

4.3.1 From hypergraph to standard graph

A hypergraph can be pulled back to a standard graph. In the following sections we will work with an undirected hypergraph. Note that this can easily be extended to a directed hypergraph by using Z, the matrix of the weights from an edge to a node, instead of W^T. W^T is the **transpose** of matrix W, obtained by turning matrix W, turning rows into columns and vice versa. Thus $w^T_{ij} = w_{ji}$.

W^T represents the weights from an edge to a node, which in an undirected hypergraph is the same as the weights from a node to an edge. In a directed hypergraph this is different, and the weights from an edge to a node is here represented as Z.

One way to represent a hypergraph is by a bipartite graph with two kinds of nodes: one sort being the nodes, the other sort being the hyperedges of the hypergraph (see figure 4.4). Two nodes of different sorts are connected in the graph representation, if the node is in the hyperedge in the original hypergraph structure.

Another way to build a standard graph out of a hypergraph is to put an edge between all nodes that are in the same hyperedge (see figure 4.4). Consider two nodes v_i and v_k which are in the same hyperedge E_j. The contribution of E_j to the weight a_{ik} between v_i and v_k is $w_{ij}w_{kj}$. The total weight is obtained by summing over all the hyperedges which contain both v_i and v_k, thus:

$$a_{ik} = \sum_{j=1}^{|E|} w_{ij}w_{kj} = (WW^T)_{ik}$$

or in matrix notation:

$$A = WW^T$$

For a directed hypergraph, we use Z instead of W^T, thus

$$A = WZ$$

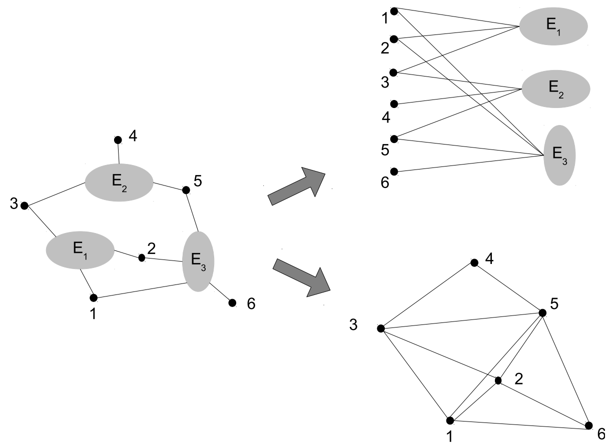

Figure 4.4: An example of two ways to represent a hypergraph.

Note that there is some information lost: it is impossible to know in this representation which nodes form a hyperedge.

4.4 Three coordinates of hierarchy, in directed graph

In *'On the origins of hierarchy in complex networks'* (Corominas-Murtra et al., 2013), three coordinates of hierarchy are given, and several networks are compared by putting them in a three-dimensional space. They work with a directed graph with unweighted links (thus there is either a link or not from one node to another). ↑ 96

The soon presented three measures spring from making a more continuous version of what is defined as a perfect hierarchy (see bottom figure 4.5). Consider a set A and a relation $R \subset A \times A$. This thus defines ordered pairs of elements of A, and is equivalent to a directed link between two nodes. $T(R)$ is the transitive closure of R. When using the representation as a directed graph, $T(R)$ is the set of all (a_i, a_j) for which a path from a_i to a_j exist. To understand this formalism better in the common usage of hierarchy, you can consider $(a_i, a_j) \in T(R)$ as a_i 'commanding' a_j. $(a_i, a_j) \in R$ is translated as that a_i is a direct commander of a_j. A **perfect hierarchy** is then defined as having following properties:

- *order*: if $(a_i, a_j) \in T(R)$, then $(a_j, a_i) \notin T(R)$

(There are no cycles. One cannot at the same time command and be commanded by a certain element.)

- *reversibility*: if $(a_i, a_k) \in R$ and $(a_j, a_k) \in R$, then $a_i = a_j$.
  (Every node is pointed to by at most one other node. Every element has only one direct commander.)
- *pyramidal structure*:
  - if $(a_i, a_j) \in R$, then $\exists a_k \neq a_j : (a_i, a_k) \in R$
    (Whenever there is a link starting from a node, there is more than one. A commander commands more than a single element.)
  - $\exists! a_i \in A : \forall a_j \in A \setminus a_i : (a_i, a_j) \in T(R)$
    (There is exactly one node, for which a path exists to all other nodes, and which is thus 'commanding' all. Since there is also 'order', this element is not commanded by any other element, and it is the only element with that property.)
  - For a_i, consider $J_i = \{a_k \in A : (a_k, a_i) \in T(R)\}$. For any a_i, a_j for which $\nexists a_k \in A : (a_i, a_k) \in R$ (nor $(a_j, a_k) \in R$), $|J_i| = |J_j|$.
    (All the elements at the bottom have a chain of commands of the same length.)

The 'order' property tells $T(R)$ should be antisymmetric. Since it is also transitive, $T(R)$ is thus a partial order (if we consider the strictness property as non-essential). 'Reversibility' is the same as condition 4.4 of the previous section (since we proved there that each element having at most one direct commander is equivalent to any two elements having at most two direct commanders). The concept of 'direct command' is used a bit differently here and in the previous section. Here it is used as belonging to R (where $T(R)$, expresses 'command'). But because of the reversibility condition, when $(a_i, a_j) \in R$, a_i is immediately above a_j (since any other element above i, thus $(a_i, a_k) \in R$, should be equal to a_j). Thus the two definitions coincide. ↑ 78

In the previous section, we proved that the second property of a pyramidal structure (that there is one greatest element), actually holds automatically when the partial order is finite, connected and reversibility holds. This property is not fundamental, but follows from the other properties (as long as we work with finite connected structures). A perfect hierarchy is thus an upper semi-lattice, with the extra conditions that a commander commands more than one element, and that the chain of command from a bottom element to the top is always of the same length. ↑ 96 ↑ 81

Applied to the set-relation $\subseteq$, reversibility means there cannot be a set A that is part of two incomparable sets, where A would be the non-inclusive intersection of the two sets. Families for which P_1 or P_2 holds fulfill the required properties for reversibility. (In these families there are no non-inclusively intersecting sets). See figure 4.2 for an illustration of families F_1 and F_2. The first property of the pyramidal structure (that a commander commands more than a single element), holds in maximal families F_2. But it does not hold in a family F_1. In general, it does not hold for a total order since, in the total order case, everything can be put on a line. Thus, if they are two elements smaller than an element i, they are comparable, with one bigger than the other. This other element is not directly commanded by i. ↑ 84

Next, Corominas-Murtra et al. generalize the properties of a perfect hierarchy into three measurements: orderability, feedforwardness, and treeness. The first measure, *orderability*, calculates the degree to which the 'order' property is fulfilled, while the *feedforwardness* measure takes into account the position of cycles relative to the top. *Treeness* considers the degree to which *reversibility* and *pyramidal structure* are met.

These measures are constructed by taking into account the **condensed graph** G_C (see figure 4.5), in which every strongly connected component (SCC) gets compressed into one node i, with a weight α_i representing the number of nodes in the original component. Thus, $G_C = (V_C, E_C)$ with $V_C \subseteq P(V)$ consisting of the sets of strongly connected components. A node v_i that does not get condensed, and is thus its own proper SCC, has a weight of 1, and $v_i \in V_C \cap V$. A strongly connected component is a subgraph for which any node is reachable by any other node in the subgraph, and which is maximal in this respect. An intuitive way to understand an SCC is to view it as a cycle, although an SCC can be more complex than a cycle. For example, two overlapping cycles can be an SCC. However, I will often use the term cycle for an SCC because this is more intuitive. If there is any link going to or starting from a node in an SCC, there will be a link created to or from the newly created node that represents this SCC. Thus, $(v_{Ci}, v_{Cj}) \in E_C$, with $v_{Ci}, v_{Cj} \in V_C$, if $\exists v_k, v_l \in V : v_k \in v_{Ci}, v_l \in v_{Cj}, (v_k, v_l) \in E$.

Orderability measures the fraction of the nodes of the graph G that does not belong to any cycle. It is defined as:

$$O(G) = \frac{|\{v_i \in V_C \cap V\}|}{|V|}$$

with V_C the set of nodes of G_C, the condensed graph. $v_i \in V_C \cap V$ is a node which is the same in the condensed and original graph. It does not belong to any cycle and has a weight $\alpha_i = 1$. The idea behind this measurement is that nodes in a cycle cannot be ordered, thus, the fewer nodes are in a cycle, the greater the orderability of the graph. Being able to order things is generally considered to be an indication of hierarchy. We work with a fraction, $O \in [0, 1]$, the bigger O, the more the structure is considered hierarchical. Since this measure assesses how many nodes are in a cycle, it is a more fuzzy version of the property of 'order' in the definition of a perfect hierarchy. When there are no cycles, there is 'order'. ↑ 101

Feedforwardness is a measure that weights the impact of cyclic modules on the feedforward structure. By feedforward structure I mean we traverse the graph from the top elements to the bottom. Here the idea is that the higher a cycle is in the structure, the more influence it will have (the more nodes below will depend on it). Thus a cycle at the top will lead to a less hierarchical structure than one at the bottom. To define this measurement, consider in G_C the set Π_M consisting of all paths starting from a **maximal node**. A node is maximal when its in-degree is zero, thus there are no links pointing to it. A node is minimal when its out-degree is zero, thus there are no links starting from it. For a maximal node $m \in M$, there are a finite number of paths $\{\pi_1, ..., \pi_n\} = \Pi_m$ that start from this node. Then for every path $\pi_k \in \Pi_M = \bigcup_m \Pi_m$, the proportion of the number of nodes it contains (in G_C) over the nodes it actually represents (in G), is calculated:

$$F(\pi_k) = \frac{|v(\pi_k)|}{\sum_{v_i \in v(\pi_k)} \alpha_i}$$

with $v(\pi_k)$ the set of nodes in π_k. The feedforwardness of a graph is now simply the average over all the paths:

$$F(G) = \frac{\sum_{\pi_k \in \Pi_M} F(\pi_k)}{|\Pi_M|}$$

Thus, if a cycle is somewhere at the top, it will be counted in more paths. This will make the cycle count more and lead to a smaller feedforwardness. Since we work with the average of fractions, $F \in [0, 1]$.

Treeness tells how pyramidal the structure is, and how unambiguous its chain of command. The idea is that in a tree, starting from the top there are multiple paths going down (expressed in the first property of a pyramidal structure). However, if you go in reverse order, going from bottom to top, there is only one possible path to follow (expressed in reversibility). Thus, this measure is defined by first considering the set of maximal nodes M, and the sets of minimal nodes μ. The **path** entropy of a maximal node $m \in M$ is then: ↑ 62

$$h_f(m) = -\sum_{\pi_k \in \Pi_m} P(\pi_k|m) log P(\pi_k|m)$$

with $P(\pi_k|m)$ the probability that the path π_k is followed, starting from node m (I will not elaborate here how this probability is calculated). This is a measure of how uncertain it is to follow a certain path. When there is only one path, the entropy is zero, there is complete certainty about which path to follow. The more paths exist, the greater the entropy will be. Then, this measurement is averaged over all maximal nodes, which is called the **forward entropy** of G_C:

$$H_f(G_C) = \frac{1}{|M|} \sum_{m \in M} h_f(m)$$

To compute the path entropy going in the other direction (from bottom to top), the direction of all links in the graph is reverted. In this new graph, the minimal nodes become maximal nodes. In this graph, $h_b(u)$, with $u \in \mu$, is calculated the same as $h_f(m)$. $H_b(G_C)$ is the average of these $h_b(u)$'s over all $u \in \mu$, and is called the **backward entropy** of G_C. The normalized difference between these two measurements is then calculated:

$$f(G) = \frac{H_f(G_C) - H_b(G_C)}{max(H_f(G_C), H_b(G_C))}$$

Finally, the treeness is calculated by averaging this measurement over W_{G_C}, the set of graphs obtained by iterative leaf-removal:

$$T(G) = \frac{1}{|W_{G_C}|} \sum_{G_i \in W_{G_C}} f(G_i)$$

W_{G_C} is obtained by starting from G_C and iteratively adding graphs obtained by either bottom-up or top-down leaf-removal of the previous graph (see figure 4.5). Leaf-removal in a top-down way means removing all the maximal nodes. This graph is then added to W_{G_C}, and the maximal nodes in this graph are now removed. This process continues until no nodes are left. With leaf-removal in a bottom-up way, the minimal nodes are removed.

When there are no links, we define $f(G) = 0$. In general, $T \in [-1, 1]$. When $T < 0$, the structure is anti-pyramidal, with, in general, a lot of nodes at the top leading to one or only a couple of nodes at the bottom. $T > 0$ is a pyramidal structure, with only a couple of nodes at the top, branching in a lot of directions. $T = 0$ corresponds to an evened structure.

Figure 4.5 shows an example of a graph that has low values for all three hierarchy measures.

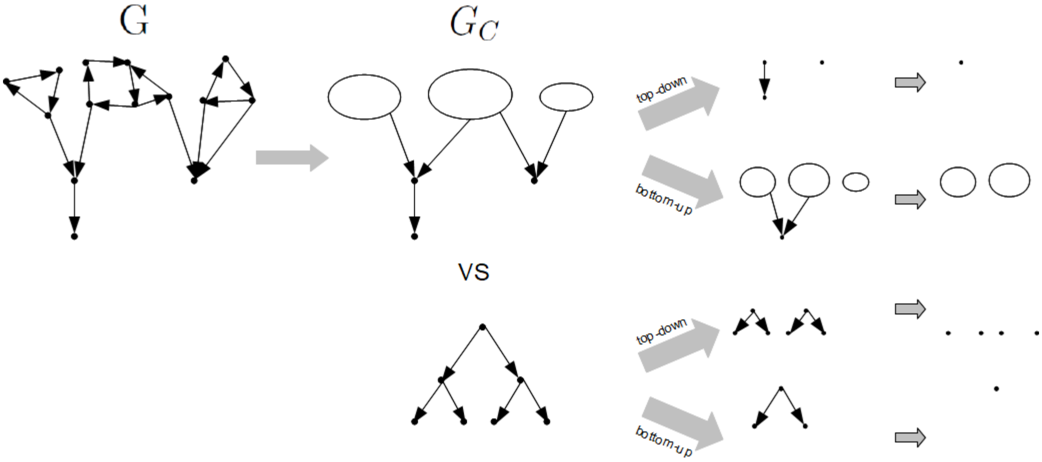

Figure 4.5: The transition from a graph to its condensed graph, followed by the graphs obtained by top-down or bottom-up leaf-removal. Top shows an anti-hierarchical structure, while below is a perfect hierarchy. Here the graph and the condensed graph is the same, since there are no cycles. In the anti-hierarchical structure, there are cycles, which are on top (low orderability and feedforwardness), there is more uncertainty about which paths to follow from top to bottom than the other way around, and chains have unequal lengths (low treeness).

↑ 101

I now discuss how this measure relates to the properties of a perfect hierarchy. Reversibility (that there is at most one node pointing to a certain node) is when the backward entropy is zero. Starting from an element at the bottom, there is only one way of going up (and this remains when nodes are removed through leaf removal). The first property of a pyramidal structure, that a commander commands more than one element, makes the forward entropy greater. In turn, this results in a greater number of paths that can be followed. The second property of a pyramidal structure is that there is only one maximal node, implying that there should necessarily be more paths starting from it (assuming we consider a connected graph). Thus, the forward entropy will be greater. The paths starting from the bottom should eventually converge to the one maximal node, making fewer paths possible and lower backward entropy. When the paths from the bottom to the top are not equal (violating the last property of a pyramidal structure), this will be punished for by considering all the graphs obtained by leaf removal. When there are paths with unequal lengths, at a certain moment, a link with the top node will be destroyed while there are still other paths, thus reducing the possible paths that can be taken from the top. This reduces the forward entropy.

4.5 Classification by degree and cluster coefficient

In this section, I classify hierarchy in a normal, undirected, unweighted graph. Here, hierarchy is understood as the unequal distribution of influence among all the nodes in a network (i.e. some nodes have more influence than others).

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2014b

4.5.1 Degree

We can look at the distribution of degree across the network—whether most nodes have the same degree or some nodes have a higher degree than others. This is done by looking at the probability distribution P, where $P(k)$ is the probability that a node has degree k. Through this distribution, two types of networks are defined: random networks and scale-free networks. A **random network** is a network that is constructed by a random process. In a random network, P follows a normal distribution: nodes vary around the mean degree, the further away from the mean, the fewer nodes of that degree. That is why in subsequent sections and chapters I will consider a network random when it has a normal probability distribution. In a **scale-free network** P follows a power-law:

$$P(k) \sim k^{-\lambda},$$

with λ some constant (Bollobás, 2001; Barabási et al., 2003; Newman, 2003). Thus, most nodes have a low degree and there are only a few nodes with a high degree. See figure 4.6 for the plots of the distributions.

A rather intuitive way to determine hierarchy in a network would be to look at the distribution of the degree across the network. The idea is that nodes with a higher degree have more influence. By extension, a hierarchical network could be defined as a scale-free network, and a non-hierarchical network could be defined as a random network. But this is a pretty naive

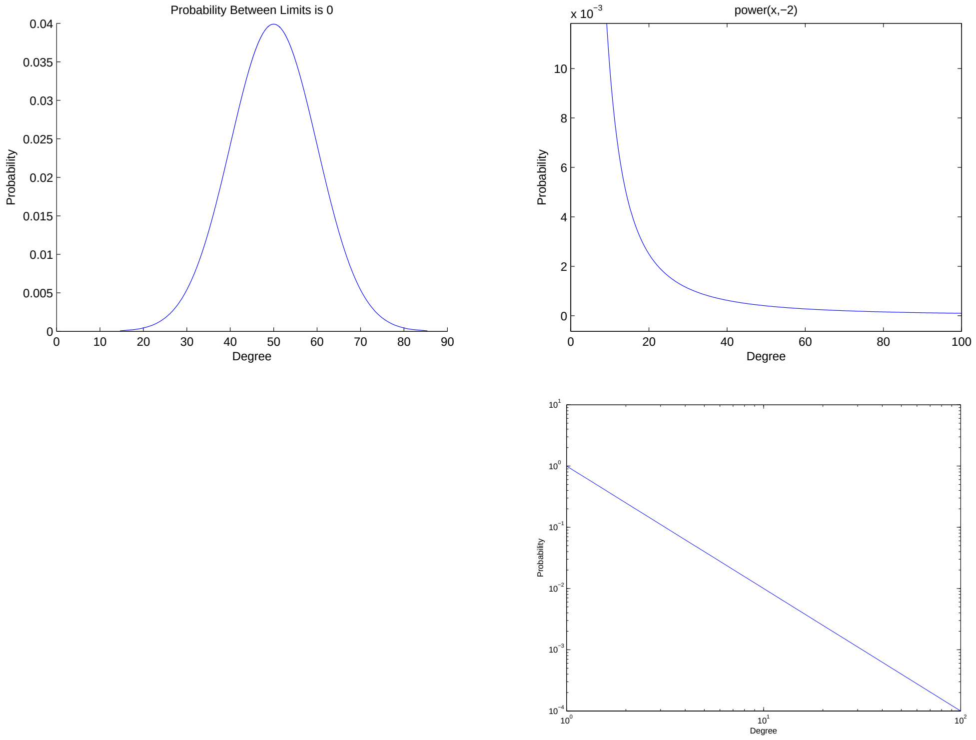

Figure 4.6: Probability distribution of random and scale-free network, lower graph on loglog-scale.

idea since a node could have only a few neighbors, and still have a lot of influence if it connects different clusters. For this reason, I will work with a better property, the cluster coefficient.

4.5.2 Cluster coefficient

The cluster coefficient is a measurement for how well the neighbors of a certain node are connected. The idea behind using it to define hierarchy is that if the neighbors of a certain node are not that well connected, they depend more on that node. For example, it's more likely that communication will have to pass through the certain node to reach a neighbor. If the cluster coefficient of a node is high, it is interchangeable with its neighbors since the connections of a neighbor are similar with the connections of the starting node. The **cluster coefficient** $cc(v)$ of a node v is defined as

$$\begin{aligned} cc(v) &= \frac{|\text{edges between neighbors of } v|}{|\text{total possible edges between neighbors of } v|} \\ &= \frac{n_v}{\frac{k(k-1)}{2}} \end{aligned}$$

with $n_v = |\text{edges between neighbors of } v|$; k =number of neighbors of v. With this measurement, we can define two different networks (Barabasi and Oltvai, 2004; Bollobás, 2001). Following definitions classifies different types of scale-free networks. A **non-hierarchical network** is a network where the cluster coefficient is independent of the degree; the averages of the cluster coefficients of all nodes with the same degree, is (approximately) the same for all the degrees. In a **hierarchical network**, the higher the degree of a node is, the lower the cluster coefficient. Here the cluster coefficient follows the scaling law

$$cc(k) \sim k^{-1}$$

with k the degree, and $cc(k)$ the average cluster coefficient of all nodes with degree k (Barabasi and Oltvai, 2004; Barabási et al., 2003).

Further in this thesis, I will explain how to construct the three network types presented, how a network can evolve from hierarchical to non-hierarchical or vice versa, and how the different networks can influence the function.

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↑ 139

4.6 Centrality

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The goal of this section is to present a general centrality measurement for the nodes of a hypergraph by using existing literature which extends eigenvector centrality from a graph to a hypergraph, and literature which give a general centrality measurement $\mathbf{c}(\alpha, \beta)$ for a graph. We'll use this measurement to say more about the number of communications in a hypergraph.

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↑ 96

The reason I am interested in a centrality measurement is to know which nodes play important roles in the network. As with the cluster coefficient, I want to investigate how centrality is distributed over the nodes of different networks to see which nodes have more control.

Bonacich's paper about general centrality is particularly interesting (Bonacich, 1987) because it generates a parameter β, which corresponds to different types of centrality. Usually, a centrality measurement looks either at the local structure (by the degree) or to the global, the whole network (by the eigenvector centrality). The parameter β in the general centrality measurement indicates whether one should look at local or global structures. The centrality of a node can be thought of as the number of communications starting from that node, with β the chance a message is passed. Thus, the greater parameter β is, the further is looked at in the network.

In the eigenvector centrality, the more central a node's neighbors are, the more central that node will be. In some real life situations, however, we see the opposite. In a market, for example, the more central your neighbors are, the more resources they will already have, and the more difficult it will be for you to trade with them. Thus, you will be less central. This case corresponds with a negative parameter value of β in the general centrality measurement.

I will now define important concepts in centrality.

4.6.1 Eigenvector centrality

The reasoning presented below comes from a paper by Volpentesta (Volpentesta and Felicetti, 2010). The idea behind eigenvector centrality is that a node is more central as its neighboring nodes are more central. It is defined as follows:

Definition **4.6.1** The **eigenvector centrality** e_i of a node v_i in a normal weighted graph is defined by:

$$\lambda e_i = \sum_j W_{ij} e_j$$

This gives several equations the centralities e_k should comply to, solving these equations gives the specific centralities. λ is merely a factor so that the equations have a solution. In matrix notation this looks as follows:

$$\lambda \mathbf{e} = W \mathbf{e}$$

This is an eigenvector equation. The solutions $\mathbf{e}$ are the eigenvectors of W, with λ its eigenvalue. Due to the theorem of Perron-Frobenius, the eigenvector associated with the largest eigenvalue has only positive entries, this eigenvector is usually taken for the centrality. The eigenvalues of W are the solutions of the equation $\det(W - \lambda I) = 0$.

To extend this to a hypergraph, we will assign a centrality to each of the nodes and each of the edges of the hypergraph. A node is more central as the edges it is contained in are more central, and analog for the edges. Thus:

***Definition* 4.6.2** The **eigenvector centrality** x_i **of node** v_i in a hypergraph is:

$$c_1 x_i = \sum_j W_{ij} y_j$$

while the **eigenvector centrality** y_j **of an edge** E_j in a hypergraph is:

$$c_2 y_j = \sum_i W_{ij} x_i.$$

Or, in matrix notation:

$$\begin{aligned} c_1\mathbf{x} &= W\mathbf{y} \\ c_2\mathbf{y} &= W^T\mathbf{x} \end{aligned}$$

or, written in equations with only $\mathbf{x}$ or $\mathbf{y}$:

$$\begin{aligned} &WW^T\mathbf{x} = \lambda\mathbf{x} \\ &W^TW\mathbf{y} = \lambda\mathbf{y} \\ &\text{with } \lambda = c_1c_2 \end{aligned}$$

This is found by searching for the eigenvectors of WW^T and W^TW (a square matrix and its transpose have the same eigenvalues).

4.6.2 General centrality

In a graph, a general centrality measurement for a node v_i is defined as follows (Bonacich, 1987):

***Definition* 4.6.3** The **general centrality** of a node v_i in a graph is:

$$c_i(\alpha,\beta) = \alpha \underbrace{\sum_j W_{ij}}_{\text{degree}} + \beta \underbrace{\sum_j c_j W_{ij}}_{\text{eig. centr.}} \tag{4.5}$$

with $c_j = c_j(\alpha,\beta)$.

or in matrix notation:

$$\begin{aligned} \mathbf{c}(\alpha,\beta) &= \alpha W\mathbf{1} + \beta W\mathbf{c}(\alpha,\beta) \\ \Rightarrow \mathbf{c}(\alpha,\beta) &= \alpha(I-\beta W)^{-1}W\mathbf{1} \end{aligned} \tag{4.6}$$

with $\mathbf{1} = (1\ldots1)^T$. Formula 4.6 is only defined if the inverse $(I-\beta W)^{-1}$ is defined, thus if $\det(I-\beta W) \neq 0$. The property below tells when this is the case:

Property 4.6.4 $(I-\beta W)^{-1}$ *is defined, if and only if* $\beta \neq \frac{1}{\lambda}$*, with* λ *an eigenvalue of* W*.*
If $\beta = \frac{1}{\lambda}$*, we got the eigenvector centrality if* $\alpha = 0$*.*

The idea behind formula (4.5) is that the centrality partially depends on the local situation, measured by the degree, and partially on the global situation, measured by the eigenvector centrality. The greater the absolute value of β, the more global the centrality is. The sign of β tells whether the neighbors of a node have a positive or a negative effect on that node. If β is positive, the more central your neighbors are, the more central you will be, while if β is negative, the more central your neighbors are, the less central you will be.

As can be seen in formula (4.6), α is just a scaling factor for the centrality, which has no effect on the distribution. Usually α is chosen such that $\sum_i c_i(\alpha,\beta)^2 = |V|$. Thus, $c_i(\alpha,\beta) = 1$ means position i has an average centrality.

↑ 110

The extension to a hypergraph is similar to the previous subsection:

Definition **4.6.5** The **general centrality of a node** v_i in a hypergraph is defined as:

$$x_i = \alpha_1 \sum_j W_{ij} + \beta_1 \sum_j W_{ij} y_j$$

And the **general centrality of an edge** E_j in a hypergraph is:

$$y_j = \alpha_2 \sum_i W_{ij} + \beta_2 \sum_i W_{ij} x_i$$

In matrix notation:

$$\begin{aligned} \mathbf{x} &= \alpha_1 W\mathbf{1} + \beta_1 W\mathbf{y} \\ \mathbf{y} &= \alpha_2 W^T\mathbf{1} + \beta_2 W^T\mathbf{x} \end{aligned} \tag{4.7}$$

Or notated with $\mathbf{x}$ and $\mathbf{y}$ in separate equations:

$$\begin{aligned} \mathbf{x} &= (I - \beta_1\beta_2 WW^T)^{-1} W(\alpha_1 \mathbf{1} + \beta_1\alpha_2 W^T \mathbf{1}) \\ \mathbf{y} &= (I - \beta_1\beta_2 W^T W)^{-1} W^T(\alpha_2 \mathbf{1} + \beta_2\alpha_1 W \mathbf{1}) \end{aligned} \tag{4.8}$$

If $\beta_1\beta_2 \neq \frac{1}{\lambda}$, with λ an eigenvalue of WW^T, the inverse is defined (4.6.4).

If $\alpha_1 = \alpha_2$, this is again a scaling factor and we will often assume this case. We'll often take $\alpha_1 = \alpha_2 = 1$ for simplicity. It is not possible to tweak the average centrality of both nodes and edges to 1.

4.6.3 As communication

A theorem we will use in the following two subsections is:

Theorem 4.6.6 $(I - cA)^{-1} = \sum_{k=0}^{+\infty} (cA)^k$, *with* A *a symmetric matrix with positive real entries, and* c *a constant fullfilling* $|c| < \frac{1}{\lambda_{max}}$, *with* λ_{max} *the biggest eigenvalue of* A.

4.6.3.1 In graph

It follows from this theorem that we can write the centrality in a graph as $\mathbf{c}(\alpha, \beta) = \alpha \sum_{k=0}^{+\infty} \beta^k W^{k+1} \mathbf{1}$, starting from (4.6). This is possible when $|\beta| < \frac{1}{\lambda_{max}}$, which we will assume from now on.

β can be seen as the chance a message is passed by a node, we presume $0 \leq \beta \leq 1$. In this interpretation, $\mathbf{c}(1, \beta) = \sum_{k=0}^{+\infty} \beta^k W^{k+1} \mathbf{1}$ is the number of communications starting from each node, where each factor accounts for the number of communications of length k:
$W\mathbf{1}$ = degree of each node = number of communications of length 1;
$\beta W^2 \mathbf{1}$= number of communications of length 2 ($\sum_j \beta w_{ij} w_{jk}$ is the number of communications from i to k; multiplying with $\mathbf{1}$ accounts for a summing over all k's); and so on.

β determines the neighborhood taken into account to calculate the centrality. $(1-\beta)^{-1}$ is the radius of this neighborhood, since the expected length of a communication is $\sum_{k=0}^{+\infty} \beta^k = \frac{1}{1-\beta}$. The last equation is a simpler case of (4.6.6), and holds since $\beta < 1$.

4.6.3.2 In hypergraph

Using theorem 4.6.6, we can write the centrality of the nodes in a hypergraph in a similar way as done above for a graph. This is possible if $|\beta_1\beta_2| < \frac{1}{\lambda_{max}}$, which we will assume from now on. Starting from (4.8), we got $\mathbf{x} = \alpha_1 \sum_{k=0}^{+\infty} (\beta_1\beta_2 WW^T)^k W\mathbf{1} + \alpha_2\beta_1 \sum_{k=0}^{+\infty} (\beta_1\beta_2)^k (WW^T)^{k+1}\mathbf{1}$.

↑ 100

We know that a hypergraph can be represented by a graph with matrix WW^T. If we take $\alpha_1 = 0; \alpha_2 = 1$ and $\beta_1 = 1$ in the above equation, we have $\mathbf{x} = \sum_{k=0}^{+\infty} (\beta_2)^k (WW^T)^{k+1}\mathbf{1}$. This is the same as the centrality of the nodes of the corresponding graph of the hypergraph.

In general, if we take $\alpha_1 = \alpha_2 = 1$, then β_1 can be interpreted as the chance an edge selects a communication (often taken as 1), and β_2 as the chance a node selects a communication. Then the centrality of a node is the number of communications from this node to a hyperedge or a node.

$$\mathbf{x} = \underbrace{\sum_{k=0}^{+\infty} (\beta_1\beta_2 WW^T)^k W\mathbf{1}}_{\text{communications to edges}} + \underbrace{\beta_1 \sum_{k=0}^{+\infty} (\beta_1\beta_2)^k (WW^T)^{k+1}\mathbf{1}}_{\text{communications to nodes}}$$

Each factor betokens communications to a node or edge with distance $k+1$:

$k = 0$ in 1st sum	$W\mathbf{1}$	communications to neighbour edge
$k = 0$ in 2nd sum	$\beta_1 WW^T\mathbf{1}$	communications to neighbour nodes
$k = 1$ in 1st sum	$\beta_1\beta_2 WW^T W\mathbf{1}$	communications to edges at distance 2
$\vdots$	$\vdots$	$\vdots$

4.7 Connections between cluster coefficient, centrality, and sets

We can link the concepts from the previous sections with sets, by looking at how much overlap there is between groups formed around nodes. I also discuss the relationship between the cluster coefficient and the centrality measurement.

4.7.1 Creating direction by writing the cluster coefficient with sets

↑ 108

The cluster coefficient can be defined using sets by defining the **neighborhood** N_i of a node as all the nodes that are connected by an edge with n_i, including n_i itself. Thus, $N_i = \{n_j | \{n_i, n_j\} \in E\} \cup \{n_i\}$. We can then write the cluster coefficient as:

$$cc(n_i) = \sum_{j \in N_i \setminus n_i} \frac{|N_i \cap N_j| - 2}{(|N_i| - 1)(|N_i| - 2)}$$

The numerator of this fraction denotes the double of the number of edges between the neighbors of n_i. This is because $n_k \in N_i \cap N_j$ implies it is a neighbor of n_i for which an edge exists to n_j, unless it is equal to n_i or n_j (which are always in $N_i \cap N_j$). The denominator denotes the double of the total possible number of edges between neighbors of n_i - since $n_i \in N_i$, the degree of n_i is $(|N_i| - 1)$.

If $N_j \subset N_i$, the neighborhood of n_j is completely contained in the neighborhood of n_i, thus n_i can directly reach all the nodes n_j can reach.
When $cc(n_i) = 0$, $|N_i \cap N_j| - 2 = 0 \ \forall j$. The only overlap between the neighborhood of such a node and a neighboring neighborhood, is the node and its neighbor itself. Thus its neighbor connects to none of the other neighbors.

When $cc(n_i) = 1$, we should have $\sum_{j \in N_i \setminus n_i} \frac{|N_i \cap N_j| - 2}{(|N_i|-1)(|N_i|-2)} = 1$. If $N_i \subset N_j$, $|N_i \cap N_j| = |N_i|$. If this holds for all j, we have $cc(n_i) = (|N_i| - 1)\frac{1}{|N_i|-1}\frac{|N_i \cap N_j|-2}{|N_i|-2} = 1$. If there would be a $j : N_i \not\subset N_j$, this would make $cc(n_i)$ decrease, thus the other direction also holds.

Both n_i and n_j have a term with $(|N_i \cap N_j| - 2)$ in the numerator, for n_i this is the term $\frac{|N_i \cap N_j|-2}{(|N_i|-1)(|N_i|-2)}$. When $(|N_i \cap N_j| - 2) \neq 0$, this term will be smaller when a node's degree is higher (the factor $\frac{1}{(|N_i|-1)(|N_i|-2)}$ in comparison with $\frac{1}{(|N_j|-1)(|N_j|-2)}$). The bigger $(|N_i \cap N_j| - 2)$, the more this factor will weight in the overall cluster coefficient, and thus the greater the possible difference will be.

We can measure the extent to which a set is contained in another by $\frac{|N_i \cap N_j|-2}{|N_i|-2} \approx \frac{|N_i \cap N_j|}{|N_i|}$, called the **containment** of N_i in N_j (the -2 here is because we do not want to take into account n_i and n_j, which are always contained in both). This measures the fraction of elements of N_i that are also in N_j. When $|N_i|$ is bigger than $|N_j|$, N_i will be less contained in N_j than the opposite. It is possible that most of the elements of N_j are in N_i, while N_i still has a lot of other elements.

The cluster coefficient measures the average containment of N_i in a neighboring N_j. Thus a high cluster coefficient means n_i's neighbhorhood is largely overlapping with neighboring neighborhoods.

The subset-relation forms a partial order. We can thus create a direction in an undirected network by creating sets around nodes via the cluster coef-

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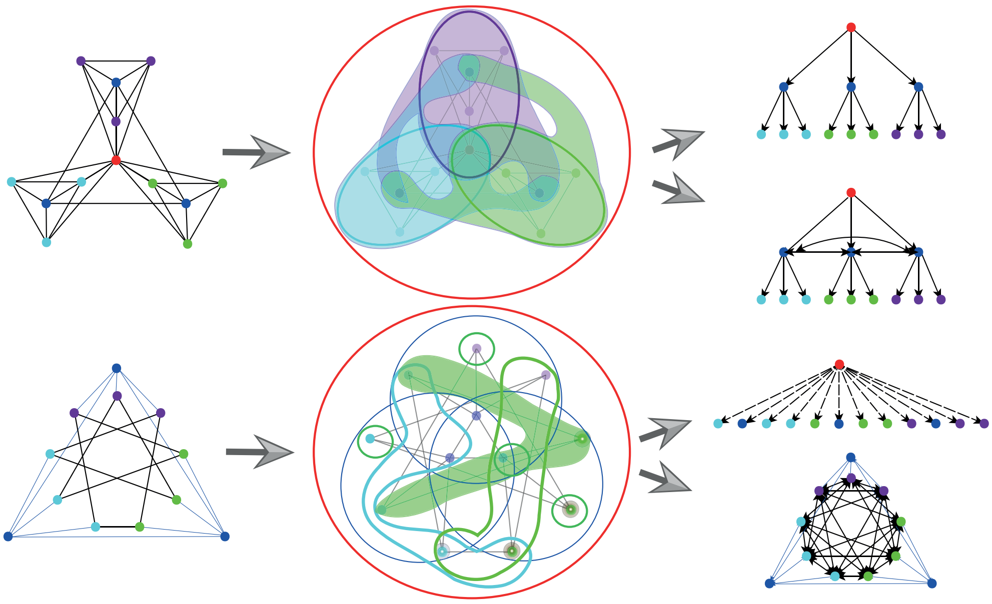

Figure 4.7: An exemplar of a hierarchical network (top) and its anti-hierarchical counterpart (bottom). The top node is only shown in the upper directed anti-hierarchical network, since it is simply connected with all nodes. Sets around nodes are shown (center), which can transform the network in a directed network in two ways (right). In the anti-hierarchical network of the center the lowest degree nodes are drawn on the outside, and only the neighborhoods of four lowest degree nodes are shown: a blue node and three green nodes. Two of them have overlap with the blue node (since it is neighbor or shares a neighbor), one hasn't. This neighborhood is depicted as four separate darker green circles.

ficient.

Figure 4.7 shows an archetype of a hierarchical network and its anti-hierarchical counterpart. Remember that a hierarchical network ↑ 109 was defined as a network in which high degree nodes have a low cluster coefficient, and vice versa (Barabasi and Oltvai, 2004; Barabási et al., 2003). The anti-hierarchical counterpart has the same degree distribution as the given hierarchical network, but nodes with a high degree now have a higher cluster coefficient than low-degree nodes. In the hierarchical network, the neighborhoods of nodes with a small degree are part of the neighborhood of the leading node(s), while they are equal to the neighborhoods of their fellow neighbors. In the next chapter, I will show how such a network is constructed ↑ 127. This is almost a family F_2 ↑ 84 (sets of the same intermediate level are still overlapping). In the hierarchical network there is more ordering, while the anti-hierarchical network only gives overlapping sets, like a family F_3 ↑ 84 (except for the neighborhood of the top node, which contains all other neighborhoods). The top node is a special case. It remains hierarchical because we cannot lower its cluster coefficient (as it has to be connected to all nodes).

If we take $n_i \to n_j$ if $N_j \subset N_i$, the hierarchical network corresponds to a perfect hierarchy (if we consider a direct commander as a minimal N_i). In the anti-hierarchical network, no nodes would be connected except for the top node which points to all other nodes. Such a relation is anti-symmetrical, and a connection can only happen uni-directionally, while in an anti-hierarchical network there is no clear direction between nodes.

We could also consider the relation $n_i \to n_j$ as when $N_i \cap N_j \neq \emptyset$ and $N_i \setminus N_j \neq \emptyset$, which does not imply anti-symmetry. There is a link from n_i to n_j when their neighborhoods are overlapping and n_i's neighborhood is not completely contained in n_j's. In the anti-hierarchical network, this results in two-directional links between the connected nodes, and between the nodes with a joint neighbor in the undirected network. The only exception is the top node, which points to all nodes, but is not pointed at. Every low-degree node connects to all but one node of every other 'cluster'.

In the hierarchical network, the directed network looks like before, except for that there are two-directional links added between the nodes on the intermediate level. Thus, the anti-hierarchical network is much more connected in its directional form while there are the same number of edges in the undirected form of both networks.

4.7.2 Similarity cluster coefficient and centrality

Sets around nodes can be created through another method, for example via the general centrality.

There is a similarity between the cluster coefficient and the centrality measurement. The cluster coefficient correlates with the number of paths of length 2 starting from a node n_i that do not return and stay within its immediate neighborhood. Since a length-2-path goes to a neighbor via another neighbor, the number of length-2-paths is twice the number of edges between neighbors (because an edge is counted twice). The cluster coefficient thus equals the number of length-2-paths divided by the total possible paths of length 2 starting from a node.

A path of length 2 can be described as a communication of length 2. The centrality can be interpreted as the number of communications with β the chance a node passes the signal. In the cluster coefficient, instead of β, the fraction of the total possible length-2-paths is considered. This can be understood as the chance of going to a neighbor being equal to $1/d_i$ (with

d_i the degree of n_i). Each edge has the same chance of being chosen, while each other neighbor has the same chance of being theoretically visited from a neighbor. This is $1/(d_i - 1)$. Thus, the cluster coefficient looks more locally, in the immediate neighborhood, to the number of communications, while the centrality measure considers the number of communications of the whole graph.

The selection of a communication,β, could be different for different agents, and could also depend on the communication. We could work with a $\beta_i(c_k)$: communication c_k will be selected with a chance $\beta_i(c_k)$ by node n_i.
In the cluster coefficient, the β_k of n_k's that are not neighbors of n_i will be zero, while the β_j of its neighbors is $\frac{1}{d_i(d_i-1)}$.

From the centrality measurement, we can obtain the number of communications from one node to the other, as an element C_{ij} of the matrix $C = \alpha(I - \beta W)^{-1}W$. The centrality is obtained by multiplying this matrix with $\mathbf{1}$, i.e. the centrality of a node is the sum of all the communications from that node to another node. This matrix can also be used when the interpretation of the centrality as the number of communications cannot be used, i.e. when β is not small enough. We can now define a neighborhood around a node i as:

$$N_i = \{n_j | c_{ij} \geq t\}$$

This considers all the nodes that have a number of communications above a certain threshold t. c_{ij} could more generally be any measurement of how connected n_i and n_j are. When $N_j \subset N_i$, all nodes n_j can reach are also reachable by n_i (where reachable means $c_{ij} \geq t$, this of course depends on the threshold). When a node n_i's centrality is greater, its c_{ij}'s will on average also be greater. Thus, it will have more c_{ij}'s that are greater than t, and $|N_i|$ will be greater. Nodes with a smaller centrality that are reachable by n_i, will have a smaller neighborhood and there is more chance that they will be contained in N_i.

As before, we can investigate how comparable the neighborhoods of a network are (whether they are closer to a family F_2 or F_3), and construct a directed network from these sets.

↑ 84

4.8 Control: number of driver nodes

I will now discuss the theory of the controllability of complex networks (Liu et al., 2011). The idea here is that a network can be controlled if certain inputs are sent to certain nodes. A person who wants to control a network then, would seek to find a minimal set of nodes to control such that he could control the whole network. In the model of Liu et al., each node j has a value x_j which is influenced by the values of the node's neighbors and the control input. We work here in a weighted, directed graph. This happens by the equation: ↑ 96

$$\frac{d\mathbf{x}(t)}{dt} = A\mathbf{x}(t) + B\mathbf{u}(t)$$

where $\mathbf{x}(t)$ is the vector of values of each node at time t, it is of length n, with n the number of nodes. A is an $n \times n$ matrix of link weights, a_{ji} is the link weight from i to j (this is the transpose from how we defined the matrix of a directed graph—we would call it a link from j to i). The network is controlled by an outside controller which has a number m of input nodes with state $\mathbf{u}(t)$. These input nodes can influence the state of some nodes. The matrix B of size $n \times m$ expresses the link weights between input nodes and normal nodes, b_{jk} is the link weight from input node u_k to node x_j. The discrete version of this equation is that at time $t+1$, the link-weighted sum of all the states at time t of the nodes that link to a specific node (input and otherwise) is added to the node's state at time t.

There is **controllability** if any desired state of $\mathbf{x}(t)$ can be reached by choosing certain input values $\mathbf{u}(t)$. We are now interested in finding the minimal set of nodes that need to be controlled (meaning having a direct link with an input node) in order to have control over the entire network. We call these nodes **driver nodes**.

Structural controllability claims that the exact values of the link weights in A and B does not matter. If the network is controllable for a certain value of the link weights, it is controllable for almost all parameter values that still keep the zero/non-zeroness of a link (where 'almost all' is defined in the mathematical sense as when the set of elements for which the property does not hold has Lebesgue measure zero).

We now try to find a minimal set of driver nodes. The general idea is that each node should have its own direct superior (a superior is a node that links to that node). If it has no superior, it is not controllable, while if it shares a superior with another node, this superior can only control one of them. This is an assumption of the theory, but the reasoning behind it is that a node cannot differentiate its output for different nodes, thus it cannot, at the same time, put two nodes at two independent values.

This implies that the minimal number of driver nodes is equal to the minimal number of input nodes. Having two input nodes linked to a node does not provide added value, and an input node linked to two nodes can only control one of the nodes.

To formalize this idea we first have to define a matching:

***Definition* 4.8.1** A **matching** of a directed graph is a subset of the edges for which no edges have a common starting node, or a common ending node. A node is matched in this matching if it is the ending node of an edge in the matching.

A matching can have connected edges in that the ending node of the one edge can be the starting node of another edge, but it cannot have two edges that start or end in the same node.

A maximal matching is a matching with maximal size. Note that there can be more than one maximal matching. A perfect matching is a matching in which all nodes are matched. Now we arrive at the following theorem:

Theorem 4.8.2 *The minimal number of driver nodes in a perfect matching is* 1*. Otherwise, a minimum set of driver nodes corresponds to the unmatched nodes in a maximal matching.*

Liu et al. prove this theorem. The idea is that a matching provides a subset of the edges in which there is, at most, one link to each node (implying only one direct controller), and from each node (since it could anyway only control one of the nodes it links to). When a node is unmatched, it does not yet have a direct controller, and needs an input node in order for the outside controller to have control.

Nodes can be split into three categories in order to distinguish two modes of control (Jia et al., 2013): redundant, intermittent, and central. **Redundant nodes** are nodes that are matched in every maximal matching, and, therefore, are never driver nodes. **Intermittent nodes** are nodes that are matched in some maximal matchings, but not in others, and are thus sometimes driver nodes, and sometimes not. **Central nodes** are unmatched in every maximal matching, and are thus always driver nodes. A minimal set of driver nodes is abbreviated as *MDS*.

Now, two different modes of control can be distinguished depending on the network structure. There is **centralized control** in a network when most nodes are redundant, thus only a couple of different MDS's are possible. When there is **distributed control** in a network, most nodes act as driver nodes in some MDS's, so most nodes are intermittent or central. The idea is that there is centralized control when there are always the same driver nodes that further direct the network.

This notion of controllability only demands that the whole state space of $\mathbf{x}$ can be reached, but it does not require that this state be maintained (i.e. that it is a stable state). This is a serious limitation of the theory since usually when one wants control, one does not simply want the desired state to be reached for only a millisecond. At the end of this thesis, I investigate the consequences further, and present the implications when a stable state is requested.

↑ 216

A different approach that I will not define here is that of network synchronizability. Here, some stability is required, but not every desired state should be reachable—only some synchrony between the nodes is demanded (Wang and Chen, 2002). The difference between these theories gives rise to a difference in the nodes to control. In controllability, driver nodes avoid hubs, while hubs can provide synchrony.

Controllability theory is also quite contrary to the discussion in 'three coordinates of hierarchy'. There we demanded that a superior commanded more than one element, while here we assume a superior can only directly command one element. A chain only needs one driver node, while in an anti-tree only the top nodes need to be driver nodes. On the contrary, a tree where a node splits into n branches needs $\frac{n-1}{n}$ of the nodes as driver nodes (since the $n-1$ nodes a driver node cannot control have to be driver nodes). A chain and anti-tree have centralized control since only the top node(s) are central nodes, the rest of the nodes are redundant, while a tree has distributed control, since all nodes except for the top node are intermittent (they can be a driver node), while only the top node is central (and there are no redundant nodes).

↑ 101

A cycle only needs one driver node, although all nodes are intermittent, thus there is distributed control. But if the cycle is part of a bigger graph and one of the nodes is pointed at, none of the nodes of the cycles need to be driver nodes, and they are all redundant (thus control is more centralized).

A reason for this difference is that in controllability, it is assumed a node can only control one of the nodes it points to, while it remains to have control over nodes further away. It has sequential control, but no parallel control. With the 'three coordinates of hierarchy', it is assumed that parallel control is possible. In reality, serial control is difficult because control is never perfect and errors further in the chain can accumulate. Nodes usually have multiple influences, and when a node is pointed at by different nodes, it will not always be the case that both nodes can be influenced by driver nodes. Controllability only looks at the perfect case, but not how much of the controllability still remains when not all necessary driver nodes can be used.

It does however also make sense to not allow parallel control because the variety of the controller should be at least as big as the variety of all ↑ 166 the controlled (this is discussed later as the law of requisite variety). Thus, it can only control one node that is as complex as itself. However, one is often not interested in complete control, but only wants to control a certain aspect. This aspect is less complex, so control of different nodes at the same time is possible. Often in hierarchical control, one is only interested in the endpoints, the nodes with no outgoing links. These are considered to be the only ones in interaction with the environment, and are the ones that create ↑ 94 the output (this is, for example, assumed by Mesarovic in the special case of a stratified system). The values of the other nodes in the chain do not matter as long as they create wanted endpoints.

4.9 Conclusion

To summarize this chapter, we built up the notion of hierarchy by starting ↑ 77 from ordered pairs, where (a, b) is different than (b, a). When at most ↑ 77 ↑ 78 one of the two is in the relation, there is antisymmetry. This was present ↑ 91 in Mesarovic's notion of vertical arrangement. We could now call this ↑ 13 ↑ 77 relation a power-over relation. When this relation is transitive, this means ↑ 78 the power-over relation is transmitted. This is therefore a partial order, and no cycles are possible (since this would contradict anti-symmetry). ↑ 103 ↑ 104 Orderability and feedforwardness measure to what extent and in which position there are cycles in a directed graph. When all elements of a partial ↑ 79 order are comparable, there is a total order.

↑ 81 I characterize a hierarchy as an upper semi-lattice—a partial order where ↑ 95 any two elements have a joint commander. This is a feature of a multi-echelon ↑ 95 hierarchy. I proved that this is equivalent with that any element can have ↑ 101 at most one direct commander, a condition for a perfect hierarchy. I showed ↑ 96 that out of this follows that there can only be one top, if the structure is connected and finite. A perfect hierarchy is an upper semi-lattice with two extra conditions: that the length of a chain of command is always the same, and that a commander should command more than one element.

↑ 118 The theory of controllability had opposite results then this last condition: here a commander could only adequately command one element. This is ↑ 121 because this theory considers sequential control, while in a perfect hierarchy control is parallel. The theory of controllability also does not demand that this control is stable.

We can construct a total order ↑ 79 from a partial order by considering every level as one class, these classes then form a total order. A structure could be considered more hierarchical when there is an easier mapping to a total order, and when the depth of the levels is bigger (a small depth means there are only a couple of levels, thus one class contains a lot of elements, and these elements are mutually incomparable). This is the connection Graeber ↑ 87 alludes on between a hierarchy of inclusion (a partial order) and a linear hierarchy (a total order).

I characterized three specific types of families ↑ 84 of sets: the total order $(F_1, \subseteq)$- a set containing another set iteratively, the partial order $(F_2, \subseteq)$- a set containing different subsets, who themselves contain different subsets, and $(F_3, \subseteq)$, where no set is fully contained in another set. This distinction can in general be made in any partial order: to either emphasize the ordered or the unordered parts. One could thus consider one main goal, thesis or struggle with some sub-goals, -thesis, or -struggle, or consider goals, ideas and struggles as interconnected, without any of them being more important. Reversibility ↑ 102 holds in F_1 and F_2, while the property that any commander commands more than one element ↑ 102, holds in F_2, but not in F_1, and in no total order.

We can extract sets from an undirected graph ↑ 114 by considering neighborhood sets around nodes (either by considering the direct neighbors, are by considering the reachable nodes, nodes where the number of communications is bigger than a certain threshold). The cluster coefficient of a node is the average containment of the neighborhood of that node in a neighboring neighborhood. We can then transform the graph into a directed network via the neighborhood sets.

Thus, while it is seldom that we have a pure F_2, with sets containing non-intersecting subsets and so on, often a set will almost completely contain the subsets, and these subsets will have almost no intersection between each other. This relates to the concept of ‘nearly decomposable’ of Simon, where the interactions between subsystems are weak. Simon’s notion of hierarchy ↑ 90 coincide with an F_2, while in a formal hierarchy there is also a functional component, there is an authority relationship between a set and it subset.

The second ↑ 92 and third ↑ 92 condition for a hierarchical system according to

Mesarovic, also includes this functional component: a higher system can change the goals of the lower system, while this lower system can only influence the results of how good the goal of the higher system is reached.

↑ 101 ↑ 102 We could derive possible functional consequences of some properties of a perfect hierarchy. When there is reversibility, an element is only directly influenced by one element, and thus it could get determined by this element. While if there are different influences, it is unpredictable how these influences would combine, and a unique constellation could emerge.

↑ 102 When one element influences more than one element, its influence will increase exponentially.

When there are no cycles, influence is always one-directional, and there is asymmetry. There are no cycles when there is antisymmetry and transitivity.

↑ 200 Hierarchy could thus be avoided by allowing cycles and more than one direct 'commander'. This will be developed in a later chapter.

Chapter 5

Change

The previous chapter discussed the structural aspects of hierarchy, but did not consider whether these structures could change. In fact, reality is dynamic and structural change does occur. This chapter describes ways in which hierarchical structures can change and how those changes occur.

First, I will repeat the argumentation of the steps that generate a hierarchical structure. Next, I will discuss how the network types from the previous chapter (random, scale-free, and hierarchical) can be constructed. I will continue by explaining how one network type can evolve into another type by adding or removing edges.

This does not say what causes a change in the system. I will present a simulation for how the emergence of a power-law (as an example of an attractor) can be counteracted by constant opposition.

I will also discuss how chemical organization theory, a framework that can be used to describe processes and how an organization arises, can be applied to model the processes by which attractors emerge and disintegrate.

These theories are quite unrelated except for that they deal with change. They are, however, necessary for further theories or provide a summary or mathematical illustration of previous discussions. Section 5.1 “Steps to hierarchical structure” summarizes arguments from the previous chapter Structure. Section 5.2.1 “Constructing networks” is used in the simulations of the next chapter and constructs the network types given in the previous chapter. Section 5.3 “A simulation of constant opposition” illustrates the concept of constant opposition, as discussed sociologically in Rise of Hierarchy, and systemically in section 3.6.2. Section 5.4 “Chemical organization theory” will be used to build an example of non-hierarchical control and formalizes the previously mentioned concept of autopoiesis.

↑ 126
↑ 76 ↑ 127
↑ 139 ↑ 107
↑ 131
↑ 25
↑ 60 ↑ 136
↑ 182
↑ 45

5.1 Steps to hierarchical structure

In this section, I will summarize step-by-step how a hierarchical structure, as described in the previous chapter, can emerge.

First, there is a directionality. We saw how a directionality can be extracted from an undirected network by considering sets. But in general, any relation that is not completely symmetric gives a directionality. Such a directionality does not have to be anti-symmetric—a link can go in two directions. A positive feedback mechanism can amplify difference in strength between the two directions, until only one direction remains, causing anti-symmetry.

↑ 78 When such an anti-symmetric relation is transitive, there is a partial order. With transitivity, we take into account the broader network. We assume the other direction in an anti-symmetric link cannot be achieved via other nodes.

↑ 96 I now offer a slightly different argument than that of the previous chapter
↑ 81 for why when any node has at most one direct commander, there is an upper
semi-lattice with only one top.

Property 5.1.1 *If in the strict version of a connected and finite partial order any node has at most one minimal upper bound, the non-strict version of this partial order is an upper semi-lattice with one greatest element.*

↑ 95 **Proof** We know from property 4.2.2 that any element having at most one minimal upper bound (one direct commander) is equivalent with any two elements having at most one joint minimal upper bound. We now have to prove that there exists a joint upper bound for any two elements in the non-strict version of the partial order.

By connected we mean that there is a path from a node to any other node, without considering the direction of the edges in this path. Consider any two elements, these can be linked through several edges, but we do not know the direction of these edges. When the direction of these edges do not change (thus we have a path $\longrightarrow\longrightarrow\longrightarrow$), one element is an upper bound of the other. Since in the non-strict version of a partial order, any element is its own upper bound, these two elements have a joint upper bound.

A change in direction cannot happen as $\longrightarrow\longleftarrow$, since then there would be an element with two minimal upper bounds (two edges pointing to the same node). Unless one of these upper bounds is bigger than the other, in which case there exist a link between these two nodes, and we can then directly take this link in the path, instead of going indirectly through the other node.

Thus, when the direction changes, this happens as ⟵⟶, two edges pointing outwards from a node. This node is an upper bound of the two elements at the end of the path (since the direction can no longer change). Thus, any two elements have a supremum, and we have an upper semi-lattice.

A finite upper semi-lattice necessarily has a greatest element. ↑ 81 Since when there would be two maximal elements, they would have a joint upper bound, which can be maximum one of these elements. Thus the other element is not maximal. There should be a maximal element, since a chain ⟵⟵⟵ of elements that still have a bigger element, cannot go on forever (since we are in a finite structure), and cannot be cyclical (since we have a partial order). ☐

5.2 Constructing and altering networks

In this section, I will discuss algorithms for how to construct the networks presented in the previous chapter. I have implemented these in Matlab.

5.2.1 Constructing networks

5.2.1.1 The three graphs categorized by degree and cluster coefficient

I will now construct examples of the random, scale-free and hierarchical networks, as introduced in section 4.5. I want the three networks to have an equal number of nodes and an approximately equal number of edges. ✎ 2014b ↑ 107

A random network is easily constructed: you start with the number of nodes desired, unconnected. Then, two nodes are selected randomly and connected by an edge. This step is repeated until the graph has the desired number of edges.

The hierarchical network is built in an iterative way. Figure 5.1 shows how this happens. The idea is to have clusters which get connected through their leaders. There is a leading node added at each next level, which connects to all the nodes. And the leaders of each cluster connect to each other. At the first level, each node is its own cluster, and its own leader. I worked with three clusters. Thus, at this step, we get a fully connected network of four nodes—three clusters and its leader. In the next step, we copy this network three times, and add a leader. The leaders of each cluster connect to each other, and the big leader connects to everyone. This can

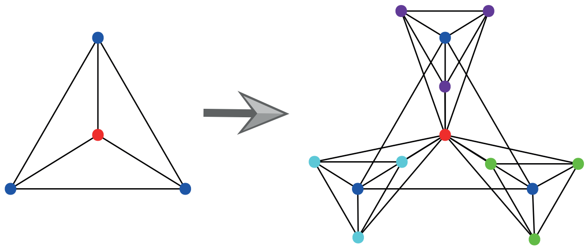

Figure 5.1: Construction of a hierarchical network.

be repeated for as long as desired. I did this procedure 4 times, creating a network of 121 nodes and 1025 edges. For this reason I also build the random network with 121 nodes and 1025 edges.

To create a non-hierarchical network, the B-A algorithm is used (Albert and Barabasi, 2001, p. 71). This works as follows: starting from a small graph, nodes are added with m edges (m is a parameter of the system, mostly between 1 and 5). But these connections don't happen randomly, there is more chance to connect with nodes that have already a high degree. This is called **preferential attachment**. Formally, the probability $\prod$ that a new node will be connected to node i with degree k_i will be

$$\prod(k_i) = \frac{k_i}{\sum_j k_j}$$

Nodes are added until the size of the graph is as desired. I will start with a graph of 5 nodes which form a line. I'll add nodes until we have 121 nodes, with $m = 4$. Then I get a graph with 928 edges.

Plotting network properties show that the constructed networks fulfill the necessary conditions. In the constructed hierarchical and non-hierarchical network, the degree frequency follows a power-law, while in our random network, the probability of a degree is normally distributed. Plotting the cluster coefficient against the degree confirms that in the hierarchical network, the cluster coefficient follows a power-law, while in the other networks the cluster coefficient is approximately the same for all degrees.

5.2.1.2 By centrality and in a hypergraph

2014a

I now want to extend this mechanism to also construct networks based on centrality, and to build hypergraphs, expanding section 4.6.

↑ 109

Since most real-life standard graphs are scale-free, we want to extend this to scale-free hypergraphs. This is understood as a hypergraph in which both the degrees of the nodes and the hyperedges follow a power-law. Thus, most nodes are contained in only a few edges, while a few nodes are contained in a lot of edges; and most edges contain only a few nodes, while a couple of edges contain a lot of nodes.

Another, more general way to look at different topologies, is to look how central a certain node or edge is in the overall network, and how the centrality distribution looks.

I will now establish methods to construct a network based on the centrality measurements. The basic mechanism will be to extend the BA-algorithm of preferential attachment. A first extension will be to use different preferences, namely the centrality, local centrality or cluster-coefficient (for a graph) instead of the degree. I also want to use the algorithm in a hypergraph. Besides building a network by adding nodes, existing networks can be strengthened by adding edges. Here, the node we start with isn't a new node, but an existing node chosen ad random or preferential by degree.

This can be written in one algorithm with two parameters to represent this different choices. The first parameter, *startingnode*, determines whether the first node is a new node, a node chosen ad random, or a node chosen by preference of degree. The second parameter states which variable decides the preference. Thus, whether it is a higher degree, general centrality or local centrality which gets more often chosen as a node or edge to connect to from the starting node. The local centrality is with respect to the starting node, it is thus the chance of communication from this node to another node. The higher this chance, the more chance to connect to the other node. This only makes sense when the starting node is an already existing node, since a new node cannot reach any other node.

When I applied this algorithm, I noticed a problem. I often received warnings that my matrix was close to singularity, probably because λ was close to $\frac{1}{\beta}$. When I looked at my eigenvalues, I saw that the biggest eigenvalue was, for some reason, often around 11. Thus, for the interpretation as communication, β should be smaller then $1/11 = 0.0909$, which is a serious restriction I did not want to follow. I saw that, indeed, there was an eigenvalue which was pretty close to $1/\beta$, the cause of the warning.

The solution I implemented for this problem is to give all edges a weight of 0.1 (where before I worked in an unweighted graph, all edges had a weight of 1), by which the largest eigenvalue drops to 1.1, and β can go until 0.9.

However, this might be a fake solution because lowering the weights makes it harder to traverse the network. Thus, we still look pretty locally. In this case the centrality is quite similar to the degree, and it isn't that useful. Thus we see that interpreting the centrality as number of communications is not that practicable.

5.2.2 Evolving networks

2014b

In this section, I develop a procedure to evolve a non-hierarchical network into a hierarchical one and vice versa by adding or deleting nodes.

To make a network more hierarchical, the idea is to decrease the cluster coefficient of high degree nodes, and increase the cluster coefficient of low degree nodes. This is done by selecting a node preferential according to its distance from the mean degree (thus nodes further away from the mean have a higher chance of being chosen). If its degree is higher than the average degree, an edge between two previously connected neighbors of the node gets deleted. On the other hand, if its degree is lower than the average degree, an edge is randomly added between two neighbors that weren't yet connected. This is done for a number of steps.

We can do the opposite to make a network less hierarchical. Thus, we add an edge between neighbors of high degree nodes, and delete an edge between neighbors of low degree nodes.

This mechanism can be seen and used in the social world in order to make a system more or less hierarchical. An authority often sets different marginalized groups up against each other, or, on a smaller scale, creates internal conflicts between people. There is some group identity created in marginalized groups so that badly connected people cluster together, increasing their cluster coefficient. The leader decreases its cluster coefficient by creating different unconnected clusters. This way people often fight between each other instead of turning against the leader.

The opposite mechanism is seen when the limits of specific struggles are transcended, when ties of solidarity are created between different struggles and communities, or when these strict categories are dismantled. For example, when links are created between several specific struggles: struggles against sexism or racism, economic and environmental struggles,...

5.3 A simulation of constant opposisition

2017

In this section I back up the hypothesis that a constant opposition in a system can be used to avoid getting stuck in a status quo where power is maintained in a hierarchical structure. I rehash the arguments developed earlier in this thesis, and do a little simulation to show how a constant opposition can work in the case of one variable.

↑ 63

This idea is very much in line with the argument in 'Rise of Hierarchy' (Gelderloos, 2005). Gelderloos argues that hierarchy did not arise because of a change in material mode (the classical Marxist view), but wherever there was no organization to prevent it. This is affirmed by the existence of hierarchical hunter-gatherer societies and egalitarian agricultural societies. In egalitarian societies, there were mechanisms to prevent hierarchy from rising.

↑ 25

↑ 34

Boehm investigates the mechanisms used in different egalitarian societies to discourage hierarchical behavior (Boehm et al., 1993; Boehm, 2009). He names this situation a reverse dominance hierarchy, as he describes the process as domination of leaders by their own followers. The mechanisms found in egalitarian communities range from public opinion through criticism, ridicule and disobedience, to extreme sanctions like exile and execution. He calls these societies intentional communities, as they consist of people who have consciously decided they want to live without hierarchy, and who take active steps to achieve this.

Generally speaking, a system will usually get into an attractor, for example, a hierarchical organizational structure. It is possible to move out of the attractor by changing the dynamics of the system with external challenges to the attractor.

The second law of thermodynamics states that a closed system will evolve toward a state of maximum entropy. This state of maximum entropy is an example of an attractor. But because the world is an open system, there are constant challenges to that attractor. An open system like the world is much more dynamic than a simple, closed system, which is why a complex phenomenon like life is possible. For further consideration, take Heylighen's (Heylighen, 2014a) interpretation of the second law of thermodynamics, wherein he states that without selection, a system will evolve to a state of maximal entropy. It is because of selection that certain states are more probable, since they have a higher chance of survival. Thus, the second law of thermodynamics implies a uniform distribution, where all states are equally probable. But selection changes the distribution, making certain states are more probable.

Analogously, there can be a mechanism to prevent power imbalances (i.e., as seen in power-law distributions caused by a positive feedback). The hypothesis here is that a power-law distribution could get flattened by a constant opposition where as soon as someone gets more than the others, he gets a headwind that restrains him from accumulating the additional resource.

To sum up, in both of these cases there is a certain law that seems difficult to avoid (either the second law of thermodynamics or a positive feedback mechanism giving rise to a power-law). In general, these laws result in the system moving into an attractor. But there can be a mechanism (selection or constant opposition) to overcome this.

To illustrate this, I built a little simulation. This can be a general model of a positive feedback phenomenon, but I made this with the present socio-technological complex in mind. Here there are several agents A_i with a certain fitness $f_i(t)$ at time t. The more fitness they have, the more fitness they will be able to gain. In a socio-technological situation, this is because more fitness means more possibilities to influence the development of technologies, and by extension, to form the environment. $f_i(t)$ can be interpreted as money, as we see that the more money someone has, the more money this person can, in general, generate. We can represent this by the following formula:

$$f_i(t+1) = f_i(t) + k \cdot f_i(t) - \sum_{j \neq i} \frac{k}{n-1} f_j(t) \tag{5.1}$$

with $k > 0$ a constant and n the total number of agents. Thus, the more fitness an agent A_i has, the more it gains (namely $k \cdot f_i(t)$), and it takes an equal amount from the fitness of all other agents to get this (agent A_i thus looses the amount $\frac{k}{n-1} f_j(t)$ due to agent A_j). We assume the total fitness remains constant (for example, with money we can assume that when some get more money, others' money will be worth less due to inflation). This can be a general model of a positive feedback mechanism (where f can represent something else than fitness).

Now we introduce a constant opposition mechanism in this model. The idea is that agents steal the fitness from the agent with the highest fitness, instead of from all agents. In the language of formulas, for all but the agent with the highest fitness, the formula changes into:

$$f_i(t+1) = f_i(t) + k \cdot f_i(t) \tag{5.2}$$

while for the agent A_j with the highest fitness, the formula becomes:

$$f_j(t+1) = f_j(t) + k \cdot f_j(t) - \sum_{i \neq j} k \cdot f_i(t) \tag{5.3}$$

I did a simulation with 1000 agents, for 100 iterations and $k = 0.1$, for both cases (either the standard case (5.1), or the one with opposition ((5.2) and (5.3))). In the two cases, the fitness values of agents started from the same normal distribution. In the simulation I ensured that the fitness did not become negative, by instead taking more from (an) other agent(s) if an agent's fitness would reach below zero (evenly in the standard case, and from the one with the second highest fitness in the opposition case).

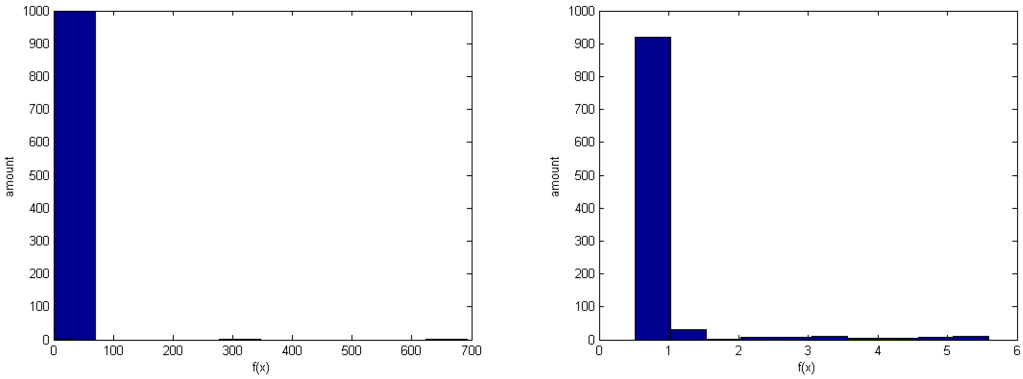

Figure 5.2: The distribution at the end of the simulation of the frequency (y-axis) of the fitness (x-axis). Left: the distribution in the standard case. Right: the distribution in the case of opposition. Note the difference in range of the x-axis between left and right: in the standard case, the highest fitness is 700, while in case of opposition, the fitness does not go higher than 6.

In the standard case, we see a power-law at the end of the simulation, as expected. Two agents have almost all the fitness, while the rest have almost none. But with opposition, we still see a power-law at certain times. Therefore, our hypothesis that we would get a flat distribution was too simplistic. Though the power-law is much less profound in the opposition case (see figure 5.2 for the distributions). The amount of (un)equality can be measured by looking at the median, the middle value if we order from small to big. At the end of the simulation, the median in the standard case was of the order 10^{-67}, while with opposition, it was 0.86, a statistically significant difference ($p = 0$). Thus, in the standard case at least half of the agents have a fitness of almost zero (actually it is all except two of the agents), while in the opposition case half of them have a fitness of more than 0.86.

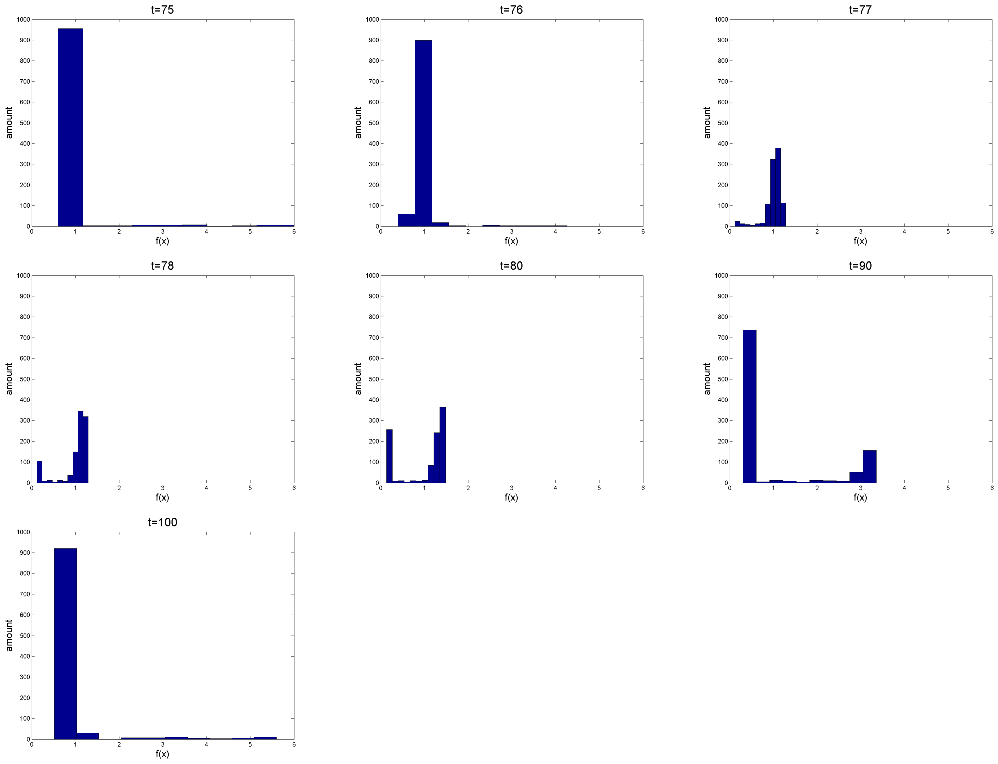

Figure 5.3: The histograms of the fitness in the case of opposition for $t = 75, 76, 77, 78$ (upper) and $80, 90, 100$ (lower). This loop takes 25 iterations, and thus repeats itself four times during the simulation of 100 iterations.

But in the case of opposition, the distribution is constantly changing. The power-law evolves to a more equal distribution, with two groups emerging, and back again to a power-law (see figure 5.3). Agents with a high fitness in the power-law, will get ‘eaten’, so that a minority emerges with a really low fitness, as agents with a high fitness lose almost all of their fitness. This causes the appearance of two groups: a smaller group with a low fitness, and a bigger group with an average fitness. But now, also agents with a fairly average fitness will loose fitness, so that the low-fitness group will become bigger. At the same time, the agents in this group will be able to grow their fitness. Some of the agents from the other group will be able to stay out of sight for a while, managing to grow their fitness above average. A power-law emerges. But eventually, these agents will will also get ‘eaten’, and the cycle repeats itself.

Which agents have the highest fitness, is constantly changing (see figure 5.4).

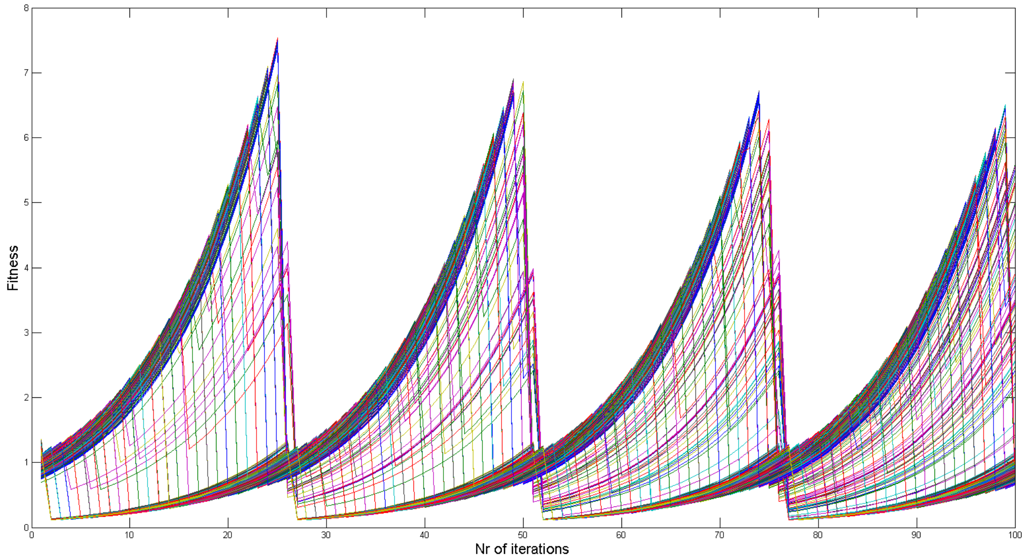

Figure 5.4: The evolution of the fitness of all agents over time (100 iterations). We see that the fitness oscillates for all agents.

Most phenomena are more complex than the manipulation of one variable. But this simulation can still be seen as a simple example showing that agency on the bigger structure is possible, even if there is a general mechanism that cannot be changed (for example a positive feedback mechanism). Different local decisions (taking from all or only the biggest) can give rise to different global behavior. In the standard case, the distribution moved to an attractor: it got stuck in one distribution, i.e. a power-law. In the opposition case, the distribution was constantly changing.

In both cases, the global also influenced the local. In the standard case, since the total fitness is kept fixed, it depends on one's position whether one's fitness will grow or decay. In the opposition case, there is no influence except for the one at the top, who feels a big influence. Constant opposition can thus make the environmental influence less profound.

With more variables there is no total order any more. There will still be a partial order, but the less the variables are correlated, and the more variables there are, the less elements will be comparable. Often when comparing two states, one variable will be bigger, while the other smaller.

5.4 Chemical Organization Theory

The framework of Chemical Organization Theory (COT) (Dittrich and Fenizio, 2007) can be used to model the emergence of a "bigger structure". It is quite different from our previous models in that it deals with processes. While it was originally used to model chemical reactions, it can be used more generally to represent any process. The terminology used, however, still comes from chemistry. The basic idea is to look at a certain **reaction network**, a set of molecules together with a set of reactions, and search for organizations formed by these reactions. Formally, a reaction network is a pair (M, R), with M a set of molecules, and R a relation $R \subseteq P_M(M) \times P_M(M)$, with $P_M(M)$ the set of all multisets with elements in M. A multiset is a collection of elements where an element can be present more than once. These molecules can be anything, thus it is not constrained to the chemical sphere. An example of a reaction is $a+b \rightarrow c+d$, where a and b are consumed, and c and d are produced.

Chemical organization theory then looks at whether a certain subset of molecules can maintain itself (the consumption of a molecule is smaller than the production) and is closed (there are no molecules produced that were not yet there). If this is the case, this subset is an organization. Formally:

***Definition* 5.4.1** A set $C \subseteq M$ is **closed**, if for all $A \rightarrow B \in R$ with all elements of the multiset A in C, all the elements of B are also in C.

We associate with a set C all the reactions that only contain elements in C. We then say that a molecule $m \in M$ is **produced** within C if there is a reaction $A \rightarrow B \in R$, with $A, B \in P_M(C)$, for which m appears less in A than in B. If there exists a reaction in C for which m appears more in A than in B, we say that m is **consumed** within C. We now define:

***Definition* 5.4.2** A set $C \subseteq M$ is semi-self-maintaining, if all $c \in C$ that are consumed within C, are also produced within C.

A **semi-organization** is a set that is semi-self-maintaining and closed. In such a set it is still possible that a molecule disappears, if it is consumed more than produced. This is why flux vectors are introduced, which tell the rate at which a reaction takes place. A set C is called self-maintaining if

there exists a flux vector for which all $c \in C$ have a positive production rate, and for which the reactions in C, and only those, have a positive flux. An **organization** is a set that is closed and self-maintaining. Everything in an organization got produced by the organization itself.
I will not elaborate how a production rate is calculated. Because of the complications involved with this, I will often only consider semi-organizations.

COT thus gives a mathematical formalism for the concept of autopoiesis (Varela et al., 1991). Luhmann uses the concept of autopoiesis to discuss how social systems can maintain themselves. Luhmann has been applied to COT (Dittrich and Winter, 2008), but in a specific rather than conceptual way: specific reactions of a political system are presented, and organizations are searched in it. ↑ 45

COT is, however, still a deterministic model—the reactions are given, and given a certain set of molecules, the system will always evolve to the same set, an attractor. Goals, and thus goal-directed behavior, are not directly defined in COT. Since I want to model how the emergent goals of the bigger structure interfere with the goals of the constituting agents, and how a local agency can influence the bigger structure, indeterminism and goal-directed behavior should be introduced.

A reaction can be seen as a certain method, and the products of this reaction as the goal of the method. Extending to multiple reactions, we can say that in an organization, the molecules involved are its 'goal' (since an organization wants to keep these molecules in existence).

To allow for indeterminism, the model can be extended by introducing agents. This will allow agents to strive for their goals, which could be represented by a set of molecules G. An agent can choose certain methods (reactions) to reach its goals. An agent is thus a **catalyst** of a reaction—it makes a reaction possible—but it is not itself consumed or produced by that reaction.

The choice of the methods by the agents, can now influence the global behavior. For example, in the constant opposition simulation, an agent could choose between two different methods to reach its goal of a higher fitness: either take it equally from other agents, or only take it from the biggest agent.

Since an organization is an attractor state that maintains itself, it can be used to model how and when an imposing structure can rise. The idea is to let certain methods emerge and disappear, by a trial-and-error of the agents. After a while an organization can emerge, where all the goals of the agents ↑ 43 are satisfied in a self-maintaining way. The question is then whether this organization will be rigid. To test this, the idea is to introduce a mutation (since the environment constantly evolves), and check whether the organization will adapt with it, or whether it will remain the same, despite not satisfying the goals of the agents anymore. A mutation can be a change in a reaction, in the goals of an agent, or by introducing or leaving out an agent.

↑ 44 The evolution from exploiter to cultivator can be explained in COT terms. We can see an exploiter as an agent that monopolizes (a) resource(s). It thus catalyzes a reaction of the form $A \to \emptyset$ (with A the resource it monopolizes). It evolves to a cultivator by building an organization that overproduces A, so that it can take A out of it in a maintaining way. A resource is overproduced when it is more produced than consumed.

Such an organization provides for the needs of some agents, but only because this leads to the production of a wanted A. This creates a dependent relationship for those agents: they can no longer provide for their goals themselves, but depend on the bigger structure to reach them. There is also only a part of the agent that the bigger structure is interested in, namely the part that provides the wanted resources. If it can find an easier way to get these resources, it will replace the agent.

These are some starting points for using COT to model this process, but elaborating this is work for the future. Some germs of possible models are presented in the supplementary information.

5.5 Conclusion

↑ 126 In this chapter, I discussed how a structure can emerge, how different (network) structures can be constructed, and how structures can become more or less hierarchical. ↑ 127 ↑ 130

↑ 131 Attractors can be avoided by a constant opposition. Networks that become less hierarchical, can be understood in this manner. Instead of randomly connecting or breaking with neighbors of neighbors, a node connects with the neighbors of a high-degree node and deletes links with neighbors of low-degree nodes. By this local choice of a node, the global behavior (a more or less hierarchical structure) can change.

While my simulation of constant opposition only worked with one variable, chemical organization theory allows more variables, and can model the emergence and disintegration of attractors. ↑ 136

Chapter 6

Influence of network on function

I will now present two simulations of a network of coordinating agents. 2014b The basic idea of both models is that agents in the network change their state depending on the state of their neighbors. I simulate this in the three different network types (hierarchical, non-hierarchical, and random), and look at what is different and the same in these networks. In the first model the agents will try to minimize the friction by using alignment, while in the ↑ 41 second model they will try to maximize the synergy by using division of ↑ 42 labor, workflow and aggregation.

I did these simulations in the beginning of my PhD. While the results are difficult to interpret and there are several shortcomings, the presented models are still interesting as examples of coordination, and the ambiguous results show that structure does not determine everything.

6.1 The networks used

In order to look at the statistical significance of the results in the simulations, we would like to generate different networks of the three different kinds. For the random network and the non-hierarchical network, this can be done by the procedure described in subsection 5.2.1.1, while the hierarchical network ↑ 127 can be constructed from a non-hierarchical network as described in section 5.2.2. ↑ 130

I first created a non-hierarchical network as before with the B-A algorithm (thus with $m = 4$), but with 1000 nodes. A hierarchical network was created by adding or deleting an edge a 1000 times, by the method described in the previous chapter. I created a random network with the number of edges equal to the average of the number of edges of the non-hierarchical and hierarchical network. Each simulation was done for these three networks. This was done 1000 times, thus each simulation was done in 3000 different networks.

Whether the properties of the networks were as desired, was checked by seeing in how far plotting the frequency of the degree and the cluster coefficient against the degree respectively, fitted a power-law. The degree of a random network clearly doesn't follow a power-law. However, the degree of the hierarchical and of the non-hierarchical network are closer to it. (Though some of the power-law behavior has been lost by hierarchizing a network with our procedure.) But the histogram of the degree of a hierarchical network still looks like a power-law.

The hierarchical network clearly follows a power-law for the plot of the degree against the cluster coefficient (a high degree node has a low cluster coefficient). A non-hierarchical network is a bit more power-law than a random network.

For both plots, the difference in power-law behavior between all networks is significant.

We observed that the strong differentiation between low and high degrees is reduced by hierarchizing. This can be explained as follows. An edge can only be deleted between neighbors that are already connected, thus on average, the neighbors selected will have a slightly higher degree than those that aren't connected. Therefore, nodes with a higher degree will decrease more in degree than other nodes. On the other side, an edge can only be added if there is no connection yet, thus nodes with a lower degree are favored for an increase in degree.

There are some existing methods that evaluate these networks, which I applied in the constructed networks. This is done by looking at the diameter, the biggest distance that exists between two nodes in a graph. Distance is measured as the length of the shortest path. The hierarchical network has a lower diameter, thus there is less distance that has to be passed than in the other networks.

Now we can look at what happens to the diameter of a network when nodes are deleted, which can happen in two ways: randomly or delete nodes with the highest degree. These two events are called failure and attack respectively (Albert et al., 2000).

With failure, in a random network the diameter increases sooner than in the other two. The hierarchical network can stay together for the longest time. But with attack, the hierarchical network falls apart the soonest, while the random network holds up the longest—it can perform almost as well under attack as it can with failure. In both cases, the diameter of the random network increases in smaller steps, while in the non-hierarchical and hierarchical network it increases with larger jumps.

The phenomenon of civil war could be explained by an attack in a hierarchical network: the node(s) with the highest degree hold the network together since they have the lowest cluster coefficient. Thus, if such a node is deleted, the network easily falls apart, which causes the leaders of the different clans/clusters to fight for the overall leadership. Civil war is often used as an argument for why hierarchical organization is necessary, since if there is no more leader, there is chaos. However, an argument could be made that any chaos could be caused by the hierarchical structure present from the beginning.

6.2 Minimize friction

In this model, there is a number n_i between zero and one assigned to each node i, which we can represent by a color on the grayscale, zero being black, one being white. We thus call n_i the color of a node. To align, so to aim at the same direction, in this case means to try to have the same color. At each time step, the number of each node will be updated towards the number of its neighbors, by the following rule:

$$n_i(t+1) = n_i(t) + \frac{\sum_{j \in N_i}(n_j(t) - n_i(t))}{2 \cdot |N_i|}$$

with N_i the set of neighbors of i.

A fitness function is assigned to each agent based on the idea that the less variation with their neighbors there is (so the less friction), the higher the fitness.

$$f(n_i) = \sqrt{\frac{|N_i|}{\sum_{j \in N_i}(n_i - n_j)^2}} \tag{6.1}$$

I will also look at the difference in color between a node and the rest of the network. This shows how well a node fits into the overall network. I call this the global alignment *ga* of a node. This is defined similarly to fitness (6.1), but applies to the whole network instead of just to the set of neighbors:

$$ga(n_i) = \sqrt{\frac{|V| - 1}{\sum_{j \in V \setminus n_i} (n_i - n_j)^2}}$$

You could see the global alignment as a measure of the influence, where influence is seen as a measurement of how much agents adapt towards node n_i. Assuming that in the beginning the colors are randomly distributed across the nodes (which is done here), the bigger the global alignment of a node, the more the other agents moved towards the color of that node, thus the bigger the influence that node has. The correspondence is not perfect though, since the global alignment doesn't differentiate whether an agent was influenced by the other agents—thus his color moved toward the color of the influencing agents—or an agent influenced other neighbors—thus the neighbors' colors moved toward the agent's color.

To differentiate between these two kinds of influence, we introduce two other measurements. The *change* of a node is the difference in color of a node between the end and the beginning. This is a measurement of how much an agent was influenced by its neighbors during the simulation. What I call the *influence* of a node, is a measurement of how much one's neighbors have moved towards that node in one step of the simulation. The node's influence at each step of the simulation thus measures how much influence a node has on other agents in that step. In a formula, this is:

$$Inf(i,t) = \frac{\sum_{j \in N_i} |n_i(t) - n_j(t)| - |n_i(t) - n_j(t+1)|}{|N_i|}$$

where $n_i(t)$ is the color of node i at time t.

6.2.1 Results

I will now run this simulation 1000 times on the three network types, where each time there are new networks generated by the algorithm explained in section 5.2.1.1 and 5.2.2. In each simulation I did 20 iterations (the behavior converged by that time). For each node, the four measurements described above were investigated: the fitness, global alignment and influence at the end of a simulation, and the change of a node. I looked at how these four measurements correlated with the degree and the clustering coefficient in the three networks, and checked with a t-test whether these four measurements were different in the three networks. The significance

↑ 127 ↑ 130

level in all the performed t-tests was 0.05.

To find correlations, I calculated the correlation coefficients for all the simulations, and checked how many of them were significant.

In all three networks, there was a negative correlation between the degree and the fitness. This is because if there are more nodes you try to adapt to, this is more difficult. But there was a positive correlation between the degree and the global alignment, and between the degree and the influence for the three networks. Thus, a high-degree node can influence the network more toward its value.

A low cluster coefficient leads to a low fitness—if your neighbors are in different clusters, its more difficult to adapt to all. Though in the random network this correlation was less clear, probably because there is less difference between cluster coefficients. A low cluster coefficient also leads to a low influence—the neighbors will be influenced more by external agents. While if the cluster coefficient is higher, the neighbors of an agent will be influenced by other neighbors of the original agent, which has values more in correspondence with that agent.

In the non-hierarchical network the global alignment is positively correlated with the cluster coefficient, while in the hierarchical and random network, it is negatively correlated (though this is less clear in the random network, where there are also quite some positive correlations). Since a high cluster coefficient makes a node more locally aligned (higher fitness), this will lead to more global alignment in the non-hierarchical network. But because in a hierarchical network a high cluster coefficient is associated with a low degree, the global alignment will here be lower (because the degree is the most significant factor). In a random network there is probably not enough difference in cluster coefficient to draw any meaningful conclusions.

There was no correlation visible between the change and both the degree and the cluster coefficient.

We now look in which networks the measurements are the largest. Per simulation, we therefore look at the mean, median and standard deviation of each measurement. The mean can give a skewed view if the distribution isn't normal because a few extremely high values can increase the mean a lot. That's why we also look at the median. We look at the standard deviation to check how much difference between the nodes there is in a network.

For the fitness, there is a significant difference between all networks for the mean and the median, where the non-hierarchical performs better than the hierarchical network, which performs better than the random network. The standard deviation is only significantly higher in the hierarchical network than the rest.

The global alignment is significantly different for the mean, median and standard deviation, where again, the non-hierarchical network performs the best, while the random network performs the worst.

There is not a significant difference between any of the measurements of the change in any of the three networks.

For the influence, the difference was also significant for all networks, but the order was different than before. For the mean and the median it was opposite—the non-hierarchical network had the lowest influence, while the random network had the highest. The standard deviation was also lowest in the non-hierarchical network, but it was higher in the hierarchical than in the random network.

Figure 6.1 shows the distribution of the different measurements of one simulation.

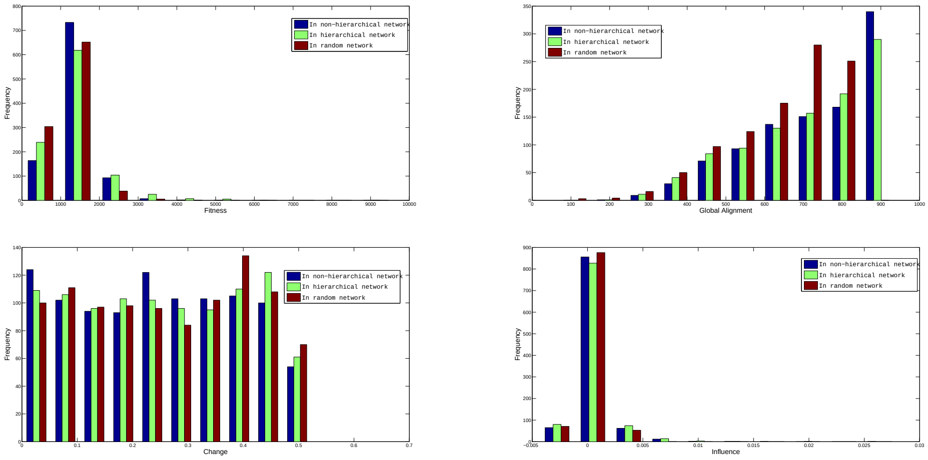

Figure 6.1: The histogram of the distributions of the 4 measurements, after one simulation. Upper left: the fitness, upper right the global alignment. Lower left the change, and lower right the influence. Blue is a non-hierarchical, green a hierarchical and red a random network.

I also checked directly whether there is a correlation between power-law behavior and the mean of the four measurements, comparing all data together, not dividing between the networks. We see that the more there is a power-law behavior, the higher the fitness and global alignment, but this correlation is higher for the degree frequency than the degree against the cluster coefficient. Because a non-hierarchical network has more power-law in the degree frequency than a hierarchical one, it performs better in these measurements. The power-law behavior with respect to the degree versus cluster coefficient causes the hierarchical network to perform better than the random one.

For the influence, we see the opposite: the more there is power-law behavior, the lower the influence, and this correlation is more profound for the degree frequency, which causes the non-hierarchical network to have the lowest influence, while the random network has the highest. There was no significant correlation with the change and power-law behavior.

When a network has a more effective structure (the correlation between the fitness and power-law behavior shows this is associated with a power-law), agents don't need to influence that much to be able to align, the network structure already enables them to align better. This is how a higher fitness can go together with a lower influence, while the change is uncorrelated.

6.3 Maximize synergy

The next model is inspired by the ecosystem. Each agent needs certain products and produces products others can use. This can be represented by assigning an n-dimensional vector to each node, with n the number of products in the system. I call this vector the need vector $\boldsymbol{n_i}$ of node i. If a node needs a certain product, I put a 1 in that place, if not, I put a 0. In the beginning, I put m 1's in a random place in each need vector.

For each system a production list $\boldsymbol{p}$ is also created. Each product an agent has, is changed by that agent into another product, a waste product. This is done by a random permutation of the products represented by the production list. This is a list containing the numbers 1 through n, but in another order. Product k becomes the product on position k in the production list, this is $\boldsymbol{p}(k)$. This production list is the same for all agents.

But which products are transformed by an agent? This is the food an agent i has, I'll represent this in its n-dimensional food vector $\boldsymbol{f_i}$. In the beginning, an agent has all the food products from his need vector, thus the two vectors are the same. In each next step, an agent obtains all the food products he needs and one of its neighbors has produced. The rest remains the same for both models. So in step 1 $\boldsymbol{f_i} = \boldsymbol{n_i}$, but from step 2 on, the vector $\boldsymbol{f_i}$ of a node is generated by:

$$\text{for } k = 1...n \quad \boldsymbol{f_i}(k) = \begin{cases} 1 & \text{if } \boldsymbol{n_i}(k) = 1 \text{ and } \exists j \in N(i) : \boldsymbol{w_j}(k) = 1 \\ 0 & \text{else} \end{cases} \tag{6.2}$$

with $N(i)$ the set of neighbors of i, and $\boldsymbol{w_j}$ the waste vector of node j as constructed below.

These food products are then transformed into waste products by the production list. The waste products are available for its neighbors. Thus, for each node i a waste vector $\boldsymbol{w_i}$ depending on the food vector $\boldsymbol{f_i}$ is defined by :

$$\boldsymbol{w_i}(\boldsymbol{p}(k)) = \boldsymbol{f_i}(k) \quad \text{for } k = 1...n. \tag{6.3}$$

The fitness of an agent i is the sum of all the waste products one of its neighbors has and he needs:

$$\text{fit}(i) = \sum_{j \in N(i)} \sum_{k | \boldsymbol{n_i}(k)=1} \boldsymbol{w_j}(k). \tag{6.4}$$

In each step, the need vector of each agent is changed by variation and selection, based on the genetic algorithm (Holland, 1992). Ten random mutations of the need vector are constructed, and the one with the highest fitness is chosen. A mutant is created by choosing two different random positions in the need vector and flip places. Thus, if we have chosen two numbers k_1 and k_2 between 1 and n randomly, and the need vector is

$$\boldsymbol{n_i} = (\boldsymbol{n_i}(1), ..., \boldsymbol{n_i}(k_1), ..., \boldsymbol{n_i}(k_2), ..., \boldsymbol{n_i}(n))$$

then our mutant is

$$\tilde{\boldsymbol{n}}_{\boldsymbol{i}} = (\boldsymbol{n_i}(1), ..., \boldsymbol{n_i}(k_2), ..., \boldsymbol{n_i}(k_1), ..., \boldsymbol{n_i}(n)).$$

10 mutants are created, and the one with the highest fitness replaces the original need vector (if this fitness is higher than the original). This assures that the fitness of an agent will rise. He will use more waste of its neighbors (by definition of the fitness), hence more food, therefore he will also produce more waste for its neighbors. Thus the fitness of its neighbors will also increase, and this effect will spread across the network. When we plot the evolution of the average fitness in time, we see that the fitness increases indeed, and is still increasing at the end of the simulation, although slower than in the beginning. This behavior is the same in all networks, though the specific values of the fitness are different.

To see how the need- and waste vector gets propagated through the network, we first look at a system where $\boldsymbol{p} = (1, ..., n)$, thus food products are transformed into themselves, $\boldsymbol{w_i} = \boldsymbol{f_i}\ \forall i$. An agent will strive to have as much waste products from its neighbors as possible, since that results in a higher fitness. It will hence aim to equalize its need vector with the waste vectors of its neighbors, by mutation of the need vector. Since $\boldsymbol{w_i} = \boldsymbol{f_i}$, it will thereby strive for a need vector which is as equal as possible with the need vector of its neighbors, especially on the positions where the need vector is equal with a neighbor of this neighbor - since then there is more chance the need will be fulfilled, and thus in this position there will be a food product. Seen from another point of view, an agent will propagate its need vector on the positions where its need is fulfilled, by sending the waste vector to its neighbors ($\boldsymbol{w_i} = \boldsymbol{f_i}$). These neighbors will evolve their need vectors towards this vector, and by doing this propagate this vector further.

In the general model where $\boldsymbol{p}$ could be any permutation list, the basic mechanism is still the same, only it is no longer true that $\boldsymbol{w_i} = \boldsymbol{f_i}$. Thus, at each agent, the vector first gets transformed before it gets further propagated. An agent evolves his need vector towards the waste vector of its neighbors, transforms this vector into the food vector and then the waste vector, and propagates this vector further, where his neighbors do the same thing. Thus the need vectors won't align towards each other anymore, but they will evolve so that the waste in the system is used as much as possible.

To summarize the model, the following steps are done for each agent i:

1. construct $\boldsymbol{f_i}$ by (6.2)
2. construct $\boldsymbol{w_i}$ by (6.3)
3. compute fitness by (6.4)
4. transform $\boldsymbol{n_i}$ by variation and selection

For the simulations, I looked at the difference in fitness and the difference in food between the end and the beginning. With the difference in fitness I mean the difference in fitness of a node between the end and the beginning of the simulation. This shows how much a node got stronger during the simulation. We could also just look at the fitness, this gives the same results, because in the beginning the fitness is approximately the same for all nodes. In the following, I will call this simply the *fitness*.

The difference in food of a node is calculated by looking at the Hamming distance between the food vector of the beginning and the end. This counts the number of positions for which the two vectors differ. I call this difference in food the *change* of a node.

This gives measurements of relatively how good a node performs and how much it got changed during the simulation (which tells how much it was influenced).

6.3.1 Results

Like the previous simulation, I will look at the correlations between network properties (degree and cluster coefficient) and the proposed measurements (fitness and change), and check which network performs the best. I did 20 iterations per simulation. I did 1000 runs of the simulation in total, over
↑ 127 three networks with 1000 nodes (constructed as described in section 5.2.1.1
↑ 130 and 5.2.2).

There is a positive correlation between the degree and the fitness. This is logical, since a higher degree means there are more neighbors you can get waste from.

There was a negative correlation between the cluster coefficient and the fitness in the non-hierarchical and hierarchical network, though in the hierarchical case, this was not significant for all simulations. In the random network there was no correlation between the cluster coefficient and the fitness, probably because there was too little difference in cluster coefficients. The negative correlation between the cluster coefficient and the fitness can be explained as follows. If a node has a low cluster coefficient, its neighbors aren't that connected. Connected nodes will be less diverse. Hence lesser connected neighbors will provide the node with more diverse resources, thus giving rise to a higher fitness.

There was no correlation between the change and both the degree and the cluster coefficient.

We now do some t-tests to check whether there is a significant difference of the fitness and change between the different networks. We again look at the mean, median and standard deviation of the fitness and the change per simulation, and use a significance level of 0.05.

Only the random network differs from the rest with respect to the mean and median fitness. For the mean, the random network performs worse, while for the median, the random network performs the best. The standard deviation is the highest in the non-hierarchical network, followed by the hierarchical network, with the random network having the lowest standard deviation.

For the change, there is no significant difference between the standard deviations. But the mean and the median change of the random network are significantly smaller than in the other networks. Figure 6.2 shows the histograms of the distribution of the fitness and the change of one simulation.

We also check the correlation between power-law behavior and the mean

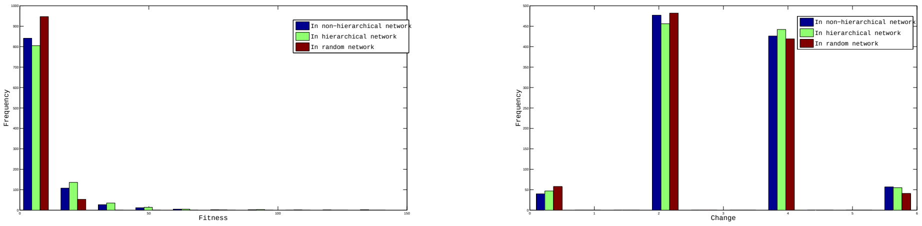

Figure 6.2: The histograms of the distribution of fitness and change, after one simulation. Left: the difference in fitness between end and beginning of the agents. On the right the change of the agents. Blue is a non-hierarchical, green a hierarchical and red a random network.

fitness and change, taking all the networks together. The correlations between all power-law behavior (both the degree frequency and the degree versus the cluster coefficient) and both the mean fitness and change are significantly positive.

Looking at the correlations separately per network type, doesn't show any significant difference.

Thus a higher degree and lower cluster coefficient correlates with a higher fitness (while it does not correlate with the change). The more there is a power-law, the more is the mean fitness and change, which explains why the random network performs the worst for both measures. But if we look at the median fitness, we see a negative correlation with power-law behavior, which is in accordance with that the random network performs best in this respect. Thus power-law behavior has a positive effect on the mean fitness, while it has a negative effect on the median fitness. Hence power-law behavior causes more inequality in fitness.

6.4 Extensions

In both models, other measurements can be taken. The second model can be extended to model some real-world phenomena. In this section several of these applications are further discussed.

The influence in the first model still looks only local, as in how much the neighbors of a node has adapted to it. We could look at a more global measure, that looks how much all the nodes have moved towards a certain node.

In both models, influence could also be understood as how much the result changes when one node changes its (begin)value.

Another fitness function could be used in the second model. Since in the above model we work only binary—either an agent has fulfilled his need for a product and thus produces it into a waste product, or he hasn't—it would be more logical to count each product only once. Thus, the fitness function of a node i would be

$$\begin{aligned} \mathrm{fit}(i) &= \sum_{k|n_i(k)=1} \max_{j\in N(i)} \boldsymbol{w_j}(k) \\ &= \sum_k \boldsymbol{f_i}(k) \end{aligned}$$

The idea to try to minimize the friction can also be introduced. In this model this could mean that it's bad for your fitness if other agents also want a product you want, and good if more neighbors have a product. This can be modeled by allowing any positive number in the food- or waste vector, not just 0 or 1. An agent divides his waste equally over the neighbors who need it. Thus, the amount of a certain food product an agent has is the sum of the amount of waste he receives from his neighbors. In this model, the fitness will be computed as the sum of all the food a node has received, thus $\sum_k \boldsymbol{f_i}(k)$.

Some biological and genetic ideas can be added. The evolution mechanism of the need vector can be made more complex by reproducing with a partner, exploiting the genetic algorithm (Holland, 1992). A partner is chosen by fitness out of the group of neighbors with enough in common. Then a crossover and mutation happens.

The notions of birth and death can be included, by adding and deleting nodes. A child can be created by the crossover and mutation mechanism, and connected with his parents. First, he gets the same fitness as his parents, he gets his mother's milk or is fed by his parents, thus he don't need to fulfill his own needs yet. As time goes by, he gets connected with more agents (by the mechanism described further), learns to be more independent, and computes his own fitness. An agent can die in two ways. First, out of hunger, thus the lower the fitness the more chance to die. Second, by being eaten by another agent. The bigger the difference is between the attacker's fitness and the prey's fitness, the more chance the prey gets killed (and thus deleted from the network).

We can use this model to try to model the evolution of life, how things become more and more complex. This can be done by assuming that the further a product is in the vector, the more complex it is. The first product could be produced into the second product, that product into the third, and so on. A more realistic model might be that the first and the second product are produced into the third, the second and third into the fourth and so on. Thus, $(12) \rightarrow 3, (23) \rightarrow 4, (34) \rightarrow 5, ...$ The first (or first two) products are always available, this could be sunlight.

We could allow transporters or men in the middle: an agent who doesn't process products, but immediately pass it over to its neighbors. Thus (some of) his food products will be the same as his waste products. The man in the middle can be seen as a catalyst in chemistry: he doesn't do anything in the process, but makes the reaction possible between two other agents. The question is now whether it is needed to give some benefit to the man in the middle to transport the goods. A catalyst doesn't have any benefit, and actually it is better to bypass the man in the middle and have direct contacts, since that's more efficient. The benefit exists if agents strive for connections which increase their fitness (thus which give them waste). You can also adapt the fitness function so that giving waste also leads to an increase in fitness.

There are three ways an agent can adapt: by changing its goals, changing its methods or changing its environment (links) (Busseniers, 2016). Which of these three happens already shows some differences in ways of control. ↑ 206

In this chapter, the first way, changing your goals, is implemented - agents change their need(vector). By giving every agent a different production list and allowing them to adapt it, the idea of changing your methods can be implemented. Changing the links can happen by letting a node connect with neighbors of neighbors, with more chance if this increases his fitness more. While a node could remove a connection if that neighbor (almost) doesn't provoke any rise in fitness. This can be done by constraining the amount of connections.

Some of these ideas can be used to build a model of a cooperative economy. The idea is that an agent has a need vector which is constant, while the production list and the links change. As before, the agents form a network and the fitness is the sum of the fulfilled needs.

Production happens if the resources coded in the production list are available. The amount of end product produced could be the amount of resources multiplied with some efficiency variable. This variable increases logarithmically as there is more produced in this manner. In the beginning, an agent isn't good at it yet, thus only a little bit is produced. But he learns fast how to do it. Once he knows how to do it, he won't get that much better in it.

The order in which products are used follows four steps. First, all agents use the products they have to fulfill their needs, directly or by first going through a production process. Next, they give their products away to the agents who need it directly or who can produce products they need with it. Third, they give it away to the agents who use it in their production process. Finally, they divide the products that are still left (if any) to all their neighbors, who just transport it.

Such an economy is organized—besides the evolution of the production list—by the links created and deleted between the agents. If A is connected to B and B to C, than A gets connected to C, with a higher chance if more needs are fulfilled. The amount of edges of an agent is limited, thus connections with certain agents are deleted. If a neighbor doesn't supply a lot of products the agent need, but takes a lot of products, he doesn't add much for the agent, and thus the connection is deleted. This is a way to avoid the development of parasites. Although giving away products doesn't have a direct profit, it is still stimulated. The details of this model still need to be worked out.

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↑ 76
↑ 97
Other structures than graphs can be used. This can be a directed graph, hierarchy could then be defined as in chapter 4. We could also work with a hypergraph, where edges can consist of more than two nodes. Here edges can be seen as a market, or for the last model more as a free shop, where products can be dropped and taken. The next step is to look what these structures do in the discussed models.

6.5 Conclusion

↑ 141
I have made some simulations which give better insight into how global organizations arise from local interactions. In the first simulation, agents tried to avoid friction by aligning their values towards each other. We saw a positive correlation between a power-law behavior and the fitness or global alignment, and a negative correlation between a power-law behavior and the influence. Thus the non-hierarchical network had the highest fitness, while the random network performed the worst.

↑ 145
In the second model, agents tried to maximize their synergy by adapting their needs to what their neighbors had to offer. The resources an agent

receives from other agents are transformed and propagated to other agents. There is a positive correlation between power-law behavior and the mean fitness and change. The random network thus performs worse than the other networks when looking at the mean fitness. For the median of the fitness however, the behavior is opposite: the random network performs the best then. There is no significant difference between the non-hierarchical and hierarchical network.

The hierarchical network is also more vulnerable to attack (while it performs better in the event of random deletions). It has in general more problems with dealing with friction.

It is difficult to draw general conclusions from these results. While part of the problem might be in the simulation setup, the results also show how the structure does not determine everything.

There might be better measures or different rules that would give clearer results, like those proposed in the extensions. Hierarchizing a non-hierarchical network made the degree power-law less profound, which obscured some of the results. Because we worked with undirected networks, a lot of the categorization of the structure chapter does not apply, including connections between the structural an functional aspects of hierarchy. But the rationale for why I used undirected networks is to show how a bigger hierarchical organization could emerge, even if the immediate connections are two-directional.

↑ 149
↑ 76

We saw that the fitness function used and the local rules influenced the results, more than the network structure. For one model, a certain network performed better, while in another model this was the opposite. This shows that there is not simply a uni-causal link from the structure to the function. This also relates to my criticism of economic determinism: the economy, the structure of society, does not simply determines society's local functioning, but there is an interaction between the two. That is why there might be more interesting results if we would let the network evolve, based on for example a fitness function.

↑ 34

Because of a lack of understanding of the functional aspects of hierarchy, like what influence and coercion are, I will develop this in the next chapter. Since simulations are not always adequate, this will be done by abstract reasoning.

Chapter 7

Autonomy and control

In this chapter I investigate formally the question of when an agent has autonomy. An autonomous agent can determine its outcomes itself as opposed to having its outcomes externally determined. I will start with an exploration of the notion of internal and external control, with a mathematically more rigorous definition provided at the end of the chapter based on theories discussed before. In between, I present existing models of control: the quantitative model of perceptual control theory and the qualitative model of the law of requisite variety. Both models are extended to a hierarchy of control: in a perceptual control hierarchy or in the law of requisite hierarchy. I posit that the argumentation for why such a hierarchy would form or be necessary, is poor. I further attempt tot clarify when there is coercion. Finally, I argue why the functional aspect of being determined often coincides with a hierarchical structure.

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The motivation to do so is to better understand what autonomy is, when exactly there is coercion, and why a hierarchical structure often coincides with determination. This investigation will provide a more mathematical definition of the difference between power-to and power-over.

↑ 13

More knowledge on the nature of coercion, can help us avoid it.

7.1 A first exploration of internal and external control

2016

A lot of people feel like they don't have control over their own life, that the path they should follow is already predetermined. They have the impression somebody or something is controlling them, and they would like to have control over their own life. But often you seem to have only two choices: either dominate or being dominated. In speaking about control, there is no difference made between being in control over your own life and controlling others. This is why I introduce the concepts of internal and external control. If one attempts to control his own situation, to fulfill his aspirations, I speak of **internal control**. **External control** refers to when one attempts to determine environmental behavior. Here, an agent wants to control everything completely.

In general, I define **control** as when one directs its acts in order to fulfill some goals, values, intents or desires. There is a cybernetic loop, where an agent sends a certain output to direct some facets. In the next section I will develop this further.

↑ 157

External control can be used in order to secure internal control. But I argue that this is not the only way: it is possible to change something without controlling others. One can find synergies in her environment so that she can develop to the fullest without standing in the way of other people's development. Thus, there are different strategies for obtaining internal control.

If one uses external control, one keeps things out of their natural state, the equilibrium state, for his own profit. People are pushed into a state where someone wants them to be, which is unnatural for them, and which they often don't want. Maintaining this state requires constant energy.

The second method is that one considers that people can take different paths in life, that there are different equilibria. How one's life develops depends on what he encounters and which acts are encouraged. Thus, there are different bifurcation points in one's life, where going one way isn't always clearly better than the other. Individuals can then align their paths so that one's path is beneficial for the other. One just interacts with others so that they have the possibility to take the other path and are empowered to do so. An example is the distribution of leaflets with information about a certain oppressive practice and suggestions for how to resist it. People have the choice to ignore the practice, but at least they now know about it, and may be empowered to do something against it. This is something completely different than telling people what they should do.

An example of the different strategies for control is the difference between traditional agriculture and permaculture. In traditional agriculture, the farmer tries to control the land completely. He removes all the organisms that don't give him food directly, often just keeping one crop which he tries to optimize to get as much as possible from it. This requires constant energy. He'll need fertilizer because the soil will get depleted from having only one crop that takes all its nutrients, and he'll need some products and machines to keep the weeds and insects away. In permaculture, the whole ecosystem is kept. One tries to interfere as little as possible in all constituent sub-ecosystems. But different kinds of ecosystems are possible, depending on small differences. The idea is that you watch and learn from how nature works, and build an ecosystem which can produce the desired outcome. The system will maintain itself. Thus, it could, in theory, be sustained indefinitely in a permanent agriculture (hence the name).

Another easier and more fictive example is the different ways you can deal with rain. You could just accept it, "It's raining, I'm getting wet, and I can do nothing about it". Or you could try to influence the weather. But you can also build a shelter. In this way, you gain control over your life without having to control the rain.

Another example can be found in **parkour**, which is a way of movement, but also a philosophy (Henry, 2017). The general principle of parkour is to get from point A to point B as efficiently as possible in a complex, often urban, environment. This is done by movements like jumping, vaulting and climbing.

With parkour, internal control is increased without altering the environment. The perception of the environment, however, changes. Walls are no longer seen as limiting your path, but become opportunities for doing new movements. A traceur (the name for a practitioner of parkour) does not follow a predestined route, but makes his own path. This philosophy can be applied, in general, to overcoming obstacles. For example, psychological ones. Obstacles become challenges, opportunities for growth and gratification. The world becomes a playground.

Parkour shows how not exercising external control does not mean you can only follow a predetermined path.

↑ 32

A related concept is 'Risk board mentality', which Gelderloos defines as the belief that contact between people who are different must result in a missionary relationship, with one converting the other (Gelderloos, 2007). Gelderloos argues that there can be a mutual influence, it's not either dominating or being dominated. You don't have to conquer the world to get control over your life.

↑ 54

The concepts of this chapter can also be related to the idea of antifragility (Taleb, 2012), in that we could make a system more antifragile without predetermining how it should behave exactly. With a strict plan, a blueprint, of how everything should be, the system will be pretty fragile. As soon as something is a little bit different than planned, everything falls apart. This is an argument for why aiming for external control could make a system more fragile.

The concept of two categories of constraints applied by management processes (Stewart, 2014) is similar to these different strategies for control. **Prescriptive constraints** specify more or less precisely the particular outcomes that occur in the managed group. Only the manager can evolve since the other entities mostly just do as they are told by the manager. With **enabling constraints**, the interests of group members get aligned with the interests of the group as a whole. Then, when an agent acts in its own self-interest, it is also in the interest of the group. The advantage of this type is that it uses the local knowledge and the diversity of the group.
Stewart also cites Salthe (Salthe, 2013) who states that constraints can arise in two ways: upper-level constraints arise external to the dynamic of entities, while lower-level constraints are fixed, internal features of the interacting entities that can influence how entities behave. Both influence the dynamic, but they aren't influenced in return, which is Stewart's definition of power. ↑ 13

In the previous paragraphs, I always somehow assumed there was one agent wanting control. It is important to keep in mind that exerting control doesn't necessarily have to happen by a single agent. In "Perceptual control ↑ 162 and social power" (McClelland, 1994), McClelland argues that social power is alignment. It is when many people align to the same goal that it is difficult to do something different. Thus, power doesn't reside in one individual alone.

In the following, I will explore how we can define internal and external control in cybernetic terms. Here, I will give a basic definition of internal and external control, and later on, I will give a definition based on entropy. ↑ 189

7.1.1 A basic definition

Some branches of systems science focus on internal control, while others aim for external control (see figure 7.1). In the **engineering approach**, ↑ 158 there is an agent (the engineer) outside the system, who assigns a goal to the system and manipulates the system so that it will reach the goal. This agent aims for external control—control over the environment. With second-order cybernetics came the **autonomous approach**, wherein the goal was re-situated as inside the agent, who acts in the environment in order to reach the goal. Subjectivity came into the picture, where systems could self-organize and pursue their own goals. From here on, internal control became more important (Heylighen and Joslyn, 2001).

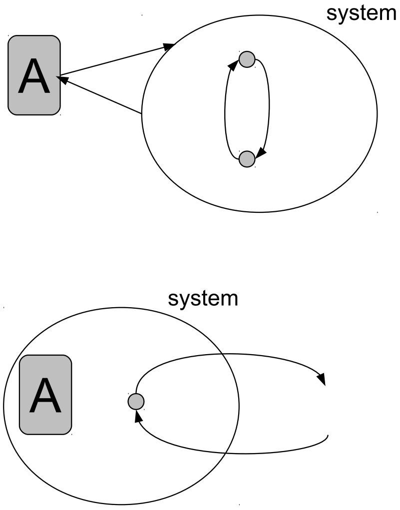


Figure 7.1: The engineering approach versus the autonomous approach. Upper figure is the engineering approach, where the agent is outside the system it tries to control. Lower figure is the autonomous approach, where the agent is now a system, with an internal goal it tries to reach by interacting with the environment.

The general scheme in cybernetics is a system that gets certain inputs (perception), which it transforms into an output (action) in order to reach goal(s). This output then changes the input the system receives from the environment. Note however, that this is completely symmetric: system and environment could simply be switched, both are transforming inputs into outputs. But in general, it is assumed there is some asymmetry—one (the system) is more powerful than the other (the environment) (Heylighen and Joslyn, 2001). But what does power mean in this context? Inputs and outputs are affected in a mutual way, and it is a priori difficult to say whether an output is changed because it is manipulated by an input, or because it wants to change this input.

To answer this question, we consider such a coupled system (see figure 7.2). We have two agents, A and B, both with goal(s) R_A and R_B. A sends a signal Y to B, which sends back a signal D. A transforms this input D into a P_A, which he then uses to create the output. All of these variables (R_A, Y, P_A,...) are objects from an unspecified form, so their values are

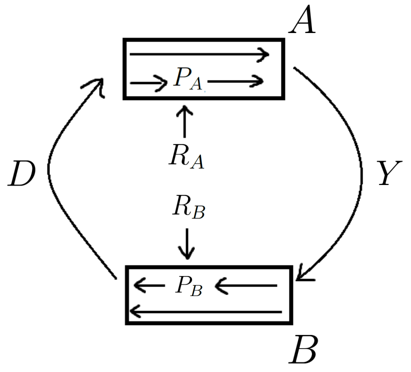


Figure 7.2: Coupled system.

not necessary numbers. When R_A is one-dimensional, P_A can be seen as the difference between the real value and the goal state, but in general, P_A can be seen as the part that matters. This means that part of the variables of the input that A cares about, in the composition he cares about. For example, A might want to have one variable bigger than another, but doesn't care how much the difference is. P_B plays the same role for B.

An agent would feel *in power* if the actions he performs can affect what he cares about, this is P_A. This can be seen as a basic definition of **internal control** (IC):

$$bIC(A) = |\frac{\delta P_A}{\delta Y}| \tag{7.1}$$

The b up front is because this is the *basic* definition of internal control, while later on I will give a definition using entropy. By the fraction on the right hand side I mean the difference in result, δP_A, given a standard difference in action, δY. Of course, this depends on the kind of action an agent performs. It might be that changing one variable doesn't have any effect, while changing another does, or that only after a certain threshold an action has effect. An agent will feel powerless if any action he does gives the same result, that is, if he doesn't have internal control. ↑ 189

This is more an abstract formula to explain what I mean by internal control, I do not specify by which measure this 'difference' is defined (this will depend on the specific form of P_A and Y) , and this formula should not be viewed as a differential equation. We do however assume there is some (unknown) measure of these differences, so that they can be compared. In the case one is simply controlling one variable, this difference can easily be defined.

On the other hand, an agent has *power over* another agent if it can influence the state of the other agent, as in the state that matters for the agent. We then say that the agent has **external control**, for which we now provide a basic definition as:

$$bEC(A) = |\frac{\delta P_B}{\delta Y}|$$

Again, this depends on what kind of actions are performed, and P_B can also be affected in different ways. There is an assumption here that we know what is important for B, since P_B is known.

However, we can also look at everything from the perspective of what happens to A. We thus look at the external control exercised on A, this is $bEC(B) = |\frac{\delta P_A}{\delta D}|$, and the internal control of A (7.1). B can here be seen just as an environment, with some unknown complex dynamics, and without the assumption that we know what is important for B (or even that there is a P_B). We can then look what a standard difference in action from A does. This δY will give rise to a difference in the input of A, this is δD. This input will change the P_A of A in an amount given by δP_A. This δP_A is the same in $bIC(A)$ and $bEC(B)$. Thus if $\delta D > \delta Y$, $bEC(B) < bIC(A)$. In words, if a small difference in action of A can lead to big changes in the input it receives, then that agent has more internal control than the external control exerted on it. On the other hand, if $\delta D < \delta Y$, then $bEC(B) > bIC(A)$. Thus if one's actions doesn't lead to a lot of results, he'll have less internal control than the external control put on him.

A similar, more specific formalism of this principle is given in (Heylighen, 1997).

Internal and external control are not necessarily related, as the following example shows (see figure 7.3). Consider a coupled system, where D and Y are two-dimensional vectors. Agent A tries to control the first variable, while B wants to control the second variable. A puts the variables at state (y_1, y_2) and sends this to B, who transforms it into (d_1, d_2). I note the updating of a variable as $F \leftarrow G$, for which I mean $F(t+1) = G(t)$: the left hand side becomes the right hand side from the previous time.

As a classical cybernetic system, each agent moves the variable it is controlling closer to its goal state. For A, this means $y_1 \leftarrow d_1 + k_1 P_A$, with $P_A = r_A - d_1$ and $k_1 \in]0, 2[$ a certain constant. If P_A is positive, d_1 is smaller than the wanted reference value, and thus this variable is increased.

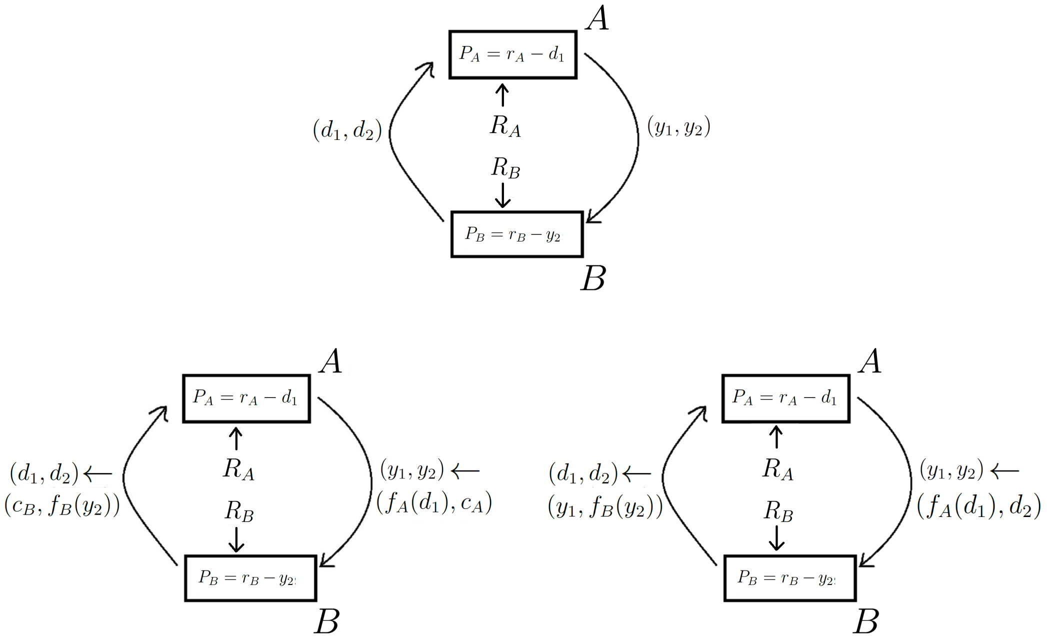


Figure 7.3: An example to show how internal and external control is not always related. Both agents control a different variable by transforming it through the functions f_A and f_B (upper). In the first case (lower left), they put the other variable to a constant, which results in external, but no internal control. In the second case (lower right), they don't affect the other variable, which result in internal, but no external control.

The opposite happens when P_A is negative. B transforms analogously the second variable by $d_2 \leftarrow y_2 + k_2 P_B$, with $P_B = r_B - y_2$ and k_2 a positive constant smaller than two. But what do the agents do with the variable that doesn't matter for them?

First, consider the case where they put this variable at a certain constant, thus $y_2 \leftarrow c_A$ and $d_1 \leftarrow c_B$. Then, both agents have no internal control at all, no matter the action they do. The difference from the goal state will remain constant, for A this is $P_A = r_A - c_B$. They do, however, have complete external control. They can completely determine the P of the other by choosing the constant they send. Thus $bEC(A) = |\frac{\delta P_B}{\delta y_2}| = |\frac{\delta(r_B - c_A)}{\delta c_A}| = 1$ (considering a change in action of the second variable). Note that this way of acting could even be, with the best intentions, to put the variable to a state the other agent prefers. But since an agent has imperfect knowledge of the goal(s) of any other agent, he won't succeed completely, and even if he does, the other agent will still feel like he's not in control.

Now, consider on the other extreme the case where the agents don't affect the variable they don't care about. Thus $y_2 \leftarrow d_2$, and $d_1 \leftarrow y_1$. Here, agents have complete internal control: a change in action will affect their P. $bIC(A) = |\frac{\delta P_A}{\delta y_1}| = |\frac{\delta(r_A - y_1)}{\delta y_1}| = 1$. However, they don't have any external control: they don't have any effect on the P of the other. Actually, we can see this as a completely decoupled system, where two agents simply try to control another variable, independent from each other.

I will now give some models of cybernetic control: perceptual control theory and the more general model in the law of requisite variety. Then, I will discuss these models and the connections between them and draw some general insights.

7.2 Perceptual Control Theory

Perceptual Control Theory was developed by W.T. Powers, but I will base my discussion of the theory on the work of McClelland (McClelland, 2004), who applies it to the social realm.

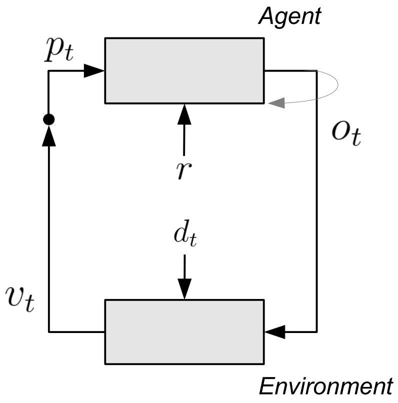


Figure 7.4: Scheme of perceptual control theory.

In perceptual control theory, an agent tries to control p_t by trying to equalize it with a certain reference value r. p_t is a perception: a part of the environment the agent receives as input, and which it wants to direct to a certain value. It does so by sending a certain output o_t. Here, the variables are one-dimensional and represent numbers. In general, this process is modeled by the following algorithm, as illustrated in figure 7.4:

$$\begin{aligned} p_t &= v_{t-1} \\ o_t &= o_{t-1} + \alpha(r - p_t) \\ v_t &= o_t + d_t \end{aligned}$$

v_t is the value of an environmental variable at time t, α a certain constant expressing the speed of adaptation, and d_t a random disturbance. Thus, at each time t the perception p_t becomes the environmental variable created at $t-1$. The agent adapts its output so that if p_t was too big before, o_t decreases, and if p_t was too small, o_t increases. d_t expresses the disturbance on the environmental variable, and is usually chosen as a random variable varying around 0. Thus the environmental variable v_t becomes the output o_t plus the disturbance. In this model, the agent will succeed in getting the perception to the desired reference value, $p_t \approx r$, except for some random fluctuations due to d_t.

But usually there are other agents who try to control the same perception, with other reference values in mind. Imagine there are n agents, then the output o_{it} an agent i creates looks like:

$$o_{it} = o_{i(t-1)} + \alpha_i(r_i - p_t)$$

The environmental variable now becomes:

$$v_t = o_{1t} + o_{2t} + ... + o_{nt} + d_t$$

In this model, the perception will converge to a weighted average of the reference values, where α_i represents the power an agent has. Assume there is convergence, then $v_t = v_{t-1}$ from a certain t on (not counting the random disturbances which will make it vary a bit). Thus

$$\begin{aligned} \sum_i o_{i(t-1)} &= \sum_i o_{i(t-1)} + \sum_i \alpha_i(r_i - p_t) \\ &\Leftrightarrow \sum_i \alpha_i(r_i - p_t) = 0 \\ &\Leftrightarrow p_t = \frac{\sum_i \alpha_i r_i}{\sum_i \alpha_i} \end{aligned}$$

Thus, the bigger α_i is, the closer an agent i can put the perception to its reference value r_i. However, the output an agent generates will explode since at every step, each agent will add $\alpha_i(r_i - p_t)$ to its output, which will be a constant different from zero (since $p_t = v_{t-1}$ won 't evolve anymore from a certain time on). While the agents are in conflict (with opposing outputs), the system will stabilize.

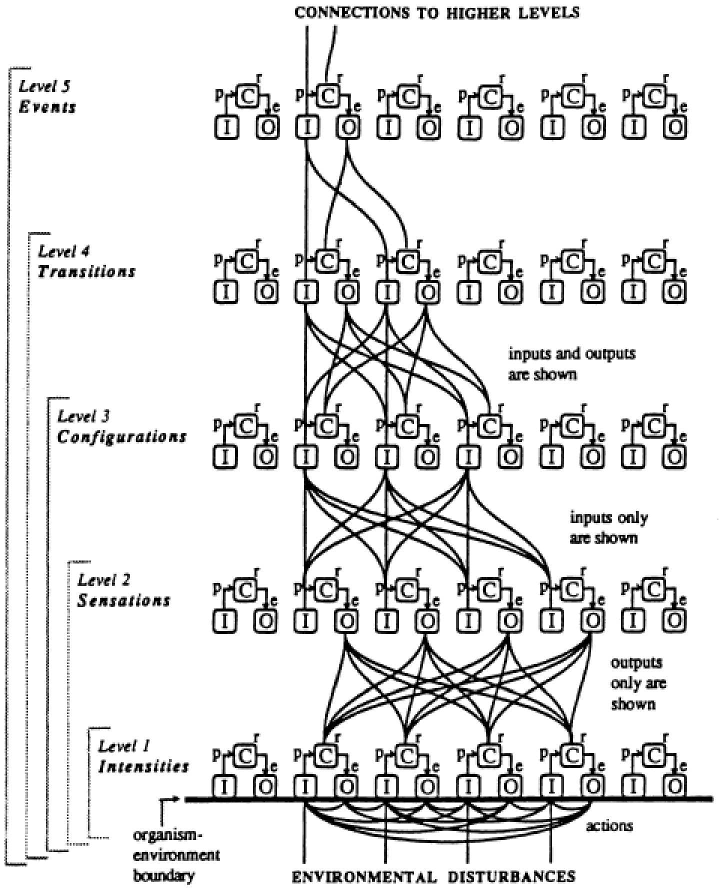
There will, however, often be a limit on the output that can be produced. We consider the case where only two agents are involved (this can be generalized to more agents). When both agents have the same limits, the environmental variable will vary around 0 ($= o_{1max} - o_{2max}$, both agents will construct their maximal output, but in opposing directions, since there is conflict), the variation only due to d_t. As soon as this disturbance exceeds one of the reference values however, this agent can relax its output, and can gain control. The perception will vary around its reference value until the disturbance is too big, so that it has to move back to its maximal output. This shows how a third party (acting as the disturbance) can have influence in a conflict by slightly favoring one party. If, however, the disturbance is so big that it is far removed from both reference values, both agents will work together by creating the same output to counteract this disturbance.

When only one of the agents has limits (or when the maximal output one agent can produce is substantially bigger than that of the other agent), the agent without limits will win the conflict—it will be able to get the environmental variable around its reference value.

But a reference value can also change. Then this one control loop is part of a **perceptual control hierarchy**, where a reference value is controlled by a higher order agent. It changes this reference value when the lower-order agent chronically fails to match its perception with its reference value (see figure 7.5). There are two reasons an agent could have a lack of control: either because the agent simply has to little power to change it (α is too low), or when there is a conflict between multiple agents.

There is no reason however why agents changing the reference value of other agents should be structured into a hierarchy, i.e. this could as well be cyclical, two agents could as well change one reference value,
↑ 182 and/or an agent might only change one reference value. Further on, I give counterexamples of such non-hierarchical configurations.

Social power can be modeled by this framework (McClelland, 1994). If
↑ 19 all the other agents have the same reference value, one or few agents with another reference value won't be able to match the reference value shared by all the other agents. This will cause a conflict, which will be resolved by a higher order agent that changes the reference value to the group's reference value. Thus, an agent will conform to the pressure of the group of all the same reference values. This implies that power resides in a group, and not in a particular individual. While any individual might feel coerced by social



Source: Adapted from Powers (1989a, p. 278).

Figure 7.5: A perceptual control hierarchy - from (McClelland, 1994).

power, as soon as it adapts, it contributes to the social power. That's how social power can be understood as alignment. It is when all agents align ↑ 41
to the same value that it is difficult to do otherwise. Thus, it is merely an illusion that the group's power is possessed by a leader. No individual can significantly manipulate which reference the group will align to.

In the next chapter, I take inspiration from this model to model agents ↑ 221
who try to gain control by changing the methods other agents use.

This model has a pretty limited view of the goal of an agent, as an agent simply wants to put a one-dimensional variable to a certain reference value.

The next section describes a more general model of control.

7.3 Law of Requisite Variety

The law of requisite variety (Ashby, 1991, 1961) encompasses a model of how an agent deals with disturbances and transforms them. While Ashby provides a general argumentation for why this law holds, I will fill in and prove the details, clarify the case of shielding, and discuss some shortcomings of the theory.

Ashby starts by assuming the following mapping:

$$D \times R \to Y \to E$$

With D, R, Y and E respectively the set of disturbances, regulatory acts, outcomes (or yields) and essential variables. A **disturbance** from D together with a **regulatory act** from R maps to an **outcome** in Y: an agent transforms an input into an output. This outcome can then be mapped into the **essential variables** E. This tells how preferred a certain outcome is by the regulator, and can be as simple as “good” or “bad”. This can be seen as the variables a system wants to keep within certain limits, for example, the temperature. There is a subset $E_0 \subset E$ which is the goal of the regulator (ie it is the goal of the regulator to get the essential variables in this set). This E_0 maps back to a set $Y_0 \subset Y$ of acceptable outcomes, those outcomes that map to E_0. This in turn maps to an $S \subset D \times R$, which defines a binary relation on $D \times R$, where $(d_i, r_j) \in S$ if it results in a wanted value.

Any input an agent gets is a disturbance, hence D encompasses all the influences from the environment. As before, the environment can be seen as an external agent. A certain d can be seen as a certain state the environment is in, and could therefore be a combination of several obstacles and opportunities.

There is a special case when the regulatory act and the outcome are just one-dimensional variables, i.e. $D \times r \to y$. This is the specific case often considered when discussing control, for example, in the previous section. The regulator can only vary one variable in order to influence the outcome. Often this outcome feeds back as input, and thus the disturbances are also one-dimensional. It typically controls these by adding a certain value to the input.

We can write the mapping $D \times R \to Y$ as a matrix, with the disturbances as rows, and the regulatory acts as columns, giving rise to a certain outcome:

$$\begin{array}{c|ccccc|} & r_1 & \cdots & r_j & \cdots & r_m \\ d_1 & z_{11} & \cdots & z_{1j} & \cdots & z_{1m} \\ \vdots & \vdots & \vdots & & \vdots & \\ d_i & z_{i1} & \cdots & z_{ij} & \cdots & z_{im} \\ \vdots & \vdots & \vdots & & \vdots & \\ d_n & z_{n1} & \cdots & z_{nj} & \cdots & z_{nm} \end{array} \tag{7.2}$$

However, this assumes that the sets D, R and Y are finite (with $|D| = n$ and $|R| = m$), and that a disturbance and a regulatory act completely determines the outcome (there is only one outcome possible given a disturbance and act). Different z_{ij}'s can correspond with the same outcome y. We can consider the equivalence class $E(y) = \{z_{ij} | z_{ij} = y\}$. This is the set of all z_{ij}'s equal to y. A z_{ij} corresponds to a couple (d_i, r_j), and it is to keep track of this source that we look at z_{ij}, and not y (z_{ij} is thus the event with outcome y, caused by d_i and r_j).

The general law of requisite variety also considers probabilities of disturbances, regulatory acts and outcomes. When I speak about D, R, Y or E further on, I mean a set where every element has a certain probability of being chosen. Sometimes I will use D as the set of disturbances, while sometimes this will signify the probability distribution. All these probabilities can be derived when probabilities $p(z_{ij})$ are known (and which z_{ij} are equal, i.e. what the equivalence classes $E(y)$ are). This description works with entropy, but we can first derive a simpler version by not considering probabilities.

We first assume that all elements from the same column are different, i.e. $z_{ij} \neq z_{kj} \forall i, k, j$. The rationale is that if the regulatory act does not change, the variety in the disturbances is the same as the variety in the outcome. We now look at the smallest possible number of different outcomes, since we want to minimize the number of outcomes to fulfill the limits of the essential variables. There can be m disturbances that can give rise to the same output, since there can be m regulatory acts dealing with them ($|R| = m$). Any more disturbances give rise to more outcomes, since then there should be a regulatory act that deals with more than one disturbance, and thus gives different outcomes (since the elements in one column are all different).

In general, we have $|Z|_{min} \geq \frac{n}{m}$, with $|Z|_{min}$ the minimal number of outcomes. When there are k times more disturbances than regulatory acts,

$n = k \cdot m$, one regulatory acts needs to deal with k disturbances, and thus will give k different outcomes. Another regulatory act (dealing with k different disturbances) can still lead to the same k outcomes.

The **variety** of a set A is $\log |A|$. The rationale is that we need $\log |A|$ bits to represent the $|A|$ elements (with the logarithmic in base 2). We have $\log |Z|_{min} \geq \log n - \log m$, thus $V_o \geq V_d - V_r$, with $V_o = \log |Z|_{min}$ the variety of the actual outcome, V_d the variety of the disturbances and V_r the variety of the regulatory acts.

Thus, the bigger the variety of the regulatory acts, the more this can reduce the variety in the disturbances.

Now assume equal elements are allowed in one column, and that a same value can occur maximum k times per column. This can be viewed as a **shield** action, where the variety of the disturbances is inhibited. Now, as long as the number of disturbances are up to $m \cdot k$, this can give rise to only one actual outcome. Since any of the m regulatory acts could deal with k disturbances, bringing them to the same value. Thus, as long as $n \leq q \cdot m \cdot k$, the actual number of outcomes can be limited to q. Thus $q \geq \frac{n}{m \cdot k}$, and $\log q \geq \log n - \log m \cdot k = \log n - \log m - \log k$. This is $V_o \geq V_d - V_r - K$, with $K := \log k$. In words, the variety of the disturbances can get reduced by a shield and regulatory acts, and the variety in the actual outcomes is equal to or greater than this reduced variety. This is a **simple** version of the **law of requisite variety**. The variety of the actual outcome, and also of the reduced variety of the disturbances, should be smaller than the allowed variety of the essential variables. Thus, the variety of the regulatory acts should be big enough so as to sufficiently diminish the variety of the disturbances.

7.3.1 General version

This simple version does not take into account the probability that a certain event occurs. There will however be more control possible when there is one disturbance with a high probability and nine disturbances with a very low probability than when there are ten disturbances with an equal probability. In the latter case, it is more important to be able to deal with all of them. This is why the general version of the law of requisite variety works with probabilities, and is expressed with entropies (Shannon, 2001). To recall,

↑ 62

***Definition* 7.3.1** The **entropy** of a variable A is defined as:

$$H(A) := - \sum_{a \in A} p(a) \log p(a)$$

where $p(a)$ is the probability that an event a occurs (or thus that the variable A is in state a).

The entropy measures the uncertainty, the variation in A. If the entropy is zero, only one event is possible with probability 1, and there is no uncertainty. The log is usually in base 2, and the rationale behind it is that $\log \frac{1}{p(a)} = -\log p(a)$ expresses the number of bits required to represent the event a. If an event needs k bits to be specified, then its probability is $\frac{1}{2^k}$ - each bit could have two values. The weighted sum is then taken over the information obtained by each event (which is equal to the number of bits required to represent it, i.e. $-\log p(a)$), where events with higher probabilities are counted more.

I will often use the term variation as a synonym for entropy, and differentiate it from the term variety as defined before. Variety only measures the number of different elements, while variation measures how probable it is that different elements would be encountered. I also use the term variation to still provide a link with the quantitative variation of a variable as in, for example, perceptual control theory.

↑ 162

The conditional entropy, which is the entropy of a variable B when we already know the state of the variable A, can now be defined:

Definition **7.3.2** The **conditional entropy** of B given A, is

$$\begin{aligned} H_A(B) := H(B|A) &:= \sum_{a \in A} p(a) H(B|A=a) \\ &= -\sum_{a \in A} p(a) \sum_{b \in B} p(b|a) \log p(b|a) \end{aligned}$$

with $p(b|a)$ the probability of b given a already occurs.

Following property will be used further on:

Property 7.3.3

$$H(B|A) = H(A,B) - H(A) \tag{7.3}$$

with $H(A,B)$ the entropy of A and B, obtained by using the probability $p(a,b)$, the probability of a and b, in the original definition of entropy.

Proof

$$\begin{aligned}
H(A,B) &= -\sum_{a\in A,b\in B} p(a,b)\log p(a,b) \\
&= -\sum_{a\in A,b\in B} p(b|a)p(a)\log\left(p(b|a)p(a)\right) \\
&= -\sum_{a\in A,b\in B} p(b|a)p(a)\log p(b|a) - \sum_{a\in A,b\in B} p(b|a)p(a)\log p(a) \\
&= -\sum_{a\in A} p(a)\sum_{b\in B} p(b|a)\log p(b|a) - \sum_{a\in A} p(a)\log p(a)\sum_{b\in B} p(b|a) \\
&= -\sum_{a\in A} p(a)\sum_{b\in B} p(b|a)\log p(b|a) - \sum_{a\in A} p(a)\log p(a) \\
&= H(B|A) + H(A)
\end{aligned}$$

□

When $H(B|A)$ is zero, as soon as A is known, the state of B is known, and thus B is a function of A: $B = f(A)$. When $H(B|A) = H(B)$, the two variables are independent in that the one variable gives no information about the other variable.

Now, we can explain Ashby's **law of requisite variety**, which claims that the larger the variation of actions to reduce disturbances, the more of the variation in the disturbances can be dealt with. The idea is to have a system that is subject to certain disturbances D, and which tries to diminish these by some regulatory actions R. The law can be stated as follows:

$$H(E_0) \geq H(Y) \geq H(D) + H_D(R) - H(R) - K \tag{7.4}$$

The variation of disturbances $H(D)$ can be diminished by a variation of regulatory acts $H(R)$, and by a constant shield K that covers some of the disturbances. But a regulator can only diminish a disturbance in so far as it knows which act to perform. Thus, the uncertainty $H_D(R)$ of which acts to perform given a specific disturbance accounts for a reduction of regulatory power. The variation in the outcome $H(Y)$ is bigger or equal to this reduced variation in disturbances (when $H(Y) = H(Y|R)$, it is equal). The regulator wants to keep some essential variables E within certain limits. $H(E_0)$ is the maximum variation in the essential variables so that the system still survives. The disturbances should thus get reduced enough so that it does not exceed the allowed variation in essential variables. We assume that there is a one-to-one mapping from Y to E, thus $H(Y) = H(E)$.

I now show how this law can be deduced. I first assume there is no shield, all elements from the same column should be different. This corresponds with

Property 7.3.4 *When all instances of a column are different, i.e.* $z_{ij} \neq z_{kj} \forall i, k \forall j$*, we have* $H_R(D) = H_R(Y)$*.*

Proof

$$\begin{aligned}
H_R(Y) &= -\sum_j p(r_j) \sum_y p(y|r_j) \log p(y|r_j) \\
&= -\sum_j p(r_j) \sum_i p(z_{ij}|r_j) \log p(z_{ij}|r_j) \\
&= -\sum_j p(r_j) \sum_i p((d_i, r_j)|r_j) \log p((d_i, r_j)|r_j) \\
&= -\sum_j p(r_j) \sum_i p(d_i|r_j) \log p(d_i|r_j) \\
&= H_R(D)
\end{aligned}$$

□

The rationale for this condition is the second law of thermodynamics—the entropy could not spontaneously decrease. When different instances of a column are the same, different disturbances would spontaneously (without a change in regulatory act) lead to the same outcome, and the entropy would decrease.

The condition that the entropy can only increase, would lead us to the less strict condition $H_R(D) \leq H_R(Y)$. Without changing a regulatory act, the variation before should be smaller or equal to the variation after. This condition would be obtained in the above proof when we leave the prerequisite that the outcome should be determined by a disturbance and regulatory act, hence when a disturbance (with a fixed regulatory act) can give rise to multiple outcomes.

We can now prove the law of requisite variety without a shield:

Theorem 7.3.5 *When all instances of a column are different, i.e.* $z_{ij} \neq z_{kj} \forall i, k \forall j$*,*

$$H(E_0) \geq H(Y) \geq H(D) + H_D(R) - H(R) \tag{7.5}$$

Proof

$$\begin{aligned} H(D) + H_D(R) &= H(R, D) && \text{(by (7.3))} \\ &= H(R) + H_R(D) \\ &= H(R) + H_R(Y) \leq H(R) + H(Y) \\ \Rightarrow H(Y) &\geq H(D) + H_D(R) - H(R) \end{aligned}$$

We further demand that $H(E_0) \geq H(Y)$. The entropy in the outcome should not exceed the maximally allowed entropy of our essential variables. We assume here that the entropy of Y is the same as that of E. □

I used in this proof the property that $H_R(Y) \leq H(Y)$, which is true for any two variables. $H_R(Y)$ can be seen as the weighted sum of the entropies of the columns of matrix 7.2. Within these columns there can be greater certainty of which outcome will occur and thus, less entropy. It is, for example, possible that within each column, only one outcome is possible, and $H_R(Y) = 0$, while different columns still give different outcomes, thus $H(Y) > 0$.

I now prove the most general version of the law of requisite variety, where there is a shield, the same outcome is allowed in one column. First, we have the following property:

Property 7.3.6 *When one outcome can occur up to k times in one column, $\forall y, j : |\{i | z_{ij} = y\}| \leq k$, we have*

$$H_R(D) \leq H_R(Y) + K \tag{7.6}$$

with $K := \log k$.

Proof For any r_j, we have

$$-\sum_i p(z_{ij}|r_j) \log p(z_{ij}|r_j) \leq -\sum_y k \frac{p(y|r_j)}{k} \log \frac{p(y|r_j)}{k}$$

since assuming y has k occurrences with equal probabilities, has the highest entropy: when there are less occurrences or some occurrences are more probable than others, the entropy is lower. Thus:

$$\begin{aligned} H_R(D) &= -\sum_j p(r_j) \sum_i p(z_{ij}|r_j) \log p(z_{ij}|r_j) \\ &\leq -\sum_j p(r_j) \sum_y p(y|r_j) \log \frac{p(y|r_j)}{k} \\ &= -\sum_j p(r_j) \sum_y p(y|r_j) \log p(y|r_j) + \sum_j p(r_j) \sum_y p(y|r_j) \log k \\ &= H_R(Y) + \log k \end{aligned}$$

□

We thus have:

Theorem 7.3.7 *When one outcome can occur up to k times in one column (thus $H_R(D) \leq H_R(Y) + K$),*

$$H(E_0) \geq H(Y) \geq H(D) + H_D(R) - H(R) - K$$

Proof

$$\begin{aligned} H(D) + H_D(R) &= H(R) + H_R(D) \\ &\leq H(R) + H_R(Y) + K \leq H(R) + H(Y) + K \\ \Rightarrow H(Y) &\geq H(D) + H_D(R) - H(R) - K \end{aligned}$$

□

While Ashby does provide this version of the law, he does not clearly define the parameter K, nor does he give a detailed proof. He discusses the law in (Ashby, 1991) and in (Ashby, 1961, p. 202-218). While he does provide the deduction in the proof of (7.5) and postulates the condition (7.6) for shielding, he does not explain how this relates to the number of times an outcome can occur in a column. It is also my contribution to provide proofs for how the allowed occurrences in a column condition the relation between $H_R(Y)$ and $H_R(D)$ (properties 7.3.4 and 7.3.6), and for property 7.3.3 about conditional entropy. Digging deeper into this theory allowed me to see some shortcomings that were not yet addressed before.

7.3.2 Shortcomings

There were some shortcomings and obstacles I encountered when digging into this theory, which I will discuss here.

It is assumed that the number of disturbances, regulatory acts and outcomes are finite and given. This is partly because of our presentation with matrices and discrete entropy, and the law might still be valid when working with continuous entropy (which measures the entropy of a continuous variable). Even then, the states of the variables and their probabilities are considered given, yet it is usually impossible to measure such events with probabilities. This assumption also has philosophical consequences. Freedom could be seen as creating new possibilities, new events which were previously unimagined. This is impossible within the framework of Shannon entropy.

Here, freedom simply entails choosing from some given choices. This corresponds to one of the perspectives on freedom discussed in section 2.1.1.

Change is not investigated even though the probabilities of D and R, and thus the outcomes, would typically evolve. An agent is often in a feedback loop with the environment, where the outcome influences the input, and thus the probability distribution of the disturbances. Moreover, an agent will change the probability of regulatory acts given a disturbance, depending on whether this act gave the desired results. Aside from this, the environment is also constantly changing, and so the probability distribution of the disturbances will also change according to other influences.

The law also only demands that the variation of the outcome is within certain limits, and not that it is in a wanted state. It is thus possible that the outcome does not vary more than the allowed variation of the essential variables, but still does not map to E_0, the wanted essential variables. An extreme case is when all disturbance-act combinations map to one outcome. If this outcome is not desired by the agent, it will not be in control, yet the law of requisite variety would still be met.

Moreover, whether the law holds depends on the categorization of the variables—what do we consider as a different y, e,...? When we consider all outcomes as the same, the law automatically holds. When we label the essential variables as just "good" or "bad", $H(E_0) = 0$, since E_0 has only one state. This is probably the reason for the conditions $H(Y) = H(E)$ and that all instances of one column should be different (unless there is a shield). But the essential variables are only part of the outcome, some outcome will also influence the environment—deliberately or by creating "waste" products. Thus, it is possible that there is a lot of variation in the outcome, while the essential variables are still within limits.

The measurements moreover depend on the choice of how to represent R, though this might be more a feature than a shortcoming. We could see an agent as a black box, transforming input into outcome. From this perspective it does not matter how we represent R. Every disturbance gives a different outcome probability distribution. How can we then say that a regulatory act working on different disturbances is the same? It does not lead to the same outcome, we could thus say that every act on a disturbance is unique. The input-output transformation does not change when we reorder matrix (7.2) by shuffling the elements of one row, and thus changing the categorization of acts. We could for example re-arrange the matrix so that elements in one column are the same (which will only be possible in specific cases). Then $H_R(Y) = 0$, while $H_D(R)$, $H(D)$ and $H(Y)$ have not changed (since the elements in one row are the same (just reshuffled), and idem for all the elements of the matrix). When we make elements of one column the same, a shielding factor should be added, but we could also rearrange the rows without invalidating the condition taken in the fist version of the law. When outcomes with high probability are put together under one regulatory act, $H(R)$ will decrease, while the other factors in the law will still remain the same. The right hand side of inequality (7.5) thus increases, while the left side does not change. The inequality still has to hold though, since all the conditions used to derive this law, are fulfilled. These considerations are important in the further sections where we want to define internal and external determination: should this change when the matrix is differently represented?

7.3.3 Link with the second law of thermodynamics

We can link the law of requisite variety with the second law of thermodynamics (Zwick, 1978), since inequality (7.5) can be written as:

$$H(Y) - H(D) + H_{eff}(R) \geq 0 \tag{7.7}$$

with $H_{eff}(R) := H(R) - H_D(R)$ the effective entropy of the regulator. We can assume that before there was no regulator transforming something, while after there is a regulator with entropy $H_{eff}(R)$. $H(D)$ can be seen as the entropy of the environment before, when there is no regulator, while $H(Y)$ is the entropy after a regulator has acted on it. The entropy change of the environment is thus $H(Y) - H(D)$, while the change in entropy of the system is $H_{eff}(R)$. The overall entropy change should be positive (7.7), which is the second law of thermodynamics: the overall entropy can only increase. $H_{eff}(R)$ is positive, while $H(Y) - H(D)$ is often negative: an agent has reduced the entropy of the environment.

This is not strictly necessary from the law of requisite variety, but it is the case when $H(E_0) < H(D)$ since $H(Y) \leq H(E_0)$. When this condition does not hold, the entropy of the environment is already within the desired range of the essential variables, and the regulator does not have to do anything. We see that in this case, the law of requisite variety also holds when $H_{eff}(R) = 0$.

This reasoning explains as well why the same elements in one columns were considered problematic. Consider an agent with just one act, a shielding act where different disturbances are mapped to the same outcome. Then $H(Y) - H(D)$ is negative (the entropy of the environment has decreased), while $H_{eff}(R) = 0$. This is in contradiction with the second law of thermodynamics. That is why there should be a constant entropy production to maintain the balance, which is expressed by K. Thus the second law of thermodynamics is now expressed as $H(Y) - H(D) + H_{eff}(R) + K \geq 0$. Hence, we can assume that shielding requires constant energy dissipation.

↑ 62

The mechanism described here can be seen as a regulator that selects certain events from the environment, thus decreasing the entropy of the environment. But it can only decrease the entropy of the environment by increasing its own entropy. This accords with the interpretation of the second law of thermodynamics as that the entropy increases when there is no selection (but that because of selection, it can decrease).

7.4 Law of Requisite Hierarchy

The law of requisite hierarchy (Aulin-Ahmavaara, 1979) builds further on the law of requisite variety, and explains how and when a system requires hierarchy to attain control. But it also contains an implicit definition of a “governor”. In this section, I will describe how this theory is developed, and in the next section, I will explain why this does not require hierarchy.

We first assume that there is no shield, $K = 0$, and that the inequality in (7.7) is an equality, thus:

$$H(Y) = H(D) - H_{eff}(R) \tag{7.8}$$

where we again define the effective variation in regulatory acts as $H_{eff}(R) = H(R) - H_D(R)$. The result still holds when these assumptions are not taken (since this term should still be smaller than $H(E_0)$, and since a shield is merely a constant), but this makes it easier to understand. Assuming the inequality is an equality, however, is a big assumption. Since the right hand side of (7.5) is equal to $H_R(Y)$ (this can be deduced from the proof), we then have $H_R(Y) = H(Y)$. This means R and Y are independent variables, and so R cannot have any influence on the outcome. But we do want do bring the right hand side of (7.8) under a certain constant $H(E_0)$. For simplicity we call this $H(Y)$.

Now, assume there is a certain regulator $R^{(1)}$ that does not succeed in bringing the outcome $H(Y_1)$ smaller than $H(E_0)$, thus it would not survive. There can then be another regulator $R^{(2)}$ which further reduces this $H(Y_1)$

to a $H(Y_2)$. Another regulator can then work on this variation until there is a regulator $R^{(m)}$ that brings the final variation in the outcome to $H(Y)$. If there is no uncertainty of how to act, this $H(Y)$ is:

$$H(Y) = H(D) - H(R^{(1)}) - ... - H(R^{(m)})$$

Each regulator has reduced the variation in disturbances by a certain amount. We can define the complete variation in regulatory acts as:

$$H(R) := H(R^{(1)}) + ... + H(R^{(m)})$$

Regulators thus work sequential on an input - see figure 7.6.

But usually there is some uncertainty about which act to do given a certain input. For $R^{(1)}$ this is $H_D(R^{(1)})$. A regulator $R^{(i)}$, $i \neq 1$, has an uncertainty about what to do with an input Y_{i-1}, this is $H_{Y_{i-1}}(R^{(i)})$. The total uncertainty is thus:

$$H^0_D(R) := H_D(R^{(1)}) + H_{Y_1}(R^{(2)}) + ... + H_{Y_{m-1}}(R^{(m)})$$

and we have:

$$H(Y) = H(D) - H(R) + H^0_D(R)$$

Because of this uncertainty, the reduction of the disturbances might still not be enough.

That is why we introduce agents that reduce the uncertainty $H_{Y_{i-1}}(R^{(i)})$ of a regulator $R^{(i)}$. We call such agents governors. While a definition of a governor remains implicit in the exposition of Aulin, I define it as follows:

***Definition* 7.4.1** A **governor** G of a regulator R is an agent that reduces R's uncertainty $H_D(R)$.

As in (7.8), a governor reduces the variation of its input. But now the input is $R^{(i)}|Y_{i-1}$, for a governor G_i of a regulator $R^{(i)}$. The governor reduces the variation of this input to $H_{Y_{i-1}}(R^{(i)}) - H_{eff}(G_i)$.

One can interpret this reduction as ‘telling what to do’ given a certain disturbance.

If we define the variation of all governors on this level as

$$H(G^{(1)}) = H_{eff}(G_1) + H_{eff}(G_2) + ... + H_{eff}(G_m)$$

the uncertainty becomes

$$H^0_D(R) - H(G^{(1)})$$

Of course, these governors still have some uncertainty about how to reduce $H^0_D(R)$ ($H_{eff}(G_i) = H(G_i) - H_{R^{(i)}|Y_{i-1}}(G_i)$), and we can build governors on the next level to reduce this uncertainty. This amounts for a total reduction of $H(G^{(2)})$. We can continue with this until we reach the top level governor $G^{(s)}$. We can now define the total reduction of uncertainty by all governors as:

$$H(G) := H(G^{(1)}) + H(G^{(2)}) + ... + H(G^{(m)})$$

The uncertainty now becomes

$$H_D(R) = H^0_D(R) - H(G)$$

The variation in the outcome is

$$H(Y) = H(D) - H_{eff}(R)$$

with now

$$H_{eff}(R) = H(R) - H^0_D(R) + H(G)$$

Governors thus introduce a vertical dimension, as one is on top of the other (see figure 7.6).

This effective variation in regulatory acts is between following limits:

$$H(R) \geq H_{eff}(R) \geq H(R) - H^0_D(R)$$

When the left is an equality, $H(R) = H_{eff}(R)$, the governors solve all the uncertainties. When $H_{eff}(R)$ is minimal, there is no governor at all. Aulin infers the law of requisite hierarchy from this theory, described into words as follows:

> *The weaker in average are the regulatory abilities and the larger the uncertainties of available regulators, the more hierarchy is needed in the organization of regulation and control to attain the same result of regulation, if possible at all.*

There is the implicit assumption in this theory that all agents (regulators sequentially reducing disturbances, with governors on top reducing uncertainty) have the same goal and representation, and that they all want to reduce the variation of D to the same values. This is seldom the case. The goal of an agent is not encoded into the theory (only that the variation should stay within certain limits, as noted before), but when another agent further diminishes the variation, there is more chance that this will not be to E_0.

↑ 173

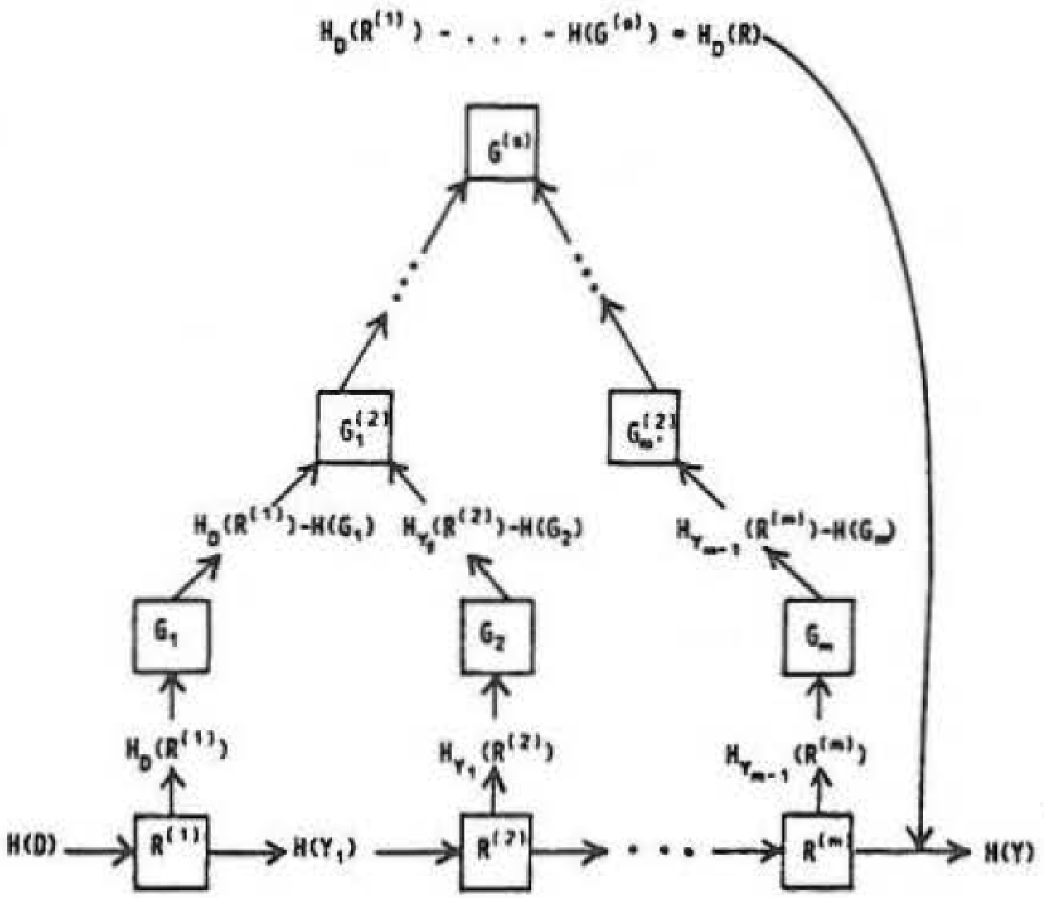


Figure 7.6: A scheme of the law of requisite hierarchy. From (Aulin-Ahmavaara, 1979).

This theory explains how a certain directionality can emerge, in that ↑ 78 there are agents that reduce the uncertainty of other agents on how to act. This defines a relation over the agents, but it does not argue why such a relation should be structured into a hierarchy. While the representation in figure 7.6 is a hierarchy, the mathematical description does not depict any governor working on multiple regulators at the same time. Each regulator has a tower of governors, but the towers can be independent, hence they would not come together into one top element. The theory does not exclude that a governor works on multiple lower level agents, which could indeed give rise to a hierarchy. But it neither excludes other configurations, like lower level regulators that also work as a governor for higher level agents.

Next, I will demonstrate how such a directionality (agents reducing the uncertainty of other agents) is possible without a hierarchical structure, and I will connect several theories from this chapter and before, putting them into a general framework.

7.5 Connecting the dots

7.5.1 Changing the method

A perceptual control hierarchy is actually just a special case of the hierarchy expressed in the law of requisite hierarchy. In both theories, there is an agent that changes the method used by another agent, namely the probability distributions of regulatory acts given a certain disturbance d, $R_d \; \forall d \in D$. With reinforcement learning, the methods of an agent are also changed, but here this happens by the agent itself.

↑ 176 In the law of requisite hierarchy, there is a governor G that brings the probability distributions R_d to R'_d, such that on average $H(R'_d) < H(R_d)$. Since it diminishes the uncertainty $H_D(R)$, this is the weighted sum of the entropies of the rows in matrix (7.2). This governor changes the probabilities of the regulatory acts an agent would choose when it encounters a certain disturbance. It changes which method the agent would use with which disturbance. $H(R)$ however, does not change. Thus, the overall probability $p(r)$ (not given a certain disturbance) of each act remains the same. The probability $p(d)$ of a disturbance can also not change, and the sum of the probabilities of each row and each column should remain the same.

Imagine now that a governor decreases the probability of z_{ij} (corresponding with the combination (d_i, r_j)) because it is below average, thus moving the probabilities further from the average, and decreasing the entropy. Since the sum in the rows and columns should remain the same, the probabilities of the other elements of the row and column z_{ij} belongs to should, on average, increase. This implies that the probabilities in other columns and rows should, on average, decrease.

↑ 164 In a perceptual control hierarchy, a higher agent changes the reference values of a lower agent, and thus the probability distribution of R. Perceptual control theory can be encoded in the framework of the law of requisite variety as follows. A disturbance $d_i = (p_t, o_{t-1})$: there is some perception coming from the environment, but we also keep some previous knowledge about which output we have sent. A certain reference value r_j can correspond with a regulatory act, and the output becomes $o_t = o_{t-1} + \alpha(r_j - p_t)$, an element of Y. This output is now transferred back to the agent, with some distortion from the environment: $p_{t+1} = o_t + d_t$ (this d_t is the distortion from the environment at time t taken from perceptual control theory, and not the input d_i from the formalism of the law of requisite variety).

Here, we assume that the distortion comes from the environment and R is a determined system with, in general, one reference value with probability 1. We could, however, also represent this by putting the error into the system, where there is some chance of choosing some neighboring reference values. For example, by a normal probability distribution around r_j. This gives the same result. Consider that a $r_i = r_j + d'_t$ is taken with a probability of $p(r_i)$, then $o_t = o_{t-1} + \alpha(r_i - p_t) = o_{t-1} + \alpha(r_j - p_t) + \alpha d'_t$. This is the same as when a $d_t = \alpha d'_t$ would be applied with a probability of $p(r_i)$ and the reference value r_j would be taken. In general, it is a combination of both—some of the distortion is because of an uncertainty and imperfection of the system on how to act, and some is caused by an influence from the environment (these influences can be random fluctuations, but also interventions from other agents, as modeled in the second case of perceptual control theory). ↑ 163

In a perceptual control hierarchy, a higher agent, whom we again call a governor, changes the reference value of a lower agent. The new reference value of this agent is the output from the governor, $r_t = V_t = O_t + D_t$ (I will put all variables of the lower level with small letters, while I represent those of the governor with capital letters). For the perception P_t the governor bases himself on the perception from the lower level (see figure 7.5). It has a certain reference value R_g, and constructs its output by $O_t = O_{t-1} + \alpha(R_g - P_t)$.

There are two possibilities of what the governor wants to control for. First, it might want to have some aggregation of lower perceptions at a certain value. Second, it might want a lower level to succeed. A possible aggregation could be the sum of lower perceptions, which it takes as P_t, and which it wants to put at a certain value R_g. If this sum is then too low, the output, and thus the reference values, would increase, which will make the lower level perceptions increase, and thus also the higher level perceptions. Here, it sends outputs to different agents.

If the governor wants the lower agent to succeed, it will adapt its reference value so that it meets the perception. Now $P_t = r_t - p_t$, and $R_g = 0$. If the reference value is now too big for the perception p_t, P_t would be positive, and $O_t = O_{t-1} - \alpha P_t$ would decrease, this is the reference value of the lower agent.

Reinforcement learning (often used in neural networks) means an agent reinforces the probability of a certain act when it gives the wanted outcome, and decreases its probability when it does not. Thus, when a disturbance d_i gives a wanted $y_0 \in Y_0$ when act r_j is chosen, the probability $p(z_{ij})$ will increase, and the probability that other acts deal with this disturbance, $p(z_{ik}), k \neq j$, will on average decrease. The opposite happens when $y_0 \notin Y_0$. This mechanism can be seen as a governor, since the probability distributions $R_d\ \forall d$ changes. But here the change of the method happens within the system, the agent is thus self-governing. In the law of requisite hierarchy, the governor was external. Such constellation can form one overall agent, wherein a governor is a component of this agent. But this scheme can also be a functional decomposition of the agent, in which the function of a governor can be described, but this governor cannot be pinpointed as one physical component of the system. This is the case with reinforcement learning as described here.

Such a change of R is thus similar to what happened in the law of requisite hierarchy. When we still demand that the overall probability of a certain act remains the same, the probability that this act deals with other disturbances, $p(z_{lj}), l \neq i$, should on average decrease. But there are other possibilities, we might increase the probability for this act to deal with similar disturbances, or we might in general increase the probability of this act, for all disturbances.

An overall increase of an act is what happened in a perceptual control hierarchy—the governor changed the reference value of the lower agent towards the perception. You could see this more continuously as that it increases the chance of a certain reference value when it matches the perception.

7.5.2 Does this imply hierarchy?

I will now investigate why an agent changing the method of another agent (by changing the probability distributions of R_d), does not imply a hierarchy, a structure discussed in chapter 4. I here understand hierarchy as an upper semi-lattice, where the relation considered is A governing (changing the method of) B. I will demonstrate this by giving counterexamples, configurations of such governors that do not comply with one of the conditions of hierarchy, i.e. governors that do not command more than one element, two governors that command the same agent, and cycles of governors.

↑ 76 ↑ 81

↑ 162 ↑ 176

In perceptual control theory as well as in the law of requisite hierarchy, a governor does not necessary command more than one element. In perceptual control theory, a governor can only control one variable, and can thus only govern several agents when this variable is an aggregation of lower level variables. The law of requisite hierarchy also only describes a governor working on one agent, although it is possible that a governor works on multiple agents, though it should be more complex then. In general, as the law of requisite variety states a regulators' acts should have at least as much variation as the disturbances, governing multiple agents will require more variation in acts, as the variation of input increases. This implies that such a governor is often a bottle neck that cannot handle all its inputs (as it is often only as complex as the other agents).

In perceptual control theory, we can imagine more than one governor that all try to influence the reference value of one agent. With two governors, the reference value of the lower level will be $r_t = V_t = O_{1t} + O_{2t} + D_t$, and as shown in the second case of perceptual control theory, none of the governors will be able to obtain control. ↑ 163 Thus, the influenced agent will be a unique constellation of the influences of the governors, and will not be controlled by them ↑ 88 (in more general cases, the combination of influences will be more complex). Only when there is just one influence, can an agent be determined by it.

To explain how there can be cycles of governors, where the method of a governor is changed by an agent implicitly governed by this governor, I build a simple example using chemical organization theory. ↑ 136 Consider following reaction network:

$$\begin{aligned} R_1 &: a + c \rightarrow b + c \\ R_2 &: b + a \rightarrow c + a \\ R_3 &: c + b \rightarrow a + b \end{aligned}$$

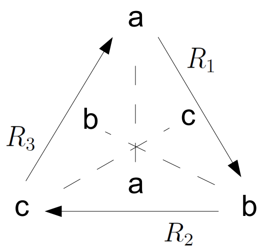


Figure 7.7: The reaction network.

The first line can be seen as an agent R_1 transforming a into b, catalyzed by c. R_2 and R_3 influence the concentration of c, and can hence influence the method of R_1. The closed sets of this reaction network are $\emptyset, \{a\}, \{b\}, \{c\}, \{a,b,c\}$, while the semi-self-maintaining sets are $\emptyset, \{a\}, \{b\}$, $\{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}$. The possible semi-organizations are thus $\emptyset, \{a\}$, $\{b\}, \{c\}, \{a,b,c\}$.

We can write this reaction network in the matrix-formalism (7.2) of the law of requisite variety as follows:

$$R_1 : \begin{array}{cc} & \begin{array}{cc} {\scriptstyle c} & {\scriptstyle \neg c} \end{array} \\ {\scriptstyle a} & \left[\begin{array}{cc} b & a \end{array}\right] \end{array} \qquad R_2 : \begin{array}{cc} & \begin{array}{cc} {\scriptstyle a} & {\scriptstyle \neg a} \end{array} \\ {\scriptstyle b} & \left[\begin{array}{cc} c & b \end{array}\right] \end{array} \qquad R_3 : \begin{array}{cc} & \begin{array}{cc} {\scriptstyle b} & {\scriptstyle \neg b} \end{array} \\ {\scriptstyle c} & \left[\begin{array}{cc} a & c \end{array}\right] \end{array}$$

When R_1 encounters a disturbance a, it will transform it to b when method c is applied, this has a probability $p(c)$, in chemistry this will be the chance c is encountered in the total of molecules, i.e. the concentration of c. Otherwise nothing will happen. When R_1 encounters another disturbance, also nothing happens (I did not encode it into the matrix for simplicity). a is encountered with a probability of $p(a)$, the concentration of a. We have here a two-directional cycle of governors: Every agent is influencing the method of the other two agents (in opposite ways), and thus an agent will indirectly influence itself. As soon as two molecules are present, the network will stabilize with all three concentrations the same, $p(a) = p(b) = p(c) = \frac{1}{3}$ (since the third molecule will be produced by the other two). We have $p_{t+1}(b) = p_t(a)p_t(c) + (1 - p_t(a))p_t(b)$: when a is encountered, R_1 will produce b when c is also encountered, but only when a is not encountered will R_2 keep the b molecules present. Similar formulas can be written for a and c. The network stabilizes when $p_{t+1}(b) = p_t(b)$, thus $p_t(a)(p_t(c) - p_t(b)) = 0$. Thus, $p_t(c) = p_t(b)$, and the other symmetrical formulas bring that this should also be equal to $p_t(a)$. Unless $p_t(a) = 0$, then either $p_t(b)$ or $p_t(c)$ should also be zero, and in this case there is only one molecule with a non-zero concentration.

There are, in general, a lot of cycles in a chemical organization since everything that is consumed should also be produced. In theory, a catalyst does not have to be produced nor consumed, but a chemical organization that produces its catalysts will be more stable since there is, in practice, always some loss of catalysts, and when these are then not reproduced, the organization will fall apart. As a catalyst can be seen as a method of an agent, producing or consuming a catalyst can be seen as a governing action (as the method is changed). When this catalyst influences its own production, there is a cycle of governors.

I've now shown how the existence of governors do not imply a hierarchy. When governing happens in a non-hierarchical structure, it will be less coercive, as an agent can have multiple influences, and/or it can influence its governor through a cycle.

7.5.3 When is coercion present?

In this section, I attempt to differentiate coercion from influence and constraint.

Coercion and constraint are often confused, yet they are not equivalent. I define **constraint**, per the discussion in 3.6.1, as a loss in the degrees of freedom. On the other hand, **coercion** occurs when an external force pushes an agent in a direction it does not really want to go. It drives the agent into a path it won't take naturally. This new path could actually have more degrees of freedom since the values of an agent usually constrain the roads it follows.

↑ 59

Take, for example, a billiard ball. When the ball lies still, it has zero degrees of freedom; its position is completely fixed. But this is its natural state, where it will end up when no unusual forces are applied. When a player hits the ball, he coerces it into a certain direction. But the degrees of freedom of the ball increase since it can now move along a line.

But how do we differentiate between coercion and influence? In the above example, the forces of gravity and the presence of the table eventually bring the ball into a still position, but since these sources were pre-existing, they are not classified as coercive. Even so, they've clearly still influenced the state of the ball. Nothing exists in a vacuum. An agent is always embedded within an environment, and is always influenced.

There are some formal definitions of coercion in the literature. In elementary theory (Willer, 2005), coercion is defined as when an agent can send negative sanctions to another agent that sends positive sanctions back. The idea behind this definition is that one agent can threaten another agent that it will send a negative sanction if it does not receive enough positive sanctions.

McClelland (McClelland, 1994) defines coercion using perceptual control theory, positing that it is the threat of using force, causing the agent to change its reference value out of prevention. Here, coercion is associated with changing a goal of an agent, whereas elementary theory identifies it as an agent allowing another agent to do something positive for it without the other agent getting anything positive in return. Here, the value system or goals of an agent are not changed. In both cases, however, there is a threat of a negative consequence that causes an agent to act in a certain way.

Coercion is easy to define when the values of an agent are considered given. In the above theories, value was expressed either by the reference value or as what is considered a 'positive' and a 'negative' sanction for an agent. But how can we define "do something you do not want", when we do not know what the values of an agent are? For this, we should derive an agent's values from the actions it performs, and see how it could be forced to do acts that contradict these values.

Heylighen (Heylighen, 2011b) defines a goal as an attractor to which an agent's actions would likely lead, an application of the 'intentional stance' (Dennett, 1989). Thus, we could equate the direction an agent is going to as its intent (where I understand direction more broadly than physical direction, but, in general, as all the states it is moving towards). This perspective accords to (McClelland, 1994), where **intention** is understood as that what is not changed, but would normally be, thus as that what is being stabilized by an agent's actions.

More generally, we can define the intention of an agent A as a change in the probability distribution when A is introduced, where the states that gain a higher probability are the wanted states, while when the probability of a state decreases, the event is unwanted.

But this definition has some difficulties when there is coercion—when an agent does certain acts which it actually does not want to do. Taking the above definition of intention, this seems impossible since we infer what one wants by what one does. This accords with the idea that methods and goals should be aligned, the method used reflects the internal goal. ↑ 31

Could we say that an agent X coerces agent A when if X would not be present, A would not go in a certain direction? In other words, coercion is when a certain disturbance d causes the probability of certain disturbances to go down while the respective agent has the preference for them to go up and vice versa.

This preference is environment-dependent, as it is derived given a certain probability distribution of the disturbances. This environment influences the agent, and it might act differently under a different distribution.

Moreover, this influence of X could also be a positive influence, as that X enables the realization of a goal of A. It is possible that only given this disturbance, the agent could move towards one of its goals, while the existing environment coerced him otherwise.

Another way to distinguish influence from coercion is by differentiating between what is still open and what was already decided, but forced otherwise. Influence could be seen as what causes an agent to end up in a certain part of its attractor, while coercion implies that the agent cannot reach its attractor. Here, influence causes one path out of different possible paths to actualize, while coercion pushes another path forward than one of the paths preferred by the agent.

7.5.3.1 In a formalism

I now clarify the formalism of how an agent can be seen as transforming a probability distribution, how coercion can be viewed when this happens, and show how we can infer an R from such a transformation.

Consider thus an agent as a transformation $D \to Y$, where R is not specified (R is internal, and cannot be viewed from the outside). We assume D and Y have the same set of possible states (namely $(d_1, ..., d_n)$), but with different probabilities. A certain disturbance d_i has some probability to be transformed into another state: $d_i \to (P_i(d_1), ..., P_i(d_n))$. This results in a general transformation of the probability distribution, where the new probability of d_j will be $P_{new}(d_j) = \sum_i P(d_i)P_i(d_j)$, with $P(d_i)$ the old probability of d_i.

The transformation an agent does can thus be represented by a matrix P: $\boldsymbol{p'} = \boldsymbol{p} \cdot P$, with $\boldsymbol{p'} = (p'_1, ..., p'_n)$ a vector of the new probabilities (those of Y), $p'_i = P_{new}(d_i)$, while $\mathbf{p} = (p_1, ..., p_n)$ represent the old probabilities of D, $p_i = P(d_i)$. Hence when we take $P_{ij} = P_i(d_j)$, we have the same transformation as before: $p'_j = \sum_i p_i P_{ij}$.

When $\boldsymbol{p}$ is considered the probability distribution of the standard, natural environment, $\boldsymbol{\Delta} = \boldsymbol{p'} - \boldsymbol{p}$ can be seen as the direction or goal of the agent within this environment. A positive Δ_i means the state d_i is wanted by the agent (it has increased its probability), while a negative Δ_i means state d_i is unwanted. The bigger $|\Delta_i|$, the more this state is (un)wanted.

When there is now a certain disturbance d_j happening (which can be represented by a probability distribution $(0, ..., 0, 1, 0, ...)$), or more in general a certain probability distribution $\boldsymbol{p^o}$ occurring, this will get transformed into a $\boldsymbol{p^{o\prime}}$. $\boldsymbol{p^o}$ represents an event or unfamiliar environment that can be potentially coercive. $\boldsymbol{\Delta^o} = \boldsymbol{p^{o\prime}} - \boldsymbol{p^o}$ then represents the direction the agent is going when confronted with this occurrence. Comparing this to the original $\boldsymbol{\Delta}$ can measure in how far this direction accords with the direction the agent wants to follow in that environment, and in far it is coerced, not an act the agent actually wants to do. This can be measured by looking at $\boldsymbol{\Delta} \cdot \boldsymbol{\Delta^o} = \sum_i \Delta_i \Delta_i^o$ (this relates to the cosine similiarity). When Δ_i and Δ_i^o are both positive or both negative, both directions agree with regard to d_i: the agent moves in the wanted direction, or away from the unwanted direction. But when one is positive and another negative, the agent moves into an unwanted direction, or away from a wanted direction. A positive term hence means the agent moves as wanted, while a negative term means it does not act as it wants. Thus the more $\boldsymbol{\Delta} \cdot \boldsymbol{\Delta^o}$ is negative, the more there is coercion.

Of course, such a measurement is highly dependent on the environment. A less environment-dependent measurement of the direction an agent wants to go, is by looking at the eigenvectors of P. For an eigenvector $\boldsymbol{p}$, we have $\boldsymbol{p}P = \boldsymbol{p}$, since we always have $||\boldsymbol{p'}||_1 = ||\boldsymbol{p}||_1 = 1$, and thus any eigenvalue $\lambda = 1$. For an eigenvector, the agent no longer changes the probability distribution, which can be considered a wanted state for the agent. An eigenvector is an attractor state.

Coercion could now be seen as moving in another direction than a linear combination of the eigenvectors (and thus away from the attractor). We could also look at the stability of eigenvectors: when most vectors end up in a certain eigenvector, this is different from an eigenvector that is only reached in a specific situation, and which might for the rest be a circumstance that is opposite to the circumstance wanted most of the times.

We can now consider n agents, and see how the emergent direction can differ from the direction wanted by any of the agents. When the transformation matrix of one agent is P^i, the overall transformation could be represented by $\frac{1}{n}\sum_i P^i$. Eigenvectors of $\frac{1}{n}\sum_i P^i$ entail the emergent direction, which differ from the average of the eigenvectors of P^i, the average of the directions of the agents. After all, the average of eigenvectors v_i of P^i is $\frac{1}{n}\sum_i v_i$ for which $v_i P^i = v_i$, and in general $(\frac{1}{n}\sum_i v_i)(\frac{1}{n}\sum_i P^i) \neq \frac{1}{n}\sum_i v_i$ (because combinations $v_i P^j$ also occur). Hence the emergent behavior could go in a different direction than what is wanted by any of the agents.

Whether and how these situations can occur, is work for further research.

You may have already noticed that matrix P closely resembles the matrix from the law of requisite variety (7.2). Indeed, when we define an r_j for each column, P expresses a probability matrix from the law of requisite variety. Thus, an act r_j is what brings an input to the outcome d_j. The probabilities of r_j are hence defined as $P(r_j|d_i) := P_i(d_j)$.

In this representation, all elements of one column are the same. But there are other ways of defining R. We could also argue that a different disturbance necessarily brings a different act. Hence, an act r_{ij} is defined as a transformation of d_i into d_j. This results in n^2 regulatory acts in the matrix representation of (7.2). This means that there are n^2 columns given that every column represents an act. Some of these acts could also be taken together and called one act. There are several ways to derive an R from a $D \to Y$ transformation, which we discussed as one of the problematics associated with the law of requisite variety. ↑ 174

Since R can be derived from a transformation, the environment can be seen as another agent transforming Y into D. We can see our scheme as a two-system model of the following transformations:

$$A : D \times R_0 \to Y$$
$$B : Y \times R_1 \to D$$

We usually call A the 'agent' and B the 'environment', but mathematically, there is no difference. This relates to how we can derive an intention from the transformation the environment performs.

7.6 Entropic internal and external control

To clarify when we can speak about coercion, I will develop a measure of determination here, and apply this to the framework of internal and external control developed in the beginning of this chapter. We can define it as: ↑ 157

***Definition* 7.6.1** The **determination** of variable Z by variable X is:

$$Det_X(Z) := \frac{H(Z) - H_X(Z)}{H(Z)} \tag{7.9}$$

When $H(Z) = 0$, we define $Det_X(Z) := 0$.

The idea behind this formula is that when $H_X(Z) = 0$, X completely determines Z ($Det_X(Z) = 1$) since when a state x of X is given, it is certain what the state of Z is. When $H_X(Z) = H(Z)$, the distribution of Z is the same no matter in what state X is, and thus X has no influence on Z, $Det_X(Z) = 0$. We have $Det_X(Z) \in [0, 1]$, and the bigger it is, the more X determines Z.

The denominator of (7.9) is the **mutual information** $I(X, Z) := H(Z) - H_X(Z)$. It measures the amount of information obtained about one variable through the other variable. $H(Z) - H_X(Z) = H(X) - H_Z(X)$ (follows from (7.3)), thus the denominators of $Det_X(Z)$ and $Det_Z(X)$ are the same. There can still be a big difference between both measurements, when $H(Z) < H(X)$, $Det_X(Z)$ will be bigger than $Det_Z(X)$, thus X can determine Z more than Z can do in return. It is thus possible that X completely determines Z, $Det_X(Z) = 1$, while Z has little influence on X. We then have $Det_Z(X) = \frac{H(X)-H_Z(X)}{H(X)} = \frac{H(Z)}{H(X)}$ (since $H_X(Z) = 0$). Thus when $H(Z)$ is considerably smaller than $H(X)$, Z will have little determination on X, and $H_Z(X)$ will be close to $H(X)$ (only differing by $H(Z)$). This can be a manifestation of a ↑ 13 power-over relation, where one variable can influence another variable, while this variable has little influence in return.

When $H(Z) = 0$, $H_X(Z) = H(Z)(= 0)$. X cannot exhibit any influence on Z since there is only one possible state of Z. This is why we defined $Det_X(Z) := 0$. It follows that $Det_Z(X) = 0$ since the denominator is zero. Z can only take one state, and so it cannot influence X by changing its state.

I will now propose definitions of internal and external control using determination and explain the rationale behind them. An agent is understood as being externally controlled when a different disturbance gives rise to a different outcome, so that if there is an agent which chooses which disturbance to send, it can control which outcome comes out. We could either define internal control as the opposite of being externally controlled, or as when choosing a different regulatory act can create a different outcome (having no internal control when any act leads always to the same result).

↑ 154 Next, I will put the example discussed in section 7.1 into this framework, and calculate the internal and external control here according to the new definitions, to see what the difference is between them and which one works best.

7.6.1 Definitions

I now use the definition of determination to define internal and external control with entropy measures. Regulatory internal control of an agent A with regulatory acts R is defined as:

$$eIC_r(A) = Det_R(Y) = \frac{H(Y) - H_R(Y)}{H(Y)}$$

The e in $eIC_r(A)$ stands for the *entropic* definition of internal control (to contrast with the basic definition given before), while the r stands for *regulatory* internal control (as I will give a complementary version further). This formula measures the extent to which the regulatory acts can determine the outcome—whether choosing a different act can influence the result. The external control exhibited on this agent by the environment B is:

↑ 157

$$eEC(B) = Det_D(Y) = \frac{H(Y) - H_D(Y)}{H(Y)}$$

This shows how much a disturbance can influence the outcome. The denominator in the formula of the determination implies that the external control from an agent on the environment can be different than the external control exerted from the environment on the agent.

The above definition depends on R. But R can be unknown, and an agent can be represented as a black box $D \to Y$. Inputs are transformed into outputs by some unknown mechanisms. When the outcome is the same for different methods, we normally do not want a difference in our measurements of internal control. That is why we introduce the following definition of complementary internal control:

$$eIC_c(A) = 1 - Det_D(Y) = \frac{H_D(Y)}{H(Y)}$$

The c in this notation stands for *complementary* internal control to differentiate it from the regulatory internal control defined before. The rationale for this formula is that the part of the outcome that is not determined by D is determined by the agent—the less D determines, the more is determined by something else. This something else can only come from the system since we still assume Y is completely determined by D and R, as D represents all possible inputs.

7.6.1.1 Discussion complementary definition

From the complementary definition of internal control, we can derive some general relation between internal and external control.

We still work with two coupled agents A and B, of which the first is usually considered the agent, while the second is considered the environment.

We already defined the external control B has over A as $eEC(B) = Det_D(Y)$. Similarly, $eEC(A) = Det_Y(D)$ is the external control of A over B. The internal control is the opposite of the external control exercised on an agent. Thus $eIC_c(A) := 1 - Det_D(Y) = 1 - eEC(B)$, and $eIC_c(B) := 1 - Det_Y(D)$.

Since $H(D) - H_Y(D) = H(Y) - H_D(Y)$, and $Det_Y(D) = \frac{H(D) - H_Y(D)}{H(D)}$, we have:

$$\begin{aligned} Det_D(Y) &= \frac{H(Y) - H_D(Y)}{H(Y)} \\ &= Det_Y(D) \cdot \frac{H(D)}{H(Y)} \end{aligned} \tag{7.10}$$

Thus, $eEC(B) = \frac{H(D)}{H(Y)} eEC(A)$. The larger $H(D)$ is in comparison to $H(Y)$, the more B has external control over A compared to the other way around. But the external control of both agents can still be large or small. The ratio $\frac{H(D)}{H(Y)}$ shows how much asymmetry there is. When $H(D)$ is close to $H(Y)$, both agents exert similar amounts of external control, and when there are big differences, one agent has substantially more external control than the other way around.

The internal control of an agent can be related to the external control it exhibits, as follows:

$$\begin{aligned} eIC_c(A) &= 1 - Det_D(Y) \\ &= 1 - Det_Y(D) \cdot \frac{H(D)}{H(Y)} \\ &= 1 - \frac{H(D)}{H(Y)} eEC(A) \end{aligned}$$

The internal control can be maximized by either minimizing the external control exhibited, or by maximizing $H(Y)$ compared to $H(D)$ (or by a combination). To minimize the determination of D on Y, either the variation of Y is made larger, or the determination of Y on D is minimized.

When A has complete external control, $Det_Y(D) = 1$. Thus, $H_Y(D) = 0$, implying $D = f(Y)$ (D is a function of Y). Thus, the outcome Y sent by A completely determines the disturbances.

When A has maximal internal control, we have $Det_D(Y) = 0$. This implies $Det_Y(D) = 0$ (from (7.10)), thus B also has maximal internal control. We have $H(D) = H_Y(D)$, and $H(Y) = H_D(Y)$, thus Y and D are independent. Y is completely determined by the acts of A, thus D does not have any influence on it.

To illustrate these formulas, the rationale when the one is more useful than the other, and the shortcomings of both, I apply them on some examples. These examples are the same as the ones given in the exploratory internal and external control section. They serve as extreme cases, where either internal or external control is maximal, and the other type is minimal.

↑ 160

7.6.2 Examples of extreme cases

- The first case was an example where an agent had no internal control since no matter which act the agent did, it always resulted in the same constant (this was conceptualized by another agent putting the outcome to a constant). When we translate this in the entropy framework of the law of requisite variety, the matrix representing the outcome given a disturbance and regulatory act, is:

$$\begin{array}{cc} & \begin{array}{cccc} r_1 & r_2 & \dots & r_m \end{array} \\ \begin{array}{c} d_1 \\ d_2 \\ \vdots \\ d_n \end{array} & \begin{bmatrix} y_1 & y_1 & \cdots & y_1 \\ y_2 & y_2 & \cdots & y_2 \\ \vdots & \vdots & & \vdots \\ y_n & y_n & \cdots & y_n \end{bmatrix} \end{array}$$

The output is completely determined by the disturbances, and the regulatory acts cannot influence it. Hence, according to the introductory section we concluded there is no internal control and complete external control.

↑ 154

We first assume $p(d_i|r_j) = p(d_i|r_k) = p(d_i)$: a regulatory act is not more probable for one disturbance than another (since the outcome is anyway already determined). Thus $H_R(Y) = H(Y)$: the entropy in one column is the same as the total entropy (since $p(y_i) = p(d_i)$). We hence have $Det_R(Y) = 0$. Since $H_D(Y) = 0$, $Det_D(Y) = 1$, and both definitions of internal control result in zero control, while external control is maximal.

But we could also assume that certain regulatory acts act more on certain disturbances compared to other acts. This means a regulator produces different acts depending on the disturbance, even though the outcome is the same. Imagine the extreme case where any regulatory act works on a different disturbance, thus $p(d_i|r_i) = 1$, while $p(d_i|r_j) = 0 \; \forall i \neq j$. This is only possible when $n = m$: every regulator works on one disturbance.

We now have $H_R(Y) = 0 \neq H(Y)$: given a regulatory act, the outcome is determined. Thus, $Det_R(Y) = 1$ and regulatory internal control is maximal, contrary to our intuition, as one disturbance always leads to the same outcome.

Since we still have $H_D(Y) = 0$, $1 - Det_D(Y) = 0$, and complementary internal control still gives zero control, while external control is still maximal.

- The second case exemplifies maximal internal control and no external control. Here, the environment did not alter the variable the agent cared about, so that the agent can completely manipulate it. Translated in the entropy framework:

$$\begin{array}{c c} & \begin{array}{cccc} r_1 & r_2 & \dots & r_m \end{array} \\ \begin{array}{c} d_1 \\ d_2 \\ \vdots \\ d_n \end{array} & \begin{bmatrix} y_1 & y_2 & \cdots & y_m \\ y_1 & y_2 & \cdots & y_m \\ \vdots & \vdots & & \vdots \\ y_1 & y_2 & \cdots & y_m \end{bmatrix} \end{array} \tag{7.11}$$

Thus the regulator can completely determine the outcome by choosing an act (having internal control), while the disturbance does not influence the outcome (thus not externally controlled).

We have $H_R(Y) = 0$ (when an r is given, the outcome is destined). Thus $Det_R(Y) = 1$.

We first assume $p(r_j|d_i) = p(r_j|d_k) = p(r_j)$: the choice of a regulatory act is not influenced by the disturbance. Then $p(y_j|d_i) = p(y_j)(= p(r_j))$, thus $H_D(Y) = H(Y)$. Hence $Det_D(Y) = 0$.

The assumption that $p(r_j|d_i) = p(r_j)$ assures there is no external control. When we leave this assumption, for example, on the other extreme that $p(r_i|d_i) = 1$, and $p(r_j|d_i) = 0 \forall j \neq i$, there is external control since a certain disturbance determines the outcome. We have $H_D(R) = 0 = H_D(Y)$, thus $Det_D(Y) = 1$. Regulatory internal control and external control are therefore maximal, while complementary internal control is zero.

In the first example, the complementary definition corresponded best with our intuition, while this last example shows why it can be worthwhile to take R into account.

In the first example, a disturbance always gave the same outcome, but when an agent used different acts with different disturbances, regulatory internal control was still maximal. An agent might feel it has internal control since a different act gives different results, but it does not know the results would be the same if it would change its acts (since it does not do that).

The second example shows why it can be worthwhile to take R into account. When we purely looked at the transformation of D into Y, a certain disturbance completely determined the outcome (with the second assumption, that $p(r_i|d_i) = 1$). Thus, the agent might look like it has little say in it. Another viewpoint is that it wants different disturbances to lead to different outcomes since it could also choose a different act so as to change the outcome. This interprets the probabilities of R as the choice an agent makes, while it could also reflect in how far the agent is able to use a certain method, and it can then not simply switch as it pleases.

The regulatory definition sees a regulatory act as a choice an agent makes for a certain result, while the complementary definition sees such acts as simply methods for obtaining a desired result without care for the internal dynamics.

Usually we understand internal control as being able to influence the outcome, regardless of how the internal dynamics work. To show how a different representation of R can influence the measurement of regulatory internal control, I create a new matrix from the last matrix (7.11) that has the same mapping $D \to Y$, thus the external dynamics are not affected, while the internal dynamics are changed.

In the law of requisite variety, it was demanded that all outcomes of one column were different, unless there was shielding. That is why we construct a new matrix by re-arranging matrix (7.11) so that all elements of one column are different:

$$\begin{array}{cc}
 & \begin{array}{ccccccc} r_1 & r_2 & \cdots & & & \cdots & r_m \end{array} \\
\begin{array}{c} d_1 \\ d_2 \\ \vdots \\ d_n \end{array} & \begin{bmatrix} y_1 & y_2 & \cdots & & & \cdots & y_m \\ y_m & y_1 & y_2 & \cdots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & & \\ y_{k+1} & \cdots & y_m & y_1 & y_2 & \cdots & y_k \end{bmatrix}
\end{array}$$

This arrangement is only possible when $n \leq m$, otherwise it is impossible to keep the elements of one column all different from one another. When we take the first assumption, we have $p(y_j|d_i) = p(y_j)$, thus $H_D(Y) = H(Y)$, and $Det_D(Y) = 0$. But now $p(r_j|d_i) = p(r_j)$ no longer holds, a certain act will act more on certain disturbances than others (depending on which outcome this gives). Thus, $H_D(R) \neq H(R)$.

$0 \neq H_R(Y) \leq H(Y)$: there is a variation of outcomes in one column. Only when $n = m$ will this be equal to the overall variation of the outcomes. Since $H(Y) - H_R(Y)$ is in between zero and $H(Y)$, we have $Det_R(Y) \in [0, 1[$. The more $n < m$ (the less disturbances there are in comparison with the regulatory acts), the closer $H_R(Y)$ will be to zero, and thus the more regulatory internal control there will be. When $n = m$, $H_R(Y) = H(Y)$, and $Det_R(Y) = 0$.

In the previous representation, regulatory internal control was maximal, allowing us to see how a different representation can completely alter the results.

7.6.3 Discussion

There are several difficulties encountered when defining internal and external control with the framework of the law of requisite variety. These partly coincide with the shortcomings discussed in section 7.3.2. ↑ 173

Some results depend on how to represent R. R can be interpreted in various ways: as the choice an agent makes, or as the available methods to deal with certain disturbances (where the agent cannot adapt R when wanted).

We do not know what the specific values are, and what the preferred states are for the regulator or the environment. We can only try to infer whether it is the agent or the environment who can manipulate the outcome to its wanted value by looking at the relative variations of the variables. But the perspective used is also that we want the freedom to vary the outcome as desired, whatever the specific value we want.

When we consider R as given, the agent has made a choice, and a lack of variation can simply mean it can put the outcome at its desired value (and does not have a lot of variation in its preferences). It is difficult to differentiate this from a determination by the environment.

In a predictable system, $H_D(Y) = 0$ and $Y = f(D)$, an outer system can send a certain disturbance to produce a specific output. Such a system is vulnerable to external control, which is why we also have $Det_D(Y) = 1$. But it does not imply external control: it is not always possible to influence the disturbances, and the agent itself might be able to affect them. None of the outcomes could be desired by the environment, while they could be desired by the agent. That is why we defined regulatory internal control as $Det_R(Y)$, which could still be maximal although external control is maximal (and thus internal control defined complementary is minimal).

$H_D(Y) = 0$ when $H_D(R) = 0$ (if given a certain disturbance there is only one act, this act can only give rise to one outcome). But in the law of requisite variety, $H_D(R)$ should be minimized since then there is less uncertainty about which act one should do given a certain disturbance, resulting in a smaller entropy. In the law of requisite variety, the goal was to limit $H(Y)$, as then the outcomes could remain within the range of desired outcomes. When $H(Y)$ is smaller, there is less chance the environment can get a desired outcome, given that the set to choose from is smaller. That is why it could make sense to define the external control of the environment as $H(Y) - H_D(Y)$, thus without dividing by $H(Y)$. The smaller $H(Y)$, the less external control is possible. But then the external control of the environment on the agent is the same as the external control of the agent on the environment since this measures the mutual information of D and Y. We divide by $H(Y)$ in the formula of the determination because a variable with a smaller entropy can be determined more. Thus, a smaller $H(Y)$ means D can more easily determine a certain outcome from the set of possible outcomes. But this set is smaller, and there is less chance one of these outcomes are wanted by the environment.

There are, however, a lot of reasons why $H_D(R)$ would be different from zero. The reality is complex, and the preferred states of an agent can evolve. Moreover, the outcome of an act can change, even under the same disturbance, because of some unknown factors. An agent can use different acts under different circumstances—certain environments, for example, can enable more acts. The distribution of the disturbances can change, for example, when an agent moves in the environment. The agent might encounter new disturbances for which it still has to find the best act. There is, furthermore, a natural entropy production (because of the second law of thermodynamics), and there are usually some errors in the process (where an agent wanted to do A, but actually did B). All of this implies that an agent is usually not completely predictable, which is sometimes even beneficial to the agent.

When an agent is not predictable, it cannot be externally controlled (or it can only be controlled by its predictive parts).

↑ 44

External control can be an example of a cultivator that is not actually best for the agent. The agent transforms the disturbance to an outcome according to its own values, but this might simply be the least bad outcome. It is an outer agent that actually wanted this outcome, and who has sent a certain disturbance in order to get it. This external agent has manipulated the agent to produce a constant flow of benefits for the external agent, making it a cultivator.

External determination (when an agent is completely externally controlled and has no internal control) means that the input can influence the output more than the agent itself. This implies that the outcome can move ↑ 185 to a value unwanted by the agent, in which case there is coercion. Determination, however, does not imply coercion. The output determined by the environment can still be desired by the agent. This would, however, be a mere coincidence, as there is no reason why this would be the case. Determination is necessary for coercion since when the environment has no influence on the outcome, it cannot push it to a value unwanted by the agent.

7.6.4 Correspondence with previous definition

↑ 154 How do these definitions relate to my basic definitions of internal and external control? In the basic definitions, an agent sent an output which was then transformed by the environment, and it was the resulting input the agent ↑ 189 wanted to control. In the entropic version, there was first a disturbance from the environment, which was acted upon with regulatory acts. It was this output the agent wanted to put within a subset of acceptable outcomes. There is no loop here (although we sometimes assume the outcome influences the disturbances).

This is the difference between feedback and feed-forward control (Heylighen, 2014a). With feedback, an agent tries to influence the input, and the output only serves this goal. With feed-forward, the agent cannot influence the disturbance, it can only transform this input into a wanted outcome. Figure 7.8 shows the difference between feedback and feed-forward in these models.

Remember my basic definition of internal control was as $bIC(A) = |\frac{\delta P_A}{\delta Y}|$. The more a standard difference in acts (δY) can lead to a difference in result (δP_A), the more internal control there is. This 'difference' was not yet well-defined. Rather than a fraction of differences, this definition can be seen as an abstract formula expressing that internal control is the extent of determination an agent's acts has on the part it wants to influence. With the definition of determination, we can now define it more clearly as $Det_Y(P_A)$. We can, in general, make a correspondence between a fraction of such differences and determination:

$$|\frac{\delta X}{\delta Z}| \leftrightarrow Det_Z(X)$$

$|\frac{\delta X}{\delta Z}|$ expresses in how far Z can determine X. We can make the basic

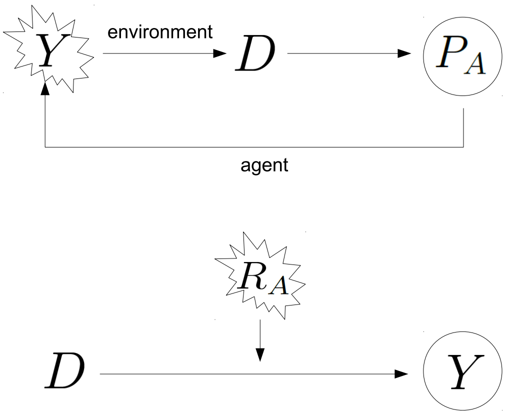


Figure 7.8: The basic model with feedback control (upper) versus the entropic version with feed-forward control (lower). The circle denotes the variable the agent wants to control, while the star-shape represents the action it does to do so.

definitions of internal and external control correspond with:

$$\begin{array}{ll} bIC(A) \leftrightarrow Det_Y(P_A) & bEC(A) \leftrightarrow Det_Y(P_B) \\ bIC(B) \leftrightarrow Det_D(P_B) & bEC(B) \leftrightarrow Det_D(P_A) \end{array} \tag{7.12}$$

These formulas differ from the newer ones. The reason is the difference between feedback and feedforward. In the original framework, an agent exhibited influence through its output Y, while in the last framework this was through its regulatory acts R_A. In the first framework, an agent tried to influence P_A, while in the recent scheme, Y was controlled (or actually E, which corresponds to the transformation from the input to the 'part that matters' P_A in the original framework). Symmetrically, the disturbances the environment sends can be understood as the regulatory acts of that environment, where the part that matters for the environment is now expressed by its outcome, the disturbances.

Thus, we can summarize the correspondence between the notations in both models as:

Basic		Entropic
Y	$\leftrightarrow$	R_A
P_A	$\leftrightarrow$	Y
P_B	$\leftrightarrow$	D
D	$\leftrightarrow$	R_B

When we substitute elements from the first column with those of the second column, formulas (7.12) for internal and external control become:

$$\begin{array}{ll} IC_A \leftrightarrow Det_{R_A}(Y) & EC_A \leftrightarrow Det_{R_A}(D) \\ IC_B \leftrightarrow Det_{R_B}(D) & EC_B \leftrightarrow Det_{R_B}(Y) \end{array}$$

Thus the formulas for internal control accord with the entropic definition of regulatory internal control. External control differs: in the basic version, an agent is externally controlled to the extent that the regulatory acts of the environment can determine the outcome, while in the entropic version, it was the extent that the disturbances could determine the outcome. Regulatory acts from the environment are considered unknown in the entropic version, where the disturbances are formed from a combination of the regulatory acts R_B from the environment, and the output Y send by the agent. In the basic version, a difference in D was actually used as a difference the environment can generate (like a difference in Y was understood as a difference in acts generated by the agent). While this definition might be more accurate, as it differs the influence coming from the environment with those from the agent, it is in practice not possible to know R_B.

7.7 Correspondence between structure and function

↑ 87
↑ 189 ↑ 185

In this section, I explain how a structural hierarchy implies functional aspects like determination and coercion, and how from such a functional definition we can derive a structural hierarchy.

↑ 189

There are three possible combinations of the influence on the input of an agent (when we use the representation of an agent coupled with the environment): the agent completely determines the input and the environment has no influence, the environment determines the input while the agent has no influence, or both have an influence on the input (we assume the input is formed by an influence and any influence that does not come from the agent is, by definition, from the environment).

↑ 13
↑ 64

We assume an output has an effect on the environment (since otherwise it would be part of an internal process inside the agent). When an agent completely determines the input, it has an influence on the environment, while the environment does not have any influence back. The agent thus has a power-over relation with the environment. The agent does not have any unwanted or uncontrolled inputs, and is self-caused. But this makes the system closed, as uncontrolled inputs could be wanted. The environment could bring unforeseen opportunities or more knowledge on how to better deal with inputs.

When the environment determines the input, it is in a power-over relation with the agent. The agent is externally controlled and cannot control its input. The effect the agent has on the environment does not get translated into its input.

In both of these cases, there is an unidirectional influence, which is not a cycle. But when both the environment and the agent can influence the input, everything is cycled. Every effect generates a cause, and every cause was triggered by an effect. Figure 7.9 illustrates these three cases.
Thus, the structural aspects of the input-output configuration of agent and environment (whether it is cyclical or not) establishes whether there is determination.

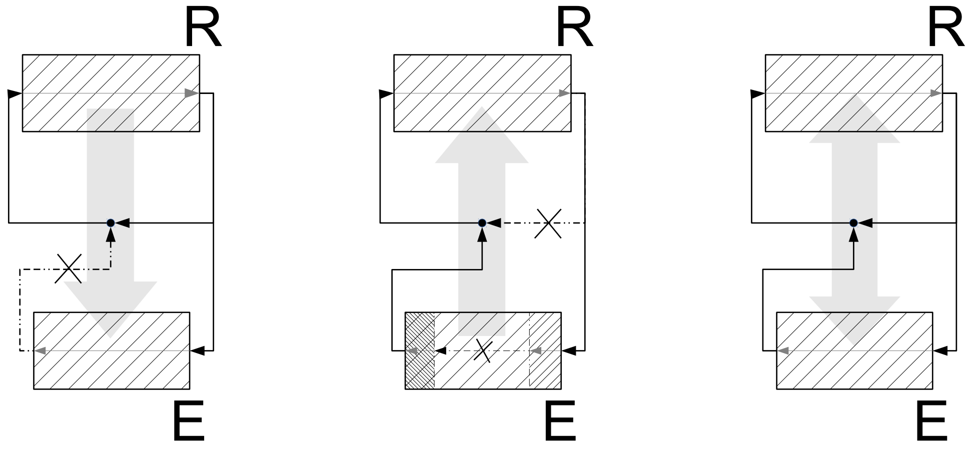


Figure 7.9: The different cases an input can be influenced. Left: the agent can determine the input, the environment has no influence. Middle: the environment can determine the input, the acts of an agent do not make it back. Right: the environment and the agent both have an influence on the input, everything is cycled.

I now show more generally how a hierarchical structure implies that an agent is more determined by its influences. Of course, this is only the case when the relation means A has some kind of influence on B. But this influence can be understood broadly. For example, as a natural number can be defined by its precursor, namely that it is $n+1$, with n its precursor, it can be said that the precursor determines this number. As a set can be seen as a union of its lower level elements, it can be said to be influenced ↑ 87 by them (while it can also be defined as an intersection, in which case the bigger sets are seen as an influence).

We assume an element x that has no influences is stable, as nothing can make it change. We further assume an element whose only influence is such a stable element, is also stable, as its influence, and therefore the result, is constant. Consider an influence that brings an element to the preceding elements plus one (the "+1"-relation). If an element is 1, an element with only this element as influence, will be and stay 2.

But when influence is cyclical, an element will change its descendant, which will eventually change the element. This way, the element won't be as stable anymore. Eventually, it is possible that all elements in a cycle are the same as a previous time—when there are no other influences, the next states will also be already encountered. For example, when we have a cycle of 3 elements and $(x_1, y_1, z_1) = (x_k, y_k, z_k)$ (the states at time 1 and k are the same), we have $(x_2, y_2, z_2) = (x_{k+1}, y_{k+1}, z_{k+1})$. Each element thus has a frequency of k: after time k, it will be in the same state again. Take for example the "+1"-relation, wherein we have $(x_{k+1}, y_{k+1}, z_{k+1}) = (z_k + 1, x_k + 1, y_k + 1)$. Assume in the beginning the elements are $(1, 2, 3)$. In the next steps the elements will become in succession: $(4, 2, 3)$, $(4, 5, 3)$, $(4, 5, 6)$,$(7, 5, 6)$... There will never be a time where the elements are the same as a previous time, and they will grow forever. But when we consider that the elements are in $\mathbb{Z}_3$ (thus working modulo 3), the elements will stay $(1, 2, 3)$, as $4 = 1(\text{mod } 3)$. In $\mathbb{Z}_6$, the frequency would be 6. In general, when there are only a finite number of states possible, eventually the states will be equal to a previously encountered state.

When an element has two unstable influences, it will itself be even more unstable. An element z under the influence of elements x and y that have a frequency of respectively k and l, will have a frequency that is the least common multiple of both frequencies, $lcm(k, l)$. As it will only be the same as a previous state when $(x_n, y_n) = (x_1, y_1)$, and this is the case when $n = i \cdot k = j \cdot l$. The variety of this element will thus often be bigger than the sum of the varieties of its inputs, and that is why we said this input is a unique constellation of its inputs, and is not determined by them. ↑ 88

When a cycle is not standing on itself, but has an element that is influenced by another unstable element (for example because this element is part of a cycle), its frequency will also increase, and be often more than the sum of the frequencies of both cycles.

We have shown how having cycles and/or more than one influence, can cause that an element is not determined by its inputs.

On the other hand, we see that when the functional aspect, i.e. determination, is present, the structural aspect follows. Determination means there is an antisymmetry: A can determine B, while B has no influence on A (as the determination would then be incomplete). There can also be only one influence, as nothing else can influence the output.

Earlier in this thesis, we discussed social power, where power was not exhibited by a top in a hierarchy, but came from the whole social system. Here, the functional aspect was present, while it seemed the structural aspect of hierarchy was not. We can, however, consider the source of the determination as an aspect system, and then this aspect system influences its components without these components having influence in return, thus leading to anti-symmetrical relations and a hierarchy. It is, however, a quite simple hierarchy (of one top connected to a lot of lower level elements), and the top cannot be grasped since it is only an aspect system. This makes it less useful to describe this case as a structural hierarchy, while there are more practical ways of describing it.

7.8 Conclusion

In this chapter, I differentiated internal from external control. Internal control is when an agent attempts to change its own situation, while external control means the agent tries to control its environment. Symmetrically, an agent is externally controlled when the environment determines the agent's output, while with internal control the agent can determine the output himself. I developed a measure of the determination of one variable by another variable. The external control exhibited on an agent is then the determination of the disturbances on the outcome. Internal control can be defined in two ways: either as the opposite of external control (the part that is not determined by the environment is determined by the agent), or as the determination of the regulatory acts on the outcome. ↑ 154 ↑ 189 ↑ 189

Both measures have their advantages and disadvantages. An agent could choose that different disturbances lead to different outcomes. Thus, internal control is not necessarily opposite to external control. But we can also see an agent as a black box transforming input into outcome, and the way to represent R should then not influence the measure of internal control. ↑ 196

Some of these problems were the same as those encountered in the law of requisite variety: the dependence on R, that we only know the variation of the variables and not whether they are in a wanted state, and so on. ↑ 173

When an agent is externally determined, it is vulnerable to coercion—the ↑ 185

outcome produced can be unwanted by the agent. It is only incidental that this outcome could be the desired outcome, as the agent has no say in it.
↑ 59 Coercion differs from constraint. Constraint limits the degrees of freedom, but this may be desired by the agent. Coercion, on the other hand, pushes the agent into an unwanted direction, which can actually increase its degrees of freedom.
↑ 200 I argued why external determination often coincides with a hierarchical structure: there can be no cycles as the agent cannot influence itself, and there can be no more than one influence, as multiple influences mean none can completely determine the outcome.
But the existence of governors, agents changing the methods of other agents,
↑ 182 does not imply a hierarchy: there can for example exist cycles of governors.

There is seldom either complete internal or complete external control. Rather, there is more of a spectrum between internal and external control, depending on in how far an agent controls its own situation, and in how far it controls the environment. We can classify diverse manifestations of control.

An agent could accept the input and make the best of it; it could change the input by sending a certain output; or it could influence methods (R) from others.

Accepting an input can be by choosing an r so that there is an outcome
↑ 166 y as wanted. This happened in the model of the law of requisite variety. But this can also go a step further, by constantly adapting one's own R so that
↑ 182 the outcome remains as wanted, as was the case with reinforcement learning.

The environment starts to be altered when an agent chooses an r so that the outcome produces a d that gives a wanted y. $H_Y(D)$ should be low for this to work, since an outcome should be able to influence the disturbances.
↑ 154 This strategy was used in our first model of internal and external control,
↑ 162 and in perceptual control theory.

An even more elaborate strategy is to influence how an outcome is transformed into a disturbance. Here the methods of other agents are changed. R, which represents to which outputs an agent transforms certain inputs,
↑ 164 is manipulated. This happened in a perceptual control hierarchy, and by
↑ 176 governors in the law of requisite hierarchy.

Influencing the input D can happen in variable ways. One could select a different $D \subset E$, with the environment E remaining the same. This could be seen as moving: the external world does not change, but one takes a more preferred part of it. Or one could change the whole environment, and thus also influence the $D \subset E$.

This can be summarized by differentiating control along two dimensions: local versus global control, and the option to change the links, methods or goals of other agents. In the next chapter several of these manifestations will be exemplified.

↑ 206

Chapter 8

Diverse manifestations of control

2016 In the following sections, I will formalize the diverse manifestations of control by using the formalization of coordination of Mesarovic (Mesarovic et al., 2000) who defines this for a hierarchical structure. I will first present this formalization, and then generalize it to any network. Next, I will apply this formalization to particular toy models where we see different manifestations of control.

There are two aspects in the difference between internal and external
↑ 154 control. Internal and external control has been discussed in the previous chapter. First, there is a difference between acting locally or globally. The other difference lies in what one tries to change, either the links, methods or goals of other agents.

↑ 118 The model of controllability (Liu et al., 2011) implies an aim for global control through the practice of adapting goals. With feedback, self-organized control can be modeled as local control with agents adapting their links. In
↑ 162 the last model, control is also local. Here I'll use Perceptual Control Theory (McClelland, 2004) to build a model of agents trying to gain control by changing the methods of their neighbors.

I will now elaborate two dimensions in the various manifestations of control.

On the one hand, the difference between these control strategies lies in their locality—one could develop local actions, or one could attempt to influence the global system.

Traditional politics still assumes people should acquire power in order to impose their societal vision. In these politics, people try to acquire global control to fulfill their needs. Alternatively, prefigurative politics assumes a

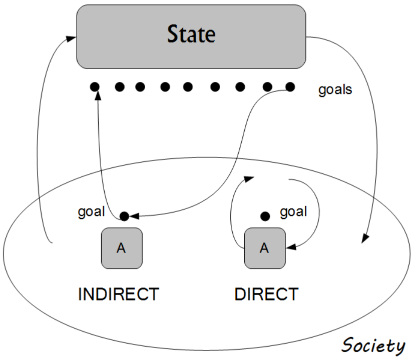


Figure 8.1: Direct versus indirect acts. A is an agent, living in a society. The agent on the left tries to reach its goals indirectly. It will try to influence the goal(s) of the state. This state tries to reach its goals by acting in the society. The problem with this approach is that some goals of the state will also influence the goal of the agent. And since the state is a much bigger structure than an individual agent, the change in the agent will be much more significant. The agent on the right acts directly in an attempt to reach its goals, by acting in the society.

person or group already strives to put their societal vision into practice in the present, i.e., goals and methods get aligned when the modes of organization reflect the future society being sought. Thus, according to prefigurative politics seeking a world where no one is controlled cannot be achieved through controlling others. Anarchists using direct action apply the principles of prefigurative politics. Direct action requires directly acting against a certain oppressive dynamic, in contrast to, for example, asking politicians to do something to create the desired change. An example is blocking an immigration detention center, so that they can't expel anyone that day.

↑ 30

With direct action, one tries to reach his goals directly. Working through the state, on the other hand, means someone tries to influence the goals of the state, so that they include his goal, or so that his goals and the ones of the state are more mutual. Figure 8.1 shows this difference.

On the other hand, the difference between several control strategies lies in the way one acts, i.e. what one tries to change. An agent could adapt its links, or try to change either the methods or the goals of its neighbors. Adapting one's links means moving to a different environment. For example, a person can try to find friends who share ideas and like what he likes, or he can try to convince his friends to do what he wants to do. Another example looks at communication. Some people spread a message with the aim of convincing. They intend to change the goals of other people. Alternatively, other people spread their ideas to create a dialogue—by getting inspiration and feedback, all participants in the discourse can improve their views. They change their connections in order to collaborate with the most interesting people, and improve the methods of their neighbors.

But these three ways are related, and we may be able to put them on a continuum. Changing one's links affects the possibilities her neighbors have, the methods they can use. And methods can be seen as putting a subgoal to reach a bigger goal. The question is then which goals are fundamental for an agent, and which are just means to an end. Probably this isn't entirely black-and-white—goals can be more or less important.

8.1 Coordination defined hierarchically

↑ 91

I now give the formalization Mesarovic (Mesarovic et al., 2000) gives of a hierarchical two-level system (which can be easily generalized to more levels). I already discussed how Mesarovic understands hierarchy in 4.2. In this model, there is a coordinator C_0, a number of infimal control systems C_i, and a process P.

This is a model of hierarchical coordination, where a top system wants to coordinate several lower level systems (called infimal control systems). Coordination is understood in the sense that the infimal systems are made to act together in order to reach an overall or top goal. They send feedback back to the top system that allows the coordinator to improve its coordination inputs. The first motivation to present this model is to show a model of hierarchical control. Mesarovic is one of the only theorists to define coordination on different levels. Further on, I will generalize the model to any structure to show that a hierarchical structure is not necessary. In this model there are still agents that send coordination inputs out in order to get a wanted result back, but this relation should no longer form a hierarchy. There can, for example, be two agents that send each other coordination inputs. Such a model can serve as a general framework to represent different agent-based network models, which gives another motivation to discuss it here. Furthermore, these applications serve as an example of diverse manifestations of control.

↑ 77

I call C_{0i} the coordination from C_0 to C_i. C_{iP} is the coordination from C_i to P. I call F_{ij} the feedback from C_i to C_j (or P if i or $j = P$). I define $C_0^+ = \times_i C_{0i}$, this is all the coordination agent C_0 sends. All the feedback C_0 receives can be represented as $F_0^- = \times_i F_{i0}$. × symbolizes the Cartesian product of sets. Coordination only happens downwards, while feedback is always in the opposite direction of a coordination input, thus upwards. Figure 8.2 shows a representation of this model.

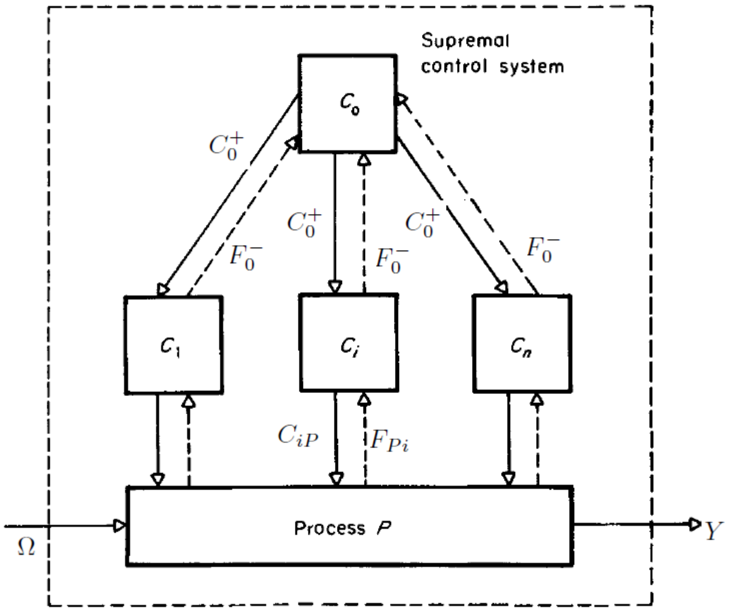


Figure 8.2: The model of Mesarovic - adapted from (Mesarovic et al., 2000).

Then each system is a set of functions transforming input signals into output signals. For the coordinator, this is:

$$C_0 : F_0^- \to C_0^+ \tag{8.1}$$

An infimal control system makes the following transformation:

$$C_i : C_{0i} \times F_{Pi} \to C_{iP} \tag{8.2}$$

While a process works as follows:

$$P : C_P^- \times \Omega \to Y$$

where Ω is the environment, and Y the output. C_P^- are all the coordination inputs C_{iP} the process gets from the infimal control systems.

We now have to define how the feedback is generated. In an infimal system, the feedback received by the system is constructed as follows:

$$f_i : C_{iP} \times \Omega \times Y \to F_{Pi}$$

The feedback the coordinator gets is:

$$f_0 : C_0^+ \times F_P^+ \times C_P^- \to F_0^-$$

What is implied here is that coordination always happens downstream, while feedback can only happen upstream. A system first takes all its inputs, which can be coordination inputs from the system above, feedback information from the system(s) below, or information from the environment, and transforms this into an output. This output can be coordination to the system below. Then, a system creates a feedback signal for itself by looking at the behavior of the system(s) below—which output did they generate given the inputs the system received? One of these inputs is the coordination the system has sent. The purpose of this feedback signal is to evaluate this coordination signal.

8.1.1 Decomposition

We can decompose these systems for a better understanding and intuition.

8.1.1.1 Decoupling

First, the system will look less complex if there is decoupling. It seems logical that the coordinator gets independent feedback signals from each subsystem. Then we get:

$$f_i0 : C_{0i} \times F_{Pi} \times C_{iP} \to F_{i0}.$$

It is possible that each subsystem controls an independent subprocess, so that the process can be decoupled.

If it isn't completely decoupled, we can add a coupling variable for such decoupling to account for the dependencies between a decoupling.

8.1.1.2 Control subsystems

We can decompose each system by regarding it as a **decision-making system**. In general, a system S is a mapping $X \to Y$. For a decision-making system, each $x \in X$ defines a decision-problem D_x. Z is the solution set of these problems. Then there is a mapping $T : Z \to Y$. Thus, (x, y) is in the system S (or equivalent, S maps x to y), if and only if there exist a z such that z is a solution of D_x and $T(z) = y$. What this basically does is split a system into a decision unit (based on D_x) and an implementer T.

The coordinator (8.1) splits into the following mappings:

$$\begin{aligned} d_0 &: F_0^- \to X_0 \\ c_0 &: F_0^- \times X_0 \to C_0^+ \end{aligned} \tag{8.3}$$

While an infimal control system (8.2) decomposes into:

$$\begin{aligned} d_i &: C_{0i} \times F_{Pi} \to X_i \\ c_i &: F_{Pi} \times X_i \to C_{iP} \end{aligned} \tag{8.4}$$

Here, the feedback signal is also used by the implementer.

8.1.2 Coordinability

Mesarovic also defines coordinability by two approaches: whether the infimal decision problems are coordinated relatively to the supremal decision problem or to a given overall decision problem.

Since Mesarovic is only interested in the command aspect, he assumes the feedback is fixed and can be left out (which I think is a big assumption, a commander can also use feedback to command better). Thus, there is only one supremal decision problem, D_0. Since the feedback is fixed, we can, without loss in generality, assume the solutions to this decision problem are the coordination inputs $c_0^+ \in C_0^+$ sent out by C_0, and that the implementer (8.3) is the identity. Further on, we define $\bar{D}(c_0^+)$ as the set $(D_1(c_{01}), ..., D_n(c_{0n}))$, the decision problems of the infimal units. Thus $\bar{x} = (x_1, ..., x_n)$ is a solution of $\bar{D}(c_0^+)$ if $\forall i : x_i$ is a solution of $D_i(c_{0i})$. Further on, Mesarovic defines the predicate $P(x, D)$ as:

$$P(x, D) \equiv x \text{ is a solution of } D$$

with D a decision problem.

We now have everything necessary to construct his definitions.

8.1.2.1 Coordinability relative to the supremal decision problem

Mesarovic defines this as:

$$\exists c_0^+ \exists \bar{x} : (P(\bar{x}, \bar{D}(c_0^+)) \wedge P(C_0^+, D_0)).$$

What this means is that there is coordinability relative to the supremal decision problem if there is a coordination input from the coordinator which is a solution to its decision problem, and which gives decision problems in the infimal control subsystems that have a solution.

Note that if the d_is and d_0 are classical functions, thus if for each c_0^+ there is one corresponding x_i, this is trivial. The decision problem D_0 then has simply one solution c_0^+. This defines the decision problems $D_i(c_0^+)$, for which each of them has the solution $x_i = d_i(c_0^+)$. Thus, in this case, the above predicate is simply always true.

In general though, a decision problem will define a set, sometimes there will be multiple solutions, sometimes it will be the empty set if there are no solutions. Thus the above is true if D_0 has some solution(s), and one of these solutions defines $D_i(c_0^+)$s that all have a solution.

8.1.2.2 Coordinability relative to a given overall decision problem

Given that we can assume that the overall decision problem depends on the process, we can also assume that we want to control the coordination signals to the process, C_P^-. Thus, the solutions of the overall decision problem D will be in the set C_P^-. Since the feedback is fixed, the implementer (8.4) of the infimal systems becomes $\pi : X \to C_P^-$, with $X = \times_i X_i$, the solutions of all infimal decision problems. Then we can define coordinability relative to an overall decision problem as:

$$\exists c_0^+ \exists \bar{x} : (P(\bar{x}, \bar{D}(c_0^+)) \wedge P(\pi(\bar{x}), D)).$$

Thus the system is coordinated if there is a coordination input from the coordinator such that the infimal decision problems have a solution and this solution transforms in a solution for the overall decision problem.

8.2 Generalization

I'd like to generalize this model to any kind of network where there isn't necessarily a 'top' and a 'bottom'. I also won't differentiate anymore between a process and a control system, everything is simply an agent. An agent gets certain input. This can come from other agents in the form of coordination or feedback, or could come from 'outside', from the environment. It transforms this into an output, which can be a coordination input for other agents, or some general output going outside.

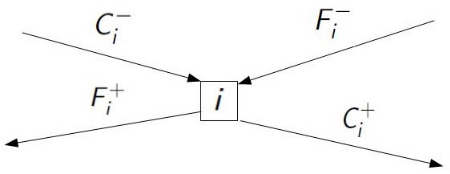


Figure 8.3: Inputs and outputs of an agent in the general model.

For the feedback, we assume the feedback an agent receives is decoupled, ↑ 210 thus it receives several feedback signals from the agents it sends a coordination input to. The function to create a feedback input will thus be constructed from the sending perspective. An agent uses his inputs and outputs to create a feedback signal.

An agent sends a certain signal, called coordination, to influence its neighbors. It gets back a feedback signal as information of how well it succeeded in its attempts.

We'll work with a directed network. The inputs and outputs of an agent i are shown in figure 8.3. A link between agent i and agent j, means i sends a coordination input $c_{ij} \in C_{ij}$ to j, where c_{ij} is the specific coordination input, and C_{ij} is the set of possible coordination inputs from i to j. Consequently, j sends a feedback signal $f_{ji} \in F_{ji}$ to i. Further on, it's possible an agent receives some input from its environment Ω_i and sends some output $y_i \in Y_i$ outside. I'll consider these also as coordination inputs and outputs, thus I define the set of coordination inputs as $C_i^- = \Omega_i \times \times_j C_{ji}$ and the coordination output as $C_i^+ = Y_i \times \times_j C_{ij}$. $F_i^- = \times_j F_{ji}$ is the feedback an agent receives, while $F_i^+ = \times_j F_{ij}$ is the feedback an agent sends. Thus the functions to create a coordination and feedback signal are the following:

$$
\begin{aligned}
C_i : C_i^- \times F_i^- &\to C_i^+ \\
F_i : C_i^- \times F_i^- \times C_i^+ &\to F_i^+
\end{aligned}
$$

An agent uses its coordination and feedback inputs to create a coordination output, and takes all of this to generate a feedback signal. The coordination output is hence taken into account to generate the feedback signal, as it can be a measure of whether the wanted result is achieved.

The toy models will give examples of how these formulas are applied, which will hopefully clarify this model. But first I would like to define coordinability in this framework, which will be a generalization of the definition of Mesarovic. For the sake of simplicity, first I will assume there is no feedback.

8.2.1 Coordinability

We can again decompose C_i into two subsystems. Since there is no feedback, this gives:

$$\begin{aligned} d_i &: C_i^- \to X_i \\ c_i &: X_i \to C_i^+ \end{aligned} \tag{8.5}$$

Now, we cannot define coordinability anymore relative to a supremal decision problem since there is no supremal unit anymore. But we can say a system is coordinated if its agents are coordinated. A definition of **weak internal coordinability** would be:

$$\exists \bar{c}, \bar{x} : \forall\, i P(x_i, D_i(c_i^-))$$

where $\bar{c} \in \times_{i,j} C_{ij} \times \times_i \Omega_i \times \times_i Y_i$, and $\bar{x} = (x_1, ..., x_n)$.

This means we can find coordination inputs so that each decision problem has a solution. If the d_is are functions (every decision problem has one solution) this is again always true. However, we would like the coordination inputs to be constructed according to our model, thus from a previous $x_i(t-1)$, by the formula $c_i^+(t) = c_i(x_i(t-1))$. But it's not necessary that the $x_i(t)$ that is a solution of $D_i(c_i^-)$ is the same as the $x_i(t-1)$ used to build this c_i^-. In a predicate, this looks like:

$$\exists \bar{x}(0), \exists t : \forall\, i \left(P(x_i(t), D_i(c_i^-(t))) \wedge (c_i^+(t) = c_i(x_i(t-1))) \right)$$

But this adds a time parameter to our model, and in practice it is difficult to check whether this holds. What we can look at though, is whether the following is true:

$$\exists \bar{c}, \bar{x} : \forall\, i P(x_i, D_i(c_i^-)) \wedge c_i^+ = c_i(x_i)$$

This assumes a stability—the same x_i that is used to build the coordination outputs c_i^+, should be a solution to the decision problem defined by the coordination inputs c_i^-. The coordination output of one agent is the coordination input of another agent. Hence when we have all the coordination outputs c_i^+, we automatically have all the coordination inputs c_i^-. Notice that

c_i^- is uniquely defined by the x_is, because the c_i^+s are. Thus we don't really have to search for the right c_i^-, which makes the $\exists \bar{c}$ part in the predicate redundant. So the following is equivalent to the above:

$$\exists \bar{x} : \forall\, i P(x_i, D_i(c_i^-)),\ \text{with } c_i^+ = c_i(x_i)$$

I call this **stable internal coordinability**. If each decision problem has one solution, d_i is a function, this is equal to:

$$\exists \bar{c}, \bar{x} : \forall\ i\ x_i = d_i(c_i^-) \wedge c_i^+ = c_i(x_i)$$

or, using the composed version again:

$$\exists \bar{c} : \forall i\ c_i^+ = C_i(c_i^-)$$

Now, we can also define coordinability relative to a given problem D. We can assume the solutions to this problem are in $Y = \bigcup_i Y_i$. There is a function $\pi : X \to Y$ (part of (8.5)). I again split up in a weak and a stable version.

There is **weak coordinability relative to a problem** D if:

$$\exists \bar{c}, \bar{x} : \forall\, i P(x_i, D_i(c_i^-)) \wedge P(\pi(\bar{x}), D)$$

While **stable coordinability relative to a problem** D is defined as:

$$\exists \bar{x} : \forall\, i P(x_i, D_i(c_i^-)) \wedge P(\pi(\bar{x}), D),\ \text{with } c_i^+ = c_i(x_i)$$

I would now like to extend these definitions so that feedback is included.

8.2.1.1 With feedback

A feedback version of **weak internal coordinability** is:

$$\exists \bar{x}, \bar{c}, \bar{f} : \forall i\ P(x_i, D_i(c_i^-, f_i^-)) \wedge f_i^+ = F_i(c_i^-, f_i^-, c_i^+)$$

The last predicate is added because otherwise the feedback could be chosen arbitrarily, and this projects to the above scenario without feedback. Here, the x_i should be a solution of the decision problem defined by the coordination and feedback inputs, while the feedback should be constructed from the feedback and coordination inputs it received, and the coordination output it sends. There is already some stability here, on the level of the feedback. The feedback an agent sends out shouldn't change the feedback signal of its neighbors. The coordination inputs can still be chosen arbitrarily, though we would like them to be constructed from a previous $x_i(t-1)$.

Stable internal coordinability is in case of feedback defined as:

$$\exists \bar{x}, \bar{f} : \forall i \; P(x_i, D_i(c_i^-, f_i^-)) \wedge f_i^+ = F_i(c_i^-, f_i^-, c_i^+), \text{ with } c_i^+ = c_i(f_i^-, x_i).$$

If the d_is are functions, this is equivalent to:

$$\exists \bar{c}, \bar{f} : \forall i \; c_i^+ = C_i(c_i^-, f_i^-) \wedge \; f_i^+ = F_i(c_i^-, f_i^-, c_i^+) \tag{8.6}$$

We can similarly speak of weak or stable coordinability relative to a problem D, by adding the condition $P(\pi(\bar{x}), D)$.

8.3 Application to various models

This generalized model can now be used to put different models into the same framework. These models show diverse manifestations of control. In these models, control is either global or local, and either the links, methods or goals of other agents will be changed. These models can be used to see which of these different strategies is the most effective. In the beginning of this chapter, the social meaning of these different strategies was discussed. ↑ 206

8.3.1 The controllability of complex networks

I would now like to apply this framework to the theory of the controllability of complex networks (Liu et al., 2011), as already discussed in 4.8. The idea here is that you try to control a network by sending certain inputs to certain nodes. This is thus a model of global control by adapting the goals of agents. Liu et al. searched for a minimal set of nodes which one has to control to have control over the whole network. In this model, each node j has a value X_j which got influenced by the values of their neighbors and the control input. This happens by the following update: ↑ 118

$$X_j \leftarrow X_j + \sum_i a_{ji} X_i + \sum_k b_{jk} u_k \tag{8.7}$$

where a_{ji} is the link weight between X_i and X_j, and b_{jk} is the link weight from the controller u_k to X_j. With $\leftarrow$ we denote that at each time step the left side is updated into the right side. Thus $A \leftarrow f$ means $A(t) = f(t-1)$, where f is a function that can contain A. Liu et al. argue that the exact values of a_{ji} and b_{jk} do not matter for the controllability. We can write this into our framework by taking X_j, the output of j, also as input of j. We take $a_{ii} = 1 \; \forall i$. An agent sends the same output to all agents. We got:

$$C_i : C_i^- \times \Omega_i \to C_i^+$$
$$X_i \leftarrow \sum_j a_{ij} X_j + \sum_k b_{ik} u_k = C_i(X, U)$$

where $X_j \in C_i^-$, $u_k \in \Omega_i$, $X = \times_j X_j$ and $U = \times_j u_j$. Thus X_i is a function of X and U.
In this model there is thus no feedback.

Liu et al. define controllability as being able to put the network in any desired state. It isn't necessary, however, that this is a steady state. You can try to steer to this state by choosing certain inputs. The problem of defining controllability is similar to the problem we faced in defining coordination. It isn't necessary that it be a stable state, but we would like the coordination inputs to be not just random, but constructed from a previous iteration. We call this version **weak controllability**, and it can be defined as a form of weak coordinability, namely when there is weak coordinability relative to all decision problems, thus

$$\forall D \exists \bar{x} : \forall \; i P(x_i, D_i(C_i^-)) \wedge P(\bar{x}, D)$$

Notice that the implementer from the decomposition in (8.5) is here the identity function; the solution x_i of the decision problem gets send out. Hence d_i (from (8.5)) is equal to C_i, which is a function. Thus the first part is actually always true, though we would like the C_i^- to come from a previous step. The second part states that we should find an $\bar{x}$ that is a solution to the decision problem, for all the decision problems. A decision problem is a subset of possible $\bar{x}$s, thus this is equivalent of stating it's true for all $\bar{x}$s.

The theory of controllability found out that you have controllability if each node has its own direct superior.

The requirement that you should reach a stable state could, however, be useful. Reaching a desired state for only a millisecond, is often not what you want. Thus, I define **stable controllability** as:

$$\forall \bar{c} \; \exists \bar{u} : \forall i \; c_i^+ = C_i(c_i^-, \bar{u}). \tag{8.8}$$

We can again define this by seeing it as an overall decision problem. The solutions of the decision problem are in $\bar{C}$. It is a solution of the problem if it is equal to our predefined desired state. Thus controllability means the following is true for all decision problems:

$$\exists \bar{c}, \bar{u} : \forall i \; c_i^+ = C_i(c_i^-, \bar{u}) \wedge P(\bar{c}, D).$$

Internal stable coordinability (from (8.2.1)) is here defined as:

$$\exists \bar{c}, \bar{u} : \forall i \; c_i^+ = C_i(c_i^-, \bar{u}) \tag{8.9}$$

which is thus less strong - there only needs to be one stable solution.

Hence, coordinability looks at whether the agents can coordinate between each other, while controllability wants them to behave in a specific externally defined way.

I'd now want to check whether there is stable controllability (8.8) in this model. Thus we consider X_i fixed for all i. We see that we want to find u_ks such that (8.7) is an equality, thus

$$\sum_{i \neq j} a_{ji} X_i + \sum_k b_{jk} u_k = 0 \; \forall j$$

Define

$$S_j = -\sum_{i \neq j} a_{ji} X_i$$

(this is completely defined, since X_i and a_{ji} are given.) Then we find

$$\sum_k b_{jk} u_k = S_j \; \forall j$$

If for a certain j, $b_{jk} = 0 \; \forall k$, then we should have $S_j = 0$. Otherwise, we should define one u_l as depending on the others by the formula:

$$u_l = \frac{S_j - \sum_{k \neq l} b_{jk} u_k}{b_{jl}}$$

Thus, each node for which $S_j \neq 0$ should have its own control input. Since when a node does not have a control input, we should have $S_j = 0$. This means almost all nodes should be controlled.

If we take $X_i = 0 \; \forall i$, we find a solution for stable internal coordinability (from (8.9)).

↑ 158

This approach fits in the engineering approach of first-order cybernetics, where the goal is appointed to the system from the outside. The aim here is thus external control. The assumption is that there is a given and completely known complex network, and one wants to control its dynamics.

8.3.2 Self-organized control

I would now like to extend the above model to allow feedback. The idea is that we see the feedback as the link weight. The link weight is changed so that the input an agent receives fulfills its desire more. The link weight w_{ij} gives the strength of the connection from i to j, this is changed depending on how useful the value of i is for j. I thus consider it as the feedback F_{ji} that j sends to i.

This is a model where an agent changes his environment in order to get control. In contrast to the previous model, the agents in this model have goals on their own. I model this by giving each agent a reference value R_i. An agent wants to move its value X_i to the reference value. The updating of a value of an agent happens as above, except that I don't allow any external input anymore (8.10). The coordination an agent sends is its value X_i multiplied with the link weight (8.11) (this operation thus happens with the sending agent instead of with the receiving agent). I consider two loops: an agent sends its updated value and the constructed feedbacks also to himself, so that an agent uses its own output as input. I thus get:

$$
\begin{aligned}
C_i : \underbrace{X_i}_{\in C_i^+} &\leftarrow \underbrace{X_i}_{\in C_i^-} + \sum_j \underbrace{C_{ji}}_{\in C_i^-} && (8.10)\\
\underbrace{C_{ij}}_{\in C_i^+} &\leftarrow \underbrace{F_{ji}}_{\in F_i^-} \underbrace{X_i}_{\in C_i^-} && (8.11)\\
F_i : \underbrace{F_{ij}}_{\in F_i^+} &\leftarrow F_{ij} + \alpha_i(R_i - X_i)X_j && (8.12)\\
&= \underbrace{F_{ij}}_{\in F_i^-} + \alpha_i(R_i - \underbrace{X_i}_{\in C_i^+}) \underbrace{C_{ji}}_{\in C_i^-} / \underbrace{F_{ij}}_{\in F_i^-}
\end{aligned}
$$

The last formula (8.12) for the updating of the link weight comes from the theory of perceptron learning (Haykin, 1994). If the total input is too big ($X_i > R_i$), the link weight (feedback) is weakened for positive inputs, and strengthened for negative ones, so that the total input becomes less. The opposite happens if the total input is too little. This is an application of reinforcement learning, as often applied in neural networks. Figure 8.4 shows how these functions work.

↑ 182

I'd now like to know whether there is stable internal coordinability in this model. This is the case if there is a solution for the above equations (from

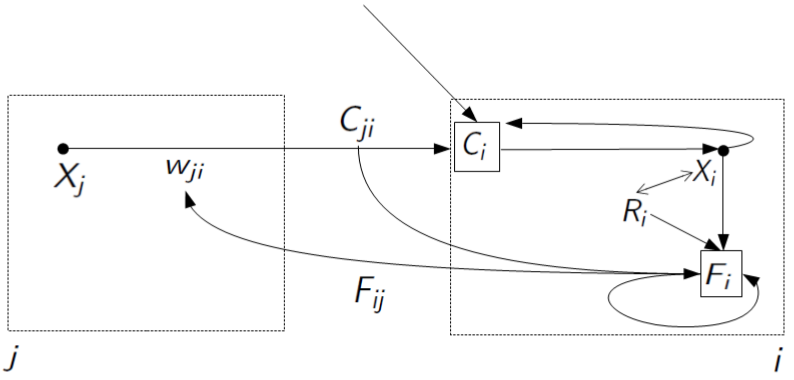


Figure 8.4: A graphical representation of the formulas (8.10), (8.11) and (8.12) of self-organized control. Agent j combines its value X_j and the link weight w_{ji} to generate a coordination signal C_{ji}. This input is used by agent i in the processes C_i and F_i. In C_i, the inputs agent i receives is combined with its value X_i to update this X_i. In F_i, this X_i is then compared with R_i to update the feedback signal F_{ij}. This feedback changes the link weight w_{ji} between j and i.

(8.6)). Thus, if:

$$\begin{aligned} \sum_j C_{ji} &= 0 \\ C_{ij} &= F_{ji} X_i \\ \alpha_i (R_i - X_i) X_j &= 0 \end{aligned}$$

for all i. A solution for this is $X_j = 0 \ \forall j$, then the feedback can be chosen at random. Another possibility is to take $X_i = R_i \ \forall i$. Then we should have

$$\sum_j F_{ij} R_j = 0 \ \forall i$$

Consider a particular i. If $R_j = 0 \ \forall j$, any feedback will satisfy. Assume there is a $k : R_k \neq 0$. For $j \neq k$, we can take F_{ij} at random, and then take

$$F_{ik} = \frac{-\sum_{j \neq k} F_{ij} R_j}{R_k}$$

Since we can do this for all i, we find another possibility for stable internal coordinability. This last solution seems most logical, namely that the agents want to have their values equal to their reference values. We can put this as an overall decision problem: $R_i = X_i \ \forall i$. Then we get that the above solution is the only possible case of stable coordinability relative to this overall decision problem.

Thus, we find that there is stable coordinability in this model if we assume the feedback (link weight) isn't bound to only positive numbers or only between 0 and 1. However, the fact that there is a solution doesn't necessarily mean that the solution will be reached by this process. It might never get into this attractor. For example, if we take $X_i = 0\ \forall i$, none of the X_is will change, thus it won't be able to reach a reference value if this value is different than zero. Also, if the learning parameter α_i is too high, agents might constantly overcompensate, thus never reach the reference value.

Also in a weak version, coordinability won't always be reached. In which circumstances coordinability is and isn't reached remains an open question, though it is plausible that it is more often reached than not, since the model is built to go to the solutions (the update of the feedback signal makes it closer to its reference value).

This model is more in line with the autonomous approach of second-order cybernetics. Each agent has its own goal, which it tries to reach by adapting its links with other agents. In a social system, these links can be friendship ties. On the internet these links can represent how strongly two people connect. For example, if you put an unwanted email in your spam folder and indicate that you don't want to receive any of these messages anymore, you are weakening your link with the sender. In this way, you provide feedback regarding how much an email is wanted. Connecting with people with whom you share interests and detaching from people who block you in reaching your goals is another example of this approach.

↑ 158

8.3.3 Control by changing the method of your neighbors

I now want to construct a model where agents try to influence the methods of its neighbors. I will base this on Perceptual Control Theory (McClelland, 2004).

↑ 162

In perceptual control theory, an agent tries to control its perception X_i, by trying to equalize it with a certain reference value R_i (8.13). But there are also other agents who try to control the same perception with other reference values in mind. The perception might also get disturbed by the environment Ω_i (8.14). Usually these disturbances are random, so this isn't much of a problem. The model can now be represented as:

$$C_i \leftarrow C_i + \alpha_i(R_i - X_i) \tag{8.13}$$

$$X_i \leftarrow \sum_j C_j + \Omega_i \tag{8.14}$$

In this model, the perception will converge to an average of the reference values, where α_i represents the power an agent has.

Inspired by this model, I now construct a new model where the idea is that one tries to influence its neighbors to send the right coordination input by sending them certain feedback. We assume the coordination an agent j sends to i is constructed as follows:

$$C_{ji} \leftarrow C_{ji} + F_{ij} \tag{8.15}$$

Each agent i has a value X_i he wants to put as close as possible to its reference value R_i. X_i is constructed as follows:

$$X_i \leftarrow \sum_j C_{ji} \tag{8.16}$$

The way an agent tries to control its neighbors to send coordination which satisfies its needs (reference value) is by sending this feedback:

$$F_{ij} \leftarrow \alpha_i(R_i - X_i) \tag{8.17}$$

A shortcoming of this model is that the coordination input is assumed to be known and of a specific form, so that it satisfies our urge to control it. That's why I want to generalize the model to assume the coordination function is unknown, and we don't even know how exactly this gets aggregated into X_i. Thus we just assume

$$X_i \leftarrow f(C_i^-)$$

with f some unknown function. Then we can still try to get control by looking how our X_i got affected by the F_i^+ we have sent out (we send the same feedback signal to all agents, thus we assume $F_i^+ = F_{ij}\ \forall j$) . If a bigger F_i^+ results in a bigger X_i, we can use the same update mechanism as above. If on the contrary a bigger F_i^+ results in a smaller X_i, we should do the opposite, subtracting instead of adding. We thus get the following formulas:

$$\begin{array}{lll} F_i^+ \leftarrow F_i^+ + \alpha_i(R_i - X_i) & \text{if} & F_i^+ \nearrow \ \Rightarrow \ X_i \nearrow \\ F_i^+ \leftarrow F_i^+ - \alpha_i(R_i - X_i) & \text{if} & F_i^+ \nearrow \ \Rightarrow \ X_i \searrow \end{array}$$

This still has some unrealistic assumptions though, because the aims and the methods got separated. The X_i and the output C_{ij} send out are completely separated. We see what kind of consequences this has when we check whether there is stable internal coordinability (8.6). It's difficult to check this for the general model, so I do this for the more specific model (thus (8.15),(8.16) and (8.17) should be equations). There is stable internal coordinability if

$$\begin{aligned} 0 = F_{ij} &= \alpha_i(R_i - X_i) \\ &\Rightarrow R_i = X_i \end{aligned}$$

And

$$R_i = X_i = \sum_j C_{ji}$$

One can easily choose C_{ji}s such that this is fulfilled.

This model is again in line with the autonomous approach, where all agents have a goal they try to reach. Here however, they do so by controlling the methods of other agents. In a social context, you can see this as someone who wants others to say what he wants to hear and do as he wants. As long as these things are independent from the goals of the person asked, this may well work. But because of this decoupling, the result is often artificial, where people are just saying what one wants to hear, but without being really committed to it. ↑ 158

8.4 Conclusion

In this chapter, I have examined different models of control. We can put this in the framework of the scope of influence and the way one acts as discussed in the end of the previous chapter and the beginning of this chapter. The scope of influence tells how local or global one acts, while the way one acts tells whether one tries to influence the links, methods or goals of neighbors. ↑ 204 ↑ 206

The first group of models tries to control a whole network, they work globally. The model of controllability is an example of this, where one tries to influence the goals of all the agents. Another example is the model of Emergent Control (Kreyssig and Dittrich, 2011) where an external force tries to achieve a global goal by adapting the local rules (methods). We saw that at least in the model of controllability, this is difficult to achieve ↑ 216 because one has to control almost all the nodes, pushing the system away from its natural state.

Other models work locally, they assume the agents want to get control.
↑ 219 In the model of self-organized control, they did this by changing the links they had with other neighbors. This worked under the assumption that the
↑ 221 feedback (link weights) wasn't bound too much. In the second model of this kind, agents tried to adapt the methods of their neighbors. This also worked, but there was the implicit assumption that goals and methods are separated, which isn't very realistic.

A general shortcoming of these models is that we assume the goal of an agent is simply to reach a certain reference value. In reality, goals are usually far more implicit and multidimensional. It might even be better to speak about certain value systems instead of certain goals, where there isn't one optimal solution. But this is more difficult to formalize, and the general principles presented in this chapter seem to be also true in this case. I.e. that it's easier to get control over one's life by acting locally and as least as possible disturbing the core values of one's environment. It might even be easier to do so in reality, because there are far more possibilities to satisfy one's values.

Chapter 9

Conclusion

The motivation behind this thesis was to better understand what power and hierarchy are, and to explore how we can live without coercion. I wanted to conduct this exploration by intersecting anarchism and mathematics. I wanted to better understand these concepts because I felt that, in contemporary society, a lack of freedom often occurs without a visible coercion or clear hierarchical structure. Alienation is widespread, and a lot of people feel as though they cannot fit in. Also, hierarchy and coercion often evolve from a previously free configuration.

I divided this issue into three main research questions. What are, on the one hand, authority and hierarchy, and, on the other hand, what are freedom and autonomy? How does hierarchy evolve in social systems? And how can we shift from hierarchical control to a more free social organization?

So, what came out of this exploration?

9.1 Summary of the thesis

In this section a summary of the thesis is given along the following lines. A structural definition of hierarchy can be related to a functional understanding of hierarchy, where concepts such as determination and coercion are investigated. In addition to questions of definition, I have investigated how hierarchy emerges and how we can move to a non-hierarchical constellation. These questions are related though, as the nature of hierarchy (how it is defined) may imply something about how it emerges and what a non-hierarchical world would look like.

Following this conclusion, a network of the full thesis is shown based on

↑ 241

this summary. Elements are grouped by paragraph and by subsection. Each subsection is taken apart in a separate network.

9.1.1 On structure, and how it relates to function

↑ 76 In chapter 4, the structural component of hierarchy is discussed. A hierarchy ↑ 81 can be understood as an upper-semi-lattice. One of the conditions for an ↑ 78 upper semi-lattice is antisymmetry. Antisymmetry means that whenever there is a link from one element to another, there is no link back. Antisymmetry ↑ 57 can arise because of a positive feedback mechanism that amplifies small differences in the direction of connections, until only one direction remains. ↑ 77 When this relation is transitive, i.e. when influence is transmitted, there can ↑ 13 be no cycles. There is a power-over relation, as an element can influence another element while this element has no influence in return. When every element has at most one direct influence, the structure is an upper-semi-lattice. I proved that in this case there is one greatest element as long as the structure is finite and connected. These steps to a hierarchical structure are ↑ 126 summarized in section 5.1.

↑ 88 Having multiple influences results in being a unique constellation of these influences, more than a sum of them. Thus, having only one influence which you cannot influence in return, leads to being determined by it, as shown in ↑ 200 section 7.7.

When we demand that an element influences more than one element or no element at all, its influence can quickly spread. Moreover, it can easily become a bottleneck, as it needs a lot of complexity to adequately control ↑ 101 all of these elements. A perfect hierarchy—when the chain of command has always the same length—is seldom the case. For this reason, these ↑ 101 properties are broadened into a three-dimensional measure, which shows how hierarchical a directed network is.

↑ 76 The above investigated when a relation is hierarchical. The set relation is a specific form of relation. By taking specific families of sets, we can ↑ 84 emphasize less or more ordered elements. Overlapping sets cannot be compared, while sets contained in each other compose a hierarchy.

↑ 114 By generating sets around nodes, we can extract a direction in undirected networks. This representation shows how there can be an asymmetry between nodes because of the difference in how they are embedded into the whole network, even if their direct relation is symmetrical. This is socially relevant in that the direct relation between two people might look symmetrical, while there is still an asymmetry because of their different social positions. The asymmetry in undirected networks was established by considering the sets of the neighbors of a node. Such a neighborhood set can be (almost) completely contained by another neighborhood set, while this set can still have a lot of other elements. This creates an asymmetry between nodes. The directed version of an undirected network is constructed by creating a new directed network where a link is drawn between nodes when the neighborhood of a node contains the neighborhood of the other node (in the old undirected network).

The cluster coefficient measures the average containment of the neighborhood of a node in the neighborhoods of its neighbors. A node with a high cluster coefficient means its neighborhood is largely contained in the neighborhood of it neighbors. Hence, most of its neighbors don't have to pass by this node to reach other nodes. The cluster coefficient measures locally, it only considers the neighborhood of a node. A general centrality measure takes a bigger part of the network into account. It measures the number of communications starting from a node, where each node has a chance β to pass on a communication. By changing the value of β, the centrality measure is more local or more global. I extended existing centrality measures to describe a general centrality measure for a hypergraph.

↑ 108
↑ 109

I defined a hierarchical network as a network in which a high degree is correlated with a low cluster coefficient, and the degree follows a power-law. I showed how the directed version of an exemplar of a hierarchical network gives a perfect hierarchy, as neighborhood sets are contained into each other. The anti-hierarchical counterpart gives rise to overlapping neighborhoods. Here there is local coherence, as nodes are better connected, while there is less ordering between the nodes.

↑ 109

Local coherence can be visualized by overlapping sets, in contradiction with a main thesis or idea that is split up in several sub-ideas, which are split into even smaller ideas, and so on. With local coherence, several elements influence each other without an ordering among them.

↑ 88

I showed how the relation of an agent changing the method of another agent does not have to fulfill any of the properties of a hierarchy. I called an agent that changes the method of another agent a governor. I showed how there can be cycles of governors, an agent with two governors, and a governor that only governs one agent. When governors are not ordered in a hierarchy, there can be a coherence among the agents, as they mutually change the method of each other.

↑ 180
↑ 182
↑ 164 ↑ 176

Perceptual control hierarchy and the law of requisite hierarchy are specific models where agents change the method of other agents. A perceptual control ↑ 162 hierarchy builds on perceptual control theory, a quantitative model where an agent attempts to steer an input to a certain reference value. In a perceptual control hierarchy, agents that change the reference value of other agents are ↑ 166 added. The law of requisite hierarchy builds further on the law of requisite variety, a qualitative model where an agent attempts to keep its output within certain limits. The law of requisite hierarchy adds agents that reduce the uncertainty of other agents regarding what to do in response to a certain disturbance. As a result of a reduced uncertainty, an agent can limit its output more.

These models simply assume that agents that change the method of other agents have to be ordered in a hierarchy. Since I showed that such agents do not have to be ordered in a hierarchy, the assumption that a hierarchy is necessary is false.

Thus, in a hierarchical structure elements can determine other elements, while in non-hierarchical structures there is more mutual influence.

9.1.2 On the functional aspect

↑ 20 The concept of determination has been discussed several times throughout this thesis. One element determining another element, for example, the ↑ 34 idea that the economy or technology completely determines how society ↑ 49 functions, was contrasted with co-evolution, where there is mutual influence.

↑ 139 In chapter 6, a simulation was built to investigate the extent to which the network structure can determine the function of the agents in it. I ↑ 107 investigated how and whether the behavior was different in different network types. The classification of these types was based on the cluster coefficient and the degree of the network. I differentiated between hierarchical, non-hierarchical and random networks. For the functioning of the agents in these networks, I examined two cases. In the first simulation, agents tried to minimize friction by bringing their value closer to the value of their neighbors. The second simulation was inspired by the ecosystem. Here agents aimed to get as many useful products as possible from their neighbors, which maximized synergy. These simulations showed that the network does not completely determine the functioning as the results were unclear and depended on the local rules applied. However, this could partly be explained by the fact that my definitions (both on the level of structure as function) were not yet well developed.

In chapter 7, a mathematical definition of determination was provided. Determination was described as the measure of the extent to which one variable A can influence another variable B. If the variation of B given the state of A is small, the determination of A on B will be large. This measure is calculated relative to the natural variation of B. Hence, the determination is large when knowing the state of A greatly reduces the variation of B. Because the natural variation of the variables is taken into account, the formula of determination is not symmetrical. A can determine B almost completely, while B could have almost no influence on A. ↑ 154 ↑ 189

This measurement is used to differentiate internal from external control. ↑ 154 Internal control means an agent can determine its own outcome. The agent then has control over its own situation, which relates to "power-to", or power ↑ 13 as the capacity to do things. An agent is externally controlled when the output is determined by the environment. Conversely, an agent has external control over the environment when it can determine the outcome of the environment. External control thus relates to a power-over relation, as one agent can influence another agent while that agent does not have influence.

I defined internal control mathematically as the determination of the acts of an agent on the result. The external control exerted on the agent is the determination of the inputs from the environment on the result.

I established that an agent does not have to externally control the environment to have internal control. On the contrary, external control often leads to less internal control.

Diverse manifestations of control can be differentiated by assessing in how far the environment is altered and in how far only the agent's own situation is changed. This differentiates the degree to which control is external from the degree to which it is internal. An agent could change its input to a wanted outcome or send an outcome to alter the input. A different outcome could simply cause the environment to create a different input for the agent, or it could change the methods or goals of the environment, i.e. the same outcome will now be transformed to a different input than before. This is similar as when a different input can simply be caused by selecting a different part of the environment (e.g. moving), or by altering the whole environment, and thus also the part that is received.

This division differentiates based on what the agent does. The focus is on the agent. Another perspective is to put the focus on how the environment is altered. To do this, we worked in a network model, as the environment should now be described in more detail. Here, control can be differentiated along two dimensions: on the one hand, local or global, and ↑ 206

on the other hand, change in the links, methods or goals of other agents.

↑ 206 In chapter 8, several models were categorized along these dimensions. This was done by putting these models all into the same framework. The developed framework was a generalization of the Mesarovic model of hierarchical coordination in which hierarchy was no longer required. This leads to a network model of agents that send coordination inputs to certain agents and receive feedback inputs from the agents they have sent coordination inputs to. I applied this framework to three different models. The theory of controllability investigates the minimum amount of input nodes needed to steer all the nodes to certain values. This is therefore a model of global control where the goals of agents are changed. In the model of self-organized control, agents want to influence their own value. This is consequently a model of local control. They exhibit control by changing the links. The last model is a model of local control where the methods of other agents are changed.

↑ 185 External determination makes an agent vulnerable to coercion. When an agent cannot determine its output itself, this output can be unwanted by the
↑ 59 agent. Coercion differs from constraint in that constraint limits the degrees of freedom of an agent, but this can be wanted by the agent, while coercion makes an agent move in an unwanted direction, which can actually increase its degrees of freedom.

Multiple notions of freedom can similarly be differentiated, as was done
↑ 59↑ 10 in section 3.6 and section 2.1.1. Freedom could be seen as when there is
↑ 64 ↑ 65 internal determination and no external determination. Or it could be seen as
↑ 185 when there is no coercion. In the latter case, the desired outcome could be
↑ 64 caused by another agent, in which case there is a dependency between agents.
↑ 11 This can be related to the concept of freedom as a right to choose from given options. Since a right to choose still implies dependency on an external agent to provide these options. The agent cannot really choose its path itself. One
↑ 196 of the drawbacks of our model of internal and external control was that the set of possible outcomes was taken as given.

But freedom could also be seen as creating possibilities that were not
↑ 10 ↑ 60 yet there before, as self-actualization, as getting out of an attractor state.
↑ 40 Freedom could be seen as a decision not to just follow the spontaneous order.
↑ 10 It is about breaking out of constraints. This relates to seeing freedom as an opposition.

However, constraints can also arise internally, and are not necessary unwanted by the agent, as selecting certain things over others is a constraint.
↑ 64 ↑ 45 Internal determination can be seen as being self-caused, and this autopoietic

state is an attractor state. There is a local coherence, but the whole is not ↑ 66
necessarily determined by one element.

9.1.3 How hierarchy emerged and how to move away from it

Hierarchy can also emerge from self-organization. An organization can ↑ 39
develop its own goals, which can become disconnected from the goal of the
agents that formed the organization, leading to the formation of a controller. ↑ 43
On an individual level, this can be an *idée fixe*, where one idea suppresses
other values of the individual, creating a hierarchy of ideas instead of a local
coherence. Such an organization can make the constituting agents do things
they did not really want to do, as the organization often needs the agents.
In chapter 7, I discussed how this was possible when the agents can only ↑ 154
choose the least bad choice from the choices provided by an external force. ↑ 196
Thus, while a controller often evolves into a cultivator that keeps the agents ↑ 44
alive because it needs certain benefits from them, this is often not the best
option for the agents.

Power can be social and does not necessarily reside in one individual. ↑ 19
Certain norms arise from the interactions of individuals. Power here cannot
be pinpointed to one structural subsystem. But we can also differentiate on
the functional level. We call a system with a specific function an aspect ↑ 47
system. One aspect system can determine another one. For example, the
economy determining how society functions.

In chapter 2, two opposing views on how hierarchy emerges were given: ↑ 9
the view that the economy and technology shape how society functions, and ↑ 26
the view that individual choices also play a role.

This caused two different views on how we can move away from
hierarchical organization. Some believe technological and economical change ↑ 33
will cause hierarchy to disappear, while others think hierarchy can only
be avoided if we organize ourselves to oppose it. This latter view was the
perspective presented in Gelderloos' Rise of Hierarchy. I called this principle ↑ 25
constant opposition, and made a simulation to show how different local ↑ 63 ↑ 131
rules can prevent a power-law from growing and becoming an attractor. ↑ 57
In this simulation I assumed that the basic mechanism 'the more an agent
has, the more he can take', cannot be changed. If agents take equally from
all other agents, this would indeed lead to a power-law. But when agents

only take from the agent having the most, a more dynamic distribution is ↑ 60 ↑ 10 encountered. Freedom as moving out of an attractor and as opposition, also fits in this view.

↑ 130 In section 5.2.2, I showed how deleting or adding certain edges can make a network less hierarchical. The idea is to increase the cluster coefficient of high degree nodes, and decrease the cluster coefficient of low degree nodes. This is done by adding edges between neighbors of high degree nodes, and deleting edges between neighbors of low degree nodes. This can be seen as a mechanism to move away from the status quo of the hierarchical network.

↑ 52 In section 3.4, I discussed several principles we can use to coordinate between ourselves: stigmergy, variation and selection, antifragility and coherence. ↑ 53 Stigmergy means traces are left in the environment where others can further build upon them, without the need for centrality or direct interaction. ↑ 54 Bringing enough variation in our actions while still being selective, is an important tradeoff to improve our functioning. ↑ 54 Antifragility means a system gets stronger after a shock. ↑ 52 Coherence means aspects are influencing each other, without one being more important than the other. This coherence plays a role in several aspects of anarchist ideas. One of the principles a lot of anarchist adhere to is that ↑ 31 aims and methods should be aligned, which is why ↑ 39 direct action is often used. This links to my discussion about intention in section ↑ 185 7.5.3, where goals could be inferred from the methods applied. The distinctions being made between attacking and building alternatives and between ↑ 52 decisions and acts, are often considered false. The latter relates to the fact that there is no need to make an ↑ 70 overall decision that determines everything, but there can be a coherence of local decisions while conflicts are still allowed to exist.

↑ 57 ↑ 58 These methods can generate a positive feedback that can cause a revolution.

9.2 Contributions of this thesis

This thesis aims to answer the anarchist question of how to have a world with as little coercion as possible. This question is not new and many answers to it have already been proposed. My contribution is unique because I have taken cybernetic and mathematical approaches to understanding the problem and proposing solutions. Likewise, the cybernetic and mathematical models and concepts I've presented are often not new, but interpreting them from an anarchist perspective, as I've done, has never been done before.

I split up this question into three research questions, which, in a nutshell, concern definitions, the past, and the future. The first question is about the meaning of concepts like authority, hierarchy, freedom and autonomy. The second question investigates how hierarchy evolves in social systems. The last question explores how we can shift from hierarchical control to a more free social organization. There are some key findings in my thesis that give non-trivial answers to these questions.

When searching for answers on how to define freedom and coercion, I found out that there is not really one unambiguous answer. However, there are several important aspects I distilled.

An important difference I distinguished, is between internal and external control. Internal control is about aiming to control your own situation, while external control is about trying to control the (whole) environment. I constructed mathematical formula to express these concepts. These concepts are similar to *power-to* and *power-over*. Power-to is the ability to do certain things, similar to internal control. Power-over assumes an asymmetry, where one party has influence without being influenced back. This relates to external control.

I introduced a formula for determination to describe internal and external control. This formula is valuable in itself. It measures the determination of one variable on another. The determination is high when knowing one variable greatly reduces the variation of another variable. The formula for determination is not symmetric, hence A can greatly determine B while B can have almost no influence on A.

I differentiated diverse manifestations of control along two dimensions. The first dimension assesses how global the control is: whether only the immediate neighborhood or the whole system is controlled. The second dimension represents how invasive the control is: whether it is the links, the methods or the goals of other agents which are changed. Different models where classified along these dimensions.

Another difference I clarified is between coercion, constraint and determination. Coercion is when one is forced to do an act one does not want to do, while constraint simply implies a reduction in the degrees of freedom. In fact, coercion can increase the degrees of freedom. Additionally, constraint can be desired, for example, when a selection is made. Determination means there is only one cause that can completely determine the result. For example, an external force that can completely determine an outcome, while an agent cannot influence it. (External) determination makes an agent vulnerable to coercion, but in principle, the outcome can be desired by the agent.

These different but related concepts give rise to different understandings of freedom. Freedom can be understood as the lack of coercion in that an agent is not manipulated to do certain things, but can determine autonomously what to do. Another understanding of freedom is as the absence of constraints. In this understanding, freedom means having many possibilities or choices. These choices could, however, still be manipulated or coerced. A type of freedom where determination is allowed as long as there is no coercion leads to dependence. The agent depends on an external force for his needs. On the other hand, being independent could be understood as 'autonomously' doing the task asked of you.

Another aspect is the structural aspect. Authority is often associated with a hierarchical structure. But how and why is a hierarchical structure necessarily connected with coercion?

I showed how a hierarchical structure implies determination. Since in a hierarchical structure, influence is one-directional, and each node has at most one direct influence. One-directional influence implies that an agent is influenced without having influence back, a definition of power-over. If an agent has only one direct influence, it cannot grow beyond this influence. When there are multiple influences, a unique constellation can emerge from the interaction of these influences.

There can be a structure that determines the function, like a hierarchical structure. This means there is a one-directional relation, as the function cannot influence the structure. An example is the idea of economic determinism, where the economic or technological circumstances determine how society functions. But the functioning could also influence the structure. In this case there is co-evolution as two systems influence each other. This is, for example, the idea that human agency plays a role in how the economy and society develop.

There can be a hierarchy on the functional level. Here, we consider different aspect systems, which are differentiated based on function. The hierarchy in such situations is less clearly visible than when the differentiation happens on the structural level, which gives different subsystems.

Another finding of the thesis is that self-organization is not necessarily the solution to hierarchy. Self-organization can lead to the emergence of a controller. Here coordination leads to the emergence of a higher-order system with its own goals. These goals could go against the goals of the agents constituting the system. Hence, this is an answer for how hierarchy could emerge but is not a possible solution to hierarchy.

Power can be social. It then does not reside in one individual. Social power is the pressure felt by an agent to move into a certain direction because everyone else moves into that direction. Power is exerted by the society as a whole.

Moreover, there are interpretations of self-organization that allow for 'guided self-organization'. Here, the local rules are influenced so as to achieve a desired global outcome. Organization can, in fact, always be seen as self-organization, depending on what is included in the 'self'.

One of the mechanisms I developed to tackle the last question about how we can move to a more free society is what I have called *constant opposition*. The idea is to constantly oppose any seed of coercion or hierarchy, so that no power can grow too big. This phenomenon has already been witnessed in some societies, where anti-authoritarian mechanisms keep such societies egalitarian. I have built a simulation to illustrate this phenomenon. This simulation demonstrates how a "take from the rich, give to the poor" strategy can counteract the "rich getting richer" effect. The result is that when agents only take from the richest agent instead of from all agents equally, the emergence of a power-law will be counteracted, giving rise to a more dynamic distribution. While money is a straightforward interpretation of what the variable used in the simulation entails, such a variable could represent anything, for example, power. The simulation is a more general illustration of how the mechanism of constant opposition can function.

Answers to how to move away from hierarchy and coercion also lay in the nature of hierarchy as discussed before. One can try to exert minimal external control, and not assume directly that this external control is necessarily for internal control. Coercion can be minimized by creating less hierarchical structures.

Another important contribution of this thesis is my clarification of the law of requisite variety and law of requisite hierarchy, well known among cyberneticians but never developed in all their details. Delving into the details allowed me to expose several implicit assumptions and shortcomings of the laws.

The main shortcoming of the law of requisite variety is that only (reduction in) variation is measured, and not the exact values. This shortcoming was specifically problematic for the law of requisite hierarchy. Here there are different agents that further reduce the variation of a variable. But agents in general do not want to bring a variable to the same value. Agents usually have different goals. The values to which one agent has reduced a variable are often unwanted by another agent. There is the implicit assumption in the law of requisite hierarchy that agents have the same goal.

This shortcoming made it difficult to interpret concepts like value, determination and coercion with this framework. But there is a general problem with defining coercion. Finding out the shortcomings in the law of requisite variety, enabled me to expose the problem with the concept of coercion. There is a paradox in 'doing things you do not want to'. Especially when you are theoretically capable of doing something else, and when your wants are not simply considered given. This paradox expresses itself in the difficulty to decide when someone is addicted. Often someone will not admit he is addicted and act as if he acts by his own choice, while it is clear for others that his deeds go against his interests.

I showed that the law of requisite hierarchy actually does not imply a hierarchical structure. I have shown in general that in models where a hierarchy was present, this hierarchy actually came from an implicit assumption. This was the case in a perceptual control hierarchy and in Mesarovic's model of hierarchical coordination. Hence, such models emerge from a hierarchical way of thinking.

We tend to think in terms of one main idea or concept that is split up into several sub-ideas, and so on. This is a hierarchical way of thinking. But we can also think in terms of a local coherence, where concepts are related to each other without any ordering among them. My thesis is structured in the latter sense, where all the ideas are linked to each other. There is not really one main thesis that can be distilled. As we are accustomed to thinking in a hierarchical way, this may make it more difficult to see the cohesion.

A hierarchical way of thinking denies human agency. Consider the argument that there is one aspect that controls all of society and all social order, for example, the Marxist view of the economy. In this and similar models, humans cannot have any influence on the ways things go. My model of constant opposition shows an alternative view, where human agency is fundamental.

By demonstrating that hierarchy is only present in models by assumption and providing alternative models and structures, I showed how to think in a less hierarchical way.

9.3 Limitations

There are some limitations of this thesis, since we can never do everything.

As concepts like power, determination, coercion and hierarchy are complicated social phenomena, not all aspects of them can be formally covered. Some of the definitions given were not yet one hundred percent clear, yet the question is whether it is possible anyway to unambiguously map such concepts to mathematical formulas. Some of the models were a bit simplistic, for example, they only considered one-dimensional variables. I have already discussed in the introduction chapter how a model can never reflect reality. ↑ 2 There will always be a trade-off between completeness and clarity.

This problem recurred in the simulations. In the simulation in chapter 6 of how the network affects the function, some of the assumptions were ↑ 139 not that well thought out. A fixed network was taken (even though the functioning often affects the network structure), the network was undirected (even though hierarchy might be more visible in a directed network), and the measure of influence to assess the functional aspect of hierarchy, was rudimentary. The simulation of constant opposition was a bit elementary, as ↑ 131 there was only one one-dimensional variable at play, and acted more as an illustration.

The shortcomings in the law of requisite variety manifested themselves also in the entropic version of internal and external control I developed. Consequently, it remained difficult to interpret concepts like value, determination and coercion with this framework.

This work is mainly theoretical, and thus, a practical understanding of how to act concretely in this world might be lacking. Many of the concrete social, political or economic questions of today's reality are not answered by this thesis.

It was one of my big ambitions to end this PhD with a big simulation. The idea was to answer the main questions (what is hierarchy, how did it emerge and how to move away from it) by integrating all the different ideas. I wanted to test or simulate different political ideas in it (for example, the different anarchist arguments) and see how they behaved. I did not succeed in this ambition due to a lack of time and tools, but also because it's a dangerous road to try to create a blueprint of how society should function. It may not be possible to integrate all of the ideas of this thesis into one big simulation or model, and this is one of the messages of the thesis: there can be a coherence between concepts without the need for these concepts to be part of a bigger model.

9.4 Future work

Still, it is possible to build parts of this model. For some of the following ideas of future work, I already have a few drafty starting points and some of these can be found in the supplementary information.

A model can be constructed of how and when a controller forms, which mechanisms can prevent that, and how those prevention mechanisms can be sustained. ↑ 136 Chemical organization theory is a good candidate as a basis, since it can already model autopoiesis. Goals should still be added or made explicit to model the way in which a formed organization can have different goals than the constituting elements. The model of the law of requisite variety could also be used for this purpose, as a first exploration has already been done in section 7.5.3. ↑ 185

A more sophisticated model of how to be less hierarchical can be built as the presented models of constant opposition ↑ 131 or of deleting and adding of edges ↑ 130, were still pretty simplistic. This can be approached by chemical organization theory or by another tool.

↑ 189 The model of determination and entropic internal and external control can be made more sophisticated by allowing different agents, this can be done by splitting the environment into several agents. In this way, a network model can be integrated with this framework.

The coherence between different ideas of the thesis, but also some of the ideas themselves, can be further developed.

↑ 91 One of the properties of hierarchy according to Mesarovic is the functional difference between the 'top' and the 'bottom'. There is a direction. In the mathematical model this was embodied by the difference between 'coordination' and 'feedback', but what the difference between these mapping actually entails can be further investigated. How do they create a functional difference? Can we relate it to the direction that emerged when one agent changes the method of another agent? ↑ 180

Some of the shortcomings of hierarchy could be analyzed in greater depth, in particular, the instability of hierarchy. The 'noise from order' principle, meaning that trying to control everything actually creates less control, could be worked out more. The (in)stability of a hierarchical structure can be investigated, in that if one connection breaks, everything falls apart (for example, in a linear text), while having a lot of connections makes the network more resilient (for example, when there are links in a text).

A mathematical model can be made of the difference between dialectics (where conflicts need to be solved into a synthesis, which often creates friction) and local coherence (in that contradictions are allowed to exist, which can overall have less friction/more coherence). For this, a definition of contradiction should be developed, probably different from ‘not being in a set’, as this simply means it does not relate to that set. This model should mathematically map to a hierarchical versus a non-hierarchical structure. This could formalize a difference between Marxist and anarchist thought. While the concept of dialectics is already well-developed, an anarchist counterpart is still quite vague. This can be related to different perspectives on competition.

Some ideas related to order relations can be developed. The difference between ‘equal’ and ‘incomparable’ can be explored and formalized. We can think and model how and whether having one way of ranking or only one goal/utility measure creates a partial order, while having multiple goals or ways of ordering can create intersecting sets. For this we can prove why a total order is linear (as in being one-dimensional), and clarify how to define dimension in this framework. A ‘dependency’ relation can be investigated (this is a relation that is reflexive and symmetric, thus not necessarily transitive). Note that a transitive relation where every node is in a cycle, is reflexive.

Creating sets in undirected networks, for example, to define the cluster coefficient, can be further researched. There is a clear similarity between the formula for containment and the one for determination, as both can make an asymmetry explicit. One can examine the difference between networks with the same centrality distribution, but where the constellation of constructed sets is different.

Especially in chapter 2, one can apply extra references and theories. For example, the definition of authority in Chinese philosophy, Kropotkin’s book on mutual aid, Bookchin’s theories, Popper’s criticism of historical materialism, the concept of permanent conflict (similar to constant opposition), and the relation between introvert and extravert thinking and internal and external control. ↑ 9

The theories developed can be applied to more concrete examples.

9.5 An end and a beginning

While I secretly hope that this thesis will be the spark that will set off a revolution, I know this will probably not be the case. It is never one person or text that can cause such things, and I don't want to be a leader of any revolution, as it won't be my kind of revolution then (that is why I only hope it secretly). Still, I hope this thesis will be inspirational and influence the way of thinking of some persons.

This thesis can help you to think in a less hierarchical way by providing different schemes than a classical hierarchical classification. It will hopefully provide a better understanding of how power works and an understanding of anti-authoritarian political ideas, based on complex systems theory.

In terms of definitions, I analyzed several aspects of power. I defined internal and external control. I clarified the difference between coercion, constraint and determination. These divisions are associated with different understandings of freedom. I furthermore demonstrated how a hierarchical structure is connected with determination.

I explained how self-organization can lead to the emergence of a controller. I proposed the mechanism of constant opposition to counteract the emergence of such hierarchy.

I showed that in several models hierarchy is only present by assumption. I illustrated how to think in a less hierarchical way by focusing on local coherence.

This thesis can be the seed of a new perspective of the world, which others can hopefully apply to their own situation, a seed that can grow and develop further by the ideas, perspectives and capacities of other people. So use this thesis as you like—as long as it is not to promote hierarchical or authoritarian ideas!

Networks of the thesis

On the following pages, I present a network showing how the most important concepts of the thesis connect, based on the summary given before. Concepts from the same paragraph of this summary are grouped together. Note that the only reason why there are no overlapping groups, is that the software I used did not allow that. Groupings are mostly there to render a better, clearer lay-out.

↑ 225

To provide more clarity, three parts of the network are taken out and shown separately. These are concepts from one specific subsection of the summary, together with all the concepts that link to one of the extracted nodes.

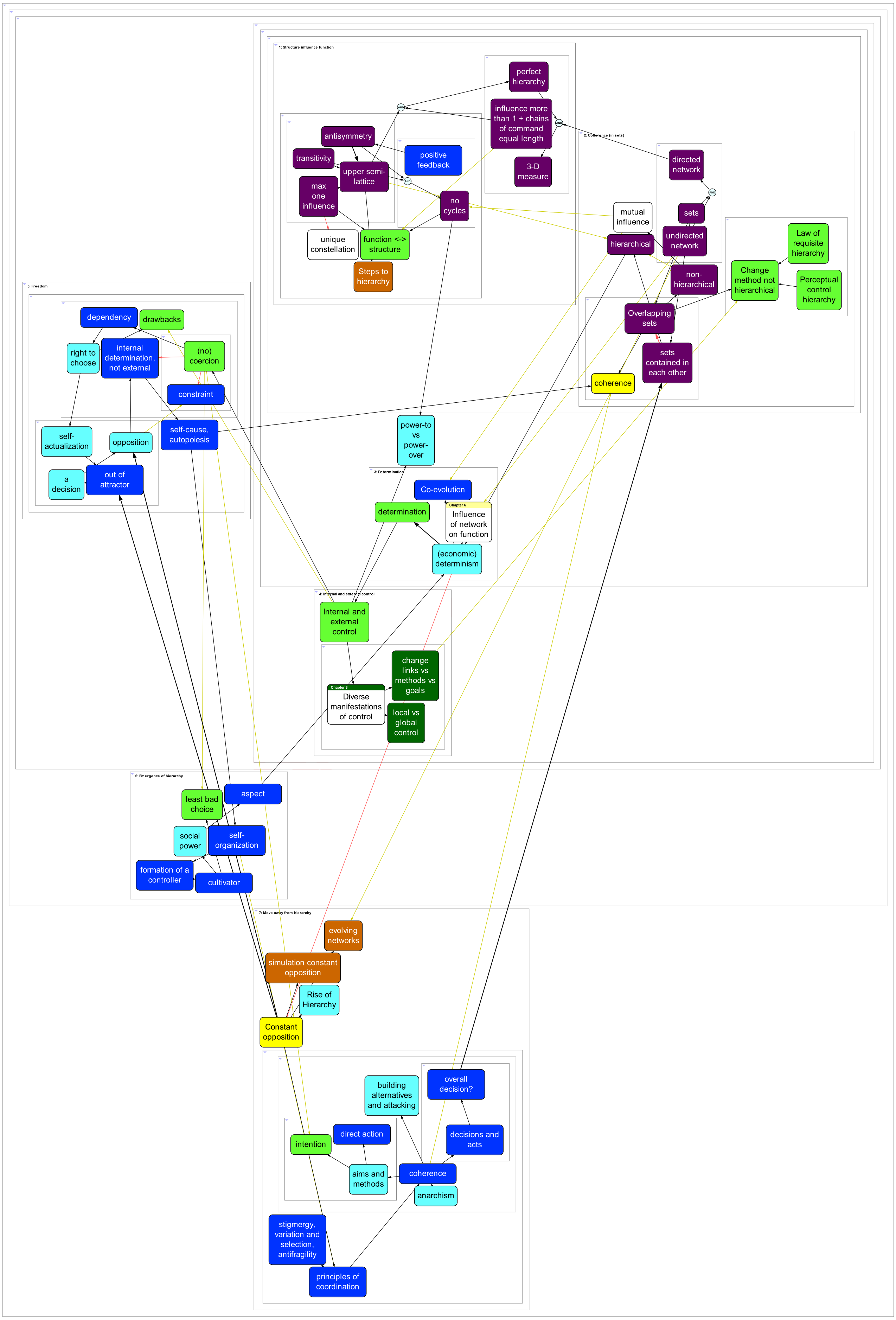
Edges shown in black mean the one concept was mentioned after or before the other concept in the summary. Yellow edges denote there is a link between these concepts made elsewhere in the thesis. Red edges signify that the concepts are opposites, whether or not these concepts are treated together in the summary. Just like in the map of the introduction, the colors of the nodes denote from which chapter a node comes. The legend used can be found below.

Next to the index, a network shows how concepts from the index relate to each other. All the index concepts mentioned on the target page of a concept from the index, are linked with this concept. As before, the colors represent the chapter to which the concept belongs to.

↑ 255

Legend:

🟦 2: Social theory
🟦 3: Understanding anarchist ideas through complex systems concepts
🟪 4: Structure
🟧 5: Change
🟨 6: Influence of network on function
🟩 7: Autonomy and control
🟩 8: Diverse manifestations of control
🟨 Important overarching concept
⬜ Overarching concept


1: Structure influence function
perfect hierarchy
influence more than 1 + chains of command equal length
3-D measure
AND
antisymmetry
transitivity
positive feedback
upper semi-lattice
max one influence
no cycles
unique constellation
function <-> structure
Steps to hierarchy
2: Coherence (in sets)
directed network
sets
mutual influence
undirected network
hierarchical
non-hierarchical
Overlapping sets
sets contained in each other
coherence
Law of requisite hierarchy
Change method not hierarchical
Perceptual control hierarchy
5: Freedom
dependency
drawbacks
right to choose
internal determination, not external
(no) coercion
constraint
self-actualization
opposition
self-cause, autopoiesis
a decision
out of attractor
power-to vs power-over
3: Determination
Co-evolution
determination
Chapter 6
Influence of network on function
(economic) determinism
4: Internal and external control
Internal and external control
change links vs methods vs goals
Chapter 6
Diverse manifestations of control
local vs global control
6: Emergence of hierarchy
least bad choice
aspect
social power
self-organization
formation of a controller
cultivator
7: Move away from hierarchy
evolving networks
simulation constant opposition
Rise of Hierarchy
Constant opposition
building alternatives and attacking
overall decision?
direct action
intention
decisions and acts
aims and methods
coherence
anarchism
stigmergy, variation and selection, antifragility
principles of coordination

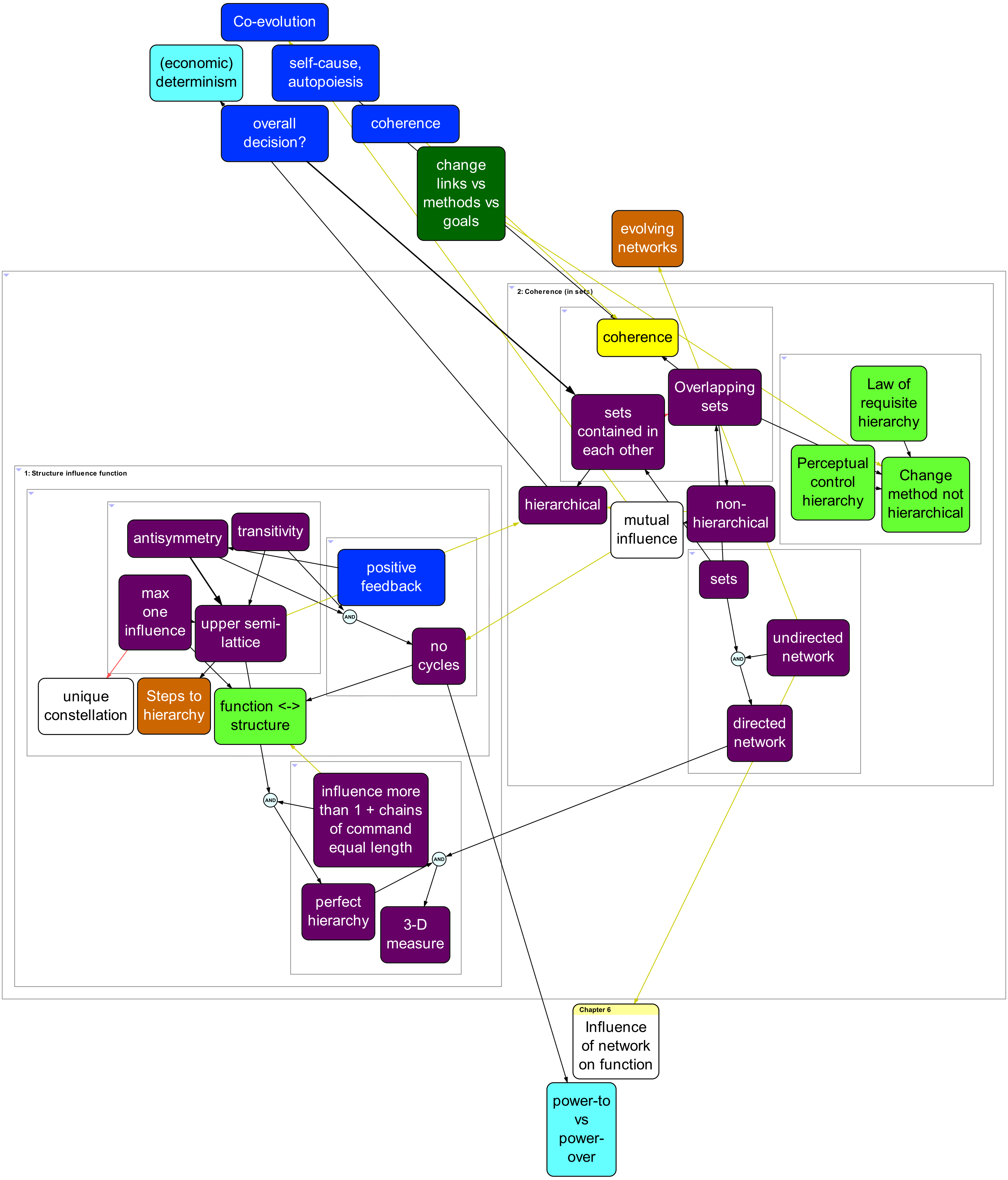


Figure 9.1: Separated network of the first subsection. These are concepts related to the structural component, how this relates to the function, and when there is coherence.

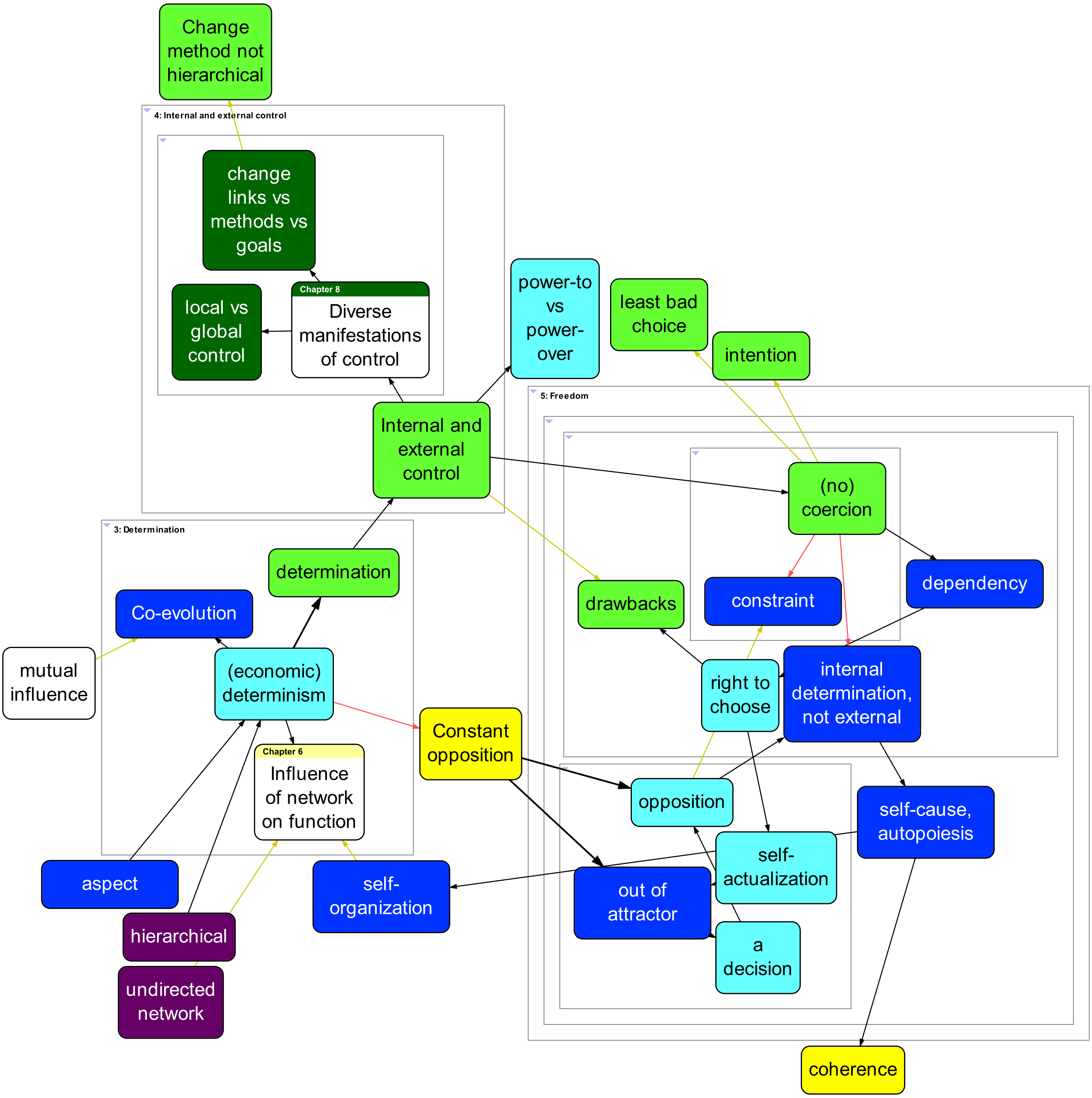


Figure 9.2: Separated network of the second subsection, focusing on the functional aspect: determination, internal and external control, freedom, ...

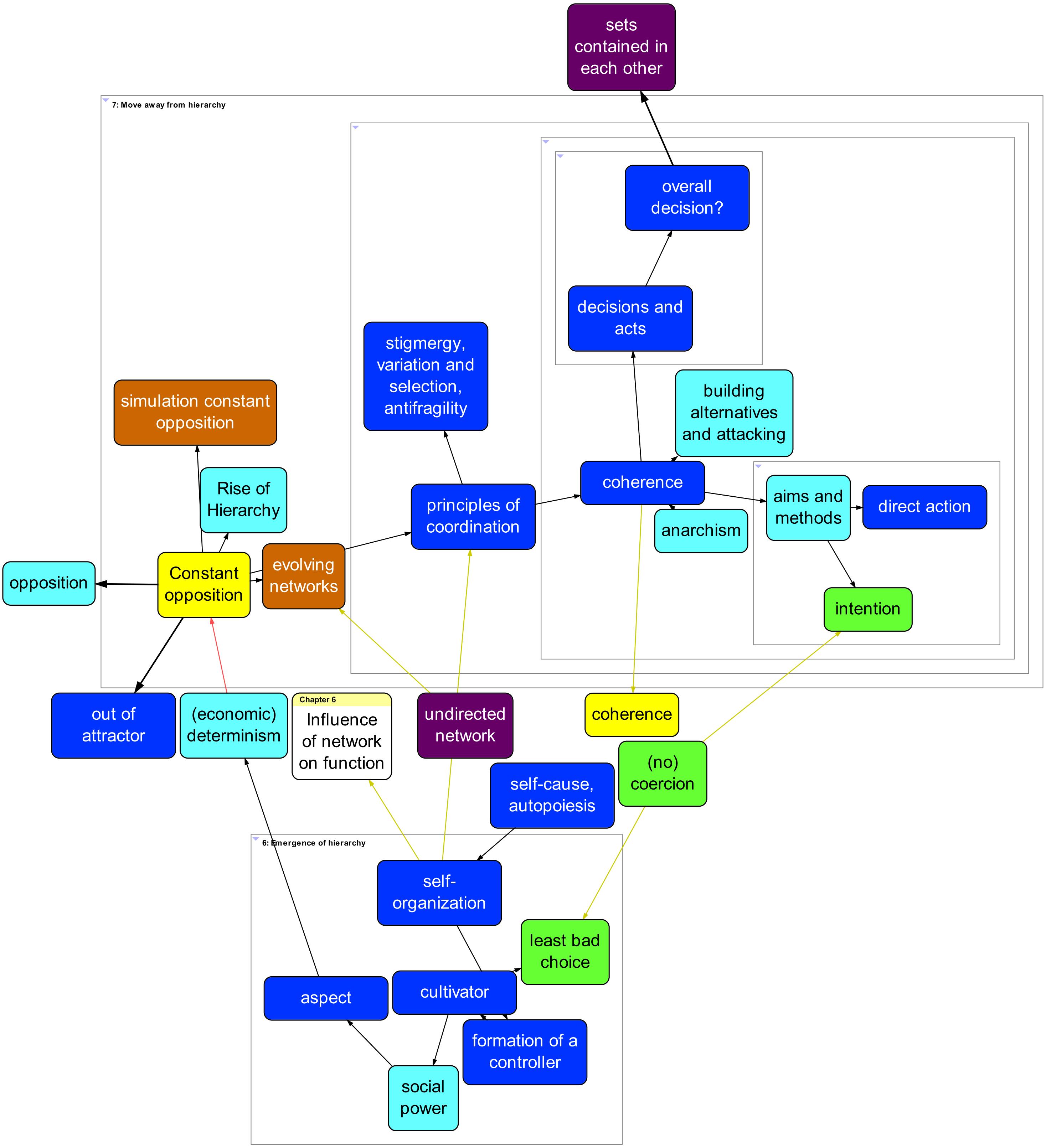


Figure 9.3: Separated network of the third subsection: the emergence of hierarchy and how to move away from it.

Bibliography

(2017). Utilitarianism. *Wikipedia*. 16

Albert, Jeong, and Barabasi (2000). Error and attack tolerance of complex networks. *Nature*, 406(6794):378–382. 140

Albert, R. and Barabasi, A.-L. (2001). Statistical mechanics of complex networks. *arXiv:cond-mat/0106096*. 128

Arendt, H. (1970). *On violence*. Houghton Mifflin Harcourt. 12

Ashby, W. R. (1961). *An introduction to cybernetics*. Chapman & Hall Ltd. 166, 173

Ashby, W. R. (1991). Requisite Variety and Its Implications for the Control of Complex Systems. In *Facets of Systems Science*, number 7 in International Federation for Systems Research International Series on Systems Science and Engineering, pages 405–417. Springer US. 166, 173

Aulin-Ahmavaara, A. Y. (1979). The law of requisite hierarchy. *Kybernetes*, 8(4):259–266. 176, 179

Bachrach, P. and Baratz, M. S. (1962). Two faces of power. *American political science review*, 56(04):947–952. 13

Bakunin, M. A. and Kenafick, K. J. (1950). *Marxism, freedom and the state*. Freedom Press London. 10, 11

Banathy, B. H. (2013). *Guided Evolution of Society: A Systems View*. Springer Science & Business Media. 40, 70

Barabasi, A.-L. and Oltvai, Z. N. (2004). Network biology: understanding the cell's functional organization. *Nature Reviews Genetics*, 5(2):101–113. 109, 116

Barabási, A.-l., Dezső, Z., Ravasz, E., and Oltvai, Z. (2003). Scale-free and hierarchical structures in complex networks. *IN SITGES PROCEEDINGS ON COMPLEX NETWORKS*, pages 1–16. 107, 109, 116

Barrett, D. (2010). *Supernormal stimuli: how primal urges overran their evolutionary purpose.* W.W. Norton & Co. 44

Bennett, J. G. (1966). *The dramatic universe. 3. Man and his nature.* Hodder & Stoughton. 64

Boehm, C. (2009). *Hierarchy in the Forest: The Evolution of Egalitarian Behavior.* Harvard University Press. Google-Books-ID: ljxS8gUlgqgC. 131

Boehm, C., Barclay, H. B., Dentan, R. K., Dupre, M.-C., Hill, J. D., Kent, S., Knauft, B. M., Otterbein, K. F., and Rayner, S. (1993). Egalitarian Behavior and Reverse Dominance Hierarchy [and Comments and Reply]. *Current Anthropology*, 34(3):227–254. 131

Bollobás, B. (2001). *Random graphs.* Cambridge University Press. 107, 109

Bonacich, P. (1987). Power and Centrality: A Family of Measures. *American Journal of Sociology*, 92(5):1170–1182. 109, 111

Bonanno, A. M. (1998). *The Anarchist Tension.* Elephant Editions. 46

Bookchin, M. (1982). *The ecology of freedom: The emergence and dissolution of hierarchy.* Cheshire Books Palo Alto, CA. 34

Bookchin, M. (1994). History, Civilization, and Progress: Outline for a Criticism of Modern Relativism. *Green Perspectives*, 29:4–5. 34

Burrell, G. and Morgan, G. (1979). *Sociological Paradigms and Organisational Analysis: Elements of the Sociology of Corporate Life.* Pearson Education. 20

Busseniers, E. (2014a). General Centrality in a hypergraph. *arXiv preprint arXiv:1403.5162.* 96, 109, 128

Busseniers, E. (2014b). Hierarchical organization versus self-organization. *arXiv preprint arXiv:1402.1670.* 39, 41, 107, 127, 130, 139

Busseniers, E. (2016). Is external control important for internal control? *Journal of Sociocybernetics*, 14(1). 6, 40, 151, 154, 206

Busseniers, E. (2017). Let's interplay! Does co-evolution enable or constrain? *Technological Forecasting and Social Change*, 114:27–34. 43, 47, 131

Castelfranchi, C. and Falcone, R. (2003). Founding autonomy: The dialectics between (social) environment and agent's architecture and powers. In *International Workshop on Computational Autonomy*, pages 40–54. Springer. 59, 64, 65

Clegg, S. R. (1989). *Frameworks of power.* Sage. 13, 20, 21

Clegg, S. R., Courpasson, D., and Phillips, N. (2006). *Power and organizations.* Pine Forge Press. 16, 21

Corominas-Murtra, B., Goñi, J., Solé, R. V., and Rodríguez-Caso, C. (2013). On the origins of hierarchy in complex networks. *Proceedings of the National Academy of Sciences*, 110(33):13316–13321. 101

Dennett, D. C. (1989). *The intentional stance.* MIT press. 186

Dittrich, P. and Fenizio, P. S. d. (2007). Chemical Organisation Theory. *Bulletin of Mathematical Biology*, 69(4):1199–1231. 45, 136

Dittrich, P. and Winter, L. (2008). Chemical Organizations in a Toy Model of the Political System. *Advances in Complex Systems*, 11(04):609. 137

Dorogovtsev, S. N., Goltsev, A. V., and Mendes, J. F. F. (2008). Critical phenomena in complex networks. *Reviews of Modern Physics*, 80(4):1275–1335. 96

Engels, F. (2004). *The Origin of the Family, Private Property and the State.* Resistance Books. 28

Feinberg, L. (1997). *Transgender Warriors : Making History from Joan of Arc to Dennis Rodman.* Beacon Press, Boston, unknown edition edition. 27, 70

Foucault, M. (1998). *The History of Sexuality(Vol-1): The Will to Knowledge.* Penguin. 63

Gelderloos, P. (2005). The rise of hierarchy. *The Anarchist Library.* 25, 40, 131

Gelderloos, P. (2007). Insurrection vs. Organization. *The Anarchist Library.* 31, 32, 40, 156

Gelderloos, P. (2010). *Anarchy Works.* 33

Gelderloos, P. (2011). Veganism: Why Not. *The Anarchist Library.* 19

Goldstein, K. (1939). The organism: A holistic approach to biology derived from pathological data in man. 10

Graeber, D. (2007). *Possibilities: Essays on hierarchy, rebellion and desire.* AK Press. 15, 32, 88

Haugaard, M. (2012). Authority. In *Encyclopedia of Social Theory*. Routledge. 18

Haykin, S. (1994). *Neural Networks: A Comprehensive Foundation*. Prentice Hall PTR, Upper Saddle River, NJ, USA, 1st edition. 219

Henry, M. (2017). *Parkour Roadmap*. MASCOT BOOKS. 156

Heylighen, F. (1997). The economy as a distributed, learning control system. *Communication & Cognition-AI*, 13(2/3):207–24. 160

Heylighen, F. (2006). Mediator Evolution: a general scenario for the origin of dynamical hierarchies. *Worldviews, Science and Us.(Singapore: World Scientific)*. 43, 44, 47

Heylighen, F. (2009). Cognitive systems: A cybernetic perspective of the new science of the mind. *Lecture Notes*, 2010. 53

Heylighen, F. (2011a). Self-organization in Communicating Groups: the emergence of coordination, shared references and collective intelligence. In *Language and Complexity*. Barcelona University Press. 39, 41

Heylighen, F. (2011b). Self-organization of complex, intelligent systems: an action ontology for transdisciplinary integration. *Integral Review*, pages 1–39. 186

Heylighen, F. (2014a). Complexity and Evolution: Fundamental Concepts of a New Scientific Worldview. 39, 55, 57, 62, 131, 198

Heylighen, F. (2014b). Cybernetic Principles of Aging and Rejuvenation: the buffering-challenging strategy for life extension. 55

Heylighen, F. (2014c). Return to Eden? promises and perils on the road to a global superintelligence. *The End of the Beginning: Life, Society and Economy on the Brink of the Singularity. Retrieved from http://pespmc1. vub. ac. be/Papers/BrinkofSingularity. pdf.* 72, 73

Heylighen, F. and Beigi, S. (2016). Mind outside Brain: a radically non-dualist foundation for distributed cognition. *Socially Extended Epistemology. Oxford University Press. Retrieved from http://pespmc1. vub. ac. be/Papers/Non-dualism. pdf.* 53

Heylighen, F. and Joslyn, C. (2001). Cybernetics and Second-Order Cybernetics. In *Encyclopedia of Physical Science & Technology (3 rd.* Academic Press. 158

Holland, J. H. (1992). Genetic algorithms. *Scientific American*, 267(1):66. 146, 150

Jia, T., Liu, Y.-Y., Csóka, E., Pósfai, M., Slotine, J.-J., and Barabási, A.-L. (2013). Emergence of bimodality in controlling complex networks. *Nature Communications*, 4. 120

Johnson, J. (2013). *Hypernetworks in the Science of Complex Systems.* World Scientific. Google-Books-ID: jQO4CgAAQBAJ. 97

Kreyssig, P. and Dittrich, P. (2011). Emergent Control. In Müller-Schloer, C., Schmeck, H., and Ungerer, T., editors, *Organic Computing — A Paradigm Shift for Complex Systems*, number 1 in Autonomic Systems, pages 67–78. Springer Basel. 223

Kropotkin, P. A. (1906). *Fields, factories, and workshops.* Swan Sonnenschein. 34

Kuhn, G. (2010). *Sober Living for the Revolution: Hardcore Punk, Straight Edge, and Radical Politics.* PM Press. 35

Laland, K. N., Odling-Smee, J., and Feldman, M. W. (2001). Cultural niche construction and human evolution. *Journal of Evolutionary Biology*, 14(1):22–33. 49

Lenartowicz, M. (2016). Creatures of the semiosphere: A problematic third party in the 'humans plus technology' cognitive architecture of the future global superintelligence. *Technological Forecasting and Social Change.* 46

Liu, Y.-Y., Slotine, J.-J., and Barabási, A.-L. (2011). Controllability of complex networks. *Nature*, 473(7346):167–173. 118, 206, 216

Lorenz, E. (2000). The butterfly effect. *WORLD SCIENTIFIC SERIES ON NONLINEAR SCIENCE SERIES A*, 39:91–94. 58

Lukes, S. (1974). Power: A radical view. *London and New York: Macmillan.* 13

Malatesta, E. (1965). *Errico Malatesta: his life & ideas.* Freedom Press. 10, 11

Maldonado, C. and Mezza-Garcia, N. (2016). Anarchy and complexity. *Emergence: Complexity & Organization*, 18(1). 59

Marshall, P. (2009). *Demanding the Impossible: A History of Anarchism.* PM Press. 30, 32

Marx, K. and Engels, F. (1970). *The german ideology*, volume 1. International Publishers Co. 33

Marx, K. and Engels, F. (2002). *The communist manifesto.* Penguin. 33

Maslow, A. H. (1943). A theory of human motivation. *Psychological review*, 50(4):370. 67

Maturana, H. R. and Varela, F. J. (1991). *Autopoiesis and cognition: The realization of the living*, volume 42. Springer Science & Business Media. 45

McClelland, K. (1994). Perceptual Control and Social Power. *Sociological Perspectives*, 37(4):461–496. 19, 157, 164, 165, 185, 186

McClelland, K. (2004). The collective control of perceptions: constructing order from conflict. *International Journal of Human-Computer Studies*, 60(1):65–99. 162, 206, 221

Mesarovic, M. D., Macko, D., and Takahara, Y. (2000). *Theory of hierarchical multilevel systems.* 91, 206, 208, 209

Mitzenmacher, M. (2003). A Brief History of Generative Models for Power Law and Lognormal Distributions. *Internet Mathematics*, 1(2):226–251. 57

MOELLER, H.-G. (2012). *The Radical Luhmann.* Columbia University Press. 45, 46

Morgan, L. H. (1877). *Ancient Society.* Transaction Publishers. 28

Newman, M. E. J. (2003). The structure and function of complex networks. *arXiv:cond-mat/0303516.* 107

Passamani, M. (2010). *"More, Much More" and other writings | The Anarchist Library.* 12, 40, 52, 69

Reclus, E., Clark, J. P., and Martin, C. (2004). *Anarchy, Geography, Modernity: The Radical Social Thought of Elisée Reclus.* Lexington Books. 58

Rifkin, J. (2014). *The zero marginal cost society: The internet of things, the collaborative commons, and the eclipse of capitalism.* Palgrave Macmillan. 27, 34

Roughgarden, J. (2013). *Evolution's rainbow: Diversity, gender, and sexuality in nature and people.* Univ of California Press. 17, 74

Ryan, M. (2005). Authority. In *Encyclopedia of Social Theory*. SAGE Publications, Inc. 18

Salthe, S. N. (2013). *Evolving hierarchical systems: their structure and representation.* Columbia University Press. 157

Serge, V. (1911). *The Individualist and society by Victor Serge 1911.* English version. 19, 43

Serge, V. (2009). *De Weerspannige - Het sociale lichaam van de autoriteit raken - Typemachine.* Dutch version. 19, 43

Shannon, C. E. (2001). A mathematical theory of communication. *ACM SIGMOBILE Mobile Computing and Communications Review*, 5(1):3–55. 62, 168

Shannon, D., Rogue, J., Daring, C. B., and Volcano, A. (2013). *Queering Anarchism: Addressing and Undressing Power and Desire.* AK Press. 22

Silberzahn, R. and Uhlmann, E. L. (2015). Crowdsourced research: Many hands make tight work. *Nature*, 526(7572):189–191. 72

Simon, H. A. (1962). The architecture of complexity. *Proceedings of the American philosophical society*, 106(6):467–482. 90

Stewart, J. E. (2014). The direction of evolution: the rise of cooperative organization. *Biosystems*, 123:27–36. 14, 44, 157

Stirner, M. (1907). The Ego and His Own. 43, 46, 53

Taleb, N. N. (2012). *Antifragile: Things that Gain from Disorder.* Penguin UK. 54, 156

Thaler, R. H. and Sunstein, C. R. (2003). Libertarian Paternalism. *The American Economic Review*, 93(2):175–179. 40

Varela, F. G., Maturana, H. R., and Uribe, R. (1991). Autopoiesis: The Organization of Living Systems, Its Characterization and a Model. In *Facets of Systems Science*, number 7 in International Federation for Systems Research International Series on Systems Science and Engineering, pages 559–569. Springer US. 137

Veitas, V. and Weinbaum, D. (2014). A World of Views: A World of Interacting Post-human Intelligences. *arXiv preprint arXiv:1410.6915*. 74

Volpentesta, A. P. and Felicetti, A. M. (2010). Eigenvector Centrality Based on Shared Research Topics in a Scientific Community. In Camarinha-Matos, L. M., Boucher, X., and Afsarmanesh, H., editors, *Collaborative Networks for a Sustainable World*, volume 336, pages 626–633. Springer Berlin Heidelberg, Berlin, Heidelberg. 110

Wang, X. F. and Chen, G. (2002). Pinning control of scale-free dynamical networks. *Physica A: Statistical Mechanics and its Applications*, 310(3):521–531. 121

Willer, D. (2005). Elementary Theory. In *Encyclopedia of Social Theory*, pages 235–239. SAGE Publications, Inc., Thousand Oaks. DOI: 10.4135/9781412952552. 185

Wrong, D. H. (2005). Power. In *Encyclopedia of Social Theory*. SAGE Publications, Inc. 14, 19

Zwick, M. (1978). Requisite Variety and the Second Law. In *Proceedings of Systems Science and Cybernetics Symposium, International Conference of Cybernetics and Society (IEE Systems, Man, and Cybernetic Society), Tokyo/Kyoto*. 175

Zwick, M. (2017). Freedom as a Natural Phenomenon. *Foundations of Science*, 22(1):1–10. 59, 63, 64, 66, 67

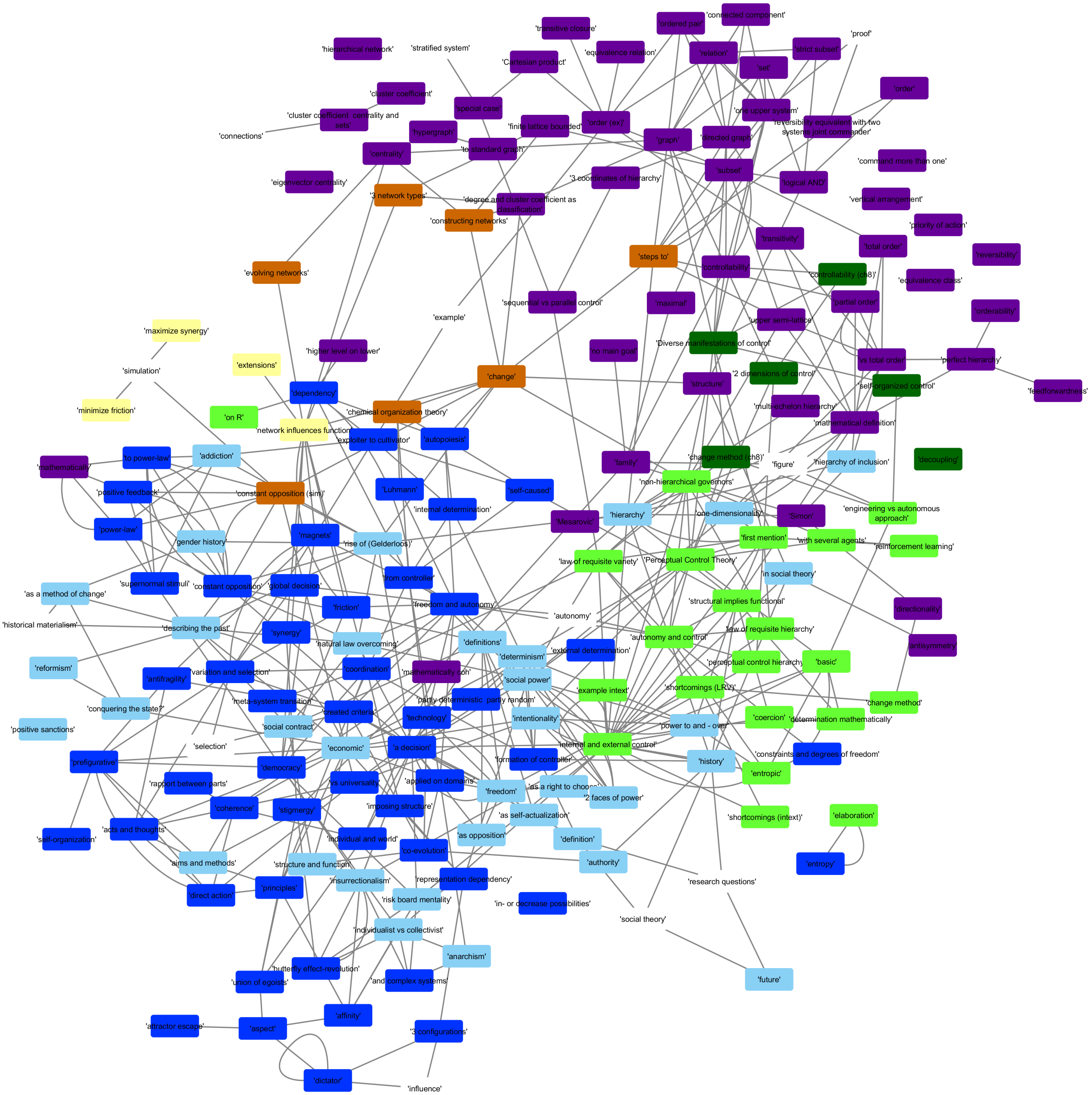


Figure 9.4: Network from the index (see next page). Every concept in this index refers to only one page: the most relevant page where this concept is discussed. Such a target page is shown in bold. If the concept occurs on other pages, it will often have a link—to this same page. These other pages will also be shown next to the concept in the index, in normal font. The network between all these concepts is shown in the above schedule. It connects every concept with all the other (linked) concepts mentioned on its target page. The colors represent the chapter where the concept is described. See p. 241 for a legenda of the colors. Note how chapters cluster together and link with other chapters.

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