Bear Networks

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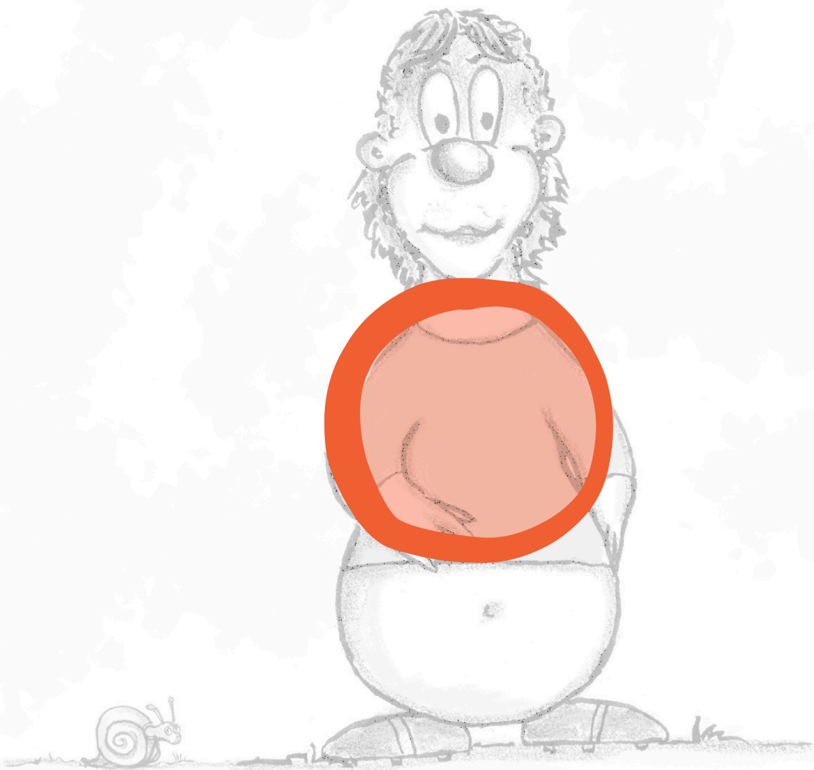
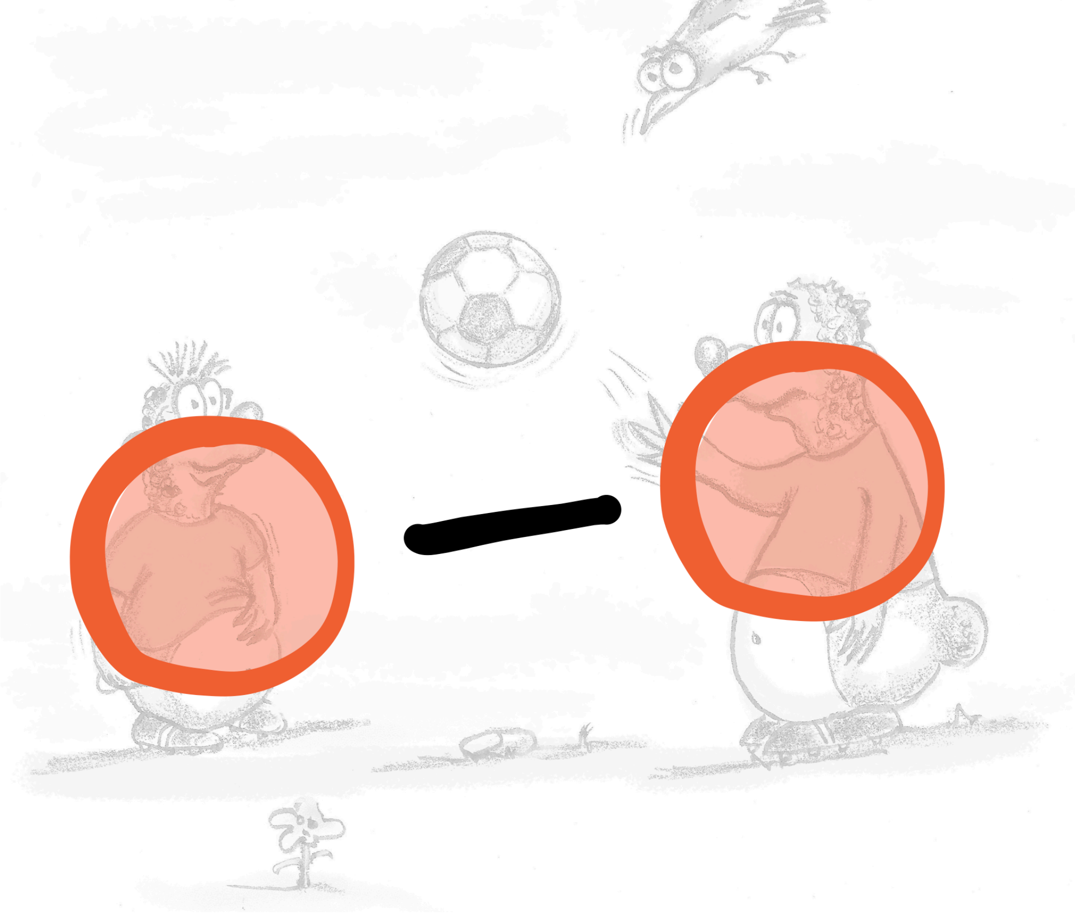
BEAR



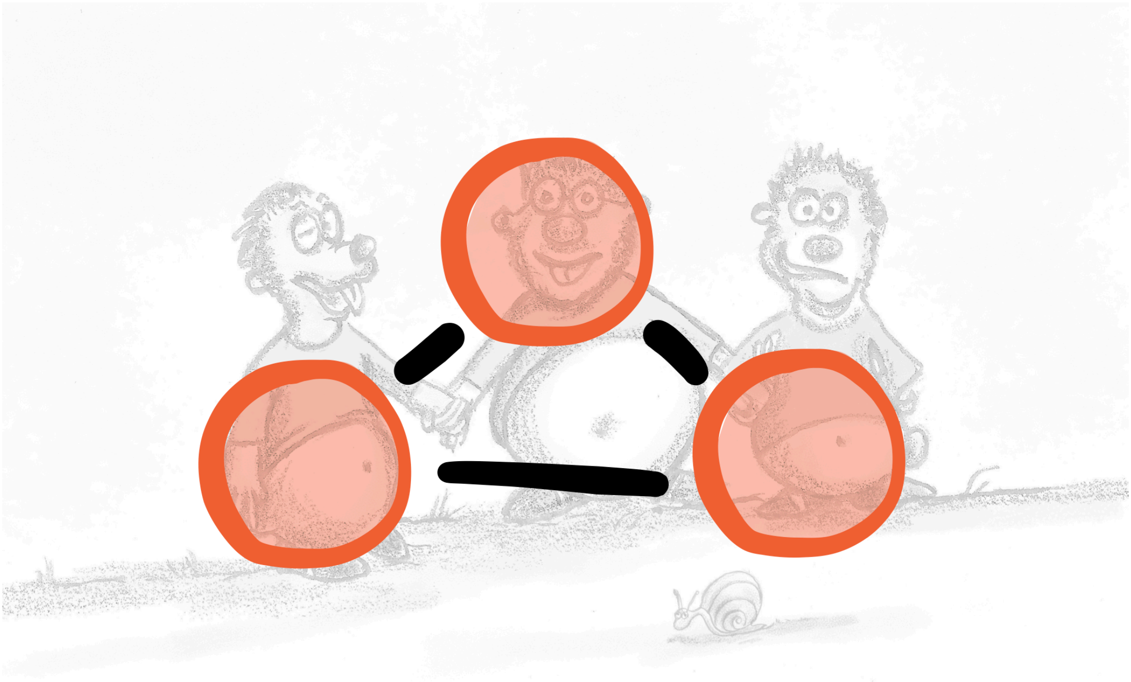
NODE

PLAYING BALL



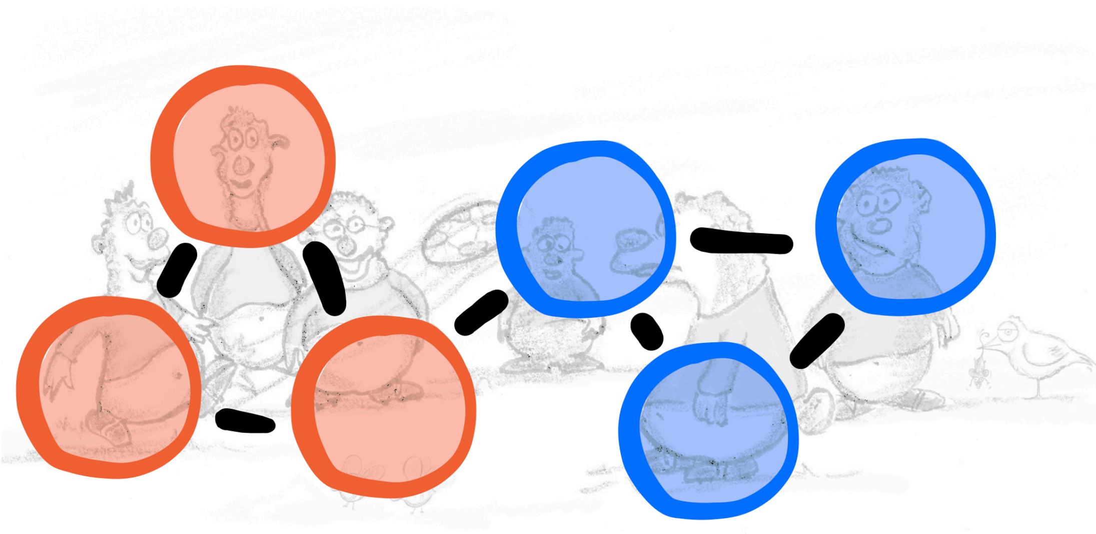
EDGE

TEAM

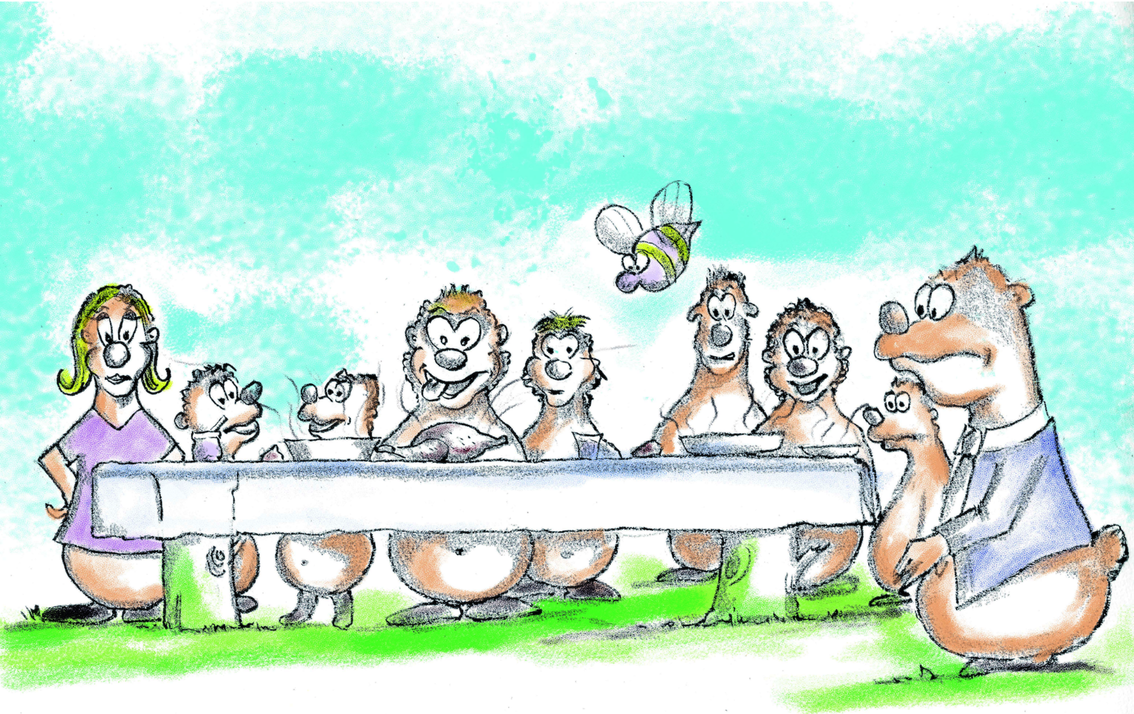


GRAPH

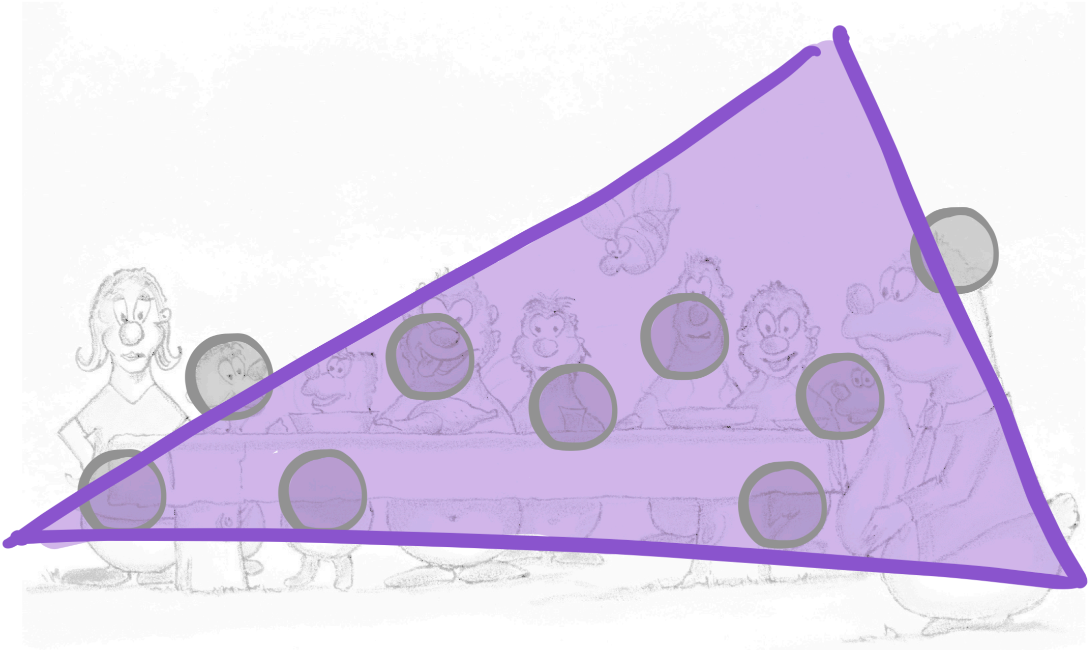
TEAM PLAY



COMMUNITY STRUCTURE

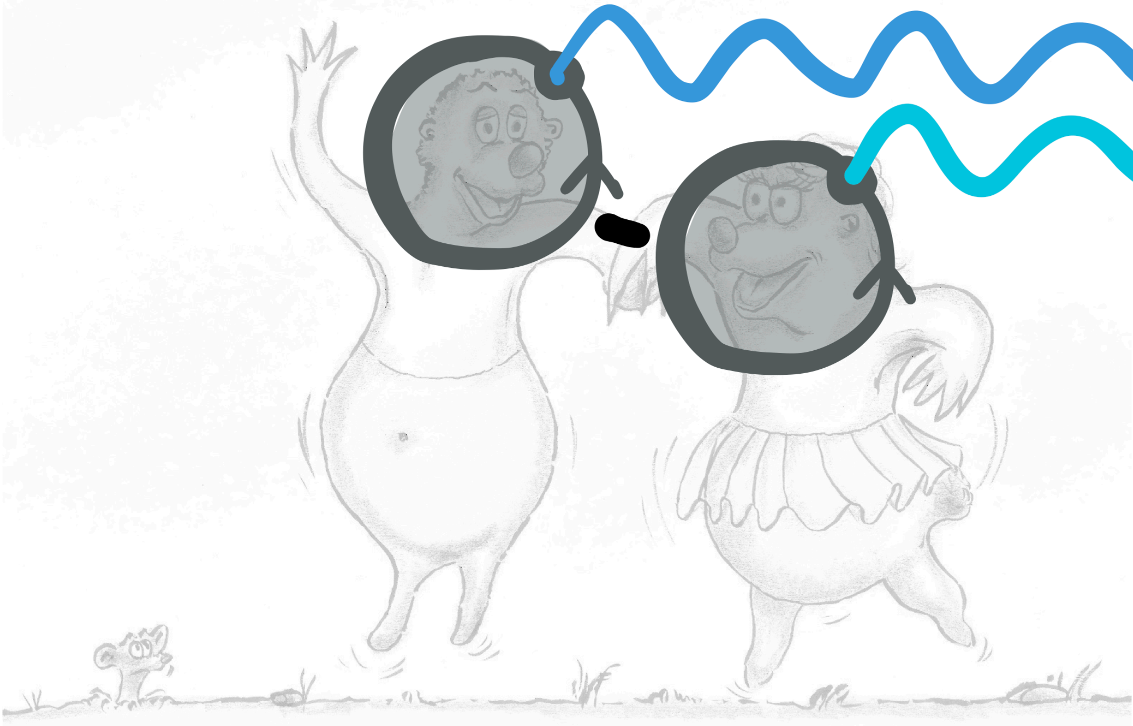


VICTORY FEAST



HYPEREDGE

DANCING THE NIGHT AWAY



SYNCHRONIZATION

Epilogue

Dear grown-up bears,

Within your paws lies a book about graph theory for baby bears. But what is graph theory and why is it important? Graphs capture our interconnected world mathematically. They consist of entities called nodes or vertices and interconnections called edges. But these seemingly simple objects give rise to mathematical puzzles:

Puzzle 1. Sketch four points on paper and then try to draw straight or curved lines between each pair of points, ensuring no lines intersect. Can you do it?

Puzzle 2. Try to do the same as in the previous problem, but with five points instead of four. Can you do it?

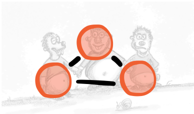
Why do mathematicians delve into the world of graphs? Because, quite simply, it's fun! Applied scientists, on the other hand, use graphs for studying real networks such as social networks, communication networks, transport networks, neural networks, biochemical networks, and epidemic networks, to name a few. For example, one can model a social network with a graph by representing people as vertices and connecting two of them with an edge if they are friends with each other. Moreover, graphs can be also used to study dynamical systems: Processes in which the behaviors of the elements (nodes) change in time, and the changes depend on the connections (edges) between elements. For instance, two bears who dance while holding hands will influence each other's movements, which might end up being synchronized.

Historically, graph theory was born in 1736, when Leonard Euler solved the Königsberg bridges Problem: "Is it possible to walk through the historical Prussian city of Königsberg in such a way that each bridge is crossed exactly once?". Euler showed that it is not possible to cross each bridge of Königsberg exactly once and established, in this way, the very first theorem of graph theory. Another beautiful result on graphs is Kuratowski's Theorem (1930), which says that one cannot draw a graph on paper with different lines not intersecting each other if the graph contains the graph made by five points all connected to each other or the graph made by three points all connected to three other points. This means that Puzzle 1 can be solved while Puzzle 2 cannot!

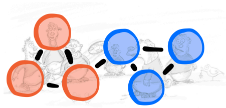
Are you starting to see the beauty of graphs? Today, despite being nearly 300 year-old, graph theory continues to thrive with myriad unsolved puzzles. Moreover, it also extends to richer structures. For instance, a generalization of graphs that is of great interest for mathematicians is that of hypergraphs. In a hypergraph, we still have nodes as in the case of graphs, but instead of having just pairwise connections, we have sets of any size which are called hyperedges. With a hypergraph we can model groups of elements, which could for instance represent communities of people. Is this not hyper-amazing?

The illustrations by Piergiorgio Mulas in this book depict a day in the life of a soccer-loving bear and, at the same time, take us on a journey through the mathematical world of graphs. Indeed, they are more than just illustrations; they extend an invitation for you to unfold the magic of graphs to any baby bear that reads this book.

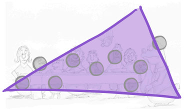
Each **bear** is a *node* in our network. And as many children, little bears like to play soccer! And if two bears **play**, they have a relationship: and *edge* between the nodes.



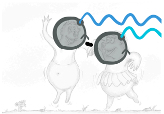
Several bears together form a **team**: A *graph* consists of several nodes connected by edges.



Soccer requires **team play**! Teams can be distinguished by their shirts. This yields a graph with *community structure*: Vertices with the same color form a community.



What's better than celebrating **victory** with a **dinner**? As a group interaction between many bears, this is a *hyperedge*!



And after dinner, the bears are **dancing the night away**! Hopping up and down and up and down and *synchronizing*.

We hope that you will enjoy exploring the realm of graphs with your baby bears, and that you will love the illustrations by Piergiorgio Mulas as much as we do!

Christian Bick and Raffaella Mulas — Amsterdam, May 2024

A nonna Cornelia — Raffaella e Piergiorgio

Für meine Kleinen — Christian

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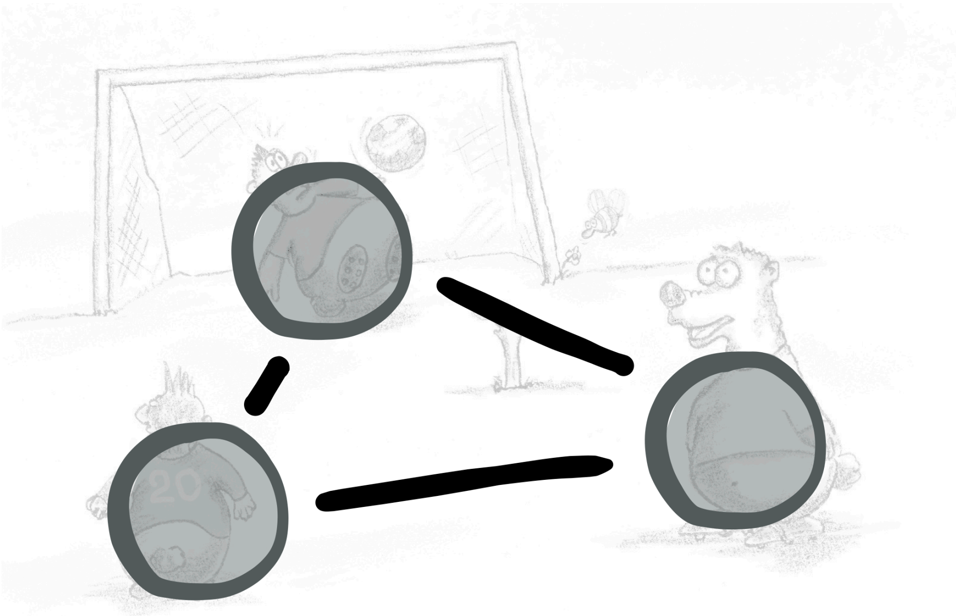

Bear Networks: Mathematics of networks introduced by soccer-loving bears.

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