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Molecular dynamics simulations in biosystems MD introduction

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Acknowledge



Based on a course by **Prof. Lukasz Cwiklik** group leader at J.Heyrovsky Institute of Physical Chemistry Czech Academy of Sciences at Prague, Czech Republic

http://cwiklik.net

Computer simulations

Physical system

(cluster of atoms, water-air interface, cell membrane, protein in water)

Experiment

(IR spectroscopy, NMR, X-ray diffraction, fluorescence microscopy, enzymatic kinetics)



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Simulations (Molecular Dynamics, Monte Carlo, Dissipative Particle Dynamics, Computational Fluid Dynamics)



Theory

(quantum mechanics,

thermodynamics,





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Simulations:

- theory based on a model
- "computational experiment"
- "computational microscope"

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The idea

Molecular simulations = **simulations of molecules/atoms**



- What they are good for:
 - Condensed phases & interfaces (bulk of water, surface of solid,...)
 - Biological systems (proteins, membranes, sugars, biomolecules)
 - Interactions between biomolecules, drugs.
 - Provide mechanistic details about the microscopic behavior

Computer simulations - History

- Monte Carlo: Ulam, von Neumann, Metropolis Los Alamos labs, 1953
- Molecular Dynamics: Alder and Wainwright Livermore labs, 1957 (hard spheres dynamics)
- Molecular Dynamics, Nobel Prize in Chemistry 2013: Karplus, Levitt, Warshel

THE JOURNAL OF CHEMICAL PHYSICS VOLUME 21, NUMBER 6

JUNE, 1953

Equation of State Calculations by Fast Computing Machines

NICHOLAS METROPOLIS, ARIANNA W. ROSENBLUTH, MARSHALL N. ROSENBLUTH, AND AUGUSTA H. TELLER, Los Alamos Scientific Laboratory, Los Alamos, New Mexico

ND

EDWARD TELLER,* Department of Physics, University of Chicago, Chicago, Illinois (Received March 6, 1953)

A general method, suitable for fast computing machines, for investigating such properties as equations of state for substances consisting of interacting individual molecules is described. The method consists of a modified Monte Carlo integration over configuration space. Results for the two-dimensional rigid-sphere system have been obtained on the Los Alamos MANIAC and are presented here. These results are compared

> Figure separate Phase Transition for a Hard region th Sphere System states w B. J. Alder and T. E. WAINWRIGHT calculati University of California Radiation Laboratory, Livermore, California denly fro (Received August 12, 1957) tions of stavs on CALCULATION of molecular dynamic motion all confir A has been designed principally to study the retheir nei laxations accompanying various nonequilibrium phethe parti nomena. The method consists of solving exactly (to the with the number of significant figures carried) the simultaneous originally

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MD simulations literature

- Alen & Tildesley "Computer Simulations of Liquids"
- Frenkel & Smit "Understanding Molecular Simulation"
- H. Martinez-Seara, L. Cwiklik, "Introductory tutorial for GROMACS" DOI: 10.5281/zenodo.1230441
- ChatGTP

Computer simulations: Hardware



- Supercomputer or cluster:
 - Allows to simulate BIGGER, but not much "longer"?



- "Super"-desktop with powerful GPU and plenty of Memory:
 - to prepare, analyze and visualize simulations

Buy the proper hardware (HW)

- Different simulations has different HW needs.
 - Amount of memory
 - Number and type of cores
 - AMD: cheaper, faster, less issues?
 - GPUs: Nvidia (Cuda) vs AMD/Lumi.
 - Local disk SSD/NVME an backups (Please).
- Fight for the proper hardware.
 - Rarely, desktops targeted to MS Office are useful for simulations.

Computer Codes

- Molecular dynamics (MD) simulation
 - GROMACS (www.gromacs.org)
 - Amber (www.ambermd.org)
 - CHARMM (www.charmm.org)
 - NAMD (www.ks.uiuc.edu/Research/namd)
 - OpenMM (http://openmm.org/)
 - LAMMPS (https://lammps.sandia.gov/)
- Molecular visualization
 - VMD (https://www.ks.uiuc.edu/Research/vmd/)
 - -Pymol (https://pymol.org/)
- Analysis
 - Python, R, Matlab
 - MDAnalysis (https://www.mdanalysis.org/)
 - MDTraj (mdtraj.org)



 Optimal simulation and working environments for MD simulations:



Pros and Cons of MD?

Advantages Atomistic resolution!

Disadvantages/limitations

System size (~10⁻⁸ m)



Classical or Quantum physics

Atoms as quantum particles or billiard balls?



Do we need to consider quantum mechanics when dealing with molecules in biosystems?



Why we can use MD at all?

Born-Oppenheimer approximation

Born-Oppenheimer approximation:

 M_e << M_{nucl} ⇒ nuclei move in an averaged field of electrons ⇒ motion of electrons can be separated from motion of nuclei

$$\Psi_{tot} = \psi_{el}(r; R) \psi_n(R)$$

for electrons:

$$H_{el}(r;R)\psi_{el}(r;R) = E_{el}(R)\psi_{el}(r;R)$$

for nuclei:

$$[T_n + E_{el}(R)]\psi_n(R) = E\psi_n(R)$$

- Nuclei move on a potential energy surface $E_{el}(R)$
- System remains in one electronic state
- Fails: near-degeneracy

Classical approximation

R

- de Broglie wavelength: $\lambda = h/p$
- Thermal de Broglie wavelength: $\lambda_{th} = h/p$, where at given temperature: $p = \sqrt{2\pi m k_b T}$ so: $\lambda_{th} = h/\sqrt{2\pi m k_b T}$
- Classical mechanics may be used for nuclei Thermal de Broglie wavelength (298K): H: $\sim 10^{-10}$ m H₂: $\sim 10^{-11}$ m He: $\sim 10^{-11}$ m

MD: equations of motion, forces and potential $V = -\int_{ref}^{r} \vec{F} \, d\vec{r}$

- In MD, classical Newton's equations of motion are solved for atoms in the considered system.
- Trajectory (positions of atoms in time) is the result of MD simulation. System properties calculated from the trajectory.
- Interaction potential must be assumed (force field).
- Initial conditions must be set (initial positions and velocities of all atoms).



Ab initio MD - Interaction potential from quantum mechanics

Born-Oppenheimer approximation:

 M_e << M_{nucl} ⇒ nuclei move in an averaged field of electrons ⇒ motion of electrons can be separated from motion of nuclei

 $\Psi_{tot} = \psi_{el}(r; R) \psi_n(R)$

for electrons:

 $\overrightarrow{F_i} = m_i \overrightarrow{a_i}$

$$H_{el}(r;R)\psi_{el}(r;R) = E_{el}(R)\psi_{el}(r;R)$$

for nuclei:

$$[T_n + E_{el}(R)]\psi_n(R) = E\psi_n(R)$$

 $V(\vec{r})$ – interaction potential, calculated on-the-fly (in each step) from the electronic wave function



$$\vec{r_i}(t=0), \vec{v_i}(t=0)$$

Limitations – computationally very expensive (short time and/or small systems only)! 14/120

 $\frac{dV}{d\vec{r_i}} = m_i \frac{d^2 \vec{r_i}}{dt^2}$

Classical MD – The force field: The molecular model (empirical)

$$V(\vec{r})$$
 - force field

The force field (V) and the environment (r) determine the force that an atom feels



The force experienced by an atom and its state determine how an atom moves

The classical MD force field

$$V_{i} (r_{1}, r_{2}, ..., r_{N}) = V_{bond} + V_{angle} + V_{dihedral} + V_{LJ} + V_{coulomb}$$

- The force field determines the force that an atom feels.
- The force field is "invariable" during a simulation.
- The force fields are usually additive.
- The force field is the collection of functions and its parameters used to compute the final force experienced by an atom.
- There are many kind of force fields to simulate biosystems, most of the use similar functional forms but different parameters
 - Is there the **ultimate** force field?

Finding an approximate interaction potential

 $V(\vec{r})$ - force field

- Force field derived by fitting to experimental and/or ab initio data
- Force fields are called "empirical potentials"
- Force fields are not 'purely' empirical, as ab initio data are also used for their derivation!

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Often they contain a lot of magic too :(

Force field: intra- and intermolecular interactions

Intramolecular interactions (*chemical*, bonding)

<u>Inter</u>molecular interactions (*physical*, non-bonding)





Force field: Intramolecular interactions



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- Intramolecular interaction can be approximated by the additive terms:
 - Bonds
 - Valence angles
 - Torsion (dihedral) angles
 - Intramolecular electrostatics
 - Intramolecular van der Waals

Intramolecular interactions – bond potential



- Bond potential usually approximated as:
 - Harmonic potential $V_b(r_{ij}) = \frac{1}{2}k_{ij}^b(r_{ij} b_{ij})^2$
 - Morse potential $V_{morse}(r_{ij}) = D_{ij}[1 \exp(-\beta_{ij}(r_{ij} b_{ij}))]^2$

Errors due to differences between quantum and classical oscillators! IN PRACTICE: it is better to constraint the bonds (it also allows larger Δt) 20/120



- Valence angle is usually approximated as:
 - Harmonic angle potential

Sometimes also as:

- Cosine-based potential (in GROMOS)
- Restricted bending potential

$$V_a(\theta_{ijk}) = \frac{1}{2}k^{\theta}_{ijk}(\theta_{ijk} - \theta^{0}_{ijk})^2$$

$$V_a(\theta_{ijk}) = \frac{1}{2} k_{ijk}^{\theta} \left(\cos(\theta_{ijk}) - \cos(\theta_{ijk}^0) \right)^2$$

$$V_{\text{ReB}}(\theta_i) = \frac{1}{2} k_{\theta} \frac{(\cos \theta_i - \cos \theta_0)^2}{\sin^2 \theta_i}$$

- Urey-Bradley potential (in CHARMM) $V_a(\theta_{ijk}) = \frac{1}{2}k^{\theta}_{ijk}(\theta_{ijk} - \theta^0_{ijk})^2 + \frac{1}{2}k^{UB}_{ijk}(r_{ik} - r^0_{ik})^2$

Intramolecular interactions – torsion angle potential



- Different (equivalent) functional forms:
 - Periodic (Fourier, proper)

$$V_d(\phi_{ijkl}) = \sum_{n=0}^{5} k_{\phi_n} (1 + \cos(n\phi_n - \phi_{sn}))$$

 k_{ϕ_n} - dihedral force constant

n – number of minimum ϕ_{sn} - angular offset (usually 0° for odd n, 180° for even n)

$$V_{rb}(\phi_{ijkl}) = \sum_{n=0}^{5} C_n (\cos(\psi))^n$$
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Ryckaert-Bellemans
 potential

Intermolecular interactions



Intramolecular interactions classification:

- Electrostatic interactions: charge-charge, charge-dipole, charge-induced dipole (or higher multipoles)
- **van der Waals interactions** (all but the above, attractive & repulsive, between uncharged molecules):
 - dipole-dipole (or higher quadrupoles, sometimes classified as part of electrostatic interactions)
 - dipole-induced dipole (Debye force, 'polarization')
 - between two instantaneously induced dipoles (London dispersion force)
 - Pauli repulsion

Electrostatic interactions – modeling via point charges

All interactions at the molecular level have their origin in electrostatics.

In non-polarizable classical MD simulation, electrostatic interactions in the force field are reduced (approximated) to **charge-charge interactions**!

In force fields, in practice, this is realized by the use of **point charges**:

- We use Point charges to generate approximate electrostatic potential
- Atomic charges are not observables, i.e., their definition is arbitrary.
- Derived usually from quantum mechanics (different schemes exist: ESP, RESP, etc.)



Hydrogen bond & electrostatics



Hydrogen bond – not longer <u>explicitly</u> included in typical force fields

Attractive intermolecular interactions: A fair approximation?

Interaction	V(r) dependence	Typical energy (kJ/mol)
ion-ion	1/r	250
ion-dipole	1/r ²	15
dipole-dipole	1/r ³	2
dipole-dipole (rotating)	1/r ⁶	0.6
London (dispersion)	1/r ⁶	2
hydrogen bond		20

based on Atkins, Phys Chem

The **always** attractive van der Waals iteractions in condensed phases can be **approximated as**:

$$V_{ij} = -\frac{C_6}{r^6}$$

Pauli repulsion

Short-range strong repulsive forces due to overlap of electronic densities

$$V_{ij} = \frac{C_{12}}{r^{12}}$$

(Atoms don't fuse easily, do they?)

Origin: Quantum mechanical exchange energy between fermions



Lennard-Jones potential

An approximate model for the isotropic part of a total (repulsion plus attraction) van der Waals force as a function of distance

$$V_{LJ}(\boldsymbol{r}_{ij}) = 4\epsilon_{ij} \left(\left(\frac{\sigma_{ij}}{r_{ij}} \right)^{12} - \left(\frac{\sigma_{ij}}{r_{ij}} \right)^6 \right) = V_{LJ}(r_{ij}) = \frac{C_{ij}^{(12)}}{r_{ij}^{12}} - \frac{C_{ij}^{(6)}}{r_{ij}^6}$$



Understanding the Lennard-Jones potential



The graph above plots the Lennard–Jones potential function, and indicates **regions of attraction and repulsion**. Atoms try to minimize their potential energy and at the lowest temperatures are sitting at the bottom of the potential curve. When the atomic separations are to the left of the minimum the atoms repel, otherwise they attract one another.

http://atomsinmotion.com/book/chapter5/md

Lennard-Jones potential – mixing rules



$$V_{LJ}(\boldsymbol{r}_{ij}) = 4\epsilon_{ij} \left(\left(\frac{\sigma_{ij}}{r_{ij}} \right)^{12} - \left(\frac{\sigma_{ij}}{r_{ij}} \right)^6 \right)$$
 atomic (not pair) property

Geometric average:

$$C_{ij}^{(6)} = \left(C_{ii}^{(6)} C_{jj}^{(6)}\right)^{1/2}$$
$$C_{ij}^{(12)} = \left(C_{ii}^{(12)} C_{jj}^{(12)}\right)^{1/2}$$

Lorentz-Berthelot rules:

$$\sigma_{ij} = \frac{1}{2}(\sigma_{ii} + \sigma_{jj})$$

$$\epsilon_{ij} = (\epsilon_{ii} \epsilon_{jj})^{1/2}$$

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Hydrophobic effect

The apparent tendency of nonpolar molecules in a polar solvent (e.g., in water) to interact with one another



- Hydrophobic effect results from electrostatic and van der Waals interactions
- The term 'hydrophobic interaction' is disputable
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An empirical force field – overall potential energy scheme



Force field in Gromacs – TIP3P water example

			HW1					_
			HW2	w				E; Br C
	Cha	rges	& int (tip3	r <mark>amo</mark> 3p.itp f	lecu ile)	lar te	rms	CA CB CC
[molec ; molna SOL	uletype me nre 2] xcl						CK CM
E atoms ; id a 1 0 Z H 3 H] t type W W W	res nr 1 1 1	r res name SOL SOL SOL	at name OW HW1 HW2	cg nr 1 1 1	charge -0.834 0.417 0.417	mass 16.00000 1.00800 1.00800	OW OH OS
#ifndef	FLEXIBL	E						02
[settl ; OW 1	es] funct 1	doh dhh 0.09572	0.15139					
[exclu 1 2 2 1 3 1	sions] 3 3 2							
#else								
[bonds ; i 1 1] j 2 3	funct 1 1	length fo 0.09572 50 0.09572 50	rce_consta 2416.0 (2416.0 (ant 0.09572 0.09572	50 50	2416.0 2416.0	

Intermolecular terms (Lennard-

Jones) (ffnonbonded.itp file)

atomtypes]					
name	at.num	mass	charge	ptype	e sigma	epsilon
	35	79.90	0.0000	A	0.00000e+00	0.00000e+00
	6	12.01	0.0000	A	3.39967e-01	3.59824e-01
	6	12.01	0.0000	A	3.39967e-01	3.59824e-01
	6	12.01	0.0000	A	3.39967e-01	3.59824e-01
	6	12.01	0.0000	A	3.39967e-01	3.59824e-01
	6	12.01	0.0000	A	3.39967e-01	3.59824e-01
	6	12.01	0.0000	A	3.39967e-01	3.59824e-01
	1	1.008	0.0000	A	0.00000e+00	0.00000e+00
	8	16.00	0.0000	A	3.15061e-01	6.36386e-01
	8	16.00	0.0000	A	3.06647e-01	8.80314e-01
	8	16.00	0.0000	A	3.00001e-01	7.11280e-01
	8	16.00	0.0000	A	2.95992e-01	8.78640e-01

Mixing rules (forcefield.itp file)

#define _FF_AMBER #define _FF_AMBER99SBILDN							
[defaults] ; nbfunc 1	comb-rule 2	gen-pairs yes	fudgeLJ fudgeQQ 0.5 0.8333				

1

funct

angle

104.52 628.02

force_constant

104.52 628.02

k

З

angles] i

j

1

Force fields – Remarks

- Based on a) experimental data (spectroscopy, density, phase transitions), b) ab initio methods, and c) trial and error.
- fast
- often system-specific (e.g., good for proteins, bad for lipids)
- there are interaction models (functional form is assumed)
- polarizable force fields sometimes used (more and more often but have their own problems)
- no chemical reactivity, problems with radicals, excited states, weakly-interacting complexes
- implicit water models, many-body effects, anharmonicity
- problematic under non-typical biological conditions (e.g., low temperature)
- steep learning curve, experience needed to use them properly

Main force fields use in biosystems

- AMBER
- CHARMM
- OPLS
- GROMOS
- Glycan
- MARTINI (coarse grained)

Solution of equations of motion

$$m_i \frac{d^2 \mathbf{r_i}}{dt^2} = \mathbf{F_i} = -\nabla_{\mathbf{r_i}} U(\mathbf{r_1}, \mathbf{r_2}, \dots, \mathbf{r_N})$$
Numerical integrators equations of motion

$$m_i \frac{d^2 \mathbf{r_i}}{dt^2} = \mathbf{F_i} = -\nabla_{\mathbf{r_i}} U(\mathbf{r_1}, \mathbf{r_2}, \dots, \mathbf{r_N})$$

- in practical systems, too many bodies for analytical solution of equations of motion
- numerical integration of Newton's equations required
 - numerical integrators are used to approximate analytical solution
- numerical integrators suffer/may suffer of various problems:
 - low accuracy
 - time irreversibility
 - numerical instability
 - low computational efficiency

- typical simulation times of biosystems: 100 ns 1 μ s
- note: 1 fs = 10⁻¹⁵ s
- if $\Delta t = 2$ fs, then 1 μ s requires 500,000,000 MD steps!

Taylor's expansion – discretization of Equations of Motion (EOMs)

Euler algorithm

(forward difference approximation)

$$\mathbf{r}_{i}(t + \Delta t) = \mathbf{r}_{i}(t) + \Delta t \ \mathbf{v}_{i}(t) + \frac{\Delta t^{2}}{2m_{i}}\mathbf{f}_{i}(t) + \mathcal{O}(\Delta t^{3})$$
$$\mathbf{v}_{i}(t + \Delta t) = \mathbf{v}_{i}(t) + \frac{\Delta t}{m_{i}} \ \mathbf{f}_{i}(t) + \mathcal{O}(\Delta t^{2})$$

NOT USED in MD:

- time irreversible (but Newton's equations are reversible)
- no phase-space preserving (Liouville's theorem violated)

Time irreversibility of integrators



t \uparrow in forward propagation forces calculated at t as a trick tosymmetrize integrator, forces can be calculated in the middle

Verlet algorithm

Derivation:

$$\mathbf{r}_{i}(t + \Delta t) = \mathbf{r}_{i}(t) + \Delta t \ \mathbf{v}_{i}(t) + \frac{\Delta t^{2}}{2m_{i}} \ \mathbf{f}_{i}(t) + \frac{\Delta t^{3}}{3!} \ \ddot{\mathbf{r}}_{i}(t) + \mathcal{O}(\Delta t^{4})$$
$$\mathbf{r}_{i}(t - \Delta t) = \mathbf{r}_{i}(t) - \Delta t \ \mathbf{v}_{i}(t) + \frac{\Delta t^{2}}{2m_{i}} \ \mathbf{f}_{i}(t) - \frac{\Delta t^{3}}{3!} \ \ddot{\mathbf{r}}_{i}(t) + \mathcal{O}(\Delta t^{4})$$

Positions from adding and velocities from subtracting the above eqs:

$$\begin{split} \mathbf{r}_{i}(t + \Delta t) &= 2\mathbf{r}_{i}(t) - \mathbf{r}_{i}(t - \Delta t) + \frac{\Delta t^{2}}{m_{i}}\mathbf{f}_{i}(t) + \mathcal{O}(\Delta t^{4}) \\ \mathbf{v}_{i}(t) &= \frac{\mathbf{r}_{i}(t + \Delta t) - \mathbf{r}_{i}(t - \Delta t)}{2\Delta t} + \mathcal{O}(\Delta t^{3}). \\ \mathbf{t} - \Delta t & \mathbf{t} & \mathbf{t} + \Delta t \\ \uparrow & \uparrow & \uparrow \\ \text{coordinates} \\ & \& \text{ forces} \end{split}$$

Verlet alg. is time reversible (symmetric when $t \rightarrow -t$)

Integrators - stability

Stability can be checked by conserved quantities: for instance, total energy (in NVE ensembles) can be used



- integrators are conditionally stable
- conserved quantities diverge if Δt too large!



In practice, the time-step must be at least one order of magnitude shorter than a characteristic time (e.g., period) of the fastest motion in the system



the fastest motion:

- typically - vibrations of O-H and C-H bonds with time scale of ${\sim}10^{\text{-14}}~\text{s}$

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• hence, typical time-step in MD $\Delta t = 1-5$ fs

Time step - tricks

- most often, O-H and C-H bonds can be constrained (SHAKE algorithm for water, LINCS for C-H), then Δt = 2 fs
- united-atom force fields typically do not include C-H bonds, if water constrained then $\Delta t = 2$ fs
- coarse-grain force fields do not include atomistic motions $\Delta t = 20 \text{ fs!}$



Simulation box



How big are biosystems of interest?



Simulation box - limitations

Simulation box – part of the physical space that is included in a simulation



- size of simulated systems limited by computational costs
- typically 10⁵-10⁶ atoms; 100-1000 nm³
- O.K. for small isolated molecules or small molecular clusters
- not proper for big molecules (e.g., big proteins) and condensed phases (e.g., small proteins in water, lipid membranes)

Solutions for condensed phases: periodic boundary conditions!



Periodic Boundary Conditions

- the simulated system is enclosed in socalled <u>simulation box</u>
- the box replicated in 3D (or 2D if needed)
- the finite system becomes infinite (but periodic!)



Benefits:

- artificial surfaces eliminated
- bulk-like behavior
- computational cost increases only slightly
- very good for periodic solid phases (crystals)
- reasonable for liquid phases

<u>Disadvantages:</u>

- artificial periodicity introduced (some interactions can display artifacts, some processes may be artificially induced)
- fails for effects with characteristic length > box size (phonons, membrane undulations, phase transitions-related phenomena)
- box has to be large enough (check!)

PBC special geometries

water-air interface (slab geometry, good for interfaces, 2x)

> 3L

Lipid bilayer ("slab-like")



Solid crystal structures with nonidealities



DaniFeri at Wikimedia Commons CC

The choice of periodic boundary condition depends on a system and the phenomena studied

Periodic Boundary Conditions – minimum image convention



L

Minimum image convention:

Only the properties of the original box should be propagated

-> only interactions within $L/_2$ radius can be considered

Benefits:

- reduced influence of periodicity
- Iower computational costs

PBC – Example problems I

Solid crystal structures with nonidealities



DaniFeri at Wikimedia Commons CC

- p.b.c. introduce artificial periodicity, so not very well suitable to model defects and nonidealities in crystal structures if a single unit cell is used
- solution: bigger box (include several unit cells)

Periodic Boundary Conditions and Cut-offs minimum image range



• To achieve minimum image convention, in practice, cut-offs are used:

if
$$r_{ij} < cutoff$$
 then $E_{ij} = E(r_{ij})$, else $E_{ij} = 0$

• with periodic boundary conditions, cut-off $<\frac{L}{2}!$



Lennard-Jones negligible (<1%) at typically 0.9-1.2nm \rightarrow cut-off can be used

decay very slowly (ch-ch 1% at ~100nm) → cut-off cannot be used (solution: Ewald –type methods)

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Particle Mesh Ewald idea

Approximate calculation of electrostatic forces in periodic systems

Assumptions:

- periodic boundary conditions
- neutral system (equal number of positive and negative charges)



infinite number of periodic images $V(r_i) = \sum_{j=1}^{\infty} \frac{q_j}{r_{ij}}$

infinite sum, conditionally convergent, slow to calculate

Particle Mesh Ewald method

Trick – divide the sum into two rapidly converging components:

- short-range (within some relatively short cutoff) solved in real space
- long-range (originating mostly from periodic images) this is mostly periodic so can be fast solved in reciprocal space)

$$V(r_i) = \sum_{j}^{\infty} \frac{q_j}{r_{ij}} = V_{short \ range} + V_{long \ range} = \sum_{k}^{J_{cutoff}} \frac{q_j}{r_{ij}} + V_{long \ range}$$

$$Prive = \sum_{k}^{J_{cutoff}} \frac{q_j}{r_{ij}} + V_{long \ range}$$

$$Prive = \sum_{k}^{N_{cutoff}} \frac{q_j}{r_{ij}} + V_{long \ range}$$

$$Prive = \sum_{k}^{N_{cutoff}} \Phi(k) |\tilde{\varrho}(k)|^2$$

$$Prive = \sum_{k}^{N_{cutoff}} \Phi(k) |\tilde{\varrho}(k)|^2$$
Fourier transform of potential and charge density

How to Solve Poisson's Equation Using Fourier Transforms

Steps



1 Begin with Poisson's equation. Recall that the electric field **E** can be written in terms of a scalar potential $\mathbf{E} = -\nabla \phi$. We can then use Gauss' law to obtain Poisson's equation as seen in electrostatics.

- $\bullet \,
 abla^2 \phi = rac{
 ho}{\epsilon_0}$
- In this equation, it is often the case that we know the charge density ρ , called the source function, and wish to know the potential ϕ . Therefore, we need to find some way to invert this equation.

2 Write out the FTs and inverse FTs of the potential and charge density. Since we are dealing with three dimensions, the FTs are adjusted accordingly, with the constant factor there for normalization purposes. The bounds will differ depending on conventions on where to set the potential to 0. Although we will not explicitly write the boundaries until evaluating the integrals, we will set the potential to 0 at infinity, so that we are integrating over all space.

$$egin{aligned} & ilde{\phi}(\mathbf{k}) = rac{1}{(2\pi)^{3/2}} \int \phi(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} \mathrm{d}^3 \mathbf{x} \ & \phi(\mathbf{x}) = rac{1}{(2\pi)^{3/2}} \int ilde{\phi}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} \mathrm{d}^3 \mathbf{k} \ & ilde{
ho}(\mathbf{k}) = rac{1}{(2\pi)^{3/2}} \int
ho(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} \mathrm{d}^3 \mathbf{x} \ &
ho(\mathbf{x}) = rac{1}{(2\pi)^{3/2}} \int ilde{
ho}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} \mathrm{d}^3 \mathbf{k} \end{aligned}$$

- Relate $\tilde{\phi}(\mathbf{k})$ with $\tilde{\rho}(\mathbf{k})$. The result will relate the potential and charge density in the **k** space, and as it will turn out, the relation is algebraic, which is considerably simpler.
- Take the Laplacian of $\phi(\mathbf{x})$. We can differentiate under the integral here because the integral is being taken with respect to \mathbf{k} , and \mathbf{x} is an independent variable.

$$\mathbf{v} \,
abla^2 \phi(\mathbf{x}) = rac{1}{(2\pi)^{3/2}} \int -k^2 e^{i \mathbf{k} \cdot \mathbf{x}} ilde{\phi}(\mathbf{k}) \mathrm{d}^3 \mathbf{k} = rac{
ho(\mathbf{x})}{\epsilon_0}$$

• FT charge density so that it is also written in the ${f k}$ space.

$${f \circ} \; rac{
ho({f x})}{\epsilon_0} = rac{1}{(2\pi)^{3/2}} \int rac{ ilde
ho({f k})}{\epsilon_0} e^{i{f k}\cdot{f x}} {
m d}^3{f k}$$

• By direct comparison, we see that the below relation holds.

$${ullet} \; k^2 ilde{\phi}({f k}) = rac{ ilde{
ho}({f k})}{\epsilon_0}$$

• If we were given charge density in the **k** space and wanted to find potential in the same space, it would be very easy. However, we are interested in finding these quantities in the **x** space. Therefore, we will need to transform a second time.

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https://www.wikihow.com/Solve-Poisson%27s-Equation-Using-Fourier-Transforms#

MD simulations

















Macroscopic properties

Newton equations of motion govern microscopic evolution and do not explicitly give macroscopic properties (e.g. temperature, pressure, surface tension)

Macroscopic properties can be obtained from trajectories via statistical mechanics (statistical thermodynamics) r_i, v_i



 $-\frac{dV}{d\overrightarrow{r_i}} = m_i \frac{d^2 \overrightarrow{r_i}}{dt^2}$



Statistical ensembles equilibrium $p_i \uparrow f$

- **Ergodic hypothesis**: all accessible microstates are equiprobable over a long period of time (different formulations, many consequences!)
 - Average over ensemble = average over time
- Macroscopic properties are given by probability distribution within an ensemble



- In equilibrium MD, we "move" within the ensemble, reproduce its probability distribution and hence we can estimate macroscopic properties, one long MD trajectory is in principle sufficient
- Macroscopic properties can be estimated along a MD trajectory
- Equilibrium dynamic properties can also be studied (e.g. self-diffusion)

Statistical ensembles

- thermodynamic statistical ensembles describe macroscopic conditions
- *NVE* microcanonical
- NVT canonical (other names: isothermal, Helmholtz canonical)
- μVT grand-canonical
- NPE isobaric
- NPT isobaric-isothermal (other name: Gibbs canonical)



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Maxwell relations: calculating thermodynamic functions



RoB at Wikimedia Commons CC

Statistical ensembles

Temperature:

$$T = \frac{2}{3k_B} \left\langle \frac{1}{2} m v^2 \right\rangle$$

Pressure:

$$p = \frac{2N}{3V} \left\langle \frac{1}{2} m v^2 \right\rangle$$



- Using statistical thermodynamics, both temperature and pressure can be related to average kinetic energy so they can be obtained from Newton equations of motion
- But Newton's equations of motion correspond to NVE (actually, even more restrictively: *E=const*)
- Therefore, fluctuations of temperature and pressure must be added by using special <u>algorithms</u>: <u>thermostat</u> and <u>barostat</u> (temperature- and pressurecoupling algorithms

Nosé-Hoover thermostat

• A friction term introduced in the eq. of motion:

$$\frac{d^2 r_i}{dt^2} = \frac{F_i}{m_i} - \lambda v_i \qquad \lambda - \text{friction coefficient}$$
$$\frac{d^2 r_i}{dt^2} = \frac{F_i}{m_i} - \frac{p_{\xi}}{Q} \frac{dr_i}{dt}$$

 ξ – friction parameter ("heat bath" variable) p_{ξ} - friction momentum Q – mass parameter

$$\frac{\mathrm{d}p_{\xi}}{\mathrm{d}t} = (T - T_0)$$
$$Q = \frac{\tau_T^2 T_0}{4\pi^2}$$

au $_{T}$ - coupling constant (by analogy to Berendsen algorithm)

- additional equation of motion solved for the new variable ξ



List of thermostats

- Nosé-Hoover
- Langevin
- Stochastic velocity rescaling (V-rescale)
 - (More stable but only in Gromacs)
- Berendsen (Do not use for production run !!)
Parrinello-Rahman barostat

- similar concept to Nosé-Hoover thermostat
- a friction term introduced in the eq. of motion:

$$\begin{aligned} \frac{\mathrm{d}^2 \boldsymbol{r}_i}{\mathrm{d}t^2} &= \frac{\boldsymbol{F}_i}{m_i} - \boldsymbol{M} \frac{\mathrm{d}\boldsymbol{r}_i}{\mathrm{d}t} \\ \boldsymbol{M} &= \boldsymbol{b}^{-1} \left[\boldsymbol{b} \frac{\mathrm{d}\boldsymbol{b}'}{\mathrm{d}t} + \frac{\mathrm{d}\boldsymbol{b}}{\mathrm{d}t} \boldsymbol{b}' \right] \boldsymbol{b}'^{-1} \end{aligned}$$

- au_P coupling constant
- β compressibility
- P_0 desired pressure
- b box vectors matrix

$$r_i \coloneqq \mu \cdot r_i$$
$$V \coloneqq \mu^3 \cdot V$$



$$\frac{\mathrm{d}\boldsymbol{b}^2}{\mathrm{d}t^2} = V \boldsymbol{W}^{-1} \boldsymbol{b}'^{-1} \left(\boldsymbol{P} - \boldsymbol{P}_{ref} \right)$$

$$\left(\boldsymbol{W}^{-1} \right)_{ij} = \frac{4\pi^2 \beta_{ij}}{3\tau_p^2 L}$$

List of barostats

- Stochastic cell rescaling (c-rescale)
 - (More stable but only in Gromacs)
- Parrinello-Rahman
- Berendsen

Temperature and pressure in MD - practicalities

 $p_x = p_y \neq p_z$

very often NPT ensemble is used (thermostat + barostat) (with p=1 atm, T=293 K or T=310 K)





 $p_x = p_y \neq p_z = 0$





- few thermostats can be introduced, e.g., 1) protein 2) water
 - surface tension ensemble can be used (N γ T)

Simulation protocol



• A) equilibrium MD

- <u>one</u> very long trajectory required if the system is ergodic
- MD trajectory corresponds to 'jumps' between microstates in a given equilibrium ensemble
- system properties calculated as averages in time (= average in ensemble)
- time not important *per se* but time-dependent phenomena can be studied (e.g., self-diffusion)

• B) non-equilibrium MD:

- <u>many</u> trajectories required
- each trajectory corresponds to one possible evolution path in the phase-space
- time is important
- system properties calculated as averages over a set of trajectories (mean values are functions of time)

The average concept

Under equilibrium

Out-of-equilibrium





$$\langle A \rangle_{time} = \frac{1}{N} \sum_{i=1}^{N} A(t_i)$$

averaging over N points taken along the trajectory each δt (usually $\delta t > \Delta t$) the result is time-independent averaging over j trajectories each δt (usually $\delta t > \Delta t$) the result is a function of time

Typical MD workflow



Stage: Defining the system



Designing /Understanding your system

- Lipids:
 - Composition
 - Mixing time
 - Asymmetric systems
- Other molecules
 - Ionic concentration
 - Experimental buffers
 - Availability of force field

- Proteins
 - Complete 3d structure?
 - Protonation state
 - Ligands (ff?)
- Not too small or too big
 - PBC effect
 - Computational cost
- Initial molecular arrangement

Force field in biosystems

$$-\frac{dV_{total}}{d\overrightarrow{r_i}} = m_i \frac{d^2 \overrightarrow{r_i}}{dt^2}$$

$$U = \sum_{bonds} \frac{k_i^b}{2} (\mathbf{r_i} - \mathbf{r_i}^{eq})^2 + \sum_{angles} \frac{k_i^a}{2} (\theta_i - \theta_i^{eq})^2 + \\ + \sum_{torsions} \sum_n k_{\phi,n} (1 + \cos(n\phi - \phi^{ref})) + \\ + \sum_{pairs} \left(\left\{ \frac{C_{ij}^{(12)}}{r_{ij}^{12}} - \frac{C_{ij}^{(6)}}{r_{ij}^6} \right\}_{vdw} + \left\{ k_e \frac{q_i q_j}{r_{ij}} \right\}_{coul} \right)$$

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Main force fields use in biosystems

- AMBER
- CHARMM
- OPLS
- GROMOS
- Glycan
- MARTINI (coarse grained)
- ProsECCo (ECC)

Choosing the force field

- Preferably choose a force field which is currently widely used
 - Berger was the best for lipids, not anymore.
- At least you will be able to have controls
- Choose a force field that describe your target properties adequately

- The force field is widely used in the community, therefore it must be good. WRONG!!!
 - Check always about know issues with the force fields.
 - Check papers
 benchmarking force
 fields

- Martini

THIS STEP WILL DEPEND IN THE USED ENGINE. (It should be the way around) Don't try to fix a bulb with a hammer

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Building the System

- Manual
 - Full control
 - Too much job
 - Error prone
 - Only for learning and debugging

Martinez-Seara, H. & Rog, T. **Molecular dynamics** simulations of lipid bilayers: Simple recipe of how to do it. Methods Mol. Biol., 2013, 924, 407-429

• Tools like CHARMM-GUI

- Important changes without warning.
- No version control (±)
 - Tomorrow might be different and you have no clue
- Save the download date
- Be aware of the atom order

THIS STEP WILL DEPEND IN THE USED ENGINE. There are no standards.

Universal builder

CHARMM-GUI (http://www.charmm-gui.org/)

CHARMM-GUI

Effective Simulation Input Generator and More

CHARMM is a versatile program for atomic-level simulation of many-particle systems, particularly macromolecules of biological interest. - M. Karplus

about us :: input generator :: Q&A :: archive :: charmm docs :: lectures :: movie gallery :: video demo :: citations :: update log :: jobs & events :: giving

Some lectures and job postings are now available. See upload log for update history and giving for donation. Contact info is given below.

Input Generator	Input Generator
Job Retriever PDB Reader	One easiest way to support CHARMM-GUI is to cite the CHARMM-GUI main paper as well as the papers of the modules used in users' publications. Please see Citations for details.
Glycan Reader & Modeler	Since most modules start with PDB Reader, it is strongly recommended to read the PDB Reader page and to see the PDB Reader demo in Video Demo.
Ligand Reader & Modeler Glycolipid Modeler	Job Retriever Facilitates recovery of jobs, when the Job ID is known
LPS Modeler Nanomaterial Modeler	PDB Reader Read a PDB file (RCSB or CHARMM formats) into CHARMM
Multicomponent Assembler Solution Builder	Glycan Reader & Modeler Read carbohydrate structures from a PDB file into CHARMM and/or model user-specified N-/O-glycan or glycan-only structure(s)
Membrane Builder Martini Maker	Ligand Reader & Modeler Generate various ligand structures using the CHARMM force field
PACE CG Builder Drude Prepper	Glycolipid Modeler Provide various glycolipid structure and PSF files
Free Energy Calculator MAP Utilizer	LPS Modeler Provide various lipopolysaccharide (LPS) structure and PSF files
DEER Facilitator NMR Structure Calculator	Nanomaterial Modeler Generate various nanomaterial systems for molecular dynamic simulation
PBEQ Solver Implicit Solvent Modeler	Multicomponent Assembler Combine PSF/CRD of non-membrane molecules into a heterogeneous system
Boundary Potential Utilizer GCMC/BD Ion Simulator	Solvator Solvate globular protein, or generate various shapes of water box
	Solution Builder (new Quick MD Simulator) Setup subsequent steps for molecular dynamics simulations of globular proteins
	Drude Prepper Prepare the systems ready for simulations with the Drude polarizable force fields from an identical system equilibrated with the CHARMM36 non-polarizable additive force fields
	Membrane/Bilayer Builder Generate a protein/bilayer complex or bilayer-only system for molecular dynamics simulations
	Membrane/Monolayer Builder Generate a protein/monolayer complex or monolayer-only system for molecular dynamics simulations
	Membrane/Nanodisc Builder Generate a lipid-only or protein-embeded nanodisc system for molecular dynamics simulations

Protein Data Bank [pdb] (https://www.rcsb.org/)

🛈 🖸 🔒 https://www.rcsb.org		♡ ☆	
	RCSB PDB Deposit - Search - Visualize - Analyze - Down	load - Learn - More -	MyPDB
	152500 Rieleziael	A REAL AND A	Mar and a start
	SPDB Labing Breakthroughs in	Search by PDB ID, author, macromolecule, sequence, or ligands	Go
	PROTEIN DATA BANK Research and Education	Advanced Search Browse by Annotations	5 A.
		ſ	ADO

	A Structural View of Biology	June Molecule of the Month
Welcome	This resource is powered by the Protein Data Bank archive-information about the 3D shapes of proteins, nucleic acids, and complex assemblies that helps students and researchers understand all aspects of biomedicine and	M
주 Deposit	agriculture, from protein synthesis to health and disease.	22
0 Search	As a member of the wwPDB, the RCSB PDB curates and annotates PDB data.	- 63 10
Visualize	The RCSB PDB builds upon the data by creating tools and resources for research and education in molecular biology, structural biology, computational biology, and beyond.	
📰 Analyze	High School Antibiotic Resistance Video Challenge	
💠 Download	2019	A STATISTICS
	WINNERS (A)	S Brand
	Vors la constant of the second	MDM2 and Cancer



- The best place to find if your target protein has a resolved 3d structure.

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Transmembrane protein structural databases

PDBTM: Protein Data Bank of Transmembrane Proteins (http://pdbtm.enzim.hu/)



PDBTM: Protein Data Bank of Transmembrane Proteins

PDBTM	version: 2019-01-04	Number of transmembrane proteins: 4007 (alpha: 3559 , beta: 426)	all 🗸 🛒	【 lzoy	$\Sigma \gg$
***	Home	Protein Structure	Sequence		
P	Search				
1	Download	\$ 6 P	GLRAAFGLSEAGFNTACVTKLFPTRSHTVA AQGGINAALGNMEEDNWRWHFYDTVKGSDW		
8	Statistics		LGDQDATHYMTEQAPASVVELENYGMPFSK TEDGKIYQRAFGGQSLKFGKGGQAHRCCCV ADRTGHSLLHTLYGRSLRYDTSYFVEYFAL		
<u></u>	Documents		DLLMENGECRGVIALCIEDGSIHRIRARNT VVATGGYGRTYFSCTSAHTSTGDGTAMVTR AGLPCQDLEFVQFHPTGIYGAGCLITEGCR		
1 and 1	Help	MARKE STA	GEGGILINSQGERFMERYAPVAKDLASRDV VSRSMTLEIREGRGCGPEKDHVYLQLHHLP PEOLAVRLPGISETAMIFAGVDVTKEPIPV		
			LPTVHYNMGGIPTNYKGQVLRHVNGQDQVV PGLYACGEAACASVHGANRLGANSLLDLVV ECRACALSTAESCORODKVRSTKPNACEES		
			VMNLDKLRFANGTIRTSELRLSMQKSMQSH AAVFRVGSVLQEGCEKILRLYGDLQHLKTF		
			Downloads		
			Cross references		
			PAOTEIN DATA BANK		

Transmembrane protein structural databases

OPM database (https://opm.phar.umich.edu)

orientations of proteins in M	(OPM) database	UNIVERSITY OF MICHIGAN COLLEGE OF PHARM Search proteins by PDB ID or nar
HOME ABOUT OPM	DOWNLOAD OPM FILES CONTACT US PPM SERVER	
Protein Classification	1uaz » Archaerhodopsin-1	
Types (3) Classes (11) Superfamilies (499) Families (958) Species (836)	 Type: Transmembrane (3 classes) Class: Alpha-helical polytopic (123 superfamilies) Superfamily: Rhodopsin-like receptors and pumps (7 families) CL0192 @ Family: Microbial and algal rhodopsins (46 proteins) 3.E.1 (TCDB) @ PF01036 @ PDBsum @ Species: Halobacterium sp. (3 proteins) Localization: Archaebacterial membrane (104 proteins) 	
Localizations (24)	1uaz >> Archaerhodopsin-1	
Proteins (4444)	Hydrophobic Thickness or Depth 31.8 \pm 1.3 Å	
	Tilt Angle 9 ± 2°	
Assembly	ΔG _{transfer} -65.3 kcal/mol	
Superfamilies (9)	Links to 1uaz PDB Sum 虙, PDB 얍, SCOP 얍, MSD 虙, MMDB 얍, Encompass 宓	
Families (19)	Topology subunit A (N terminus extracellular side)	000000000000000000000000000000000000000
Localizations (8)	Resolution 3.40	
Assemblies (207)	Primary PDB represention 1uaz	
	Other PDB entries representing this structure none	
Protein Links	Number of TM Secondary Structures 7	
PDB Sum PDB R MPKS R	Membranome none	Download File: 1uaz.pdb
PDB Sum 딸, PDB 딸, MPKS 딸, MPDB 룹	Comments: Light-driven proton pump. It may interact with bacterioruberin in the claret membrane.	PDB Sum &, PDB &
PPM Server		Topology in Archaebacterial membrane extracellular side cytoplasmic side
		3D view in GLMol or Jmol 🗗
	Subunits: 1	
	A - Tilt: 8 - TM segments: 1(15-38),2(48-68),3(86-104),4(111-133),5(138-160),6(179-197),7(207-230)	view assembly page

Stage: Defining MD parameters



Choose the statistical ensemble for membrane simulation

- thermodynamic statistical ensembles describe macroscopic conditions
- NVE microcanonical
- NVT canonical (other names: isothermal, Helmholtz canonical)
- μVT grand-canonical
- NPE isobaric
- NPT isobaric-isothermal (other name: Gibbs canonical)



NPT baths

- Thermostats
 - Nosé-Hoover
 - Langevin
 - V-rescale
 - Berendsen (Do not use for production run !!)
- Barostats:
 - Parrinello-Rahman
 - c-rescale
 - ??Berendsen??

Temperature and pressure:Summarymembrane
 $p_x = p_y \neq p_z$ Mono
 $p_x = p_y$









- Use NPT ensemble with proper thermostat and barostat combination
 - e.g., p=1 atm, T=310 K
 - Use semi-isotropic barostat
- Few thermostats can be introduced
 - e.g., 1) membrane 2) protein 3) water
 - Separate groups have to be big enough

Choosing MD parameters

- Cutt-off schemes
- Time step
- PME

You CANNOT choose them. They are part of the force field

Stage: Simulation



Simulation software

- Molecular dynamics (MD) simulation
 - GROMACS (www.gromacs.org)
 - Amber (www.ambermd.org)
 - CHARMM (www.charmm.org)
 - NAMD (www.ks.uiuc.edu/Research/namd)
 - OpenMM (http://openmm.org/)
 - LAMMPS (https://lammps.sandia.gov/)
- MD software are like religions, better belong to many.
 - Then you can pick the best of each
 - Communities hardly mix

Open science

Open source is not an option; it is a MUST.

What you cannot see, you cannot comprehend

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Software considerations

- Use always the last stable version:
 - At least check if
 there are bugs
 affecting you?
- If your results change between versions, it is likely that you have hit a bug. At least is worth researching

- Using black boxes usually leads to one place. Wrong data
 - Close source codes :(
- Expensive codes?
 - Why to learn
 something you
 won't have in the
 future

Perform the simulations

- Equilibration time
- Long enough
- Replicas (Ergodicity)
- What can I do when resources are limited? My answer:
 - Search for the required resources
 - Don't do it
- For people of Finland you should be thankful of having CSC.
- For the others: http://www.hpc-europa.eu/

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Have you simulated long enough? Likely not

- You need to sample enough.
 If not do not bother to make the simulation
- Make replicas to be sure that your simulation is indeed converged



- Protein membrane dynamics is slow
 - One simulation of 1 microsecond not enough

MD simulations



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Stage: Validation and Analysis



Validate your results

- Always compare your results with experiments.
 - It is tedious but we do science not science fiction.
- Ask someone else to check your simulation files.
 - We are humans, aka, we make errors.

- There is not a perfect force field
 - That you get something in one force field does not make it true
- Use a couple of force fields.
 - They are usually parametrized independently so if they share the same results, it might be that is some sense in the underlying physics

Analysis

Any data you will obtain by analyzing a simulated system, no matter how sophisticated analysis method is, will only be as good as the simulation allows.

The " in" " out" principle

Visual analysis of MD simulations

- employs visualization software (e.g., VMD)
- basic properties of the system can be qualitatively analyzed
- useful for simulation monitoring but also as a basis for further detailed numerical analysis
- fast, powerful, often underrated



Example: orientation of peptide in lipid membrane

Numerical analysis of MD simulations

MD only provides instantaneous positions, velocities, forces on each atom at each time step. Also box size.

Typically analyzed quantities:

- macroscopic properties: temperature, pressure, volume, energy, polarization, surface tension
- microscopic properties: atomic/molecular ordering, distribution, conformations, hydrogen bonding



These quantities are either static (e.g., atomic distribution) or dynamics (e.g., diffusion coefficient)

Numerical analysis of MD simulations

Example: contacts of membrane groups with transmembrane helix, and deuterium order parameter of lipid tails



Radial distribution function (RDF, g(r))



$$g_{AB}(r) = \frac{\langle \rho_B(r) \rangle}{\langle \rho_B \rangle_{local}} \\ = \frac{1}{\langle \rho_B \rangle_{local}} \frac{1}{N_A} \sum_{i \in A}^{N_A} \sum_{i \in B}^{N_B} \frac{\delta(r_{ij} - r)}{4\pi r^2}$$

- a measure of the probability of finding a particle at a distance of r away from a given reference particle, relative to that for an ideal gas
- RDF is related (via Fourier transform) with so-called structure factor (S(q)) which can be determined experimentally via X-ray diffraction or neutron diffraction

$$S(\mathbf{q}) = \mathbf{1} +
ho \int_V \mathrm{d} \mathbf{r} \, \mathrm{e}^{-i \mathbf{q} \mathbf{r}} g(\mathbf{r})$$

RDF example 1: ideal lattice (crystal)



Christopher Rowlley at Wikimedia Commons CC BY-SA
RDF example 2: water





- hydration structure analysis
- comparison with experiment



RDF example 3: Lipid Monolayer







- 2D RDF for studying lateral arrangement of molecules
- phase transition in monolayer can be analyzed

Density profiles

density of selected atoms, groups of atoms, molecules as a function of position



- typical units: kg/m³, number density (nm⁻³), electron density, arbitrary unit
- density profile can provide structural information but also for orientation of molecules (e.g., at interfaces)

Changes on the molecular properties due to the environment

Probability distribution of bond, angle, dihedral value (It does not have to be the same as in the potential provided)



- for analysis of average molecular conformations
- distances can be used for binding analysis (also time-dependent)
- other, indirect, phenomena can be captured (e.g., phase transitions)

Orientation distributions



We can follow the orientation distributions or with time (for example peptide orientation respect to bilayer normal)

Root mean square deviations (RMSD) in structure

$$RMSD(t_1, t_2) = \left[\frac{1}{M} \sum_{i=1}^{N} m_i \|\mathbf{r}_i(t_1) - \mathbf{r}_i(t_2)\|^2\right]^{\frac{1}{2}}$$



- usually used for proteins, monitoring time evolution of protein structure
- fitting may be done to t=0 or t=t_i-Δt
- fitting may be done to different structure than RMSD calculation (e.g., fitting to C-alpha, RMSD of whole structure)
- very good for <u>equilibration monitoring</u>!

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Secondary structure of proteins

Residue

Residue

time evolution of protein secondary structure

- only for proteins
- secondary structure with individual residues influence in time
- external software (DSSP) needed (free)
- very good for <u>equilibration</u> monitoring!



la in POPC

00000

Secondary structure

400000

50000

in Gromacs: gmx do dssp

Molecular diffusion

Mean square displacement, diffusion coefficient



- diffusion coefficient calculated based on Fick's law (linearity of MSD assumed!)
- Problems: MSD non-linearity, center of mass movement during in simulation

Time correlation functions

time and memory characteristics of a process



 $C_f(t) = \langle f(\xi)f(\xi+t) \rangle_{\xi}$

averaged over time origins ξ

different variants: contact acf, velocity acf (for IR spectra), dipole acf (permittivity), pressure acf (for viscosity)

Error analysis: block averaging

- Statistics over thermodynamically fluctuating data is difficult
- Issues: fluctuations due to limited size of the system, time correlations, trajectory sampling etc.
- Good estimation of the error of a mean: block averaging (see Allen & Tildesley for more details)



in Gromacs: gmx analyze -ee

The best review for membrane and membrane protein systems

Javanainen, M. & Martinez-Seara, H.

Efficient preparation and analysis of Membrane And Membrane Protein Systems

Biochimica et Biophysica Acta (BBA) -Biomembranes, Elsevier BV, 2016, 1858, 2468-2482