Knowledge Representation and Reasoning

Logic and Reasoning in Uncertain Conditions

Reasoning in Uncertainty

- Probabilities and Uncertainty
- The Bayes rule
- Measures of Belief and Disbelief
- Certainty Factors (CF)
- Hypotheses and Evidences
- Logical operators and CF
- Inference rules and CF

Probability Theory

- In real world, events happen with a certain **probability**, usually the result of many factors
- Some of these factors are independent with each other, i.e. happen independently
- Others usually happen (or not happen) together at the same time, correlated or not (by chance).
- Example: the event “rain” happens with a probability that depends on dependent factors “cloud” and “cold”

The Bayes Theorem

- In order to calculate "joined" probabilities we use the Bayes rule:

P(H|Ek) = P(Ek|H)*P(H) / SUM{ P(Ei|H)*P(H) }

H = hypothesis (e.g. H="rains")

Ek = evidence k (e.g. Ek="cloud")

H|Ek = propability of "rain" when there is "cloud"

Ek|H = probability of "cloud" when "rain" (past history)

Certainty Factors

- Bayes rule becomes too complex when there are many evidences Ei that are dependent
- Instead, we often use approximate models for describing levels of certainty
- **Certainty Factors** (Shortliffe, 1976): first used in the expert system MYCIN (implem. in LISP)
- They describe hypotheses and evidences in terms of “certainty” about their truth

MYCIN: “500 rules, roughly the same level of competence as human specialists in blood infections” (Britannica)

Measures of Belief and Disbelief

Measure of Belief (MB):

$$MB(H|E) = 1 \quad , \quad \text{if } P(H)=1$$
$$= \max\{P(H|E),P(H)\} / (1-P(H)) \quad , \quad \text{if } P(H)<1$$

Measure of Disbelief (MD):

$$MD(H|E) = 1 \quad , \quad \text{if } P(H)=0$$
$$= (P(H) - \min\{P(H|E),P(H)\}) / P(H) \quad , \text{if } P(H)<1$$

Certainty Factors

Certainty Factor (CF):

$$CF(H|E) = MB(H|E) - MD(H|E)$$

$$0 <= MB(H|E) <= 1 \quad , \quad 0 <= MD(H|E) <= 1$$

$$\text{thus:} \quad -1 <= CF(H|E) < +1$$

CF: Multiple evidences

What if new evidence Ek becomes available?

MB(H|E1,E2) = MB(H|E1) + MB(H|E2)*[1-MB(H|E1)]

MD(H|E1,E2) = MD(H|E1) + MD(H|E2)*[1-MD(H|E1)]

CF(H|E1,E2) = MB(H|E1,E2) - MD(H|E1,E2)

CF: Uncertain evidences

What if evidence E is not 100% certain to be true ?

MB(H|E) = normal MB value for H based on E

- If E comes from "measurements" M then:

  CF(E|M) = certainty of E based on M

- The updated value for MB(H|E) now becomes:

  $MB(H|E)_{new} = MB(H|E) * \max\{ 0, CF(E|M) \}$

CF: Composite hypotheses

What if combosite hypotheses are to be evaluated?

MB(H1 **AND** H2 |E) = min{ MB(H1|E), MB(H2|E) }
(similarly for MD)

MB(H1 **OR** H2 |E) = max{ MB(H1|E), MB(H2|E) }
(similarly for MD)

MB(**NOT** H|E) = 1 - MB(H|E)
(similarly for MD)

CF: IF-THEN inference rules

How do we calculate CF for logical IF-THEN rules?

rule(R): IF "E" THEN "H" with certainty CF(R)

- Case: CF(H)>0,CF(R)>0 then:

  $$CF(H|E) = CF(H) + CF(R)*(1-CF(H))$$

- Case: CF(H)<0,CF(R)<0 then:

  $$CF(H|E) = CF(H) + CF(R)*(1-CF(H))$$

- Case: CF(H)*CF(R)<0 (i.e. different signs) then:

  $$CF(H|E) = (CF(H)+CF(R)) / (1-\min\{|CF(H)|,|CF(R)|\})$$

P.C. – Readings

- S. J. Russell, P. Norvig, “Artificial Intelligence: A Modern Approach”, 2nd/Ed, Prentice Hall, 2002. [see: ch.19]