World Academy of Science, Engineering and Technology
International Journal of Economics and Management Engineering
Vol:9, No:3, 2015

The Impact of Transaction Costs on Rebalancing an Investment Portfolio in Portfolio Optimization

B. Marasović, S. Pivac, S. V. Vukasović

***Abstract*—**Constructing a portfolio of investments is one of the most significant financial decisions facing individuals and institutions. In accordance with the modern portfolio theory maximization of return at minimal risk should be the investment goal of any successful investor. In addition, the costs incurred when setting up a new portfolio or rebalancing an existing portfolio must be included in any realistic analysis.

In this paper rebalancing an investment portfolio in the presence of transaction costs on the Croatian capital market is analyzed. The model applied in the paper is an extension of the standard portfolio mean-variance optimization model in which transaction costs are incurred to rebalance an investment portfolio. This model allows different costs for different securities, and different costs for buying and selling. In order to find efficient portfolio, using this model, first, the solution of quadratic programming problem of similar size to the Markowitz model, and then the solution of a linear programming problem have to be found. Furthermore, in the paper the impact of transaction costs on the efficient frontier is investigated. Moreover, it is shown that global minimum variance portfolio on the efficient frontier always has the same level of the risk regardless of the amount of transaction costs. Although efficient frontier position depends of both transaction costs amount and initial portfolio it can be concluded that extreme right portfolio on the efficient frontier always contains only one stock with the highest expected return and the highest risk.

***Keywords*—**Croatian capital market, Fractional quadratic programming, Markowitz model, Portfolio optimization, Transaction costs.

I. Introduction

IN 1952 H. M. Markowitz [12] developed the first model for portfolio optimization and with that model he laid the foundation of the modern portfolio theory. His model is based upon only two criteria: return and risk. The risk is measured by the variance of returns' distribution. Markowitz shows how to calculate portfolio which has the highest expected return for a given level of risk, or the lowest risk for a given level of expected return (the so-called efficient portfolio). The problem of portfolio selection, according to this theory, is a problem of quadratic programming which consists of minimizing risk while keeping in mind an expected return which should be guaranteed. The importance of Markowitz's work is affirmed by the Nobel Prize for Economics he won in 1990. However, parallel to introducing the Markowitz model in the common usage its limitations and drawbacks were being noticed. One of disadvantages of Markowitz model is the fact that it doesn't take into consideration transaction cost although costs incurred when setting up a new portfolio or rebalancing an existing portfolio must be included in any realistic analysis.

In this paper, we apply a method for finding an optimal portfolio with proportional transaction costs on the Croatian capital market and analyze the same. These costs vary linearly with the amount of a security bought or sold. This method allows different costs for different securities, and different costs for buying and selling. This model captures the feature that transaction costs are paid when a security is bought or sold and the transaction cost reduces the amount of that particular security that is available. In particular, both the risk and the return in our model are measured using the portfolio arising after paying the transaction costs.

The portfolio rebalancing problem has similarities to the index tracking problem [1], [5]. See [20] for a discussion of portfolio optimization models. Portfolio optimization models with alternative risk measure have been investigated in [8], [10], [16]. Contrary to the expectations of the modern portfolio theory, the tests carried out on a number of financial markets (AMEX, NYSE, TSE, Paris' Stock Exchange, etc.) have revealed the existence of other indicators, besides return and risk, important in portfolio selection. The most important anomalies discovered to date are the size measured by stock market capitalization and the Price Earning Ratio (PER) [4]. Considering the importance of variables other than return and risk, selection of the optimal portfolio becomes a multi-criteria problem which should be solved by using the appropriate techniques. The multi-criteria nature of the portfolio selection was well presented in the paper of Khoury et al. [9] and today an arsenal of multidimensional and multicriteria methods such as factor analysis, goal programming, AHP, ELECTRE, MINORA, ADELAIS, etc. have been already applied in portfolio selection [4], [7], [11], [15], [21].

An application of the portfolio optimization model with transaction costs on the Croatian capital market is presented in [17]. In that paper authors used the model in which measure of risk (variance) of efficient portfolio wasn't calculated in an appropriate manner. In order to properly represent the variance of the resulting portfolio, it is necessary to rescale by the funds available after paying the transaction costs [14]. In the model applied in this paper the variance is calculated on proposed way.

The paper is organized in five sections. After this introductory section, in the second section the elements of the

B. Marasović is with the Faculty of Economics, Department of Quantitative methods, University of Split, 21000 Split, Croatia (phone: 00385914430697; fax: 0038521430601; e-mail: branka.marasovic@ efst.hr).

S. Pivac is with the Faculty of Economics, Department of Quantitative methods, University of Split, 21000 Split, Croatia (e-mail: snjezana.pivac@ efst.hr).

S. V. Vukasović is graduated student with the Faculty of Science, Department of Mathematics, University of Split, 21000 Split, Croatia (e-mail: stjvuk@pmfst.hr).

Modern (Markowitz's) Portfolio Theory is presented. Portfolio Rebalancing Problem is exposed in the third section. In the fourth section portfolio model with transaction cost is applied on the Croatian capital market and it is investigated the properties of the obtained effective portfolios. Finally, some concluding remarks are given.

II. The Elements of the Modern (Markowitz's) Portfolio Theory

Between two or more portfolios of risky assets, the investors will choose the one that gives the lowest variance of return of all portfolios having the same expected return, or the one that has the highest expected return of all portfolios having the same variance, i.e. the investors will choose an efficient portfolio [2]. The efficient frontier is the set of all efficient portfolios. Now we show how to calculate efficient portfolios and the efficient frontier [12].

We use the following notation: There are N risky assets, each of which has expected return $E(r_i)$. The variable R is the column vector of expected returns of these assets:

$$R = \begin{bmatrix} E(r_1) = \overline{r}_1 \\ E(r_2) = \overline{r}_2 \\ . \\ . \\ . \\ E(r_N) = \overline{r}_N \end{bmatrix},$$

and S is the $N \times N$ variance-covariance matrix:

$$S = \begin{bmatrix} \sigma_{11} & \sigma_{21} & \dots & \sigma_{N1} \\ \sigma_{12} & \sigma_{22} & \dots & \sigma_{N2} \\ . & & & \\ . & & & \\ . & & & \\ \sigma_{1N} & \sigma_{2N} & \dots & \sigma_{NN} \end{bmatrix}.$$

A portfolio of risky assets is a column vector x whose coordinates sum to 1:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_N \end{bmatrix}, \ \sum_{i=1}^{N} x_i = 1.$$

Each coordinate x_i represent the proportion of the portfolio invested in risky asset i. The expected portfolio return $E(r_x)$ of a portfolio x is given by the product of x and R:

$$E(r_x) = x^T R = \sum_{i=1}^{N} x_i E(r_i) \quad (1)$$

The variance of portfolio x's return, $\sigma_x^2 \equiv \sigma_{xx}$ is given by the product:

$$\sigma_x^2 = x^T S x = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} \quad (2)$$

The covariance between the return of two portfolios x and y, $Cov(r_x, r_y)$, is defined by the product:

$$\sigma_{xy} = x^T S y = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i y_j \sigma_{ij} . \quad (3)$$

Mathematically, we may define an efficient portfolio as follows [13]. For a given portfolio variance σ_x^2 (or standard deviation), an efficient portfolio x is one that solves:

$$\max E(r_x) = x^T \cdot R = \sum_{i=1}^{N} x_i \cdot E(r_i) \quad (4)$$

subject to:

$$x^T S x = \sigma_x^2 \quad (5)$$

$$\sum_{i=1}^{N} x_i = 1 \quad (6)$$

$$x_i \geq 0, \ \ \forall i = 1, 2, ..., N. \quad (7)$$

Last conditions mean that short sales of assets are restricted [3].

III. Portfolio Rebalancing Problem

Portfolio rebalancing problem in the presence of transaction costs was investigated by [14]. What we consider is an extension of the basic portfolio optimization problem in which transaction costs are incurred to rebalance a portfolio? That is, transactions are made to change an already existing portfolio, $\overline{x}$, into a new and efficient portfolio, x. A portfolio may need to be rebalanced periodically simply as updated risk and return information is generated with the passage of time. Further, any alteration to the set of investment choices would necessitate a rebalancing decision of this type.

In addition to the obvious cost of brokerage fees/commissions, here are two examples of other transaction costs that can be modeled in this way [14]:

1. Capital gains taxes are a security-specific selling cost that can be a major consideration for the rebalancing a portfolio.
2. Another possibility would be to incorporate an investor's confidence in the risk/return forecast as a subjective „cost". Placing high buying and selling costs on a security would favor maintaining the current allocation $\overline{x}$. Placing a high selling cost and low buying cost could be used to express optimism that a security may outperform its forecast.

Let u_i and v_i represent the amount bought and sold (respectively) of security i. The amount invested in each of the securities will be:

$$x = \bar{x} + u - v \,. \tag{8}$$

We assume proportional transaction costs. Let C_{Bi} and C_{Si} denote the transaction cost of buying and selling one unit of security *i*, respectively. We assume, $0 \le C_{Bi} < 1, 0 \le C_{Si} < 1$ and $\exists i$ for which is $C_{Bi} + C_{Si} > 0$.

We let x_0 denote the total amount spent on transaction costs, so

$$x_0 = C_B^T u + C_S^T v \,. \tag{9}$$

The total amount invested in the securities, after paying transaction costs, will be $1 - x_0$. We obtain the constraint

$$e^T x = 1 - C_B^T u - C_S^T v \,. \tag{10}$$

Exploiting the fact that, $e^T \bar{x} = 1$, (9) immediately gives

$$ex^T = e^T x - e^T u + e^T v - C_B^T u - C_S^T v \tag{11}$$

The resulting equation is:

$$\left(C_B + e\right)^T u + \left(C_S - e\right)^T v = 0 \,. \tag{12}$$

This equation can be used to give a model for minimizing the variance of the resulting portfolio subject to meeting an expected return of E_0 in the presence of proportional transaction costs. The resulting model is:

$$Min\, x^T S x \tag{13}$$

subject to:

$$x^T \cdot R \ge E_0 \tag{14}$$

$$x - u + v = \bar{x} \tag{15}$$

$$\left(C_B + e\right)^T u + \left(C_S - e\right)^T v = 0 \tag{16}$$

$$u, v, x \ge 0 \,. \tag{17}$$

To this point, we have been optimizing the standard risk measure for efficient frontiers, that is:

$$x^T S x = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} \,.$$

When there are no transaction costs to be paid, one dollar is always available for investment, i.e. $(\sum_{i=1}^{N} x_i = 1)$. This assumption is implicit in the standard risk measure. However, for nonzero transaction costs that implicit assumption is no longer valid. One dollar is not available for investment; costs will be paid to rebalance. The appropriate objective is therefore

$$\frac{x^T S x}{\left(1 - x_0\right)^2} \tag{18}$$

Here x_0 is again the amount paid in transaction costs. Therefore, $(1 - x_0)$ is the actual amount available for investment, so we are choosing to scale the standard risk measurement by the square of the dollar amount actually invested. This gives the fractional quadratic programming problem (FQP) which we will solve to find the optimal portfolio for a given expected return.

$$Min \frac{x^T S x}{\left(1 - x_0\right)^2} \tag{19}$$

subject to:

$$x^T \cdot R \ge E_0 \tag{20}$$

$$x - u + v = \bar{x} \tag{21}$$

$$\left(C_B + e\right)^T u + \left(C_S - e\right)^T v = 0 \tag{22}$$

$$u, v, x \ge 0 \tag{23}$$

The fractional objective f(x) can be made quadratic using the technique of replacing the denominator by the square of the reciprocal of a variable. This is a straightforward extension of the technique of [6] for fractional programs where the objective is a ratio of linear functions and the constraints are linear. Let

$$t := \frac{1}{1 - C_B^T u - C_S^T v} \tag{24}$$

and then define

$$\hat{u} := tu \,,\ \hat{v} := tv \,,\quad \hat{x} := tx \,. \tag{25}$$

Note that since *u* and *v* are constrained to be nonnegative, we must have $t \ge 1$. Note that we now have $t - C_B^T u - C_S^T v = 1$. The constraints (20)-(22) can be multiplied through by t. Thus, the fractional quadratic program (FQP) is equivalent to the quadratic programming problem (QP)

$$Min\, \hat{x}^T S \hat{x} \tag{26}$$

subject to:

$$\hat{x}^T \cdot R - E_0 t \ge 0 \tag{27}$$

$$\hat{x} - \hat{u} + \hat{v} - \bar{x}t = 0 \tag{28}$$

$$\left(C_B + e\right)^T \hat{u} + \left(C_S - e\right)^T \hat{v} = 0 \tag{29}$$

$$t - C_B^T \hat{u} - C_S^T \hat{v} = 1 \tag{30}$$

$$\hat{u}, \hat{v}, \hat{x}, t \ge 0 \,. \tag{31}$$

Once we find a solution $\left(\hat{u}^*, \hat{v}^*, \hat{x}^*, t^*\right)$ to (QP), we can obtain a solution $\left(u^*, v^*, x^*\right)$ to the original problem (FQP) by rescaling $\hat{x}, \hat{u}$ and $\hat{v}$, so $x^* = \frac{\hat{x}^*}{t^*}$, $u^* = \frac{\hat{u}^*}{t^*}$ and $v^* = \frac{\hat{v}^*}{t^*}$. The efficient frontier

is found by optimizing (QP) for different values of E_0. If we do not take into the consideration (27) for solution of the problem we get global minimum variance portfolio.

Extreme right portfolio on the efficient frontier can be obtained as the solution of linear programming problem:

$$Max\, x^T R \tag{32}$$

subject to:

$$x - u + v = \bar{x} \tag{33}$$

$$\left(C_B + e\right)^T u + \left(C_S - e\right)^T v = 0 \tag{34}$$

$$u, v, x \geq 0\,. \tag{35}$$

In [14] authors have introduced variables to both buy u and sell v each security. We have not imposed an explicit constraint requiring that if a certain security is bought then it cannot also be sold. Both buying and selling a security would not be a desirable strategy in practice, but it might decrease the risk measure $\hat{x}^T S\hat{x}$. Solution $(\hat{u}, \hat{v})$ is called complementary if it satisfies $\hat{u}^T\hat{v} = 0$, that is, if no stock is both bought and sold. In the paper [14] authors shown that if the return constraint $\hat{x}^T \cdot R - E_0 t \geq 0$ is active at the optimal solution to (QP) then the optimal solution must be complementary. If the return constraint is not active at the optimal solution, then it is possible that an optimal solution will not be complementary. However, authors also shown that a complementary solution can always be found efficiently even in this situation.

IV. Analysis of Mean-Variance Portfolio Optimization Model with Included Transaction Costs Through the Application on the Croatian Capital Market

Through the application of the presented mean-variance portfolio optimization model with included transaction costs on the Croatian capital market we conduct analysis of efficient portfolios obtained by presented model. From the total number of securities quoted on the Zagreb stock exchange in 2013 and 2014 a sample of ten stocks from CROBEX10 index has been separated. Stocks included in that index are ten the most liquid stocks with the highest free float, turnover and market capitalization on Zagreb Stock Exchange.

Companies included in CROBEX10 index are: AD Plastik (ADPL-R-A), Adris grupa (ADRS-P-A), Atlantic grupa (ATGR-R-A), Ericsson Nikola Tesla (ERNT-R-A), HT (HT-R-A), INA (INA-R-A), Končar-elektroindustrija (KOEI-R-A), Ledo (LEDO-R-A), Podravka (PODR-R-A), Petrokemija (PTKM-R-A) [19].

For each security from the sample we take the closing price at the end of each two-week period from January 1st 2013 to November 5th 2014. First we calculate the two-week returns for each security. We choose two-week returns because the most of the stocks from the sample have normal distribution of two-week returns and in this case variance is adequate measure of risk.

For period t and security A, two-week return r_{At} is defined as: $r_{At} = \ln\left(\frac{P_{A,t}}{P_{A,t-1}}\right)$ [18]. First we calculate efficient frontier using Markowitz model based on two-week returns during period from January 1st 2013 to October 22nd 2014. Obtained efficient frontier and ten efficient portfolios are shown on Fig. 1 and Table I.

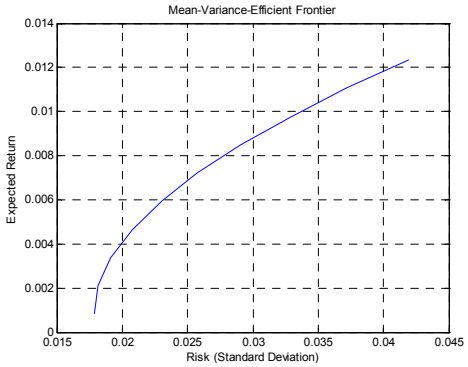

Fig. 1 Efficient frontier on the date October 22nd 2014

TABLE I
Efficient Portfolios on October 22nd 2014

ADPL-R-A	ADRS-P-A	ATGR-R-A	ERNT-R-A	HT-R-A	INA-R-A	KOEI-R-A	LEDO-R-A	PODR-R-A	PTKM-R-A	E(R)	σ
3.17%	4.42%	0.00%	10.26%	7.05%	25.32%	11.60%	37.92%	0.00%	0.26%	0.08%	1.79%
2.75%	10.03%	3.52%	9.06%	0.11%	23.03%	12.02%	39.47%	0.00%	0.00%	0.21%	1.81%
1.25%	12.36%	13.40%	7.42%	0.00%	17.00%	9.07%	39.51%	0.00%	0.00%	0.34%	1.91%
0.00%	14.59%	23.36%	5.74%	0.00%	10.85%	6.00%	39.46%	0.00%	0.00%	0.47%	2.08%
0.00%	16.52%	33.32%	4.00%	0.00%	4.41%	2.63%	39.12%	0.00%	0.00%	0.59%	2.30%
0.00%	17.81%	43.91%	2.06%	0.00%	0.00%	0.00%	36.22%	0.00%	0.00%	0.72%	2.57%
0.00%	17.38%	56.04%	0.00%	0.00%	0.00%	0.00%	26.58%	0.00%	0.00%	0.85%	2.91%
0.00%	16.29%	68.94%	0.00%	0.00%	0.00%	0.00%	14.77%	0.00%	0.00%	0.98%	3.29%
0.00%	15.20%	81.85%	0.00%	0.00%	0.00%	0.00%	2.95%	0.00%	0.00%	1.11%	3.71%
0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.23%	4.19%

Portfolios from Table I are efficient on October 22nd 2014. However when we include in analysis stocks return on November 5th 2014 (next two-week returns) those portfolios are no more efficient and it is necessary to conduct portfolio rebalance. During portfolio rebalance we assume that an

investor wants to keep the same return as in initial portfolio in previous period with risk minimization.

Most of Croatian brokerages charge "all-in" type of transaction fees. It means that Zagreb Stock Exchange and SKDD (Central Depository and Clearing Company) fees are included in brokerage fees. Also most of Croatian brokerages charge the same transaction fees both for selling and buying orders. During year 2013 those fees was between 0.35% and 1.25% [22]. Therefore, in this paper we conduct portfolio rebalance for the highest and the lowest value of brokerage fees on the Croatian Capital Market.

Results of portfolio rebalance with 1.25% brokerage fees are given in Table II and with 0.35% brokerage fees are given in Table III.

The first row in Table II represent new efficient portfolio which we obtained by rebalancing the first portfolio from Table I with transaction costs of 1.25% while the first row in Table III represent new efficient portfolio which we obtained by rebalancing first portfolio from Table I with transaction costs of 0.35%. From Tables II and III we can notice that rebalanced portfolios with the same return have higher risk if transaction costs are higher. So, we can conclude that if investor wants to achieve given rate of return he has to accept higher rate of risk for higher transaction costs.

TABLE II
EFFICIENT PORTFOLIOS ON NOVEMBER 5TH 2014 OBTAINED BY REBALANCING OF PORTFOLIOS FROM TABLE 1 WITH BROKERAGE FEES 1.25%

ADPL-R-A	ADRS-P-A	ATGR-R-A	ERNT-R-A	HT-R-A	INA-R-A	KOEI-R-A	LEDO-R-A	PODR-R-A	PTKM-R-A	E(R)	σ
2.51%	6.84%	0.00%	10.00%	5.89%	24.12%	11.92%	38.63%	0.00%	0.00%	0.08%	1.78347%
1.64%	11.33%	5.44%	9.12%	0.00%	21.11%	11.81%	39.47%	0.00%	0.00%	0.21%	1.82194%
0.04%	13.70%	14.68%	7.66%	0.00%	15.28%	9.07%	39.51%	0.00%	0.00%	0.34%	1.91882%
0.00%	15.62%	23.94%	6.09%	0.00%	8.95%	6.00%	39.36%	0.00%	0.00%	0.47%	2.07997%
0.00%	17.47%	33.32%	4.57%	0.00%	2.82%	2.66%	39.12%	0.00%	0.00%	0.59%	2.29252%
0.00%	18.31%	43.91%	2.59%	0.00%	0.00%	0.00%	35.17%	0.00%	0.00%	0.72%	2.54918%
0.00%	17.90%	55.60%	0.41%	0.00%	0.00%	0.00%	26.06%	0.00%	0.00%	0.85%	2.86633%
0.00%	17.54%	67.66%	0.00%	0.00%	0.00%	0.00%	14.77%	0.00%	0.00%	0.98%	3.23145%
0.00%	17.19%	79.80%	0.00%	0.00%	0.00%	0.00%	2.96%	0.00%	0.00%	1.11%	3.63299%
0.00%	2.07%	97.88%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.23%	4.09590%

TABLE III
EFFICIENT PORTFOLIOS ON NOVEMBER 5TH 2014 OBTAINED BY REBALANCING OF PORTFOLIOS FROM TABLE 1 WITH BROKERAGE FEES 0.35%

ADPL-R-A	ADRS-P-A	ATGR-R-A	ERNT-R-A	HT-R-A	INA-R-A	KOEI-R-A	LEDO-R-A	PODR-R-A	PTKM-R-A	E(R)	σ
2.51%	6.84%	0.00%	10.01%	5.90%	24.13%	11.93%	38.66%	0.00%	0.00%	0.08%	1.78346%
1.60%	11.40%	5.42%	9.17%	0.00%	21.16%	11.84%	39.37%	0.00%	0.00%	0.21%	1.82188%
0.00%	13.73%	14.66%	7.71%	0.00%	15.26%	9.11%	39.51%	0.00%	0.00%	0.34%	1.91864%
0.00%	15.72%	23.92%	6.19%	0.00%	8.97%	6.00%	39.19%	0.00%	0.00%	0.47%	2.07972%
0.00%	17.63%	33.23%	4.68%	0.00%	2.71%	2.87%	38.86%	0.00%	0.00%	0.59%	2.29218%
0.00%	18.85%	43.73%	2.79%	0.00%	0.00%	0.00%	34.62%	0.00%	0.00%	0.72%	2.54877%
0.00%	18.78%	55.31%	0.72%	0.00%	0.00%	0.00%	25.17%	0.00%	0.00%	0.85%	2.86570%
0.00%	18.27%	67.37%	0.00%	0.00%	0.00%	0.00%	14.35%	0.00%	0.00%	0.98%	3.23068%
0.00%	17.50%	79.67%	0.00%	0.00%	0.00%	0.00%	2.81%	0.00%	0.00%	1.11%	3.63166%
0.00%	2.15%	97.84%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.23%	4.09401%

TABLE IV
GLOBAL MINIMUM VARIANCE PORTFOLIOS ON NOVEMBER 5TH 2014 OBTAINED BY REBALANCING PORTFOLIOS FROM TABLE 1 WITH BROKERAGE FEES 1.25%

ADPL-R-A	ADRS-P-A	ATGR-R-A	ERNT-R-A	HT-R-A	INA-R-A	KOEI-R-A	LEDO-R-A	PODR-R-A	PTKM-R-A	E(R)	σ
2.54%	5.11%	0.00%	10.45%	7.42%	24.58%	11.69%	37.81%	0.00%	0.35%	0.0466%	1.7820%
2.53%	5.10%	0.00%	10.43%	7.40%	24.53%	11.67%	37.72%	0.00%	0.35%	0.0465%	1.7820%
2.53%	5.09%	0.00%	10.40%	7.38%	24.46%	11.63%	37.61%	0.00%	0.35%	0.0464%	1.7820%
2.52%	5.07%	0.00%	10.37%	7.36%	24.38%	11.60%	37.50%	0.00%	0.35%	0.0462%	1.7820%
2.51%	5.06%	0.00%	10.34%	7.33%	24.31%	11.56%	37.39%	0.00%	0.35%	0.0461%	1.7820%
2.50%	5.04%	0.00%	10.31%	7.32%	24.25%	11.54%	37.30%	0.00%	0.35%	0.0460%	1.7822%
2.50%	5.03%	0.00%	10.28%	7.29%	24.18%	11.50%	37.19%	0.00%	0.35%	0.0459%	1.7820%
2.49%	5.02%	0.00%	10.25%	7.27%	24.11%	11.47%	37.08%	0.00%	0.34%	0.0457%	1.7820%
2.48%	5.00%	0.00%	10.22%	7.25%	24.03%	11.43%	36.97%	0.00%	0.34%	0.0456%	1.7820%
2.48%	4.99%	0.00%	10.20%	7.24%	23.98%	11.41%	36.89%	0.00%	0.34%	0.0455%	1.7820%

TABLE V
EXTREME RIGHT PORTFOLIOS ON NOVEMBER 5TH 2014 OBTAINED BY REBALANCING PORTFOLIOS FROM TABLE 1 WITH BROKERAGE FEES 1.25%

ADPL-R-A	ADRS-P-A	ATGR-R-A	ERNT-R-A	HT-R-A	INA-R-A	KOEI-R-A	LEDO-R-A	PODR-R-A	PTKM-R-A	E(R)	σ
0.00%	0.00%	97.53%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.2166%	4.1505%
0.00%	0.00%	97.62%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.2177%	4.1505%
0.00%	0.00%	97.86%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.2207%	4.1505%
0.00%	0.00%	98.11%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.2238%	4.1505%
0.00%	0.00%	98.35%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.2268%	4.1505%
0.00%	0.00%	98.61%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.2301%	4.1505%
0.00%	0.00%	98.91%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.2338%	4.1505%
0.00%	0.00%	99.23%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.2378%	4.1505%
0,00%	0,00%	99,55%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	1,2418%	4,1505%
0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.2474%	4.1505%

Furthermore, in this paper we analyze global minimum variance portfolio. We conduct rebalance of portfolios from Table I with transaction costs of 1.25 % with aim to find global minimum variance portfolio. From Table IV we can observe that global minimum variance portfolio has always the same level of risk (variance) regardless of initial portfolio. However, rebalancing with higher volume and number of transactions (higher costs) cause lower return.

Finally, solving problem of linear programming (32)-(35) we get portfolios with the highest return but with the highest risk. From Table V we can observe that all portfolios consist of only one stock and also have the same variance. Again, return depends of volume and number of transactions during rebalancing and can be calculated from: $\left(1-C_B^T u - C_S^T v\right)\cdot \max\left\{E(r_i): i=1,2,...,N\right\}$.

V. CONCLUSION

The results show that efficient frontier is always positioned in the same risk interval regardless both of the amount of transaction costs and initial portfolio. Efficient portfolio return is negative correlated with number and volume of transactions. Finally, it can be concluded that efficient frontier obtained by presented model is always positioned below efficient frontier obtained by Markowitz model i.e. Markowitz efficient portfolio always have higher or equal return than return of efficient portfolio obtained by presented model.

REFERENCES

[1] C. J. Adcock, and N. Meade, "A simple algorithm to incorporate transactions costs in quadratic optimization," *European Journal of Operational Research*, vol. 79, no. 1, 1994, pp. 85-94.

[2] Z. Aljinović, B. Marasović, and N. Tomić-Plazibat, "The selection of the optimal portfolio on the Croatian capital market," in *Proceedings of Sixth International Conference on Enterprise in Transition*, May 2005. pp. 299-301.

[3] Z. Aljinović, B. Marasović, B. Šego, *Financijsko modeliranje*. Ekonomski fakultet u Splitu, Split, 2011.

[4] A. Bouri, J. M. Martel and H. Chabchoub, "A Multi-criterion approach for selecting attractive portfolio," *Journal of Multi-Criteria Decision Analysis*, vol. 11, 2002, pp. 269-277.

[5] T. J. Chang, N. Meade, J. E. Beasley and Y. M. Sharaiha, "Heuristics for cardinality constrained portfolio optimization," *Computers and Operations Research*, vol. 27, 2000, pp. 1271-1302.

[6] A. Charnes and W. W. Cooper, "Programming with linear fractional functionals," *Naval Research Logistics Quarterly*, vol. 9, 1962, pp. 181-186.

[7] O. L. V. Costa and A. C. Paiva, "Robust portfolio selection using linear-matrix inequalities," *Journal of Economic Dynamics and Control*, vol. 26, 2002, pp 889-909.

[8] E. De Giorgi, "A Note on Portfolio Selection under Various Risk Measures", *Working Paper*, no. 9, *National Centre of Competence in Research Financial Valuation and Risk Management,* 2002. http://www.nccr-finrisk.unizh.ch/media/pdf/wp/WP009_3.pdf

[9] N. Khoury, J. M. Martel and M. Veilleux, "Methode multicritere de selection de portefeuilles indiciels interantionaux," *Acualite Economique* vol. 69, no. 1, 1993, pp. 171–190.

[10] H. Konno, H. Waki and A. Yuuki, "Portfolio optimization under lower partial risk measures," *Financial Engineering and the Japanese Markets*, vol. 9, no. 2, 2002, pp. 127-140.

[11] B. Marasović, Z. Babić, "Two-step multi-criteria model for selecting optimal portfolio," *International Journal of Production Economics*, vol. 134, 2011, pp. 58-66.

[12] H. M. Markowitz, "Portfolio Theory," *Journal of Finance*, vol. 7, 1952, pp. 77-91.

[13] H. M. Markowitz and P. Todd, *Mean-Variance Analysis in Portfolio Choice and Capital Markets*. John Wiley & Sons, 2000.

[14] J. E. Mitchell and S. Braun, Rebalancing an Investment Portfolio in the Presence of Transaction Costs, *Working Paper*, *Department of Rensselaer Polytechnic Institute*, 2002.

[15] W. Ogryczak, "Multiple criteria linear programming model for portfolio selection," *Annals of Operations Research*, vol. 97, 2000, pp. 143-162.

[16] R. T. Rockafaller and S. Uryasev, "Optimization of Conditional Value-at-Risk," *Journal of Risk*, vol. 2, 2000, pp. 21-41.

[17] B. Škarica and Z. Lukač, "A Comparison of Basic and Extended Markowitz Model on Croatian Capital Market", *Croatian Operational Research Review*, vol. 3, 2012, pp. 236-245.

[18] P. Venkataraman, *Applied Optimization with MATLAB Programming*, John Wiley & Sons, 2002.

[19] The Zagreb Stock Exchange. History Trading 2011 and 2012. http://zse.hr/default.aspx?id=44101&index=CROBEX10 [Accessed July 2013].

[20] S. A. Zenios, "Asset/liability management under uncertainty for fixed income securities," *Annals of Operations Research*, vol. 59, 1995, pp. 77-98, reprinted in *World Wide Asset and Liability Modeling*, editors: W. T. Ziemba and J. M. Mulvey, Cambridge University Press, 1998.

[21] C. Zopounidis, "Multicriteria decision aid in financial management," *European Journal of Operational Research*, vol. 119, 1999, pp. 404–415.

[22] Poslovni dnevnik. http://www.poslovni.hr/burze/brokeri-u-hrvatskoj-su-konkurentniji-od-slovenskih-evo-zasto-222728 [Accessed Oct. 2013].