World Academy of Science, Engineering and Technology
International Journal of Materials and Metallurgical Engineering
Vol:9, No:2, 2015

Effects of Pipe Curvature and Internal Pressure on Stiffness and Buckling Phenomenon of Circular Thin-Walled Pipes

V. Polenta, S. D. Garvey, D. Chronopoulos, A. C. Long, H. P. Morvan

International Science Index, Materials and Metallurgical Engineering Vol:9, No:2, 2015 waset.org/Publication/10000513

Abstract—A parametric study on circular thin-walled pipes subjected to pure bending is performed. Both straight and curved pipes are considered. Ratio D/t, initial pipe curvature and internal pressure are the parameters varying in the analyses. The study is mainly FEA-based.

It is found that negative curvatures (opposite to bending moment) considerably increase stiffness and buckling limit of the pipe when no internal pressure is acting and, similarly, positive curvatures decrease the stiffness and buckling limit. For internal pressurised pipes the effects of initial pipe curvature are less relevant. Results show that this phenomenon is in relationship with the cross-section deformation due to bending moment, which undergoes relevant ovalisation for no pressurised pipes and little ovalisation for pressurised pipes.

Keywords—Buckling, curved pipes, internal pressure, ovalisation, pure bending, thin-walled pipes.

I. Introduction

THIN-WALLED components have an increasingly important role in engineering as allow a better exploitation of material properties. Their use is increasing in parallel with development of high performance materials and improvement of manufacturing technologies. Applications range from mechanics to marine industry and assume an essential function in aeronautical and aerospace technologies wherein employment of composite materials is extremely common. Here circular thin-walled pipes subjected to bending moment and internal pressure are studied.

Thin-walled pipes are naturally subjected to buckling failure when compressive or bending load are acting. The benefits of internal pressure on buckling limit have long been known. Many authors have studied buckling phenomenon [1], [2], [3] and [4]. Some found that internal pressure enhance the behaviour of circular pipes against buckling for different load conditions such as axial load [5], torsion [6] and bending [7]. Others focused on curved pipes and their results show that a curved pipe buckles earlier than the corresponding straight pipe [8], [9] and [10]. Besides that, studies on cross-section deformation of circular pipes under bending have been conducted demonstrating that the cross-section flattens with bending [11] and [12]. However, these issues have been mainly analysed singularly due to their complexity. The aim of the present paper is to study simultaneously the effects of pipe curvature and internal pressure to find the consequent changes in stiffness and buckling limit and to check the relationship between such changes and the cross-section deformation.

V. Polenta and H. P. Morvan are with the Institute for Aerospace Technology, Aerospace Technology Centre, University of Nottingham, Nottingham NG7 2TU, United Kingdom (e-mail: valerio.polenta@nottingham.ac.uk).

S. D. Garvey, D. Chronopoulos and A. C. Long are with the Faculty of Engineering, University Park, The University of Nottingham, Nottingham NG7 2RD, United Kingdom.

TABLE I: FE Model Parameters

Parameter	Value
Pipe diameter, D	100 mm
Pipe length, L	700 mm
Wall thickness, t (D/t)	0.25 mm (400) – 1.00 mm (100)
Radius of curvature[1], R (R/D)	-10 m (-100) – 10 m (100)
Young Modulus, E	200 GPa
Poisson's ratio, ν	0.3
Internal pressure, p	0, 1.5 MPa

II. Methodology

Investigation of the problem is conducted by taking advantage of Finite Element Analysis. The software used is Ansys 14.5. The analysis is performed on models of thin-walled pipes ($D/t \geq 100$) meshed with SHELL181 which is a four-nodes shell element implementing Kirchhoff-Love plate theory; after a sensitivity analysis the element size is set to be equal to 3.5 mm in the axial direction and 5.236 mm in the circumferential direction. Bending moments are applied to the two ends which are constrained to behave as rigid planes. The pipe diameter is fixed to 100 mm and the total pipe length is 7 times the pipe diameter which allows to have the middle part along the span not affected by the ends constraints. The parametric analysis is conducted by varying the ratio D/t, the initial pipe curvature and internal pressure. In order to reduce the computational power required by the simulations just a quarter of cylinder is modeled taking advantage of the double symmetry of the problem (Fig. 1). The material model is linear with typical properties of a steel. Table I summarises the model parameters.

Each simulation consists in a non-linear analysis in which large displacements are taken into account. The bending moment is applied step by step until the maximum allowable load is reached, i.e. buckling limit. The examined results are rotation of the ends and cross-section deformation for each

[1]This refers to the pipe centreline curvature (see Fig. 5). Negative values represent pipe whose initial curvature is opposite to the curvature deriving from bending moment.

International Science Index, Materials and Metallurgical Engineering Vol:9, No:2, 2015 waset.org/Publication/10000513

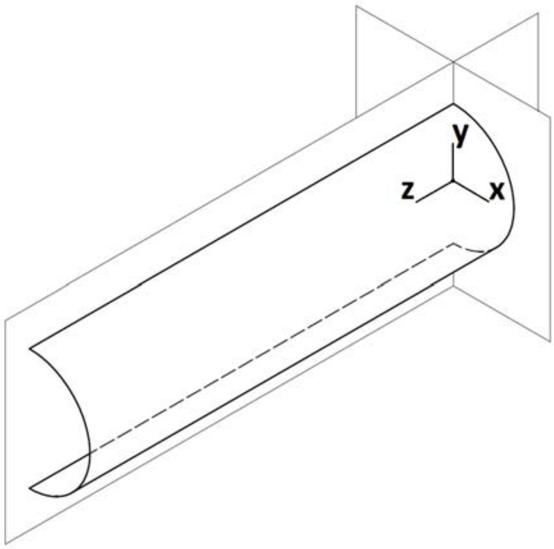


Fig. 1: Quarter of pipe and the two planes of symmetry (XY and YZ). The applied moment acts in X direction and rotates the pipe end downward

load step. These are then expressed as load-deflection curves and ovalisation-load curves to highlight how stiffness and cross-section deformation change with internal pressure and pipe curvature.

III. Numerical Results

The following figures show the results for pipes with wall thickness equal to 1.00 mm ($D/t = 100$), 0.50 mm ($D/t = 200$) and 0.25 mm ($D/t = 400$) and internal pressure p equal to 0 and 1.5 MPa. Specifically, Fig. 2 shows the load-deflection curves and Fig. 3 shows the ovalisation-load curves referred to the mid-section. Ovalisation θ is defined as difference between horizontal diameter D_h (normal to the plane of curvature) and vertical diameter D_v (in the plane of curvature) divided by the initial diameter D (Fig. 4):

$$\theta = \frac{D_h - D_v}{D} \tag{1}$$

Each graph consists of 5 curves, one for each examined pipe curvature. Curvature k is defined as the ratio of pipe diameter D over radius of curvature R, i.e. radius of pipe centreline (Fig. 5):

$$k = \frac{D}{R} \tag{2}$$

Negative values represent curvatures opposite to the bending moment action. The curves of Figs. 2 and 3 are interrupted at the point corresponding to pipe collapse.

Table II lists the values of ovalisation of the mid-section right before collapse.

IV. Results Analysis

The influence of pipe curvature and internal pressure on stiffness and buckling limit can be studied by analysing Fig. 2. It can be noted that pipes with no internal pressure have a behaviour much sensitive to the pipe curvature k (Figs. 2 (a), 2 (c) and 2 (e)). Positive curvature decreases the critical buckling moment and negative curvature has the opposite effect. Moreover, pipe curvature affects the *slope* of load-deflection curves, i.e. pipe stiffness. Positive curvatures lead to a load-deflection curves with a decreasing slope.

Internal pressure alters these graphs. Load-deflection curves of internally pressurised pipes ((Figs. 2 (b), 2 (d) and 2 (f)) have a slope almost constant. Internal pressure enhances and evens out performance of pipes with different curvature in terms of stiffness and buckling limit.

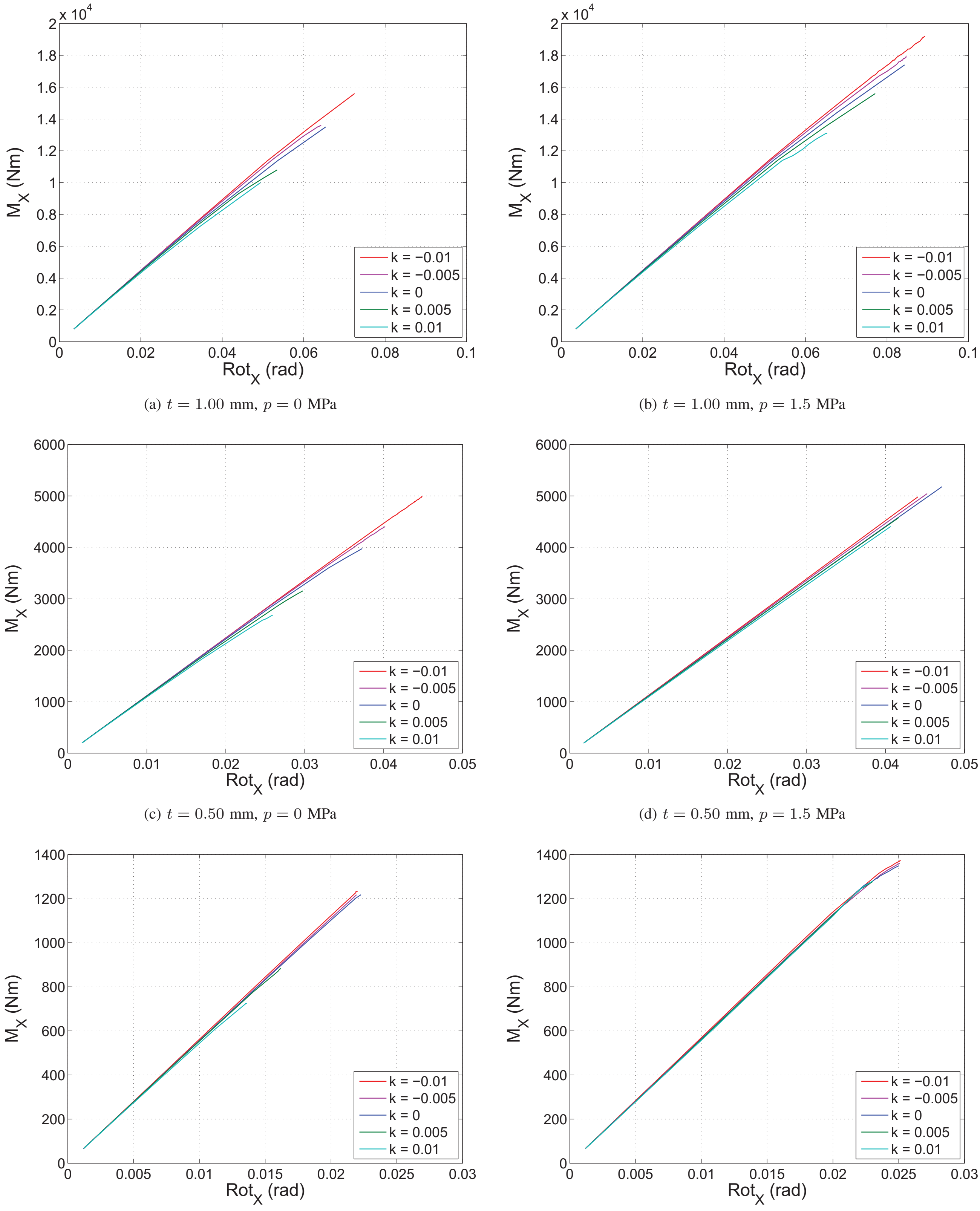
This difference in behaviour of non-pressurised and pressurised pipes is in relationship with the cross-section ovalisation. Cross-section of a thin-walled pipe under bending flattens with load application, with a consequent reduction of inertial moment and stiffness. The higher is the pipe curvature, the more the cross-section flattens. However, the action of pressure on the internal surface tends to keep the cross-section circular. Fig. 3 and values in Table II show that mid-section ovalisation is reduced for the internally pressurised pipes respect to the corresponding non-pressurised ones.

As final remark, note that it has been used the same value of internal pressure, $p = 1.5$ MPa, for all the pressurised models, regardless of the wall thickness. If we expressed such value in an opportune dimensionless way, such as the ratio internal pressure over critical internal pressure which results in pipe failure, then the thinnest pipe (in terms of wall thickness) would be the most internally pressurised. Indeed, the present case study confirms that the pipe with $t = 0.25$ mm is the most sensitive to pressure: load-deflection curves in Fig. 2 (f) are almost overlapping and ovalisation values in Fig. 3 (f) are very reduced.

TABLE II: MID-SECTION OVALISATION

t	k	$p = 0$ MPa	$p = 1.5$ MPa
0.25 mm	0.01	$4.85 * 10^{-2}$	$1.46 * 10^{-2}$
	0.005	$3.44 * 10^{-2}$	$9.09 * 10^{-3}$
	0	$4.03 * 10^{-2}$	$5.46 * 10^{-3}$
	-0.005	$4.92 * 10^{-3}$	$1.39 * 10^{-3}$
	-0.01	$-1.72 * 10^{-2}$	$6.16 * 10^{-4}$
0.50 mm	0.01	$1.04 * 10^{-1}$	$5.25 * 10^{-2}$
	0.005	$9.47 * 10^{-2}$	$3.92 * 10^{-2}$
	0	$1.04 * 10^{-1}$	$3.15 * 10^{-2}$
	-0.005	$6.15 * 10^{-2}$	$1.93 * 10^{-2}$
	-0.01	$1.87 * 10^{-2}$	$5.93 * 10^{-3}$
1.00 mm	0.01	$1.63 * 10^{-1}$	$1.90 * 10^{-1}$
	0.005	$1.84 * 10^{-1}$	$1.98 * 10^{-1}$
	0	$1.81 * 10^{-1}$	$1.76 * 10^{-1}$
	-0.005	$1.53 * 10^{-1}$	$1.45 * 10^{-1}$
	-0.01	$1.14 * 10^{-1}$	$1.15 * 10^{-1}$

International Science Index, Materials and Metallurgical Engineering Vol:9, No:2, 2015 waset.org/Publication/10000513



(a) $t = 1.00$ mm, $p = 0$ MPa

(b) $t = 1.00$ mm, $p = 1.5$ MPa

(c) $t = 0.50$ mm, $p = 0$ MPa

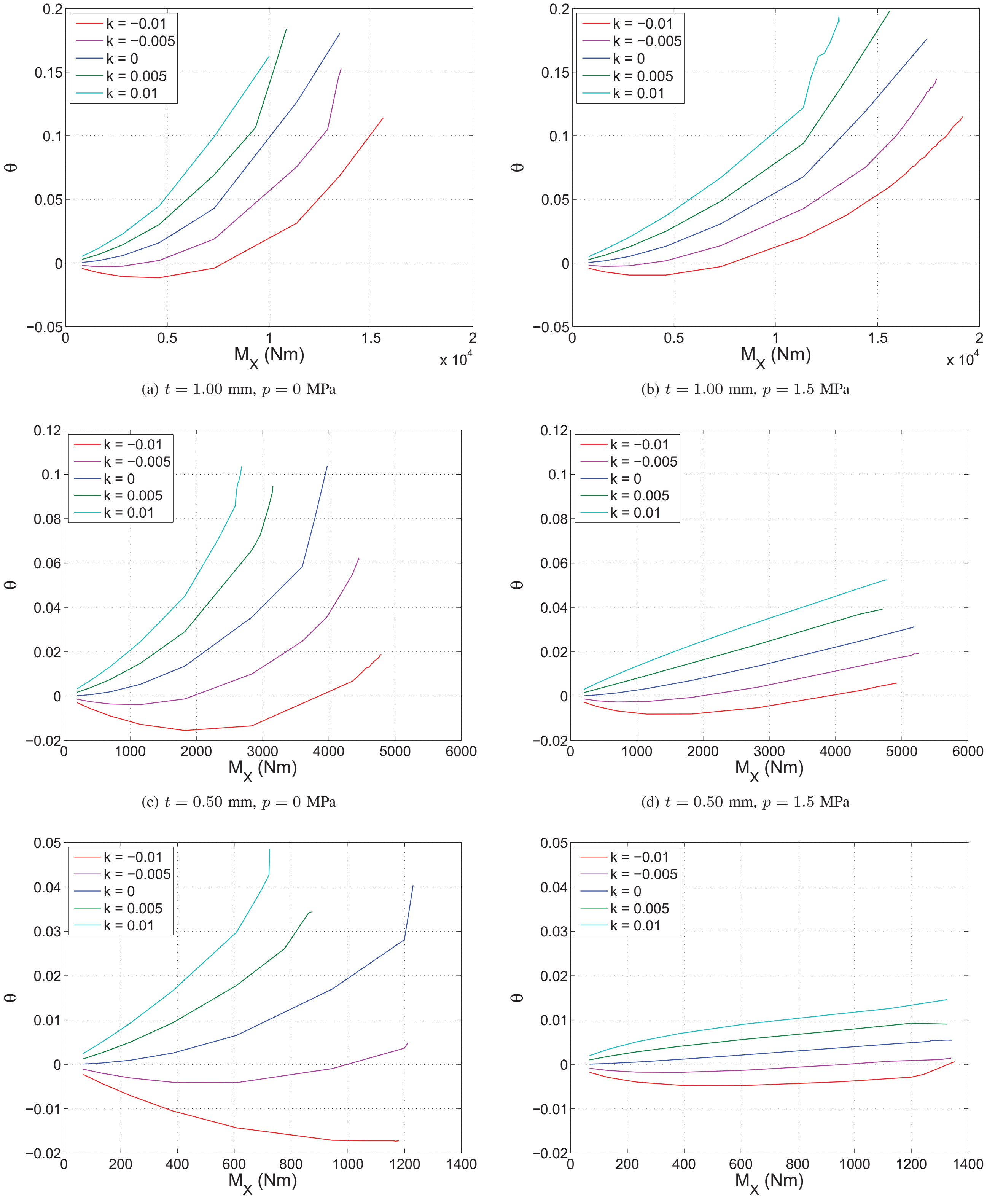
(d) $t = 0.50$ mm, $p = 1.5$ MPa

(e) $t = 0.25$ mm, $p = 0$ MPa

(f) $t = 0.25$ mm, $p = 1.5$ MPa

Fig. 2: Load-deflection curves

International Science Index, Materials and Metallurgical Engineering Vol:9, No:2, 2015 waset.org/Publication/10000513



(a) $t = 1.00$ mm, $p = 0$ MPa

(b) $t = 1.00$ mm, $p = 1.5$ MPa

(c) $t = 0.50$ mm, $p = 0$ MPa

(d) $t = 0.50$ mm, $p = 1.5$ MPa

(e) $t = 0.25$ mm, $p = 0$ MPa

(f) $t = 0.25$ mm, $p = 1.5$ MPa

Fig. 3: Ovalisation-load curves. Ovalisation is referred to the mid-section

International Science Index, Materials and Metallurgical Engineering Vol:9, No:2, 2015 waset.org/Publication/10000513

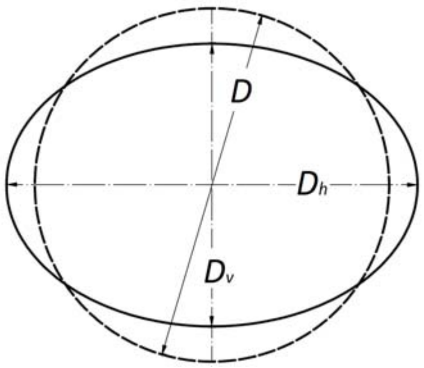


Fig. 4: Cross-section ovalisation. Dashed and solid line represent the undeformed and the ovalised pipe, respectively

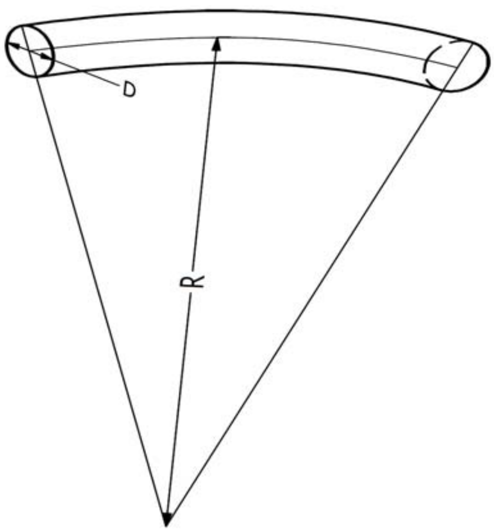


Fig. 5: Curved pipe. The ratio pipe diameter D over pipe centreline radius R defines the pipe curvature k

V. Conclusion

Results shown in the previous section confirm the already known beneficial effect of internal pressure on thin-walled pipes in terms of critical buckling moment. Moreover, they highlight a clear relation between cross-section deformation and pipe stiffness. No pressurised pipes with positive curvature flattens with bending load, their stiffness decreases and their critical buckling load is low. Pressurised pipes exhibit a much smaller ovalisation, their stiffness is almost constant throughout moment application and their critical buckling load is higher. Furthermore, internal pressure leads to results less sensitive to pipe curvature.

The present study suggests the possibility to use internal pressure to enhance structural performance as well as to change the components' stiffness. In cases wherein strength is not the only important design parameter, combination of internal pressure and component curvature can provide best results in terms of maximum allowable load and displacement. Most attractive applications concerns industries wherein high mechanical performance and lightness are key factors, such as structural optimisation of wings in aircrafts with high wingspan.

Acknowledgment

This work is funded by the European Commission under the project titled INNOVATE, The systematic Integration of Novel Aerospace Technologies, FP7 project number 608322 which is part of the FP7-PEOPLE-2013-ITN call. The authors would like to gratefully acknowledge the Marie Curie Innovative Doctoral Programme for the financial support.

References

[1] S. P. Timoshenko and J. M. Gere, "Theory of elastic stability," 1961.
[2] G. Bryan, "On the stability of a plane plate under thrusts in its own plane, with applications to the buckling of the sides of a ship," *Proceedings of the London Mathematical Society*, vol. 1, no. 1, pp. 54–67, 1890.
[3] S. R. Hauch and Y. Bai, "Bending moment capacity of pipes," *Journal of Offshore Mechanics and Arctic Engineering*, vol. 122, no. 4, pp. 243–252, 2000.
[4] K. Mørk, J. Spiten, E. Torselletti, O. Ness, and R. Verley, "The superb project and dnv'96: Buckling and collapse limit state," in *Proceedings of the International Conference on Offshore Mechanics and Artic Engineerinng*. American Society of Mechanical Engineers, 1997, pp. 79–90.
[5] J. Hutchinson, "Axial buckling of pressurized imperfect cylindrical shells," *AIAA Journal*, vol. 3, no. 8, pp. 1461–1466, 1965.
[6] H. Crate, S. Batdorf, and G. W. Baab, "The effect of internal pressure on the buckling stress of thin-walled circular cylinders under torsion," DTIC Document, Tech. Rep., 1944.
[7] A. Robertson, H. Li, and D. Mackenzie, "Plastic collapse of pipe bends under combined internal pressure and in-plane bending," *International Journal of Pressure Vessels and Piping*, vol. 82, no. 5, pp. 407–416, 2005.
[8] H. Yudo and T. Yoshikawa, "Buckling phenomenon for straight and curved pipe under pure bending," *Journal of Marine Science and Technology*, 2014.
[9] V. Polenta, S. Garvey, D. Chronopoulos, A. Long, and H. Morvan, "Optimal internal pressurisation of cylindrical shells for maximising their critical bending load," *Thin-Walled Structures*, vol. 87, pp. 133–138, 2015.
[10] H. M. Mourad and M. Y. Younan, "Limit-load analysis of pipe bends under out-of-plane moment loading and internal pressure," *Journal of pressure vessel technology*, vol. 124, no. 1, pp. 32–37, 2002.
[11] A. Hilberink, *Mechanical behaviour of lined pipe*. Ph. D. Thesis, Delft Technical University, ISBN 978-94-6186-012-5, 2011.
[12] C. Thinvongpituk and S. Poonaya, "A study of the ovalisation of circular tubes subjected to pure bending," in *Current Themes in Engineering Science 2008: Selected Presentations at the World Congress on Engineering2008*, vol. 1138, no. 1. AIP Publishing, 2009, pp. 129–140.

Valerio Polenta graduated with honors in Mechanical Engineering, with a masters thesis developed at The University of Bristol. During his final year of study he took part in the European project, EGPR 2013 (European Global Product Realisation), which have seen the collaboration of five universities and an industrial partner in order to develop a small aircraft for disable pilots from the concept to the prototyping.

Valerio is currently a Marie Curie Fellow at The University of Nottingham and his PhD activity studies the exploitation of internal pressurisation as a means of enhancing structural properties. He is working to the Marie Curie Innovative Doctoral Programme "INNOVATE" and is part of the WP2 (Airframe and Control Technologies), therefore his main role in the project is to set out methods whereby internal pressurisation can contribute strength, stiffness and stability in aerospace structures.