Available online at www.eccomasproceedia.org
Eccomas Proceedia SMART (2023) 1851-1862

ECCOMAS
Proceedia

X ECCOMAS Thematic Conference on Smart Structures and Materials
SMART 2023
D.A. Saravanos, A. Benjeddou, N. Chrysochoidis and T. Theodosiou (Eds)

HYBRID TWIN APPLIED TO STRUCTURAL HEALTH MONITORING

Sebastian Rodriguez[*,1], **Daniele Di Lorenzo**[1,2], **Francisco Chinesta**[1,2], **Eric Monteiro**[1], **Marc Rebillat**[1] **and Nazih Mechbal**[1]

[1]PIMM, ENSAM Institute of Technology
151 Boulevard de l'Hôpital, 75013, Paris, France
e-mail: sebastian.rodriguez iturra@ensam.eu, francisco.chinesta@ensam.eu, eric.monteiro@ensam.eu, marc.rebillat@ensam.eu, nazih.mechbal@ensam.eu

[2] ESI Group Chair, PIMM, ENSAM Institute of Technology
151 Boulevard de l'Hôpital, 75013, Paris, France
e-mail: daniele.dilorenzo@esi-group.com

Abstract. To ensure the proper functioning of a structure, a monitoring during its life cycle is necessary, with the objective of detecting in time any possible anomalies or damage of the structure. To accomplish this, high fidelity numerical models that correctly capture the physics of the system should be developed, so that this model can be further used to achieve a desired design goal, as well as to ensure the longevity of the structure. However, the complexity of real phenomena often makes it impossible for physics-based models to deliver a correct prediction of reality, which limits their use. To overcome this limitation, one solution consists in building a model based on experimental data to ensure correct predictability. Nevertheless, this imposes technical limitations, since obtaining data is often scarce due to limited number of sensors or high costs of experimental campaigns. In this context, hybrid twins emerge as an attractive solution to this problem. Hybrid twins consists in enriching a physics-based model by building an ignorance model, which corrects the predictions of the numerical model. This allows to build a representative model of reality by using a limited number of sensors, since the global behavior of the system is reproduced by the physical model, making the ignorance model to be constructed in a coarse way. In this sense, the present work shows the implementation of a hybrid twin, applied to the monitoring of a structure using Structural Health Monitoring techniques. The performance of the developed hybrid twin is tested on synthetic data, where the hybrid twin built from a simplified physics-based model allows to correct the latter and can be used later to accurately predict damage location on a more complex structure.

Key words: Parametric dynamics, Structural health monitoring, Real-time simulations, Model-order reduction, Hybrid twin

doi: 10.7712/150123.9955.444924

1 Introduction

In recent years, Structural Health Monitoring (SHM) has proven its effectiveness in monitoring structures for localized damage detection. This technique consists in using the information of the gap between an undamaged and a damaged response of the structure to identify where the damage is located. The undamaged response is predicted by a physics-based model, involving the use of well-known techniques such as the Finite Element Method (FEM) [1], Finite Difference Method [2], to best capture the actual phenomena under study. SHM works very well, however a limitation of this method consists in the fact that the numerical model used to simulate the undamaged system must be very well calibrated, if not, numerical errors in the identification can appears.

To reduce these errors, data-based models built using machine learning [3] or deep learning techniques [4], are used. These data-based models are denoted as *digital twins*, making possible to approximate a real process and also gives real-time predictions in function of a parameter set. However, its implementation requires large amount of data for their construction, something not always possible in real-life, especially when experiment realizations are expensive.

For this reason, a new twin have been proposed in [5], the hybrid twin. The main idea consists in using the fact that the physics-based model even if it doesn't fit the real phenomena, it captures most of the physics and behavior of it, making possible to enrich it by the use of an *ignorance model* built using measured data. This allows the hybrid twin to gives close predictions of a real phenomena by using less amount of measured data [6, 7, 8], since the enrichment can be constructed in a coarse way, which is one of the key advantages of this technique. In details, the main idea consists in first fitting the physics-based model with respect to the data measured by sensors, allowing the numerical simulation to best predict the experience, such that the difference between the numerical predictions and reality is as coarse as possible. To next build a parametric regression of the ignorance model once some experimental realization are performed. One of the limitation for the construction of the hybrid twin is the fact that this model must be constructed and enriched "on the fly", and for this purpose the numerical model employed must give feedback in real-time for any parametric set.

This last is one of the big limitation in the context of SHM, especially when dealing with dynamics problems, since the computation of the dynamic response of the physic-based model can be very costly, even more, when large number of parameters are considered. Due to the reasons mentioned above, this paper propose at first, a solver able to compute at low-cost this response called here harmonic-hybrid algorithm, secondly a parametric model of this physics-based model is constructed using the model-order reduction technique sPGD when considering as parameters the location of the damage. The parametric approximation is able to predict in real-time the system's response for any set of these parameters, which allows further developments in order to enrich this model under real-time constraints, following a hybrid twin methodology. Once the hybrid twin is built, efficient deep learning techniques are used along this twin to be able to later predict damage location in the real structure by considering then as input only experimental data.

The present paper is structured as follows, first, Section 2 introduces the reference problem considered in this work. Section 3 introduces the harmonic-hybrid algorithm that helps to reduce simulations costs. Following Section 4 presents the construction of the hybrid twin, which main objective consists in construct a model that gives close predictions with respect to a real process. Section 5 introduces the deep learning architecture applied to the hybrid twin to construct a surrogate model able to predict damage location using experimental data. In the following, Section 6 introduces a numerical example that illustrate the performance of the proposed method. Finally Section 7 gives conclusions and perspectives.

2 Reference problem

Let us consider a domain $\Omega \subset \mathbb{R}^l$ with $l \in \{1, 2, 3\}$, on a time domain $I = [0, T]$ and with constant boundary $\partial\Omega = \partial_N\Omega \oplus \partial_D\Omega$ over time, where $\partial_N\Omega$ and $\partial_D\Omega$ are the boundaries related to the imposed Neumann and Dirichlet conditions respectively. This structure is submitted to surface forces $\underline{f}^N$ on $\partial_N\Omega \times I$ (Neumann boundary conditions), to imposed displacements $\underline{u}^D$ on $\partial_D\Omega \times I$ (Dirichlet boundary condition) and to volumetric forces $\rho\underline{f}$ on $\Omega \times I$, where one considers the hypothesis of small deformations.

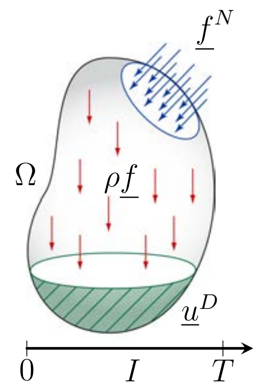


Figure 1: Mechanical domain under study.

The reference problem consists in finding a displacement field $\underline{u}(\underline{x}, t) \in \mathcal{U}$ and a stress field $\underline{\underline{\sigma}}(\underline{x}, t) \in \mathcal{F}$ verifying:

- Initial conditions:

on Ω,
$$\begin{aligned} \underline{u}|_{t=0} &= \underline{u}_0 \\ \dot{\underline{u}}|_{t=0} &= \dot{\underline{u}}_0 \end{aligned} \tag{1a}$$

- Dynamic equilibrium equation:

on $\Omega \times I$,
$$\rho\ddot{\underline{u}} = \text{div}(\underline{\underline{\sigma}}) + \rho\underline{f} \tag{1b}$$

- Neumann and Dirichlet boundary conditions:

$$\text{on } \partial_N\Omega \times I, \qquad \underline{\underline{\sigma}} \cdot \underline{n} = \underline{f}^N \tag{1c}$$

$$\text{on } \partial_D\Omega \times I, \qquad \underline{u} = \underline{u}^D \tag{1d}$$

- Constitutive relations:

  Since one considers small deformations the strain tensor is given as $\underline{\underline{\varepsilon}}(\underline{u}) = \frac{1}{2}\left(\nabla\underline{u} + \nabla\underline{u}^T\right)$ (where $(\bullet)^T$ denotes the transpose of $\bullet$). In this paper one focus on a localized damage, therefore one has $\Omega = \Omega_{\text{und}} \cup \Omega_{\text{dam}}$, where Ω_{und} denotes the domain that is not damaged and Ω_{dam} the domain where it exists damage respectively. From the above, the domain without damaged verifies a classic Hooke's linear constitutive relation while on the damaged zone one represent the damage as a scalar value $d \in [0,1]$, which simulate the loss of rigidity on the damaged zone. In this sense, one has:

$$\begin{aligned} \underline{\underline{\sigma}}(\underline{x}) &= \mathbb{K} : \underline{\underline{\varepsilon}}(\underline{x}) && \forall \underline{x} \in \Omega_{\text{und}} \\ \underline{\underline{\sigma}}(\underline{x}) &= (1-d)\mathbb{K} : \underline{\underline{\varepsilon}}(\underline{x}) && \forall \underline{x} \in \Omega_{\text{dam}} \end{aligned} \tag{1e}$$

with $\mathbb{K}$ the undamaged fourth order Hooke's tensor. Additionally, in the present work different parameters are considered in order to present a general framework. For instance, parameters related to the structure $\underline{\nu}$, parameters related to the external loading $\underline{\mu}$ and parameters related to the damage location denoted $\underline{\chi}$.

In order to mathematically formalize the presented reference problem, the following Section 2.1 presents the necessary formulations for its resolution.

2.1 Governing Equations

In order to solve the reference problem introduced in Section 1, the Finite Element Method (FEM) [1] is applied to its weak formulation, where the semi-discretized equation given below is obtained:

$$\underline{\underline{\boldsymbol{M}}}(\underline{\chi})\,\underline{\ddot{\boldsymbol{u}}}(\underline{\mu},\underline{\chi};t) + \underline{\underline{\boldsymbol{D}}}(\underline{\chi})\,\underline{\dot{\boldsymbol{u}}}(\underline{\mu},\underline{\chi};t) + \underline{\underline{\boldsymbol{K}}}(\underline{\chi})\,\underline{\boldsymbol{u}}(\underline{\mu},\underline{\chi};t) = \underline{\boldsymbol{F}}(\underline{\mu};t) \tag{2}$$

where $\underline{\boldsymbol{u}}(\underline{\mu},\underline{\chi};t)$ represents the nodal values of the displacement for time instant t, $\underline{\underline{\boldsymbol{M}}}(\underline{\chi})$, $\underline{\underline{\boldsymbol{D}}}(\underline{\chi})$, $\underline{\underline{\boldsymbol{K}}}(\underline{\chi})$ and $\underline{\boldsymbol{F}}(\underline{\mu};t)$ corresponds to the mass, damping, stiffness matrices and vector of external imposed forces respectively. Here a Rayleigh damping is used, i.e $\underline{\underline{\boldsymbol{D}}} = \alpha\underline{\underline{\boldsymbol{M}}} + \beta\underline{\underline{\boldsymbol{K}}}$, where α and β are constants that must be chosen appropriately in function of the material considered.

Equation (2), which is written in space-time, can also be treated in the space-frequency domain by applying a Fourier transform, giving:

$$\left[-\omega^2\underline{\underline{\boldsymbol{M}}}(\underline{\chi}) + i\omega\underline{\underline{\boldsymbol{D}}}(\underline{\chi}) + \underline{\underline{\boldsymbol{K}}}(\underline{\chi})\right]\underline{\hat{\boldsymbol{u}}}(\underline{\mu},\underline{\chi};\omega) = \underline{\hat{\boldsymbol{F}}}(\underline{\mu};\omega) \tag{3}$$

where we denote $\underline{\hat{\boldsymbol{u}}}(\underline{\mu}, \underline{\chi}; \omega) = \mathcal{F}(\underline{\boldsymbol{u}}(\underline{\mu}, \underline{\chi}; t))$ the Fourier transformation in time of the nodal displacements.

The direct resolution of (2) or (3) can be very costly especially if the number of spatial DOFs is large, even more, this computational effort is exacerbated if a parametric resolution must be performed. To overcome this limitation, a new resolution technique is proposed in this paper. This technique is presented in the following Section.

3 Low-cost solver in dynamics: a modal-harmonic frequency approach

If one considers the presence of localized damage in the structure, equation (3) can be rewritten as follows:

$$\left[-\omega^2\left(\underline{\underline{\tilde{\boldsymbol{M}}}} + \Delta\underline{\underline{\boldsymbol{M}}}(\underline{\chi})\right) + i\omega\left(\underline{\underline{\tilde{\boldsymbol{D}}}} + \Delta\underline{\underline{\boldsymbol{D}}}(\underline{\chi})\right) + \left(\underline{\underline{\tilde{\boldsymbol{K}}}} + \Delta\underline{\underline{\boldsymbol{K}}}(\underline{\chi})\right)\right]\underline{\hat{\boldsymbol{u}}}(\underline{\mu}, \underline{\chi}; \omega) = \underline{\hat{\boldsymbol{F}}}(\underline{\mu}; \omega) \quad (4)$$

where $\underline{\underline{\tilde{\boldsymbol{M}}}}$, $\underline{\underline{\tilde{\boldsymbol{D}}}}$ and $\underline{\underline{\tilde{\boldsymbol{K}}}}$ denotes the undamaged mass, damping and stiffness matrices respectively, in addition one defines $\Delta\underline{\underline{\boldsymbol{M}}}(\underline{\chi})$, $\Delta\underline{\underline{\boldsymbol{D}}}(\underline{\chi})$ and $\Delta\underline{\underline{\boldsymbol{K}}}(\underline{\chi})$ the corrective matrices that must be added in order to take into account the localized damage.

The proposed technique of resolution to reduce computational costs is performed on three principal steps: (i) a regularized solution $\underline{\hat{\boldsymbol{u}}}_{reg}$ is computed, whose objective consists in verifying the imposition of external charges, (ii) a modal-base approximation $\underline{\hat{\boldsymbol{u}}}_{dyn}$ is determined which allows to approximately catch the dynamic behavior of the system at low-cost and (iii) an intrusive PGD solution $\underline{\hat{\boldsymbol{u}}}_{PGD}(\underline{\mu}, \underline{\chi}; \omega)$ which allows to finally correct and reach the desired solution. This can be mathematically expressed as follows:

$$\underline{\hat{\boldsymbol{u}}}(\underline{\mu}, \underline{\chi}; \omega) \approx \underline{\hat{\boldsymbol{u}}}_{reg}(\underline{\mu}; \omega) + \underline{\hat{\boldsymbol{u}}}_{dyn}(\underline{\mu}, \underline{\chi}; \omega) + \underline{\hat{\boldsymbol{u}}}_{PGD}(\underline{\mu}, \underline{\chi}; \omega) \quad (5)$$

where each term is computed one after the other. In the following lines, the determination of each term is presented.

• **Regularization solution**:

Since the regularization solution exists to verify the external loads on the structure, it is therefore determined by solving:

$$\underline{\underline{\tilde{\boldsymbol{K}}}}\,\underline{\hat{\boldsymbol{u}}}_{reg}(\underline{\mu}; \omega) = \underline{\hat{\boldsymbol{F}}}(\underline{\mu}; \omega) \quad (6)$$

which corresponds to a quasi-static problem. By using an intrusive approach of the PGD, the resolution of (6) can be obtain at low-cost, where the solution can be approximated as follows:

$$\underline{\hat{\boldsymbol{u}}}_{reg}(\underline{\mu}; \omega) \approx \underline{\underline{\boldsymbol{\Psi}}}\,\boldsymbol{\varphi}(\underline{\mu}; \omega) \quad (7)$$

with $\underline{\underline{\boldsymbol{\Psi}}} = [\underline{\boldsymbol{\psi}}_1, ..., \underline{\boldsymbol{\psi}}_m]$ and $\boldsymbol{\varphi}(\underline{\mu}; \omega) = [\varphi_1(\underline{\mu}; \omega), ..., \varphi_m(\underline{\mu}; \omega)]^T$ the discretized spatial and frequency PGD functions respectively, with m the number of PGD modes of the decomposition.

• **Modal-base solution**:

On the other hand, the dynamic counterpart is approximated as best as possible using the modal basis of the undamaged structure, such as:

$$\hat{\underline{\boldsymbol{u}}}_{dyn}(\underline{\mu}, \underline{\chi}; \omega) \approx \underline{\underline{\boldsymbol{\Phi}}}\, \underline{\boldsymbol{\xi}}(\underline{\mu}, \underline{\chi}; \omega) \tag{8}$$

with $\underline{\underline{\boldsymbol{\Phi}}} = [\underline{\boldsymbol{\phi}}_1, ..., \underline{\boldsymbol{\phi}}_{m_{dyn}}]$ and $\underline{\boldsymbol{\xi}}(\underline{\mu}, \underline{\chi}; \omega) = [\xi_1(\underline{\mu}, \underline{\chi}; \omega), ..., \xi_{m_{dyn}}(\underline{\mu}, \underline{\chi}; \omega)]^T$ the discretized spatial modal functions and the modal variables in frequency respectively, where m_{dyn} denotes the number of modal basis modes considered. This approximation is valid for the cases where a localized damage is considered, such as the damage doesn't influence as much the modal basis of the structure. This approach is considered due to its advantages in reducing numerical costs as will be seen below. By using expression (6) in (4) one obtains:

$$\left[-\omega^2 \left(\tilde{\underline{\underline{\boldsymbol{M}}}} + \Delta \underline{\underline{\boldsymbol{M}}}(\underline{\chi})\right) + i\omega \left(\tilde{\underline{\underline{\boldsymbol{D}}}} + \Delta \underline{\underline{\boldsymbol{D}}}(\underline{\chi})\right) + \left(\tilde{\underline{\underline{\boldsymbol{K}}}} + \Delta \underline{\underline{\boldsymbol{K}}}(\underline{\chi})\right)\right] \hat{\underline{\boldsymbol{u}}}_{dyn}(\underline{\mu}, \underline{\chi}; \omega) = \hat{\underline{\boldsymbol{G}}}(\underline{\mu}, \underline{\chi}; \omega) \tag{9}$$

with:

$$\hat{\underline{\boldsymbol{G}}}(\underline{\mu}, \underline{\chi}; \omega) = -\left[-\omega^2 \underline{\underline{\boldsymbol{M}}}(\underline{\chi}) + i\omega \underline{\underline{\boldsymbol{D}}}(\underline{\chi}) + \Delta \underline{\underline{\boldsymbol{K}}}(\underline{\chi})\right] \hat{\underline{\boldsymbol{u}}}_{reg}(\underline{\mu}; \omega) \tag{10}$$

Now by using the approximation (8) of the dynamic term, and by projecting the equilibrium equation by $\underline{\underline{\boldsymbol{\Phi}}}^T$, one obtains:

$$\left[-\omega^2 \left(\underline{\underline{\boldsymbol{m}}} + \Delta \underline{\underline{\boldsymbol{m}}}(\underline{\chi})\right) + i\omega \left(\underline{\underline{\boldsymbol{d}}} + \Delta \underline{\underline{\boldsymbol{d}}}(\underline{\chi})\right) + \left(\underline{\underline{\boldsymbol{k}}} + \Delta \underline{\underline{\boldsymbol{k}}}(\underline{\chi})\right)\right] \underline{\boldsymbol{\xi}}(\underline{\mu}, \underline{\chi}; \omega) = \underline{\bar{\boldsymbol{f}}}(\underline{\mu}; \omega) \tag{11}$$

with:

$$\underline{\bar{\boldsymbol{f}}}(\underline{\mu}; \omega) = \underline{\underline{\boldsymbol{\Phi}}}^T \underline{\bar{\boldsymbol{F}}}(\underline{\mu}; \omega) \quad ; \quad \underline{\underline{\boldsymbol{m}}} = \underline{\underline{\boldsymbol{\Phi}}}^T \tilde{\underline{\underline{\boldsymbol{M}}}}\, \underline{\underline{\boldsymbol{\Phi}}} \quad ; \quad \underline{\underline{\boldsymbol{d}}} = \underline{\underline{\boldsymbol{\Phi}}}^T \tilde{\underline{\underline{\boldsymbol{D}}}}\, \underline{\underline{\boldsymbol{\Phi}}} \quad ; \quad \underline{\underline{\boldsymbol{k}}} = \underline{\underline{\boldsymbol{\Phi}}}^T \tilde{\underline{\underline{\boldsymbol{K}}}}\, \underline{\underline{\boldsymbol{\Phi}}}$$

and:

$$\Delta \underline{\underline{\boldsymbol{m}}}(\underline{\chi}) = \underline{\underline{\boldsymbol{\Phi}}}^T \Delta \underline{\underline{\boldsymbol{M}}}(\underline{\chi})\, \underline{\underline{\boldsymbol{\Phi}}} \quad ; \quad \Delta \underline{\underline{\boldsymbol{d}}}(\underline{\chi}) = \underline{\underline{\boldsymbol{\Phi}}}^T \Delta \underline{\underline{\boldsymbol{D}}}(\underline{\chi})\, \underline{\underline{\boldsymbol{\Phi}}} \quad ; \quad \Delta \underline{\underline{\boldsymbol{k}}}(\underline{\chi}) = \underline{\underline{\boldsymbol{\Phi}}}^T \Delta \underline{\underline{\boldsymbol{K}}}(\underline{\chi})\, \underline{\underline{\boldsymbol{\Phi}}}$$

By rearranging the terms of (11), one obtains:

$$\left[-\omega^2 \underline{\underline{\boldsymbol{m}}} + i\omega \underline{\underline{\boldsymbol{d}}} + \underline{\underline{\boldsymbol{k}}}\right] \underline{\boldsymbol{\xi}}^n(\underline{\mu}, \underline{\chi}; \omega) = \underline{\bar{\boldsymbol{f}}}(\underline{\mu}; \omega) - \left[-\omega^2 \Delta \underline{\underline{\boldsymbol{m}}}(\underline{\chi}) + i\omega \Delta \underline{\underline{\boldsymbol{d}}}(\underline{\chi}) + \Delta \underline{\underline{\boldsymbol{k}}}(\underline{\chi})\right] \underline{\boldsymbol{\xi}}^{n-1}(\underline{\mu}, \underline{\chi}; \omega) \tag{12}$$

where "n" denotes the index of the iterative procedure. The resolution stops until we reach a certain threshold of an indicator ϵ expressed as:

$$\epsilon = \left\| \underline{\boldsymbol{\xi}}^n - \underline{\boldsymbol{\xi}}^{n-1} \right\| / \left\| \underline{\boldsymbol{\xi}}^{n-1} \right\| \tag{13}$$

The main advantage of the above formulation is that matrices $\underline{\underline{\boldsymbol{m}}}$, $\underline{\underline{\boldsymbol{d}}}$ and $\underline{\underline{\boldsymbol{k}}}$ are diagonal, therefore the resolution of (12) is complete decoupled for each modal-basis mode, drastically reducing in this sense the computational times for its resolution.

• **Intrusive PGD solution**:

Once the procedure stops, a good approximation of the solution is obtained, however, since a fixed modal-basis computed from the undamaged structure is considered, the quality of the solution can have some errors due to this approximation. In this sense, a last step is applied, which consists on an intrusive PGD in order to ensure the quality of the final solution. Since the use of the fixed modal-basis capture most of the solution, the additional number of PGD modes added at the end are few. This problem can therefore be formulated as follows:

$$\left[-\omega^2 \underline{\underline{\boldsymbol{M}}}(\underline{\chi}) + i\omega \underline{\underline{\boldsymbol{D}}}(\underline{\chi}) + \underline{\underline{\boldsymbol{K}}}(\underline{\chi})\right] \underline{\hat{\boldsymbol{u}}}_{PGD}(\underline{\mu}, \underline{\chi}; \omega) = \underline{\hat{\boldsymbol{H}}}(\underline{\mu}, \underline{\chi}; \omega) \tag{14}$$

where:

$$\underline{\hat{\boldsymbol{H}}}(\underline{\mu}, \underline{\chi}; \omega) = -\left[-\omega^2 \underline{\underline{\boldsymbol{M}}}(\underline{\chi}) + i\omega \underline{\underline{\boldsymbol{D}}}(\underline{\chi}) + \underline{\underline{\boldsymbol{K}}}(\underline{\chi})\right] \left(\underline{\hat{\boldsymbol{u}}}_{reg}(\underline{\mu}; \omega) + \underline{\hat{\boldsymbol{u}}}_{dyn}(\underline{\mu}, \underline{\chi}; \omega)\right) \tag{15}$$

The details of the determination of this solution are not presented in this paper for the seek of simplicity, however, the reader can refer to [9] for technical details.

Once $\underline{\hat{\boldsymbol{u}}}(\underline{\mu}, \underline{\chi}; \omega)$ is computed following (5), its temporal counterpart $\underline{\boldsymbol{u}}(\underline{\mu}, \underline{\chi}; t)$ is simply determined by an inverse FFT. This temporal response is the capital information used all along the present paper.

4 Hybrid twin applied to SHM

As explained in the introductory Section, the objective of hybrid twins consists in enriching a physics-based model using measured data. In the context of this work, the hybrid twin is used for two principal objectives: (i) improving the prediction of deformation on sensor position, in order to be the closest possible to the real process, and (ii) to use its predictions to build a surrogate model in order to be able to predict a posteriori the damage location of the real process using as input measured data.

To do so, the hybrid twin construction is separated into two steps, the first consists in the parametrization of the physics-based model, which is further used to fit it with respect to the measured data. Later, the ignorance model is built using measured data, in where both the physics-based and ignorance model are sum up to gives the hybrid twin, whose aims consists in giving accurate predictions with respect to reality, this is:

$$\text{Real process} \approx \text{Hybrid twin} = \text{Physics-based} + \text{Ignorance model} \tag{16}$$

The parameterization of the physics-based model is presented in Section 4.1 and the construction of the ignorance model in Section 4.2.

4.1 Real-time prediction of the physics-based model over a parametric domain

The algorithm presented in this section has the great advantage of being able to predict a dynamic response of the structure for any parameter considered at low-cost, or in real-time. This property is of utmost importance as mentioned in the introductory Section in order to implement a robust hybrid twin, specially in the sense of model fitting, capital for the construction of the ignorance model introduced in Section 4.2. The idea consists in building a parametric model of the dynamic response on selected sensors in the structure. To do so, here the sparse Proper Generalized Decomposition (sPGD) [10, 11] is considered, which allows to interpolate the response over a parametric domain in a separate variable multiplication format. In this sense, the sPGD approximation of the deformation tensor on sensor i is defined as follows:

$$\underline{\underline{\varepsilon}}^{[i]}(\underline{\nu}, \underline{\mu}, \underline{\chi}) \approx \underline{\underline{\varepsilon}}^{[i]}_{m_i}(\underline{\nu}, \underline{\mu}, \underline{\chi}) = \sum_{k=1}^{m_i} \left[\prod_{j=1}^{n_\nu} X_j^{k,i}(\nu_j) \right] \left[\prod_{j=1}^{n_\mu} Y_j^{k,i}(\mu_j) \right] \left[\prod_{j=1}^{n_\chi} Z_j^{k,i}(\chi_j) \right] \tag{17}$$

where m_i denotes the number of PGD modes of the decomposition [9]. In addition, n_μ, n_ν and n_χ denotes a general number of parameters related to material properties of the structure, parameters related to the applied external force and parameters related to damage respectively.

4.2 Construction of the ignorance model

Here, the ignorance model is constructed. The first step for its construction consists in fitting an undamaged structure against the parametric physics-based model when considering no damage and where the parameters related to the external force are known (i.e amplitude of the force, frequency content, etc). This, in order to obtain the material parameters that best approximate the measured response and the real process. This inverse analysis is performed using the Levenberg–Marquardt algorithm [12, 13]. This procedure is expressed mathematically as follows:

$$\{\underline{\hat{\nu}}\} = \arg\min_{\{\underline{\nu}\}} \sum_{i=1}^{\text{Nsensors}} \left[\frac{\underline{\underline{\varepsilon}}^{[i]}_{\text{mes}} - \underline{\underline{\varepsilon}}^{[i]}_{m_i}(\underline{\nu})}{\underline{\underline{\varepsilon}}^{[i]}_{\text{mes}}} \right]^2 \tag{18}$$

where $\underline{\underline{\varepsilon}}^{[i]}_{\text{mes}}$ denotes the strain tensor measured at sensor i. Let us note in (18) that the dependence of $\underline{\underline{\varepsilon}}^{[i]}_{m_i}$ on $\underline{\mu}$ and $\underline{\chi}$ is removed, because these parameters are not considered for the minimization problem.

Once some experiment realizations are performed, where damage location of the experience is known, the information of the gap between the physics-based prediction strain and the measurements is stored, which is given by:

$$\Delta\underline{\underline{\varepsilon}}^{[i]}(\underline{\hat{\nu}}, \underline{\hat{\chi}}) = \underbrace{\underline{\underline{\varepsilon}}^{[i]}_{\text{mes}}(\underline{\hat{\chi}})}_{\text{Measurement}} - \underbrace{\underline{\underline{\varepsilon}}^{[i]}_{m_i}(\underline{\hat{\nu}}, \underline{\hat{\chi}})}_{\text{sPGD}} \tag{19}$$

Once a sufficient amount of data have been collected, the ignorance model is interpolated along the parametric space corresponding to the damage location. The reader may note that these points are potentially unstructured in nature. In this sense, here the sPGD is again considered for the construction of the ignorance model due to its robustness in providing an accurate prediction. Therefore one defines the interpolation of the ignorance $\Delta\underline{\underline{\varepsilon}}^{[i]}_{\tilde{m}_i}(\underline{\hat{\nu}}, \underline{\chi})$ following a similar approximation as (17), such as:

$$\Delta \underline{\underline{\varepsilon}}^{[i]}(\underline{\hat{\nu}}, \underline{\chi}) \approx \Delta \underline{\underline{\varepsilon}}^{[i]}_{\tilde{m}_i}(\underline{\hat{\nu}}, \underline{\chi}) \tag{20}$$

where for this case $\tilde{m}_i$ corresponds to the number of modes considered for the sPGD approximation of the ignorance corresponding to sensor i. From all the above, the hybrid twin is simply constructed by adding the contribution of the physics-based model (17) and ignorance model (20), giving:

$$\underline{\underline{\varepsilon}}^{[i]}_{\text{HT}}(\underline{\chi}) = \underbrace{\underline{\underline{\varepsilon}}^{[i],m_i}_{\text{ph}}(\underline{\nu}, \underline{\mu}, \underline{\chi})}_{\text{physics-based}} + \underbrace{\Delta \underline{\underline{\varepsilon}}^{[i]}_{\text{ig}}(\underline{\hat{\nu}}, \underline{\chi})}_{\text{ignorance model}} \tag{21}$$

5 Identification of damage location using the hybrid twin and deep learning techniques

Here, the main idea consists in determining the localized damage properties, which here corresponds to damage location. These parameters can be determined using any inverse technique, however, in order to perform high efficient identification of these parameters, in this work one considers a Deep Learning approach consisting on a feedfordward neural network. As input for the NN one considered the difference between an *undamaged* and *damaged* response of the total deformation. Here the strain comming from the undamaged structure ($\underline{\underline{\varepsilon}}_{\text{und}}(t)$) could come from the physics-based model when considering no damage or a direct measure and the strain corresponding to the damaged structure comes from the hybrid twin ($\underline{\underline{\varepsilon}}_{\text{HT}}(\underline{\chi}; t)$), such as:

$$\Delta \underline{\underline{\varepsilon}}(\underline{\mu}, \underline{\chi}; t) = \underline{\underline{\varepsilon}}_{\text{und}}(t) - \underline{\underline{\varepsilon}}_{\text{HT}}(\underline{\chi}; t) \tag{22}$$

and as output one considers the damage location $\underline{\chi}$. The hidden layers considers hyperbolic tangent as activation functions and linear activation function at the output layer [4]. The quantity of hidden layers depends on the problem to be tested. In this sense, for the identification of damage location one considers the following neural network mapping NN$(\cdot)$:

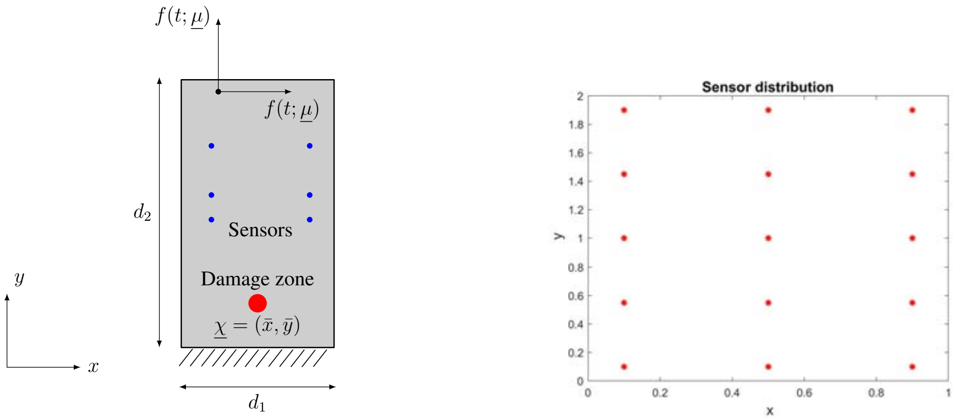
$$\text{NN}(\Delta \underline{\underline{\varepsilon}}(\underline{\mu}, \underline{\chi}; t)) \rightarrow \underline{\chi}_{\text{pred}} \tag{23}$$

where $\underline{\chi}_{\text{pred}}$ denotes the prediction of the NN, such as $\underline{\chi}_{\text{pred}} \approx \underline{\chi}$.

Remark: It should be noted that experimentally it is not always possible to measure all components of the strain tensor, so here the term $\Delta \underline{\underline{\varepsilon}}(\underline{\mu}, \underline{\chi}; t)$ denotes only the information of the experimentally available components.

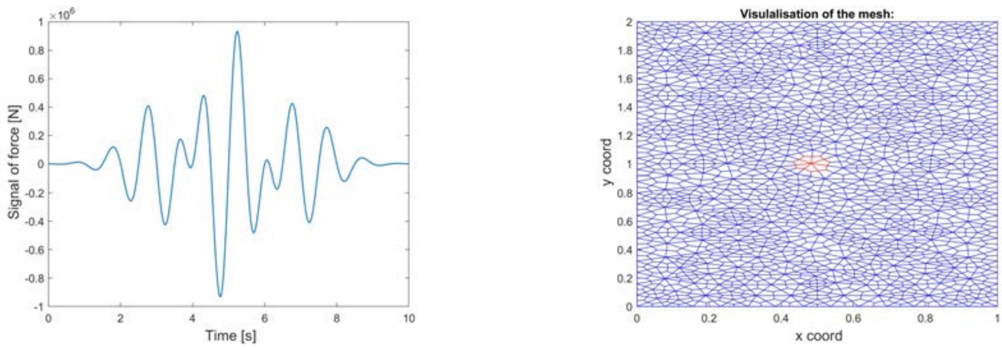
6 Numerical example

The numerical test considered here consists on a rectangular plate, on which a time-dependent point force is imposed as illustrated in Figure 2a, in addition to the above, a sensor arrangement as shown in Figure 2b is considered. The plate is modeled under a plane stress condition. The dimensions of the plate are $d_1 = 1\ [m] \quad ; \quad d_2 = 2\ [m]$. For the sake of simplicity no parameters for the loading term are considered, however, its consideration can be taken into account without



(a) (b)

Figure 2: Illustration of the reference problem (2a) and sensor distribution considered (2b) where the sensors are represented by red dots.

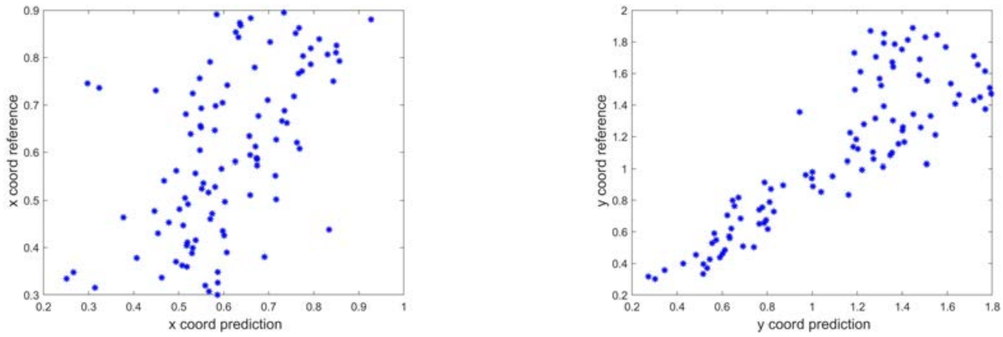


(a) (b)

Figure 3: External force $f(t)$ considered (3a) and representation of the localized damage by red elements (3b).

difficulty. The applied force is defined as a combination of sinusoids at different frequencies, the time evolution of the force is illustrated in Figure 3a. Localized damage is also considered in the structure, with a unique severity $d = 0.9$. The damaged zones in the structure are introduced as groups of finite elements of the mesh, as represented in Figure 3b.

In order to test the hybrid twin methodology presented in this work, one considers a physics-based model consisting on an isotropic structure, while an anisotropic structure is considered for the synthetic "real experience". After the construction of the hybrid twin by using a total of 50 experimental realizations, the strain tensor of the real process is approximated following (21). With this information, the damage detection algorithm of Section 5 is applied, obtaining the results presented in Figures 4a and 4b for the prediction of the x and y location of the damage, where $17[\%]$ error of prediction is achieved.



(a) (b)

Figure 4: Prediction versus reference for the x coordinate of the damage location (4a) and y coordinate (4b).

7 Conclusions and perspectives

This paper shows the implementation of a hybrid twin applied to the Structural Health Monitoring technique when dealing with structures dynamics problems. The main idea of the hybrid twin consists in approximating a real process throw the use of a physics-based model enriched with measured data throw the use of an ignorance model. The physics-based simulations are obtain by means of a hybrid-harmonic technique, which allows to decrease the computational burden associated to the resolution of the dynamic response, making possible the perform of parametric studies at low-cost. Later a parametric model is built by applying model-order reduction technique sparse-Proper Generalized Decomposition (sPGD). The ignorance model is also built after some experimental realizations are perform by using the sPGD.

Here the hybrid twin is applied to a structure in dynamics in order to delivers accurate strains predictions, which are later used to built a surrogate model throw the use of deep learning techniques able to predict damage localization in the structure. Such as this surrogate model can be later used to predict damage location in the structure only using measured data of the real process. The principal objective of building a surrogate model throw the use of a hybrid twin is to reduce the number of experimental realizations, which in general are limited due to costly experimental campaigns.

As a perspective, the introduction of more optimized deep learning techniques than the one used in this work is considered to improve the prediction of damage localization.

REFERENCES

[1] O. C. Zienkiewicz and R. L. Taylor, *The finite element method for solid and structural mechanics*. Elsevier, 2005.

[2] G. D. Smith, G. D. Smith, and G. D. S. Smith, *Numerical solution of partial differential equations: finite difference methods*. Oxford university press, 1985.

[3] S. J. Russell, *Artificial intelligence a modern approach*. Pearson Education, Inc., 2010.

[4] I. Goodfellow, Y. Bengio, and A. Courville, *Deep learning*. MIT press, 2016.

[5] F. Chinesta, E. Cueto, E. Abisset-Chavanne, J. L. Duval, and F. E. Khaldi, "Virtual, digital and hybrid twins: a new paradigm in data-based engineering and engineered data," *Archives of computational methods in engineering*, vol. 27, no. 1, pp. 105–134, 2020.

[6] A. Sancarlos, M. Cameron, J.-M. Le Peuvedic, J. Groulier, J.-L. Duval, E. Cueto, and F. Chinesta, "Learning stable reduced-order models for hybrid twins," *Data-Centric Engineering*, vol. 2, 2021.

[7] A. Sancarlos, M. Cameron, A. Abel, E. Cueto, J.-L. Duval, and F. Chinesta, "From rom of electrochemistry to ai-based battery digital and hybrid twin," *Archives of Computational Methods in Engineering*, vol. 28, no. 3, pp. 979–1015, 2021.

[8] V. Champaney, F. Chinesta, and E. Cueto, "Engineering empowered by physics-based and data-driven hybrid models: A methodological overview," *International Journal of Material Forming*, vol. 15, no. 3, pp. 1–14, 2022.

[9] F. Chinesta and E. Cueto, *PGD-based modeling of materials, structures and processes*. Springer, 2014.

[10] R. Ibáñez, E. Abisset-Chavanne, A. Ammar, D. González, E. Cueto, A. Huerta, J. L. Duval, and F. Chinesta, "A multidimensional data-driven sparse identification technique: the sparse proper generalized decomposition," *Complexity*, vol. 2018, 2018.

[11] A. Sancarlos, V. Champaney, J.-L. Duval, E. Cueto, and F. Chinesta, "Pgd-based advanced nonlinear multiparametric regressions for constructing metamodels at the scarce-data limit," *arXiv preprint arXiv:2103.05358*, 2021.

[12] C. Ghnatios, F. Masson, A. Huerta, A. Leygue, E. Cueto, and F. Chinesta, "Proper generalized decomposition based dynamic data-driven control of thermal processes," *Computer Methods in Applied Mechanics and Engineering*, vol. 213, pp. 29–41, 2012.

[13] H. P. Gavin, "The levenberg-marquardt algorithm for nonlinear least squares curve-fitting problems," *Department of Civil and Environmental Engineering, Duke University*, vol. 19, 2019.