COST ESTIMATION OF NEW CARS AND EQUIPMENT UNDER CONDITIONS OF UNCERTAINTY

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Abstract. *Methods used in the valuation of assets are usually referred to either a market, or income, or cost approach. Meanwhile, correct valuation methods can be obtained by applying several approaches together. One of these methods is based on the formula proposed by D.S. Lvov at one time, which links the market values of the assessed machine and its analogue and takes into account differences in the operating characteristics of machines and their useful lives. However, when deriving the formula, it is assumed that the characteristics of the machine do not change during the useful life. We propose a generalization of this formula for a situation where the operating characteristics of machines change during their operation, and the useful life is random.*

Keywords: *machinery, equipment, valuation, benefits, useful life, probability distribution, market value, cash flow discounting.*

1.Introduction

The cost estimation of various objects is carried out by professional appraisers in accordance with International Valuation Standards (IVS), national valuation standards, and current legislation. Machines and equipment (hereinafter referred to as machines) are among the objects subject to cost estimation. Market value assessment methods for machines are not extensively covered in valuation literature. Some of them are described in (Asaul et al., 2011; Fedotova, 2018). However, most of the practical methods used are based on overly rigid (insufficiently realistic) assumptions.

There are many types of costs, but the primary one is market value (MV). In IVS, the definition of this concept takes up several pages. Therefore, we will not provide such a definition, but we will note that the MV of an object on the valuation date reflects both the price of the object in a transaction conducted on that date between typical and economically rational market participants, and the benefits obtained by the owner of the object from its future use.

This article is dedicated to developing one of the well-known methods for estimating MV of machines in order to more fully account for factors such as physical wear and uncertainty in the term of their useful life.

Market value estimation methods usually fall into the cost, comparison, or income approach. Most often, the market value of machines is evaluated using the comparison approach, utilizing information on market prices of its analogs. Adjustments, either additive or multiplicative, are made to the analog price to account for differences between the evaluated machine and the analog in certain characteristics. Income approach methods do not require information on analogs of the evaluated machine. They utilize information on the economic benefits obtained by the owner of the machine from its subsequent use.

Machines produced under the same project are divided into new (not yet in operation) and used machines. The market value of used machines is usually estimated by properly adjusting the cost of similar new ones (Smolyak, 2016; Fedotova, 2018). The methods for such adjustments deserve separate consideration, and we will not dwell on them. We will instead focus on the assessment of new machines. In most cases, their market value is estimated using the comparative approach based on manufacturer or dealer prices. However, it is not uncommon that the assessed machines are not available on the market at the valuation date. This is precisely the situation discussed in this article. It turns out that there is an opportunity to assess the market value of a machine using a combination of ideas from comparative and income approaches. One method of this type is based on a formula proposed earlier by Lvov in 1969, which was widely used in the Soviet era to assess the efficiency of new technology and establish prices for new engineering products. In this article, we first provide necessary definitions of key concepts, briefly summarize Lvov's formula, and then demonstrate how it can be improved to more accurately account for the characteristics of the machines' operation process.

2.Methods.

2. 1 Key Concepts. Lvov's Formula

Machines are typically used for their intended purpose, performing specific tasks. This work has utility for market participants and therefore has a certain market value. The cost of some tasks (such as painting surfaces or transporting goods) can be estimated using market data, however, many tasks are intermediate operations in the technological process of producing the final product, and assessing their market value presents certain difficulties.

The main characteristics of a machine's use in a certain period are its productivity and operational costs (excluding depreciation and taxes). By benefits from the use of the machine in a certain period, we mean the market value of the work performed by it in this period minus the operational costs. In connection with this definition, three circumstances need to be noted:

IVS does not provide a clear definition of the concept of benefits, but our definition is not inconsistent with other provisions of these standards.

The benefits indicator is similar in content to the net income and EBITDA metrics used in business valuation.

The amount of benefits generated by the machine in a period simultaneously reflects the market value of renting out the machine for that period. In the System of National Accounts (SNA 2008), where the market value of capital assets is also assessed using comparative and income approaches, benefits are interpreted as services [embedded in the asset] capital.

We will also assume that upon the end of the useful life span (ULS), the machine is disposed of, yielding zero benefits. This assumption is justified, as costs and revenues associated with disposal usually differ insignificantly.

Considering that the ULS for machines may not be a whole number of years, and their characteristics change continuously, the process of using the machine will be considered in continuous time. In this case, it is convenient to use indicators of the intensity of operational costs and benefits, reflecting the costs and benefits of using the machine over a small unit of time.

In the situation considered by D.S. Lvov, inflation is assumed to be absent, time is measured in years and fractions of a year, and the influence of random factors is absent. In this context, the machine is assumed to have a known useful life span of T years, productivity of W, and intensity of operational costs (excluding depreciation and taxes) Z. Let's denote the market value of one unit of work performed by the machine as p. In this case, the intensity of benefits generated by it (D) can be defined as the value of work performed in a unit of time minus the corresponding operational costs. This leads to the following formula:

$$D=pW-Z. \tag{1}$$

The relationship between the asset's value and the benefits from its use is established using the principle of anticipated benefits. It is mentioned in IVS among the basic principles, and its essence is elaborated in a series of provisions of the standard describing valuation methods. However, a specific formulation of the principle is not provided in IVS. In Smolyak (2016), this principle is presented in the following form, applicable even in conditions of uncertainty: the asset's value on the valuation date is equal to the expected sum of discounted benefits from its subsequent economically rational use in the forecast period (including the asset's value at the end of the period). The forecast period can be chosen arbitrarily.

Since during the design of the machine, efforts are made to select economically rational values for its characteristics, and typical machine owners use them for their intended purpose, we will consider that the machine's characteristics mentioned above correspond to its economically rational use. In this section, the benefits generated by the machine are deterministic, so from the principle of anticipated benefits, it follows that the value of a new machine V is equal to the sum of discounted (at the market to pre-tax continuous rate r, 1/year) benefits from its use:

$$V=\int_0^T De^{-rt}dt=D\frac{1-e^{-rT}}{r}=\left(pW-Z\right)m_c\left(T;r\right), \tag{2}$$

Where $m_c(T;r)$ is the time-constant income multiplier defined by the formula:

$$m_c\left(T;r\right)=\int_0^T e^{-rt}dt=\frac{1-e^{-rT}}{r}. \tag{3}$$

Note that assessors typically determine market pre-tax discount rates (E) in annual percentage terms. The equivalent rate in continuous time is calculated using the formula: $r=\ln\left(1+E/100\right)$.

When the cost of work p is known, the machine could be evaluated using formula (2). However, as noted, this is not always possible. It is precisely for such situations that Lvov's formula was proposed. Essentially, it is based on a comparative approach to assessing the market value. It is assumed that a similar new machine (performing the same tasks) can be found on the market, for which not only its productivity Wa, operational cost intensity Za, and useful life span Ta are known, but also its market value Va. For this machine, formula (2) will be written as follows: $V_a=\left(pW_a-Z_a\right)m\left(T_a;r\right).$ (4)

From this equality, an unknown unit of work can be found:

$$p=\frac{V_a+Z_am\left(T_a;r\right)}{W_am\left(T_a;r\right)}. \tag{5}$$

It is easy to see that the obtained cost p equals the ratio of the total discounted one-time and current costs of performing the work over the entire period of operation of the counterpart to the total discounted volume of work, which fully complies with the requirements of the cost approach to valuation (IVS; Fedotova, 2018). In other words, the cost of one unit of work when using a combination of the comparative and income approaches precisely coincides with the specific discounted costs of producing these works using the counterpart machine.

Substituting the value of p from (4) into formula (1), we get:

$$V = V_a \cdot \frac{W}{W_a} \cdot \frac{m(T;r)}{m(T_a;r)} + \left[Z_a \frac{W}{W_a} - Z \right] \cdot m(T;r). \tag{6}$$

It was this formula (in a slightly different form and for discrete time) that was proposed by D.S. Lvov in his doctoral dissertation and book (Lvov, 1969), and then widely used in the Soviet Union for assessing the efficiency of new technology (Methodology, 1977) and establishing prices for new industrial products. In (Asaul, 2011) and several other sources, the corresponding evaluation method is referred to as the equally effective analog method.

As seen from formula (6), the cost of the machine can be found by adjusting the cost of the counterpart considering its differences in productivity, operational costs, and lifespan, which corresponds to the comparative approach to valuation.

Note that formula (6) is also applicable to used machines, however, obtaining information about productivity, operational costs, and lifespan for such machines can be challenging. Let us specify two significant drawbacks of Lvov's formula:

Machines that experience changes in their technical and economic characteristics during operation do not exist. However, sometimes, for illustrative purposes, examples of evaluating similar objects are provided in appraisal literature (referred to as "one-hossshay" in English-language literature).

The lifespan of the machine is assumed to be known. However, machines of the same make, released simultaneously and operating under identical conditions, do not retire from operation simultaneously. In reliability theory, the lifespans of machines (which may differ from the assumed lifespan) are considered random.

Next, we will try to address these shortcomings by incorporating relevant adjustments into Lvov's formula, using the same approach as in (Smolyak, 2018).

2.2 Incorporating the dynamics of the technical and economic characteristics of the machine

Previously, the technical and economic characteristics of the machine were considered independent of its age. In this section, we will consider such dependency. Usually, as a machine ages, there is a general tendency for its productivity to decrease and its operational costs to increase. Deviations from this trend are typically caused by factors that are difficult to account for in practical assessments. Nevertheless, by analyzing the operational performance of machines of different ages, a general trend for a range of machines can be identified. In (Smolyak, 2016), information on the dependency of these indicators on age for certain types of machines is presented, based on a small number of available publications on this matter. It turns out that in most cases, the decrease in productivity and increase in operational costs with age follow a linear law. Accordingly, according to formula (1), the decrease in the intensity of benefits brought by the machine with age should also occur following a linear law. It is worth noting that a typical market participant who owns a machine will use it as long as it is profitable. Therefore, the service life of the machine ends when the intensity of benefits it brings does not turn to zero. Such a term in valuation standards (IVS), International Financial Reporting Standards (IFRS 16), and appraisal literature is referred to as the economic service life.

Consider a machine whose benefit intensity at the beginning of operation is D, and the service life under rational operation is T years. In this case, due to the assumption made, the benefit intensity will decrease at a rate of D/T over the service life, and therefore, after t years from the start of operation, it will amount to D(1-t/T). In this case, the sum of discounted benefits from using the machine will be:

$$\int_0^T D\left(1-\frac{t}{T}\right)e^{-rt}dt = Dm_l\left(T,r\right), \tag{7}$$

where $m_l\,(T; r)$ – the multiplier of income linearly decreasing to zero, determined by the formula:

$$m_l\left(T,r\right)=\frac{rT+e^{-rT}-1}{r^2T}. \tag{8}$$

As we can see, formula (7) differs from (2) only in that it includes a different multiplier ml instead of mc. All subsequent calculations in section 2 remain unchanged, and we obtain the same formula (5) for the estimated value of the machine, only with the multiplier ml. However, even after such a substitution, formula (5) will not adequately reflect the real processes of operating machines. The point is that in all previous reasoning, it was assumed that all new machines (as well as their new analogs) have the same ULS. However, this assumption is not confirmed in practice. In the next section, we will try to eliminate this drawback.

2.3 Consideration of the probabilistic nature of the service life.

In reliability theory, it is assumed that the operation of a machine ends at the end of its service life when the machine reaches a certain ultimate technical state (thus, the service life generally exceeds the ULS). Additionally, it is taken into account here that the operation processes of machines have a probabilistic nature, so the service lives of machines will be random. There are many publications dedicated to mathematical modeling of machine operation processes, as well as determining the mean values and coefficients of variation of their service lives, for example (Bekker, 1991; Erumban, 2008; Lin et al., 2014; Wang et al., 2011). Some specialists conduct accelerated reliability tests on machines or collect information on their resource failures, approximating the obtained data with a normal or another known distribution. In the national accounting systems of different countries, mean values and probability distributions of ULS for capital assets are used and published, but they relate to excessively large groups of assets (for example, machines used in the energy sector). It seems that the coefficients of variation of service lives found by the mentioned methods can be used and applied to useful life spans (ULS).

As we can see, the available information is insufficient to take into account the probabilistic nature of the ULS of a specific machine in practical evaluation. In this regard, methods for approximate estimation of the mean value and coefficient of variation of ULS have been proposed, based on information available to appraisers. For example, recommendations for determining the ULS of machines based on their depreciation, normative, or assigned terms are given in the handbook (Leifer, 2019), and recommended values of coefficients of variation of service lives are in (RD 26-01-143-83, table 1-2 of Appendix 3; Ostryakov, 2003). In (Smolyak, 2021), it was suggested to classify machines into three categories based on the values of the ULS coefficient of variation and criteria were provided for experts to assign a specific machine to a particular class. In this article, it is assumed that the ULS of machines follows a gamma distribution, as is common, for example, in the national accounting system of Germany. The gamma distribution is concentrated on the positive part of the numerical axis and is determined by two parameters - the shape parameter alpha and the scale parameter L. The density of this distribution is: $p(x) = x^{\alpha-1}\frac{e^{-x/L}}{L^{\alpha}\Gamma(\alpha)}$, where □ – Euler's gamma function. A random variable with this distribution has an average value of L □ and the coefficient of variation □$^{-1/2}$.

It is assumed that the available information allows the appraiser to estimate the average value (S) and the coefficient of variation of the ULS of the evaluated machine (v). In this case, the parameters of the gamma distribution will be L=Sv^2и □= $1/v^2$.

The probabilistic nature of the ULS machine can be taken into account as follows. If the ULS of the machine T were a deterministic quantity, then its MV, by virtue of formula (7), would be equal to $Dm_l(T,r)$, where D - is the intensity of the benefits brought by the machine at the beginning of operation. However, in case of accidental ULS, according to the evaluation standards, the MV of the machine should be determined by the expected value of the total discounted benefits from its operation. It can be estimated approximately by taking the mathematical expectation of a random variable $Dm_l(T,r)$. This leads to the following expression for the MV machine:

$$V = \int_0^{\infty} D\frac{rT + e^{-rT} - 1}{r^2 T} T^{\alpha-1}\frac{e^{-T/L}}{L^{\alpha}\Gamma(\alpha)}dT = \frac{D}{r^2}\left[r - \frac{1-(1+rL)^{1-\alpha}}{(\alpha-1)L}\right].$$

If you substitute it here $L=Sv^2$, □= $1/v^2$и $D=pW-Z$ from formula (1), this expression will take the form similar to (2):

$$V = Dm_{lg}(S;v;r) = (pW - Z)m_{lg}(S;v;r), \tag{9}$$

where $m^*(S;v;r)$ – the multiplier of linearly decreasing income corresponding to the gamma distribution of the ULS , determined by the formula:

$$m_{lg}(S;v;r) = \frac{1}{r^2}\left[r - \frac{1-\left(1+rSv^2\right)^{1-v^{-2}}}{\left(1-v^2\right)S}\right]. \tag{10}$$

Further reasoning proceeds in the same way as in the derivation of the Lvov formula (5).As a result, the following formula is obtained, similar to (5), but taking into account the dynamics of its main technical and economic characteristics and the probabilistic nature of its useful life:

$$V = V_a\cdot\frac{W}{W_a}\cdot\frac{m_{lg}(S;v;r)}{m_{lg}(S;v;r)} + \left[Z_a\frac{W}{W_a} - Z\right]\cdot m_{lg}(S;v;r). \tag{11}$$

In contrast to the "original" Lviv formula, instead of the specified machine and analog ULS, (11) includes the average values and coefficients of variation of these terms.

As you can see, even with a random match of the machines, the form of the formula has been preserved, only the expression for the income multiplier has changed.

Figure 1 shows the dependencies of the multiplier m $mlg(S;v;r)$ from S for $r = 0.1$ and different v, на , in Fig. 2 dependencies of $m_{lg}(S;v;r)$ on r for different S и v.

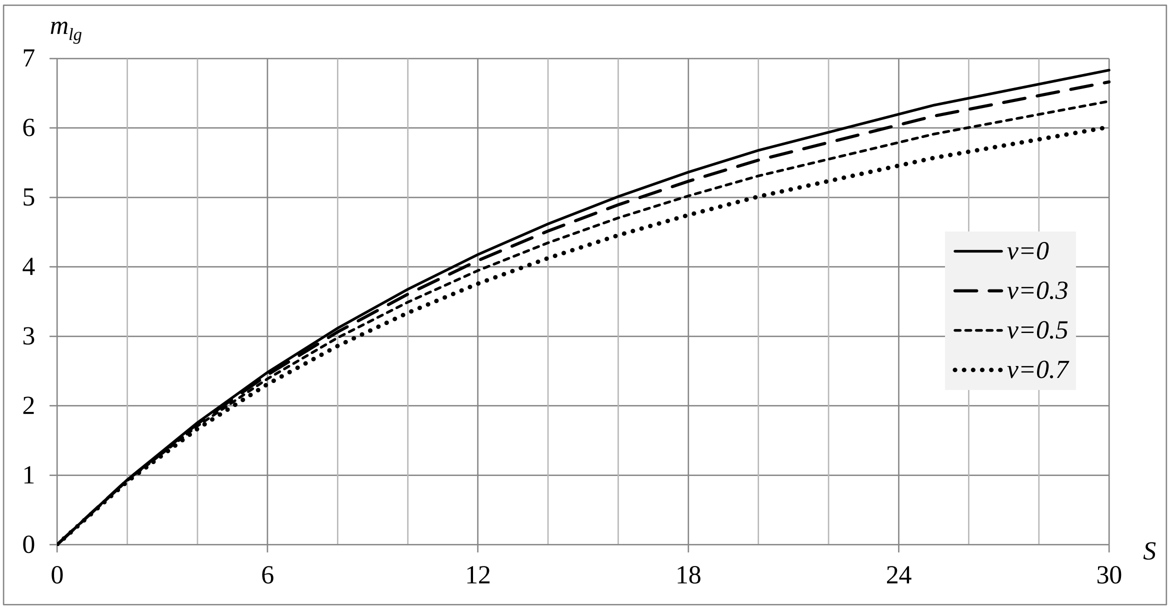

Fig. 1. The dependence of the m_{lg} multiplier on the average service life (S) for r = 0.1 and different v.

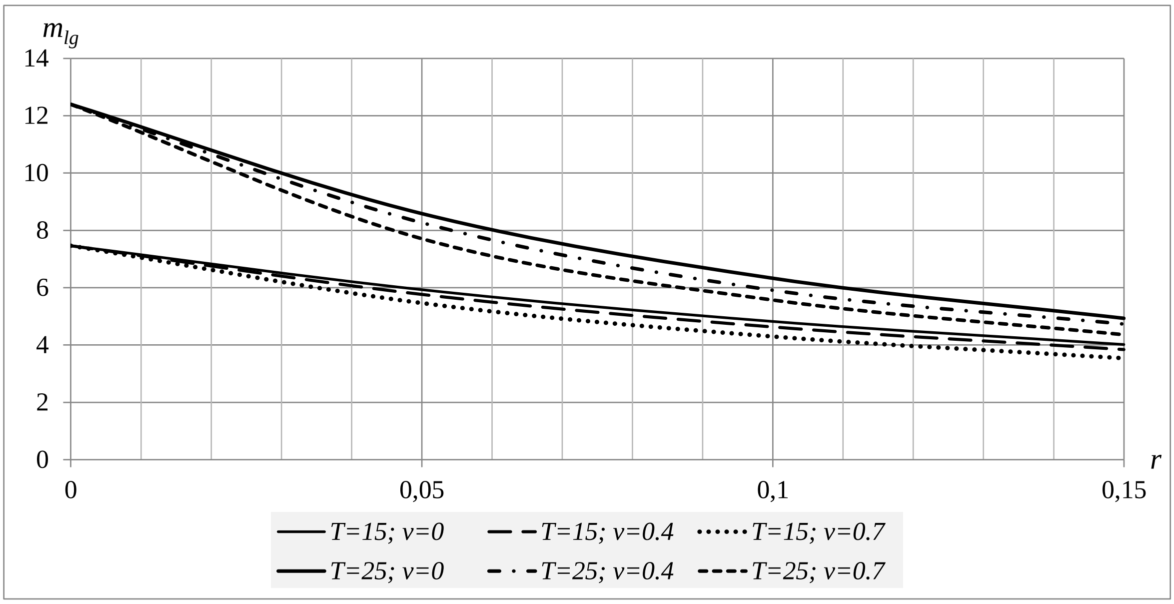

Fig. 2. The dependence of the m_{lg} multiplier on the discount rate (r) for different average values (S) and coefficients of variation (v) of the service life.

Example. The machine's performance is 17% higher than that of the analog. The cost of the analog is 10 million rubles. The operating costs of the machine and the analog are 86 and 79 million rubles, respectively, and their standard service lives are 8 and 11 years. The machine and the analog are both mass-produced equipment of a broad profile. According to (Leifer, 2019), the average ULS of such machines will be 1.84 times longer than the standard service life. In this case, the average ULS of the machine and analog will be 14.72 and 20.24 years, respectively. The coefficient of variation of the service life for both machines, taking into account recommendations (Ostryakov, 2003), is assumed to be 0.4, with a discount rate of 0.10. In that case $m_{lg}(14.72; 0.4; 0.1)=4.577$,$m_{lg}(20.24; 0.4; 0.1)=5.467$, and the MV of the car will be equal to:

$$V = 10 \cdot 1.17 \cdot \frac{4.577}{5.467} + [79 \cdot 1.17 - 86] \cdot 4.577 = 39.2 \ \text{mln. Rubl}$$

It should be noted that calculations using the "usual" Lvov formula, i.e., using formula (2) and the average service lives of both machines, will result in a cost 1.5 times higher: V=59.9 million rubles.

In this and the previous sections, it was assumed that the main characteristics of machines deteriorate with age. However, some authors believe that with proper maintenance, these characteristics can be kept constant. Examples include equipment such as aircraft engines, which maintain consistent performance over a set number of operating hours. Nevertheless, the probability of failure increases over time for such objects, leading to expected damages from potential failures. To properly account for this, the expected damages from possible failure should be included in the operational costs, and these damages increase with age/usage.

2.4 The Influence of More Realistic Assumptions

In the discussions conducted above, several unrealistic assumptions were made for the sake of simplification. These can be eliminated by applying a general method detailed in (Smolyak, 2016).

Including inflation and accident risk. In deriving formula (11), it was assumed that there is no inflation and that the machine is not subject to accidents that would take it out of operation (these are referred to as resource failures in reliability theory). It turns out that to account for these factors, the discount rate r used in the calculation formulas must be determined differently: r = r0 + rf - i, where r0 is the pre-tax nominal risk-free rate, rf is the probability of the machine having an accident within a year, and i is the rate of price growth for machines of that type.

Considering disposal value. Until now, the disposal value of machines was considered to be zero. In reality, for certain types of machines, it can be relatively significant (up to 15% of the market value). It appears that in such instances, the Lvov formula and its aforementioned extensions should use the machines' "depreciated value" (the difference between market value and disposal value). In particular, formula (11) will then take the following form:

$$V=\left(V_a-U_a\right)\cdot\frac{W}{W_a}\cdot\frac{m_{lg}\left(S;v;r\right)}{m_{lg}\left(S;v;r\right)}+\left[Z_a\frac{W}{W_a}-Z\right]\cdot m_{lg}\left(S;v;r\right)+U,$$

where U и U_a – recycling costs of the machine and its equivalent.

Other Service Life Distributions.

In deriving formula (11), it was assumed that the service life of the machine follows a gamma distribution. However, the distribution of service lives could be different. For example, many experts believe that machine based on gamma distribution versus Weibull distribution (where the expression for m_{lg} turns out to be considerably more complex). It turns out that in typical ranges of parameter changes S, v, and r for real machines, the difference is insignificantservice lives follow a Weibull distribution. In such cases, the formula for the m_{lg} multiplier will change. Nevertheless, we have examined how significantly the values of this multiplier will differ if it is determined.

Results.

Within this study, a method for assessing the cost of new machinery and equipment under conditions of uncertainty has been developed. This method is based on the formula proposed by D.S. Lvov and takes into account changes in the operational characteristics of machinery during their use, as well as the randomness of their useful life.

It has been demonstrated that the proposed method allows for a more adequate consideration of the features of machinery operation compared to traditional cost assessment methods, which assume constant operational characteristics of machinery throughout their useful life.

Computational experiments were conducted, confirming the effectiveness of the proposed method. It was shown that in some cases, the proposed method may lead to a higher valuation of machinery compared to traditional methods, which can be particularly relevant for assessing the value of new and innovative machinery and equipment.

Furthermore, a literature review was conducted, revealing that the proposed method is novel and has no analogs in domestic or foreign literature. Thus, the results of this study can be utilized in the development of new methods for assessing the cost of machinery and equipment under conditions of uncertainty.

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