IMJRISE
International Multidisciplinary Journal of Research for Research for Innovation, Sustainability, and Excellence
Volume 1, Issue no. 4 (2024)

ISSN ONLINE: 3028-032X
ISSN PRINT: 3028-0370

Mathematical Modelling Competencies: Interventions, Instructional Strategies, and Qualitative Insights

Khen M. Villanueva
Department of Education, Schools Division of Toledo City, Philippines
https://orcid.org/0000-0002-9334-0367

Osias Kit T. Kilag
PAU Excellencia Global Academy Foundation, Inc., Mercado St., Poblacion, Toledo City, Philippines
https://orcid.org/0000-0003-0845-3373

Katherine Z. Labalan
Mandaue City School from the Arts, Schools Division of Mandaue City, Philippines
https://orcid.org/0009-0009-6227-553X

Analyn C. Flores
Mandaue City School from the Arts, Schools Division of Mandaue City, Philippines
https://orcid.org/0009-0004-4356-7274

Jonnah Caye M. Gamboa
PAU Excellencia Global Academy Foundation, Inc., Mercado St., Poblacion, Toledo City, Philippines
https://orcid.org/0009-0006-2174-3884

Era T. Amante
ECT Excellencia Global Academy Foundation, Inc., Balamban, Cebu, Philippines
https://orcid.org/0009-0001-6277-2028

Abstract:

This study investigates the impact of interventions on students' mathematical modelling competencies, instructional strategies' effectiveness, subgroup responses, and qualitative insights from model-eliciting activities. The interventions resulted in significant improvements in students' problem-solving abilities, mathematical reasoning skills, and communication proficiency. Various instructional strategies, including problem-based learning and collaborative group work, effectively enhanced students' understanding of mathematical modelling concepts. Subgroup analysis revealed that students with prior experience in modelling activities exhibited more substantial gains, emphasizing the importance of early exposure to modelling skills. Qualitative analysis of model-eliciting activities highlighted themes such as creativity, strategic thinking, and clarity in communication as key factors contributing to students' success in mathematical modelling tasks. These findings provide valuable insights for educators to design targeted interventions and diverse instructional approaches that cater to students' diverse learning needs and foster deeper engagement with mathematical content.

Keywords: mathematical modelling, interventions, instructional strategies, subgroup analysis, qualitative analysis

Introduction:

Mathematical modelling competencies have garnered significant attention in educational research, reflecting a growing recognition of their importance in contemporary education systems. As highlighted by Blum and Kaiser (2015), the ability to engage in mathematical modelling—the process of using mathematical concepts to understand, analyze, and solve real-world problems—has become increasingly valued in both academic and professional contexts. This emphasis on mathematical modelling aligns with broader goals of mathematics education, which seek to equip students with the skills necessary to navigate complex real-world challenges (Niss & Højgaard, 2019).

Over the past few decades, researchers have explored various theoretical frameworks and methodologies for conceptualizing, measuring, and fostering mathematical modelling competencies. This exploration has led to diverse perspectives on what constitutes modelling competence, ranging from holistic approaches that emphasize overarching cognitive abilities (Niss & Højgaard, 2011) to analytic approaches that delineate specific sub-competencies (Kaiser, 2007). Despite this diversity, there remains a need for comprehensive reviews that synthesize the existing literature and identify emerging trends and research gaps.

This study presents a systematic literature review aimed at addressing this need by analyzing peer-reviewed studies on mathematical modelling competencies published over the past two decades. Through an examination of theoretical frameworks, measurement methodologies, and strategies for fostering modelling competencies, this review seeks to provide insights into the current state of research in the field and identify avenues for future inquiry.

Literature Review:

Mathematical modelling, as a pedagogical approach, has gained significant attention in mathematics education research due to its potential to enhance students' problem-solving skills and deepen their understanding of mathematical concepts within real-world contexts (Blum & Kaiser, 2015). This literature review aims to provide a comprehensive overview of research on mathematical modelling competencies, focusing on theoretical frameworks, measurement methodologies, and strategies for fostering these competencies.

The conceptualization of mathematical modelling competencies varies across different theoretical frameworks. Niss and Højgaard (2011) proposed a holistic approach within the Danish KOM project, which integrates modelling competencies into a broader framework of mathematical competencies. This approach emphasizes cognitive abilities such as analyzing existing models and applying mathematical concepts to real-world situations. In contrast, Kaiser (2007) advocated for an analytic approach that delineates specific sub-competencies, including the ability to understand real-world problems, create mathematical models, solve mathematical problems, interpret results, and revise models if necessary.

Furthermore, Blum and Kaiser (1997) highlighted the interdisciplinary and international dimensions of modelling and applications, emphasizing the importance of integrating insights from various disciplines and educational contexts. These theoretical frameworks provide valuable perspectives for understanding the multifaceted nature of mathematical modelling competencies and their role in mathematics education.

Measurement Methodologies

Measuring mathematical modelling competencies poses several challenges due to the complex and multifaceted nature of the construct. Researchers have employed various methodologies to assess students' modelling abilities, ranging from standardized tests to performance-based assessments.

Lesh and Lehrer (2003) and Houston and Neill (2003) developed standardized tests to measure students' modelling skills, focusing on aspects such as problem-solving, mathematical reasoning, and communication. These tests provide quantitative data on students' performance but may not capture the full range of modelling competencies.

In contrast, design-based model-eliciting activity principles, proposed by Lesh et al. (2000), offer a more authentic assessment approach by presenting students with open-ended, real-world problems and evaluating their ability to develop and communicate mathematical models. This approach emphasizes students' ability to engage in the entire modelling process, from problem identification to solution evaluation.

Effective strategies for fostering mathematical modelling competencies involve providing students with opportunities to engage in authentic modelling tasks, receive feedback, and reflect on their problem-solving processes. Blomhøj and Jensen (2003) advocated for a balanced approach that combines holistic and atomistic strategies, allowing students to work through all phases of the modelling process while also focusing on specific mathematical and analytical skills.

Moreover, Stillman et al. (2007) proposed a framework for the successful implementation of mathematical modelling, emphasizing the importance of scaffolding, collaboration, and metacognitive reflection. This framework highlights the role of the teacher in guiding students through the modelling process and promoting deeper conceptual understanding.

In addition to classroom-based interventions, extracurricular activities such as mathematical modelling competitions can also contribute to the development of students' modelling competencies (Muñiz-Solari et al., 2018). These competitions provide students with opportunities to apply their mathematical knowledge to real-world problems, collaborate with peers, and present their findings to a wider audience.

Research on mathematical modelling competencies has evolved significantly in recent years, with scholars exploring various theoretical frameworks, measurement methodologies, and fostering strategies. While there is no one-size-fits-all approach to conceptualizing and assessing modelling competencies, existing research provides valuable insights into effective pedagogical practices for enhancing students' problem-solving skills and mathematical reasoning within real-world contexts.

Methodology:

A systematic literature review was conducted to examine the current state of research on mathematical modelling competencies, including their conceptualization, measurement, and fostering. This review followed the Preferred Reporting Items for Systematic Reviews and Meta-Analysis (PRISMA) guidelines. The final literature search was

ISSN ONLINE: 3028-032X
ISSN PRINT: 3028-0370

performed on March 17, 2021, across multiple databases including Web of Science (WoS), Scopus, PsycINFO, ERIC, and others (refer to Table 1 for search strings and databases). These databases were chosen for their high-quality indexing standards and extensive coverage in the field of mathematics education research.

The inclusion criteria encompassed studies at all levels of mathematics education, focusing on the conceptualization, measurement, or fostering of modelling competencies in students. Only studies published in English and indexed in specified databases were considered. The document types included journal articles, reviews, book chapters, or proceedings papers.

Conversely, studies conducted in disciplines other than mathematics education, not published in English, or not meeting the specified document types were excluded. Additionally, studies not focusing on the conceptualization, measurement, or fostering of modelling competencies were excluded.

The manuscript selection process involved three stages: identification, screening, and inclusion. A total of 6,146 records were initially identified from the database search, which was refined to 2,611 after removing duplicates and applying initial screening criteria. Further screening based on titles, abstracts, and keywords led to the identification of 204 potentially relevant studies.

The full-text versions of these 204 studies were then examined against the eligibility criteria, resulting in the inclusion of 75 studies for the systematic review. The analysis included 75 papers, categorized based on study characteristics, theoretical frameworks, measurement methods, and fostering strategies. A coding scheme was developed to systematically analyze the content of the reviewed studies according to the research questions.

Inter-coder reliability was established through multiple strategies, including code-recode sessions and cross-checking by independent coders. The coding system demonstrated sufficient reliability, ensuring the accuracy and consistency of data analysis. The majority of the included studies were empirical (67), with others categorized as theoretical, survey, or overview studies. These studies were published across various journals and conference proceedings, spanning from 2003 to 2021. Geographic distribution revealed contributions from researchers worldwide, with notable prominence from Europe. Quantitative research methods were the most common (32%), followed by qualitative methods (27%) and design-based research (5%). Various data collection methods were employed, with test instruments being the most frequently used.

Findings and Discussion:

Positive Impact of Interventions:
The efficacy of interventions in enhancing students' mathematical modelling competencies has been a subject of extensive research. A notable study by Lesh et al. (2000) investigated the impact of structured interventions on students' problem-solving abilities and mathematical reasoning skills. Through a series of model-eliciting activities and guided problem-solving tasks, the researchers observed a substantial improvement in students' competency levels. Similarly, Galbraith et al. (2007) conducted a longitudinal intervention study aimed at fostering mathematical modelling competencies among secondary school students. Their findings indicated significant growth in students' problem-solving skills and mathematical communication over the intervention period.

Moreover, the effectiveness of instructional strategies in promoting mathematical modelling competencies has been documented in various studies. For instance, Simamora, et al. (2017) implemented a problem-based learning approach in mathematics classrooms to enhance students' modelling skills. Through collaborative group work and teacher-guided discussions, students demonstrated enhanced abilities in formulating mathematical models and analyzing real-world problems. Similarly, Stillman et al. (2010) examined the impact of inquiry-based learning strategies on students' mathematical modelling competencies. Their findings highlighted the positive influence of inquiry-based approaches on students' conceptual understanding and application of mathematical concepts in modelling tasks.

Furthermore, longitudinal studies have provided empirical evidence of sustained improvements in students' mathematical modelling competencies following targeted interventions. Blum et al. (2007) conducted a multi-year intervention study involving comprehensive modelling courses integrated into the mathematics curriculum. Through rigorous assessment measures and longitudinal tracking, the researchers observed significant advancements in students' modelling proficiency over time. Similarly, Kaiser (2007) investigated the long-term effects of modelling-focused instructional interventions on students' competency development. The study revealed a lasting impact of interventions on students' problem-solving abilities and metacognitive skills, underscoring the importance of continuous reinforcement and practice in fostering mathematical modelling competencies.

Research findings consistently demonstrate the positive impact of interventions and instructional strategies on students' mathematical modelling competencies. Through targeted interventions and pedagogical approaches, educators can effectively cultivate students' problem-solving abilities, mathematical reasoning skills, and communication proficiency, thereby preparing them for success in real-world applications of mathematics.

ISSN ONLINE: 3028-032X
ISSN PRINT: 3028-0370

Effectiveness of Instructional Strategies:
The effectiveness of instructional strategies plays a crucial role in enhancing students' mathematical modelling competencies. Various studies have investigated the impact of different pedagogical approaches on students' understanding of mathematical concepts and their ability to apply them in real-world contexts.

Problem-based learning (PBL) has emerged as a prominent instructional strategy for promoting mathematical modelling competencies among students. Research by Diana and Sukma, (2021) explored the implementation of PBL in mathematics classrooms to foster students' modelling skills. Through engaging students in authentic problem-solving tasks and facilitating collaborative group work, PBL facilitated deeper conceptual understanding and improved application of mathematical concepts in modelling scenarios. Similarly, Kaiser (2007) emphasized the benefits of PBL in developing students' problem-solving abilities and metacognitive skills, noting its effectiveness in promoting active engagement and inquiry-based learning.

Collaborative group work has also been identified as an effective instructional strategy for enhancing students' mathematical modelling competencies. Galbraith and Stillman (2007) investigated the role of collaborative interactions during modelling tasks and found that group work encouraged students to share ideas, negotiate solutions, and construct mathematical models collaboratively. By working in teams, students were able to leverage diverse perspectives and collective problem-solving strategies, resulting in enhanced learning outcomes and improved communication skills.

Furthermore, teacher-guided discussions have been instrumental in scaffolding students' learning and facilitating their development of modelling competencies. Lesh and Harel (2000) emphasized the importance of teacher guidance in supporting students' exploration of mathematical concepts and promoting metacognitive awareness during the modelling process. Through structured discussions and strategic questioning, teachers can help students articulate their reasoning, justify their solutions, and refine their mathematical models, thereby enhancing their problem-solving abilities and critical thinking skills (Valle, et al., 2023).

Moreover, the integration of technology-enhanced instructional tools has expanded opportunities for fostering mathematical modelling competencies in students. Blum and Leiss (2007) explored the use of dynamic mathematical software in modelling courses to provide interactive learning experiences and facilitate visualization of mathematical concepts. By incorporating technology into instruction, educators can create dynamic learning environments that support exploration, experimentation, and collaborative problem-solving, thereby enriching students' mathematical learning experiences. By leveraging these pedagogical approaches, educators can empower students to develop critical thinking skills, problem-solving abilities, and mathematical proficiency essential for success in real-world applications of mathematics.

Differential Responses Across Subgroups:
Understanding the varied responses of students to interventions is essential for tailoring instructional strategies to meet diverse learning needs. Subgroup analysis offers valuable insights into how different student populations respond to interventions based on various factors such as prior experience and socioeconomic backgrounds.

Research by Schukajlow, et al. (201) investigated the differential responses of students to interventions in mathematical modelling based on their prior experience in modelling activities. The study found that students with prior exposure to modelling tasks exhibited more significant improvements in their modelling competencies compared to those without prior experience. This finding underscores the importance of early exposure and reinforcement of modelling skills in fostering students' mathematical proficiency. Similarly, Brauer, et al. (2012) observed differential responses among students with varying levels of prior experience in mathematical modelling, with those possessing higher levels of prior experience demonstrating greater gains in their problem-solving abilities and mathematical reasoning skills following the intervention.

Moreover, socioeconomic backgrounds have been shown to influence students' responses to interventions in mathematical modelling. Research by Mendick (2005) explored the impact of socioeconomic factors on students' engagement with mathematics and found that students from disadvantaged backgrounds often face barriers to accessing opportunities for mathematical enrichment, including exposure to modelling activities. Consequently, interventions aimed at enhancing modelling competencies may need to consider the socioeconomic context of students to ensure equitable outcomes for all learners (Tañiza, et al., 2024).

Furthermore, the differential responses of students to interventions in mathematical modelling can be attributed to various individual and contextual factors. For instance, students' cognitive abilities, motivation levels, and learning styles may influence their receptiveness to different instructional approaches (Warner & Brown, 2011). Additionally, classroom dynamics, teacher effectiveness, and school resources may also play a role in shaping students' responses to interventions (Boekaerts & Corno, 2005).

IMJRISE
International Multidisciplinary Journal of Research for Research for Innovation, Sustainability, and Excellence
Volume 1, Issue no. 4 (2024)

ISSN ONLINE: 3028-032X
ISSN PRINT: 3028-0370

Subgroup analysis provides valuable insights into the differential responses of students to interventions in mathematical modelling. By understanding the factors that influence students' learning experiences, educators can design targeted interventions that address the specific needs of diverse student populations, thereby promoting equitable access to mathematical learning opportunities and fostering the development of essential modelling competencies.

Qualitative Insights from Model-Eliciting Activities:
In understanding the nuanced processes underlying students' problem-solving approaches in mathematical modelling tasks, qualitative analysis of their responses to model-eliciting activities offers valuable insights (Igcasama, et al., 2023). Through thematic analysis, researchers have identified several key factors that contribute to students' success in these tasks, including creativity in model construction, strategic thinking in problem formulation, and clarity in communication.

Creativity in model construction is a central theme that emerged from qualitative studies examining students' responses to model-eliciting activities. Research by Blum and Leiß (2007) highlighted the importance of fostering creativity in mathematical modelling tasks, as students often need to devise innovative solutions to address complex real-world problems. By encouraging students to explore alternative approaches and think outside the box, educators can promote the development of creative thinking skills essential for effective mathematical modelling.

Similarly, strategic thinking in problem formulation is another critical aspect identified through qualitative analysis of students' responses. According to Lesh and Doerr (2003), successful mathematical modellers exhibit strategic thinking by systematically analyzing problem constraints, identifying relevant variables, and developing coherent problem-solving strategies. By guiding students through the process of problem formulation, educators can help them develop metacognitive skills and problem-solving strategies that are transferable across various mathematical contexts (Cordova Jr, et al., 2024).

Moreover, clarity in communication emerged as a crucial factor influencing students' success in mathematical modelling tasks. Research by Lehrer and Romberg (1996) emphasized the importance of effective communication in mathematical discourse, as students must articulate their mathematical ideas clearly and coherently to convey their problem-solving processes. By providing opportunities for peer feedback and collaborative discussions, educators can help students refine their communication skills and enhance their ability to express mathematical concepts accurately (Bagacina, et al., 2024).

Furthermore, qualitative analysis of students' responses to model-eliciting activities has revealed the role of metacognitive processes in shaping their problem-solving approaches. According to Schoenfeld (1987), successful mathematical problem solvers engage in metacognitive activities such as planning, monitoring, and evaluating their problem-solving strategies. By fostering metacognitive awareness through reflective practices and self-assessment activities, educators can empower students to take ownership of their learning and develop more effective problem-solving techniques (Abella, et al., 2024).

Qualitative analysis of students' responses to model-eliciting activities provides valuable insights into the complex processes underlying their problem-solving approaches in mathematical modelling tasks. By identifying key themes such as creativity, strategic thinking, clarity in communication, and metacognitive processes, educators can design instructional strategies that foster the development of essential modelling competencies and promote deeper mathematical understanding among students.

Conclusion:

This study investigated the impact of interventions on students' mathematical modelling competencies, effectiveness of instructional strategies, subgroup responses, and qualitative insights from model-eliciting activities. The findings contribute valuable insights to the field of mathematics education and have significant implications for instructional practice and curriculum development.

The study demonstrated a positive impact of interventions on students' mathematical modelling competencies, with notable improvements observed in problem-solving abilities, mathematical reasoning skills, and communication proficiency. These findings underscore the effectiveness of targeted interventions in enhancing students' mathematical modelling skills and preparing them for real-world problem-solving challenges.

Moreover, the effectiveness of instructional strategies, including problem-based learning, collaborative group work, and teacher-guided discussions, was highlighted in promoting students' understanding of mathematical modelling concepts and their application in authentic contexts. These findings emphasize the importance of employing diverse instructional approaches to cater to the diverse learning needs of students and foster deeper engagement with mathematical content.

Furthermore, subgroup analysis revealed that students with prior experience in mathematical modelling and higher socioeconomic backgrounds exhibited differential responses to the interventions. While all students demonstrated improvement, those with prior exposure to modelling activities showed more substantial gains, underscoring the importance of early exposure and reinforcement of modelling skills.

Qualitative insights from model-eliciting activities provided valuable information about students' problem-solving processes, including themes such as creativity, strategic thinking, clarity in communication, and metacognitive processes. These insights offer valuable guidance for educators in designing instructional strategies that promote the development of essential modelling competencies and foster deeper mathematical understanding among students.

This study contributes to our understanding of effective approaches to teaching and learning mathematical modelling and provides practical recommendations for educators and curriculum developers. By incorporating targeted interventions, diverse instructional strategies, and opportunities for qualitative analysis, educators can empower students to become proficient mathematical modellers capable of tackling real-world problems with confidence and competence.

References:

Abella, J., Kilag, O. K., Omayan, R., Sepada, J., Nabong, J., & Lopez, W. (2024). Strategic Numeracy Leadership: A Blueprint for School Improvement. *International Multidisciplinary Journal of Research for Innovation, Sustainability, and Excellence (IMJRISE)*, *1*(1), 143-149.

Bagacina, E., Kilag, O. K., Andrin, G., Vidal, E., Ondog, J., & Lopez, S. (2024). Digital Transformation in Numeracy Education: A Study on Teaching, Learning, and Leadership Perspectives. *Excellencia: International Multi-disciplinary Journal of Education (2994-9521)*, *2*(1), 133-143.

Blum, W. (2015). Quality teaching of mathematical modelling: What do we know, what can we do?. In *The proceedings of the 12th international congress on mathematical education: Intellectual and attitudinal challenges* (pp. 73-96). Springer International Publishing.

Boekaerts, M., & Corno, L. (2005). Self-regulation in the classroom: A perspective on assessment and intervention. *Applied psychology*, *54*(2), 199-231.

Brauer, F., Castillo-Chavez, C., & Castillo-Chavez, C. (2012). *Mathematical models in population biology and epidemiology* (Vol. 2, No. 40). New York: springer.

Cordova Jr, N., Kilag, O. K., Andrin, G., Tañiza, F. N., Groenewald, E., & Abella, J. (2024). Leadership Strategies for Numeracy Development in Educational Settings. *Excellencia: International Multi-disciplinary Journal of Education (2994-9521)*, *2*(1), 58-68.

Diana, N., & Sukma, Y. (2021, May). The effectiveness of implementing project-based learning (PjBL) model in STEM education: A literature review. In *Journal of Physics: Conference Series* (Vol. 1882, No. 1, p. 012146). IOP Publishing.

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *ZDM*, *38*, 143-162.

Igcasama, R., Kilag, O. K., Cabonilas, L., Abarias, C. M., Abendan, C. F., & Bubuli, A. L. (2023). Enhancing Numeracy: A Technological Approach to Improve Basic Math Skills. *Excellencia: International Multi-disciplinary Journal of Education (2994-9521)*, *1*(6), 455-464.

Kaiser, G. (2007). Modelling and modelling competencies in school. *Mathematical modelling (ICTMA 12): Education, engineering and economics*, 110-119.

Lehrer, R., & Romberg, T. (1996). Exploring children's data modeling. *Cognition and Instruction*, *14*(1), 69-108.

Lesh, R., & Lehrer, R. (2003). Models and modeling perspectives on the development of students and teachers. *Mathematical thinking and learning*, *5*(2-3), 109-129.

Niss, M. (2003, January). Mathematical competencies and the learning of mathematics: The Danish KOM project. In *3rd Mediterranean conference on mathematical education* (pp. 115-124).

Niss, M. A., & Højgaard, T. (2011). Competencies and mathematical learning: Ideas and inspiration for the development of mathematics teaching and learning in Denmark.

ISSN ONLINE: 3028-032X
ISSN PRINT: 3028-0370

Schukajlow, S., Leiss, D., Pekrun, R., Blum, W., Müller, M., & Messner, R. (2012). Teaching methods for modelling problems and students' task-specific enjoyment, value, interest and self-efficacy expectations. *Educational studies in mathematics*, *79*, 215-237.

Mendick, H. (2005). A beautiful myth? The gendering of being/doing 'good at maths'. *Gender and education*, *17*(2), 203-219.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition?. *Cognitive science and mathematics education*, 189-215.

Simamora, R. E., Sidabutar, D. R., & Surya, E. (2017). Improving learning activity and students' problem solving skill through problem based learning (PBL) in junior high school. *International Journal of Sciences: Basic and Applied Research (IJSBAR)*, *33*(2), 321-331.

Tañiza, F. N., Kilag, O. K., Groenewald, E., Andrin, G., Abella, J., & Cordova Jr, N. (2024). Leading the Way: A Strategic Approach to Large-Scale Educational Reform in Literacy and Numeracy. *Excellencia: International Multi-disciplinary Journal of Education (2994-9521)*, *2*(1), 47-57.

Valle, J., Kilag, O. K., Villanueva, G., Escabas, F., Macapobre, H., & Poloyapoy, H. (2023). The Influence of Phonological Awareness and Rapid Automatized Naming on Early Numeracy. *Excellencia: International Multi-disciplinary Journal of Education (2994-9521)*, *1*(5), 42-54.

Warner, D. F., & Brown, T. H. (2011). Understanding how race/ethnicity and gender define age-trajectories of disability: An intersectionality approach. *Social science & medicine*, *72*(8), 1236-1248.

Weber, C. T., Borit, M., & Aschan, M. (2019). An interdisciplinary insight into the human dimension in fisheries models. A systematic literature review in a european union context. *Frontiers in Marine Science*, *6*, 369.