Statistical Assessment of Models for Determination of Soil – Water Characteristic Curves of Sand Soils

S. J. Matlan, M. Mukhlisin, M. R. Taha

***Abstract*—**Characterization of the engineering behavior of unsaturated soil is dependent on the soil-water characteristic curve (SWCC), a graphical representation of the relationship between water content or degree of saturation and soil suction. A reasonable description of the SWCC is thus important for the accurate prediction of unsaturated soil parameters. The measurement procedures for determining the SWCC, however, are difficult, expensive, and time-consuming. During the past few decades, researchers have laid a major focus on developing empirical equations for predicting the SWCC, with a large number of empirical models suggested. One of the most crucial questions is how precisely existing equations can represent the SWCC. As different models have different ranges of capability, it is essential to evaluate the precision of the SWCC models used for each particular soil type for better SWCC estimation. It is expected that better estimation of SWCC would be achieved via a thorough statistical analysis of its distribution within a particular soil class. With this in view, a statistical analysis was conducted in order to evaluate the reliability of the SWCC prediction models against laboratory measurement. Optimization techniques were used to obtain the best-fit of the model parameters in four forms of SWCC equation, using laboratory data for relatively coarse-textured (i.e., sandy) soil. The four most prominent SWCCs were evaluated and computed for each sample. The result shows that the Brooks and Corey model is the most consistent in describing the SWCC for sand soil type. The Brooks and Corey model prediction also exhibit compatibility with samples ranging from low to high soil water content in which subjected to the samples that evaluated in this study.

***Keywords*—**Soil-water characteristic curve (SWCC), statistical analysis, unsaturated soil.

I. INTRODUCTION

SOIL-WATER characteristic curves are important for the analysis of groundwater recharge, agriculture, and soil chemistry, as they are used to predict soil-water storage, water supply to plants (field capacity), and soil aggregate stability [1]. These relationships are also of considerable value to geotechnical and geoenvironmental engineering practices as an important input in their application. The characterization of the engineering behavior of unsaturated soil is dependent on the SWCC, which is a graphical representation of the relationship between water content or degree of saturation and soil suction [2], [3]. Understanding the behavior of water in unsaturated soil is a challenge for researchers [4]. It has been found that a reasonable description of the soil-water characteristic curve is important for the accurate prediction of unsaturated soil parameters [5]. The measurement procedures employed to determine the SWCC, however, are difficult, expensive, and time-consuming. During the last few decades, a major focus has been laid on developing empirical equations for SWCC prediction. Several researchers have stressed the need for extensive work to be carried out in this direction in order to simplify and improve existing model concepts and to facilitate easy practice of unsaturated soil concepts. Past studies have shown that the SWCC can also be empirically correlated to other unsaturated soil properties, such as hydraulic conductivity and shear strength [6]-[9].

A large number of empirical equations have been proposed to best fit laboratory data for SWCCs, including those of Gardner [10]; Brooks and Corey [11]; Brutsaert [12]; Tani [13]; McKee and Bumb [14], [15]; Van Genuchten [16]; Burdine [17]; Mualem [6]; Kosugi [18], [19]; and Fredlund and Xing [20]. These equations can be classified as either two-parameter or three-parameter SWCC equations [9]. One of the most crucial questions surrounds how precisely existing equations can represent the SWCC [21]. Leong and Rahardjo [22] undertook a review of several empirical SWCC equations by applying each equation to a database of laboratory measurements. It was noted that the SWCC equations with the highest number of soil fitting parameters provided the closest fit to the data sets. The Fredlund and Xing [20] SWCC equation has been found to perform marginally better than the Van Genuchten [16] equation. Cornelis et al. [23] also compared nine closed-formed unimodal analytical functions in order to describe the SWCC. In the latter study the Van Genuchten [16] and Kosugi [18] models exhibited the best fits to the observed data, specifically at high and medium water content. A number of different factors influence the uniqueness of the SWCC for each particular type of soil. As is well known mineral composition and pore structure are basic effective factors which affect the soil water characteristic curve. Leong and Rahardjo [22] have stated that most equations developed thus far have limited success depending on the analyzed soil type, with Cornelis et al. [23] also concluding that the performance of all models (in terms of their match with data) improves with increasing clay content

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and decreasing sand content. Due to the fact that different models have different ranges of capability depending on soil mineral compositions and pore structures, it is therefore essential to evaluate the precision of the SWCC models used for each particular type of soil. It is expected that better SWCC estimation would be achieved via a thorough statistical analysis of its distribution within a particular soil class.

With this in view, a statistical analysis was conducted in order to evaluate the reliability of the SWCC prediction models against laboratory data. Optimization techniques were used to obtain the best-fit of the model parameters in four forms of SWCC equation using laboratory data for relatively coarse-textured (i.e., sandy) soil. In this case, a soil samples were obtained from UNSODA, comprising 14 samples of sand were used in the analysis. The four most prominent SWCC models (those of Gardner [10]; Brooks and Corey [11]; Van Genuchten [16]; and Kosugi [19]) were evaluated and computed for each sample. These four existing models were selected based on their relative simplicity and practicality (all contain two fitting soil parameters in their equations), as well as their widespread adoption and citation. The produced results will be valuable to practitioners in deciding which empirical relationship to use for modeling the SWCC of a sandy soil.

II. Soil-Water Characteristic Curve Models

The models developed by Gardner [10], Brooks and Corey [11], and Van Genuchten [16], as well as the lognormal distribution model of Kosugi [19], are some of the most notable examples found in the literature. All are parametric models based on a pore size distribution function and the capillary theory, and all also contain two fitting parameters in their model equations. The equations representing each model along with definitions of the variables used are given as follows.

In general, the normalized water content or a dimensionless water content term, Θ, also known as the effective saturation, Se, will be used to represent the equations associated with the soil-water characteristic curve models.

$$\Theta = \frac{(\theta - \theta_r)}{(\theta_s - \theta_r)} \tag{1}$$

where θ is volumetric water content, and θ_s and θ_r are the saturated and residual volumetric water content, respectively.

A. Gardner's Model

One of the first equations used to model the soil-water characteristic curve, the Gardner [10] equation is a continuous function originally intended as a means of modeling the unsaturated coefficient of soil permeability [24]. The equation has since been adapted to model the soil-water characteristic curve via the use of two fitting parameters, a and n, related to the inverse of the air entry value and the pore size distribution, respectively.

$$\Theta = \frac{1}{1 + a\psi^n} \tag{2}$$

where ψ is the soil suction with a unit of kilopascals (kPa). By substituting (1) into (2), we obtain the volumetric water content form of the Gardner model as follows:

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{1 + a\psi^n} \tag{3}$$

The Gardner model has a particularly simple form with few parameters; it is thus convenient to use and has a wide range of applications. However, it cannot accurately describe the soil-water characteristic curve for saturated and near-saturated soils [25].

B. Brooks and Corey's Model

Brooks and Corey's [11] model is amongst the earliest equations proposed for the soil-water characteristic curve and remains a popular model - albeit in the form of a power-law relationship. The model is given by the following equation:

$$\Theta = \left[\frac{\psi}{h_b}\right]^{-\lambda} \qquad \psi > h$$
$$\Theta = 1 \qquad \psi \le h_b \tag{4}$$

The equation uses two fitting parameters, namely h_b and λ. Parameter hb is related to the air entry value of the soil, while the λ parameter is termed the pore size index and is related to the pore size distribution of the soil. The model is assumed to be constant for suctions less than the air entry value. The soil-water characteristic curve is assumed to be an exponentially decreasing function at soil suctions greater than the air entry value [24].

The volumetric water content form of the Brooks and Corey model can be written as follows:

$$\theta = \theta_r + (\theta_s - \theta_r)\left[\frac{\psi}{h_b}\right]^{-\lambda} \tag{5}$$

Although the Brooks and Corey model is relatively simple and thus widely used [25], it does not provide a continuous mathematical function for the entire soil-water characteristic curve [24].

C. Van Genuchten's Model

The most widely-adopted alternative to the Brooks and Corey model is that proposed by Van Genuchten [16]. This model uses three fitting parameters, namely α*,* n, and m. The Van Genuchten model can be mathematically described as follows:

$$\Theta = \frac{1}{\left[1 + (\alpha\psi)^n\right]^m} \tag{6}$$

To simplify and derive closed form equation for unsaturated conductivity based on Mualem [6], the n and m parameters in the SWCC equation can have a fixed relationship in which *m=(1-1/n).* This suggestion therefore reduces Van Genuchten's three-parameter equation to a two-parameter SWCC equation:

$$\Theta = \frac{1}{\left[1 + (\alpha\psi)^n\right]^{(1-1/n)}} \quad (7)$$

where parameter α is related to the inverse of the air entry value, the n parameter is related to the pore size distribution of the soil, and the m parameter is related to the asymmetry of the model. By substituting (1) into (7), we can write the volumetric water content form of the Van Genuchten model as:

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{\left[1 + (\alpha\psi)^n\right]^{(1-1/n)}} \quad (8)$$

The Van Genuchten model is complex in form and relies on a greater number of fitting parameters than the models discussed above. However, it produces a continuous output in the unsaturated zone and provides a good description of the soil-water characteristic curve under most circumstances [25].

D.Lognormal Distribution Model

The last SWCC model considered here is based on the model suggested by Kosugi [19]. This model was developed by applying a lognormal distribution law and its parameters are directly related to the soil pore radius distribution. The lognormal distribution model developed by Kosugi can be described as follows:

$$\Theta = Q\left[\frac{\ln \psi / h_m}{\sigma}\right] \quad (9)$$

where is related to the complementary error function, erfc, defined as:

$$Q(x) = erfc\ \frac{(x/\sqrt{2})}{2} \quad (10)$$

The model uses two fitting parameters, namely h_m and σ. Parameter h_m is a capillary pressure head related to the median pore radius, and σ is a dimensionless parameter related to the width of the pore radius distribution. By substituting (1) into (9), we can write the volumetric water content form of the lognormal distribution by Kosugi [19]:

$$\theta = \theta_r + (\theta_s - \theta_r)Q\left[\frac{\ln \psi / h_m}{\sigma}\right] \quad (11)$$

The lognormal distribution model has a more complex form because of the complementary error function present and is thus more difficult to use. However, the model does exhibit greater flexibility in terms of representing the soil-water characteristic curve in the wet and dry regions for all soil types [24].

All four models contain two fitting parameters other than θ_s and θ_r. Therefore, for ease of scheduling, the fitting parameters are marked as P1 and P2. Table I summarizes these parameters for the SWCC models evaluated. The parameter marked as P1 is a suction value related to the inflection point on the SWCC, which is also related to the air entry value. The parameter marked as P2 is a dimensionless parameter which affects the slope of the SWCC in the desaturation zone. Further details are available in Sillers et al. [24] and Leong and Rahardjo [22]; both of these papers provide examples regarding the influence of the parameters in each model.

TABLE I
PARAMETERS OF THE SWCC MODELS

Model	Parameter 1 (P1)*	Parameter 2 (P2)**
Gardner [10]	a	n
Brooks and Corey [11]	h_b	λ
Van Genuchten [16]	α	n
Kosugi [19]	h_m	σ

III. MATERIALS AND METHODS

A. Sources of Soil-Water Characteristic Curve Data

A total of 14 undisturbed soil samples represent sand textural class with soil-water characteristic data selected from the Unsaturated Soil Hydraulic Database (UNSODA 2.0) [26] were used to evaluate the performance of the four SWCC models. Fig. 1 displays the textural distributions of the sand soil. Soil samples were selected from the database based on availability of their basic soil properties (notably bulk density, porosity, and organic matter content). Table II summarizes the properties of the soil samples used in this study.

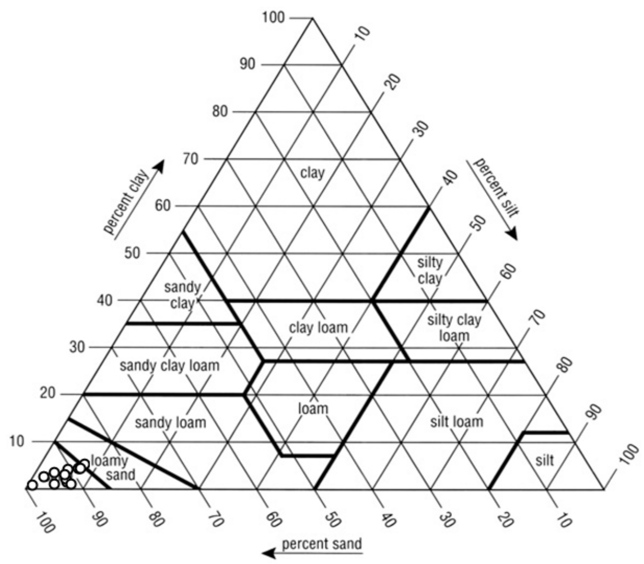


Fig. 1 Textural distribution of the 14 sand samples used in this study

TABLE II
BASICS PROPERTIES OF THE SOIL SAMPLES

Number of sample	14
Bulk density (g/cm^3)	1.15 - 1.622
Porosity (cm^3/cm^3)	0.377 - 0.55
Organic matter content (%)	0.2 - 3.9
Soil textural class (USDA classification)	Sand

B. Model Analysis

The soil-water characteristic curves obtained in this study are presented in terms of volumetric water content, with θ plotted on an arithmetic scale and soil suction ψ on a logarithmic scale. Optimization techniques were used to obtain best-fit parameters for each soil-water characteristic curve data set. The curve-fitting routine determines model parameters such that the mathematical function passes as close as possible to the experimental data points, without necessarily going through any of them. Fitting for all 14 soil data sets was performed via nonlinear least-square analysis based on a trust-region algorithm method carried out using the curve fitting tool in the MATLAB software program. The latter is an iterative method which starts with some initial values of the parameters. In the curve fitting tool program, the initial values of parameters for the Gardner, Brooks and Corey, Van Genuchten, and Kosugi models were obtained by using reported literature values [23] for the analyzed soils.

Various statistical measures can be employed to compare the fitting accuracy of SWCC models. In the present study, the root-mean-square error (RMSE) and the coefficient of determination (R^2) were used to help determine model best fit. The RMSE (m^3m^{-3}) statistic is an indicator of the overall error of the evaluated model function, with a value closer to zero indicating a better fit. In other words, the RMSE is a measure of the precision of the predicted parameters, and should be as small as possible for unbiased precise prediction. Generally the best indicator of fit quality, the R^2 statistic is a measure of the linearity between observed and fitted data, with a value approaching unity indicating that the fit explains most of the variability in the observed data.

The RMSE is here expressed as:

$$RMSE = \sqrt{\frac{1}{N}(SSE)} \tag{12}$$

where the SSE statistic is the least-squares error of the fit, defined as:

$$SSE = \sum_{i=1}^{N}\left[\theta_i^{obs} - \theta_i^{fit}(j)\right]^2 \tag{13}$$

where j is a parameter vector containing the unknowns that need to be estimated; i = 1, 2, …, N; N is the number of soil-water characteristic data values for each soil sample; θ_i is the soil water content value corresponding to the i data pair for each soil sample; and *obs* and *fit* denote observed and fitted values, respectively.

The value of R^2 reflects the proportion of the total sum of squares (SST) that is partitioned into the model sum of squares (SSM), since SST is equal to SSM plus SSE:

$$R^2 = \frac{SSM}{SST} = 1 - \frac{SSE}{SST} \tag{14}$$

IV. Results and Discussion

The four models with fitting parameters proposed by Gardner (GD), Brooks and Corey (BC), Van Genuchten (VG) and Kosugi (LN) were compared for the fourteen soil samples. Table III summarizes the models' fitted parameters value for the sand soil samples evaluated in this study. When considering the θ_s and θ_r values between each model, the LN and GD models produce slightly higher values (i.e. mean, upper and lower) than the VG and BC models. However, the BC model is a model that produces the lowest prediction values. This discrepancy is likely due to sample variability which influenced by the aggregation or distribution of individual SWCC data and also the models flexibility. As regards to the P1 and P2 values, one difficulty in comparing model parameters arises due to the fact that the function of the model parameter values is not in line with the other model parameters and thus cannot be compared directly. This uniqueness means that the grouping of soils with typical fitting parameters also becomes difficult. Nonetheless, the parameter values shown in Table III can still serve as useful initial values for researchers attempting to use one of the models to analyze a sandy soil type.

TABLE III
Model Parameter Values for Sand Soil

Model	Value	θ_s	θ_r	P1	P2
GD	Mean	0.4421	0.0551	0.2540	2.2864
	Upper	0.8896	0.1013	1.5450	9.5200
	Lower	0.3245	0.0342	1.83E-07	0.5143
BC	Mean	0.3853	0.0380	1.5569	0.9875
	Upper	0.4557	0.0971	3.9240	2.9600
	Lower	0.3070	3.08E-08	0.5580	0.2659
VG	Mean	0.4048	0.0487	0.5153	2.7881
	Upper	0.4765	0.0999	1.5020	9.7310
	Lower	0.3222	0.0246	-0.4450	1.4720
LN	Mean	0.4454	0.0578	3.7504	1.1761
	Upper	0.9910	0.1030	11.810	3.4170
	Lower	0.3210	0.0345	0.2542	0.1721

Table IV shows the statistical measure values of the four models for the evaluated sandy soils. When considering the RMSE values (i.e. mean, upper and lower), the BC model is associated with the lowest values, meaning that the fitted curve produced via this model had the highest correlation with the observed SWCC. The LN model performed the least well, with intermediate results obtained using the VG and GD models. A similar trend was observed in terms of mean R^2 values, with the BC model again exhibiting the strongest linearity, closely followed by the VG, GD and LN models. Overall, the results indicate that the BC model scored highest in the analysis of sandy soils in terms of goodness of fit. Intermediate results were achieved by the GD and VG models and the lowest by the LN model.

TABLE IV
STATISTICAL MEASURE VALUES OF THE MODELS FOR SAND SOIL

Model	Value	R^2	RMSE
GD	Mean	0.9862	0.0171
	Upper	0.9980	0.0264
	Lower	0.9418	0.0066
BC	Mean	0.9877	0.0145
	Upper	0.9995	0.0265
	Lower	0.9388	0.0042
VG	Mean	0.9876	0.0160
	Upper	0.9966	0.0256
	Lower	0.9450	0.0086
LN	Mean	0.9848	0.0182
	Upper	0.9970	0.0266
	Lower	0.9406	0.0080

A more detailed analysis was carried out by considering the best-fit curves produced by the best-fitted models for each soil-water characteristic dataset. In this case, the accuracy statistical criterion (i.e., RMSE) was used to define the best models for each data set. Fig. 2 illustrates the best-fitting models for each soil-water characteristic according to soil type. As shown in Fig. 2, the BC model produced encouraging results, being associated with the best-fitting curve for almost all sand samples available. BC model predictions were also successful for a range of values from low to high soil water content. The GD model shows worked best only for the analysis of moderate soil water content, and the VG model for moderate and higher levels of soil moisture. The LN model did not manage to better represent any of the sand samples analyzed in the present study.

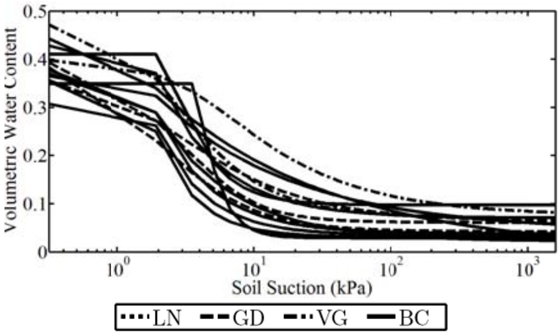


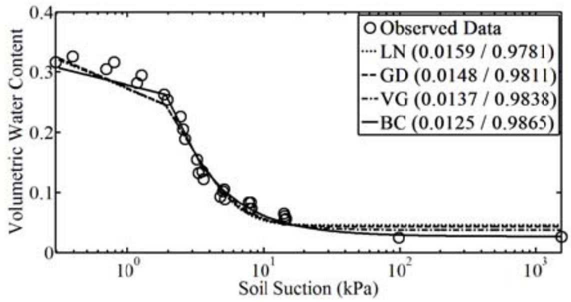
Fig. 2 Best-fit models for each SWCC dataset of sand soils

Table V enables a comparison to be made regarding the number of best-fit curves produced by the four models within each soil type. Regarding the 14 analyzed sandy soils, the BC model fit the data better in 9 samples, the GD model in 3 and the VG model in 2 samples. Although the LN model did not fit the data as well as the other three models, a reasonable fit was still produced for all samples.

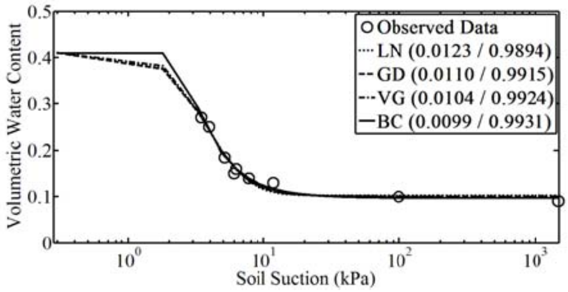
TABLE V
SUMMARY OF BEST-FIT CURVE PRODUCED BY THE FOUR MODELS

Model	Number of Best-fit samples
GD	3
BC	9
VG	2
LN	0
Total of samples	14

To further illustrate the behavior of the four models when fitted to the SWCC data for the analyzed sand soil, the observed data and fitted curves are compared in Fig. 3.



(a)



(b)

Fig. 3 Comparison of two sets of soil-water characteristic curve data fitted using the four models for sand soil. Figures in parentheses '( )' are the RMSE and R^2 values of the fitted curves according to the SWCC models (i.e., RMSE / R^2)

Analysis of the sand sample results in Fig. 3 (a) reveals that the BC model produced the best fit over the entire range of data compared to the other models - although it did not accurately match some points near soil saturation. The GD, LN and VG model graphs do not match the shape of the observed data near the saturation and residual regions, decreasing the flexibility of their respective curves. However, the three models still provided relatively accurate and realistic fits. With regard to the sand sample results shown in Fig. 3 (b), all models performed very well in that they produced relatively good and realistic fits. Indeed, the discontinuous character of the BC model did not seem problematic in terms of evaluating the sand soils analyzed in the present study. It should also be noted that the GD, LN and VG models exhibited almost the same fitting behavior, with the difference between them only marginal.

In summary, the statistical results show that the BC model led to the best fit for 9 of the 14 analyzed sand soil samples shown in Fig. 2 and Table V, with the BC model performing better than the GD, VG and LN models in terms of goodness of fit. Note that the mean fitting errors for the BC model were smaller than those for the three other models, as well as being the best model in terms of linearity (Table IV). The BC model also exhibited the better fit over the entire range of available observed data, as illustrated in Fig. 3, with this model's discontinuous character apparently unproblematic for sand soil evaluation. These results strongly support previous research in which similar observations were obtained by Cornelis et al. [23] for sandy soil.

V. Conclusion

The estimated and measured Soil-Water Characteristic Curve, (SWCC), of sand soils is compared in this study. The measured SWCCs data were obtained from UNSODA 2.0. The SWCC parameters of Gardner, Brooks and Corey, Van Genuchten and Kosugi models were estimated by using computer program MATLAB. The study indicates that the measured SWCCs of sand soils used in this investigation match satisfactorily with the estimated SWCC by Brooks and Corey model, which able to better represent almost all the samples, including those ranging from low to high soil water content. Therefore, it can be concluded that the Brooks and Corey model is the most suitable for describing the SWCC of a sandy soil.

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