

Relativistic addition of velocities in a five-dimensional spacetime

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Abstract

Another method of adding relativistic velocities is shown. It uses a fifth dimension of spacetime rotating the framework of the Minkowski diagram into it and thus is an indication of the existence of a fifth dimension. As proof of correctness of the result of the rotation the addition theorem of velocities is derived from it. Photographs of a hardware model illustrate how to find the resulting velocity by the rotation into the five-dimensional spacetime. Alongside the paradox " $c + u' = c$ " is resolved.

1. Introduction

In Minkowski diagrams relativistic addition of velocities according to the addition theorem of velocities can be illustrated [1]. For example $v = 0,6 c$ and $u' = 0,65 c$ results in $u = 0,899 c$

and in the according Minkowski diagram (fig.1) $u = \Delta x / \Delta t = (15,5 \text{ Ls}) / (17,2 \text{ s}) = 0,901 c$ can be read, which means the same numerical value within the accuracy of drawings.

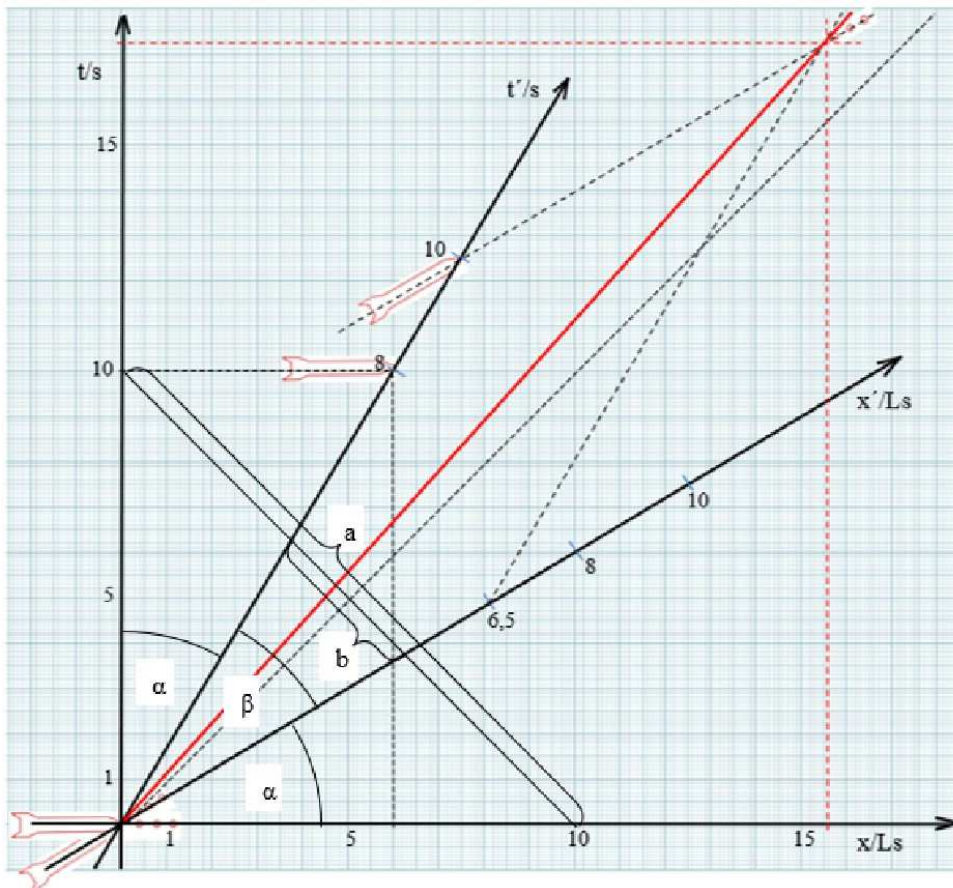


Fig. 1: Minkowski diagram of a uniformly moved rocket in space at a speed of $v = 0.6 c$, which shoots protons at the coordinate origin in the direction of the movement at a speed of $u' = 0.65 c$. In the framework S of the observer the protons move on the red worldline.

One can imagine the oblique angled framework S' with the coordinate axes x' and t' being originated by turning the rectangular framework S around the angle bisector by an angle φ into the fifth dimension (out of the drawing level), with the orthogonal projections of the axes x and t into the drawing level lying on the axes x' and t' .

Hereinafter is shown that the projection of the worldline of the body with the speed u' (a proton) in the rest system of the body with the speed v (the rocket) into the drawing level coincides by the turning with its (red) worldline in System S . This means that velocities can be added relativistic assuming the existence of a fifth dimension and using it.

2. Adding velocities within the five-dimensional spacetime

The angle φ depends on the velocity v or on v/c in this way:

$$\cos\varphi = \tan\left(\frac{\pi}{4} - \arctan\frac{v}{c}\right) .$$

Proof:

Figure 1 shows that

$$\beta = \frac{\pi}{2} - 2\alpha$$

and

$$\tan\alpha = \frac{v}{c} ,$$

$$\text{so} \quad \beta = \frac{\pi}{2} - 2 \arctan\frac{v}{c} .$$

Now β is substituted by the rotation angle φ . For that purpose in figure 2 is shown the spatial representation of the rotation of the line a in figure 1 around the angle bisector w by angle φ . By the rotation the line a becomes the line b .

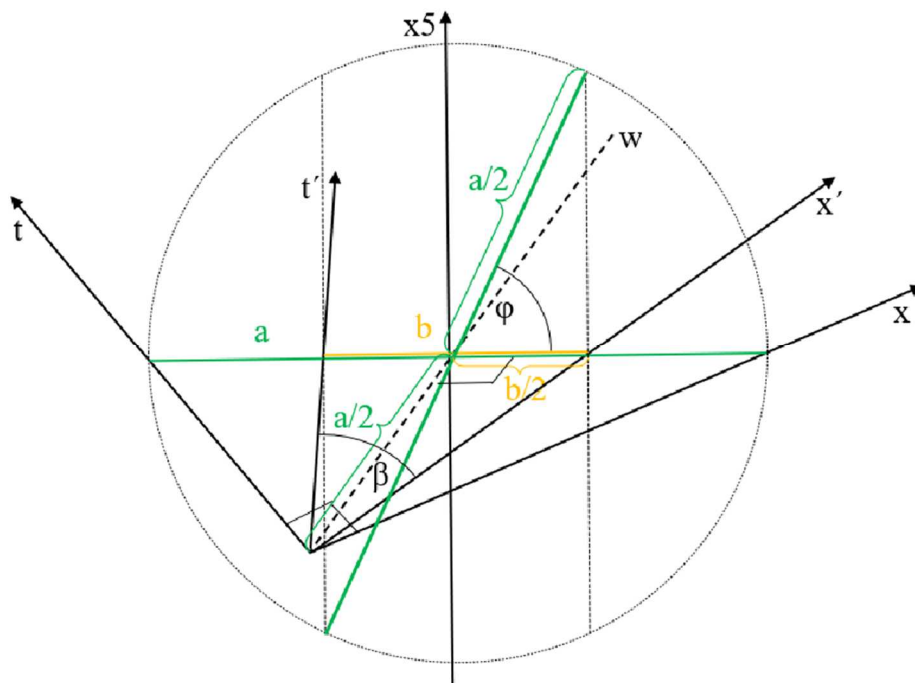


Fig. 2: Spatial representation of the rotation of the line a in fig. 1 around the angle bisector w by angle φ .

In the horizontal triangle can be seen

$$\tan\frac{\beta}{2} = \frac{b/2}{a/2} ,$$

in the vertical triangle

$$\cos\varphi = \frac{b/2}{a/2} .$$

Thus $\cos\varphi = \tan\frac{\beta}{2}$

and so $\cos\varphi = \tan\left(\frac{\pi}{4} - \arctan\frac{v}{c}\right)$ (1)

or $\cos\varphi = \tan(45^\circ - \alpha)$.

In fig. 3 a rocket at the speed of $v = 0.2 c$ fires a proton with the speed $u' = 0.3 c$ in the rest system of the rocket. The t' -

axis is the worldline of the rocket in system S, the green line is the worldline of the proton in S' before the turning and

the red line is the worldline of the proton in S after the turning respectively the projection of its worldline in S' . So the resulting velocity can be found turning the green worldline by the angle φ .

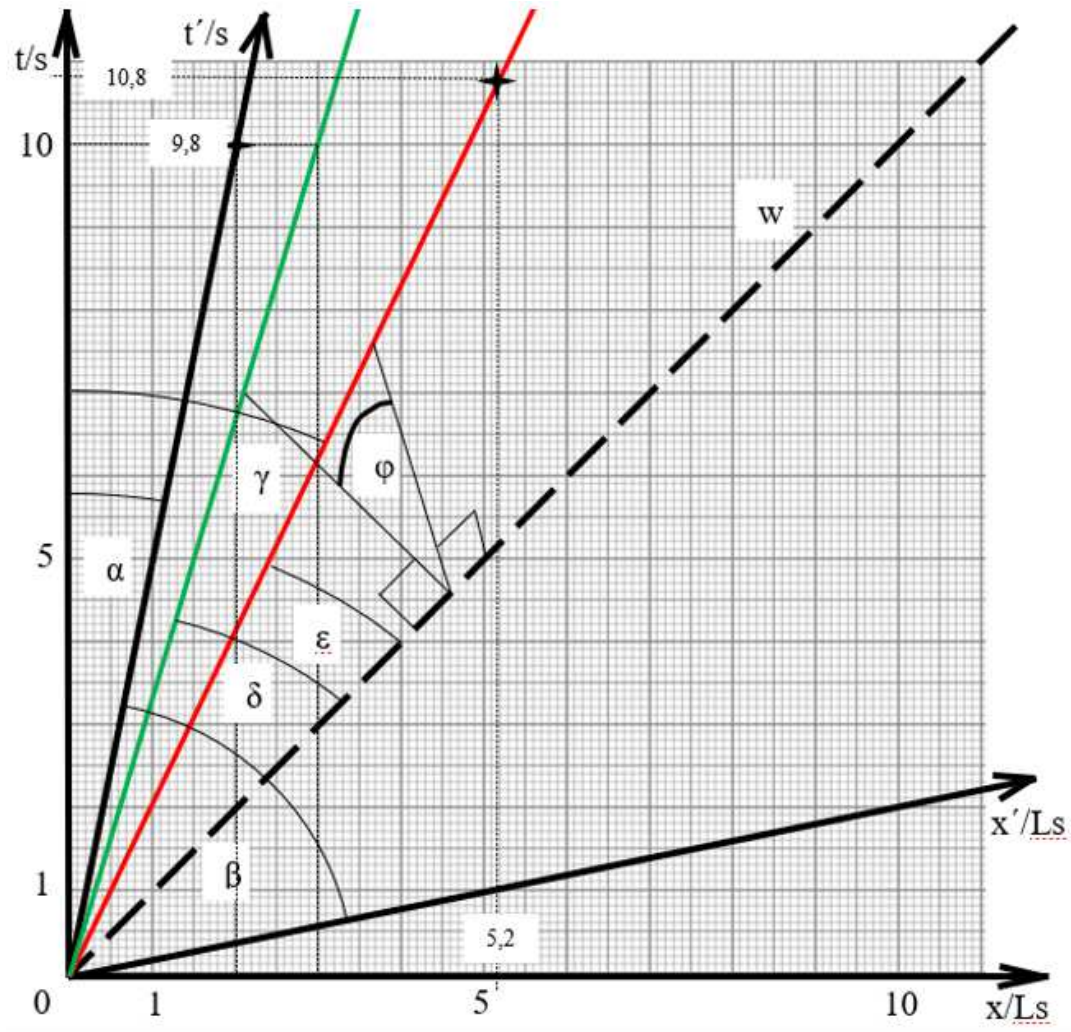


Fig. 3: Minkowski-diagram for the relativistic addition of the velocities $v = 0.2 c$ and $u' = 0.3 c$; The addition theorem of velocities results in $u = 0.4717 c$, the drawing in $u=5.2 Ls / 10.8 s = 0.48 c$. (The rotation angle is $\varphi = 48.19^\circ$. With $\tan\gamma = \frac{u}{c} = 0.4717$ follows $\gamma = 25,25^\circ$, see illustration by example 2 in the attachment).

3. Derivation of the addition theorem of velocities

As a proof that the turning of S' into the fifth dimension actually results in the same relativistic sum of velocities as the

Minkowski diagram and as the addition theorem of velocities now the addition theorem of velocities is deduced using

the angle φ and the turning into the fifth dimension.

Rotating the framework S' into the fifth dimension by the angle φ produces the angle α between the time axes (see fig. 3) and the projection of the green worldline of a proton in S' becomes the red worldline in S which includes the angle γ with the t -axis. The angle bisector includes the angle δ with the green worldline and the angle ε with the red worldline.

Figure 3 shows $\tan \alpha = \frac{v}{c}$,
 $\tan \gamma = \frac{u}{c}$, $\tan (45^\circ - \delta) = \frac{u'}{c}$ (2)

By the rotation the projection of the angle δ becomes the angle ε with the equation

$$\tan \varepsilon = \tan \delta \cdot \cos \varphi .$$

Proof:

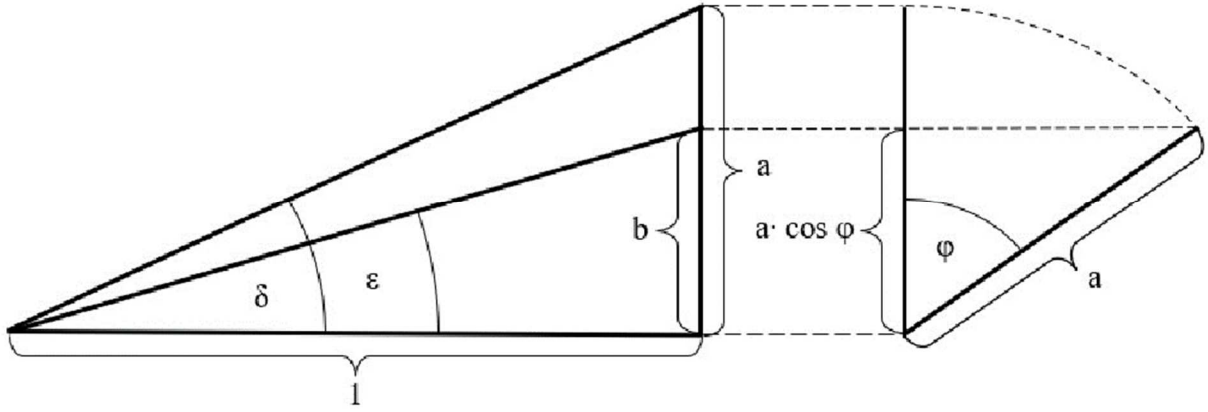


Fig. 4: Downside view (left) and side view (right) of the rotation of the angle δ out of the drawing level; ε is the orthogonal projection of δ into the drawing level.

According to figure 4 $b = a \cdot \cos \varphi$

as well as $\tan \delta = a$

and $\tan \varepsilon = b$.

So $\tan \varepsilon = a \cdot \cos \varphi$

and $\tan \varepsilon = \tan \delta \cdot \cos \varphi$.

With $\varepsilon = 45^\circ - \gamma$ follows

$$\tan (45^\circ - \gamma) = \tan \delta \cdot \cos \varphi . \quad (3)$$

From the addition theorem of the tangent

follows

$$\tan(45^\circ - \gamma) = \frac{1 - \tan \gamma}{1 + \tan \gamma} = \frac{1 - u/c}{1 + u/c} = \frac{c - u}{c + u} .$$

For $\tan \delta$ follows from (2)

$$\tan(45^\circ - \delta) = \frac{1 - \tan \delta}{1 + \tan \delta} = \frac{u'}{c}$$

so

$$1 - \tan \delta = \frac{u'}{c} (1 + \tan \delta)$$

and

$$\tan \delta = \frac{1 - u'/c}{1 + u'/c} = \frac{c - u'}{c + u'} .$$

For $\cos \varphi$ follows from

$$\cos \varphi = \tan (45^\circ - \alpha)$$

$$\cos \varphi = \frac{1 - \tan \alpha}{1 + \tan \alpha} = \frac{1 - v/c}{1 + v/c} = \frac{c - v}{c + v} .$$

From (3) then follows

$$\frac{c-u}{c+u} = \frac{c-u'}{c+u'} \cdot \frac{c-v}{c+v} \quad , \quad (4)$$

then

$$(u' + v) \cdot c^2 = u(c^2 + u'v)$$

and so

$$u = \frac{u'+v}{1+\frac{u'v}{c^2}} \quad ,$$

the addition theorem of velocities.

4. Examples

This result shall be illustrated on the basis of the second example with

$$v = 0.2 c \quad \text{and} \quad u' = 0.3 c.$$

If a straight line is given by the unit

vector $\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$ then any point in \mathbb{R}^3 is

turned by the rotation matrix

$$\begin{pmatrix} n_1^2 (1 - \cos \alpha) + \cos \alpha & n_1 n_2 (1 - \cos \alpha) - n_3 \sin \alpha & n_1 n_3 (1 - \cos \alpha) + n_2 \sin \alpha \\ n_2 n_1 (1 - \cos \alpha) + n_3 \sin \alpha & n_2^2 (1 - \cos \alpha) + \cos \alpha & n_2 n_3 (1 - \cos \alpha) - n_1 \sin \alpha \\ n_3 n_1 (1 - \cos \alpha) - n_2 \sin \alpha & n_3 n_2 (1 - \cos \alpha) + n_1 \sin \alpha & n_3^2 (1 - \cos \alpha) + \cos \alpha \end{pmatrix}$$

by the angle α around that straight line [2]. For $\alpha := \varphi = 48,19^\circ$ and the unit vector of the angle bisector between the

coordinate axes x and t $\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$ follows

the rotation matrix

$$\begin{pmatrix} 0,8333 & 0,1666 & 0,5270 \\ 0,1667 & 0,8333 & -0,5270 \\ -0,5270 & 0,5270 & 0,6667 \end{pmatrix} .$$

The point (3|10|0) of the worldline of the proton in framework S' before the turning is mapped by this matrix to the point (4,167|8,833|3,629) which is positioned exactly over the worldline of the proton in framework S . For with $\tan \gamma = 4,167/8,833$ follows $\gamma = 25,26^\circ$, which is the same angle as calculated at fig. 3 (the difference of 0.01° is a rounding error).

This means, that instead of the addition theorem of velocities or a Minkowski diagram a rotation matrix can be used, for with $\tan \gamma = u/c$ follows $u = 0.4718 c$ (the difference of 0.0001 is a rounding error).

Moreover the angel γ and from that the velocity u can be calculated with analytical geometry using the rotating by the angle φ too (see the lines d and e in fig. 5).

The 5d-model is confirmed at a hardware-model, see the photographs in figures 5, 6, 8, 9. Cameras do not produce parallel projections but central projections. That is why there are slight distortions and the right point of taking the pictures had to be chosen from a distance as large as possible. So the pictures had to be enlarged which caused fuzziness.

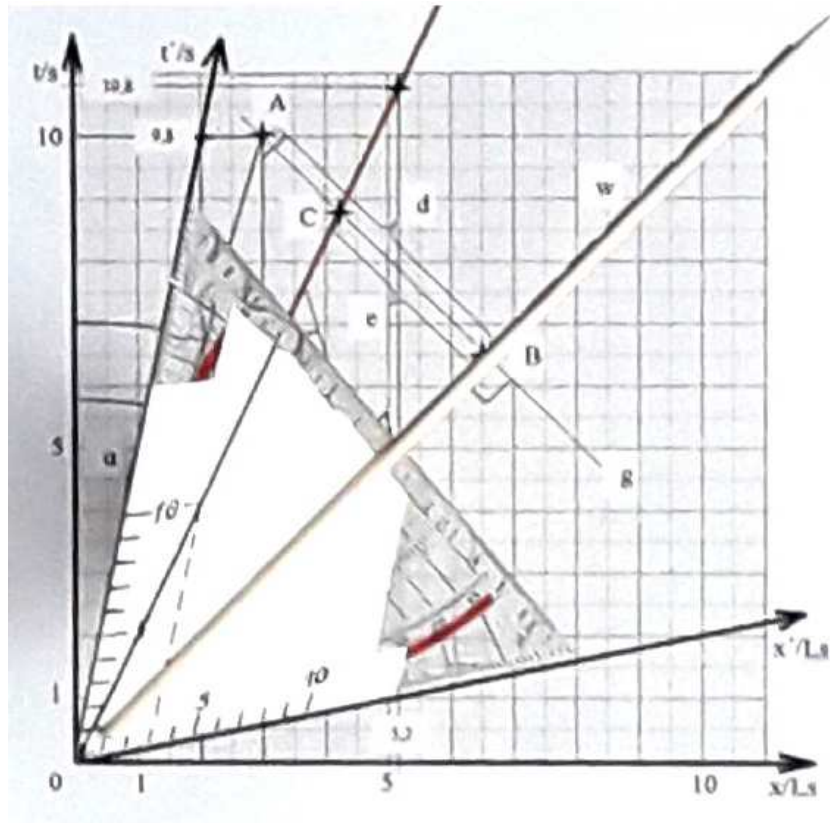


Fig. 5: Right-angled geo-triangle rotated by the angle φ at $v = 0.2 c$ and $u' = 0.3 c$. The dark worldline of the proton on the geo-triangle transitions differentially into its red worldline in framework S (the gap is caused by the flattened edge of the geo-triangle).

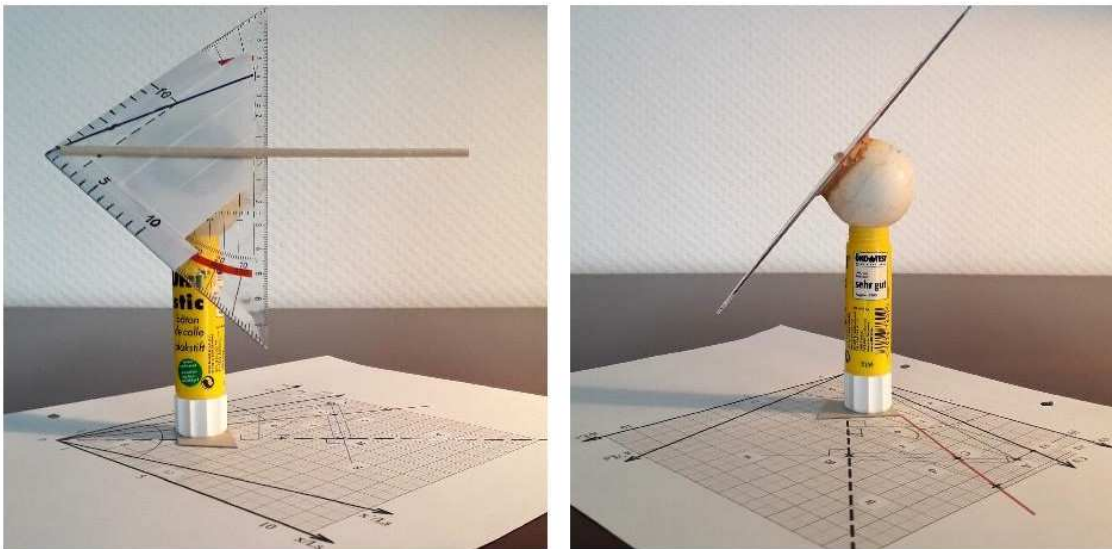


Fig. 6: The same object as in fig. 5 in side views which are orthogonal to each other; because of the central projection the angle φ is a bit bigger than $48,19^\circ$ ($50,5^\circ$). The parallel projection in fig. 10 shows an angle $\varphi = 48.2^\circ$.

Parallel projections are produced by a computer code written with GNU Octave (see attachment and fig. 7).

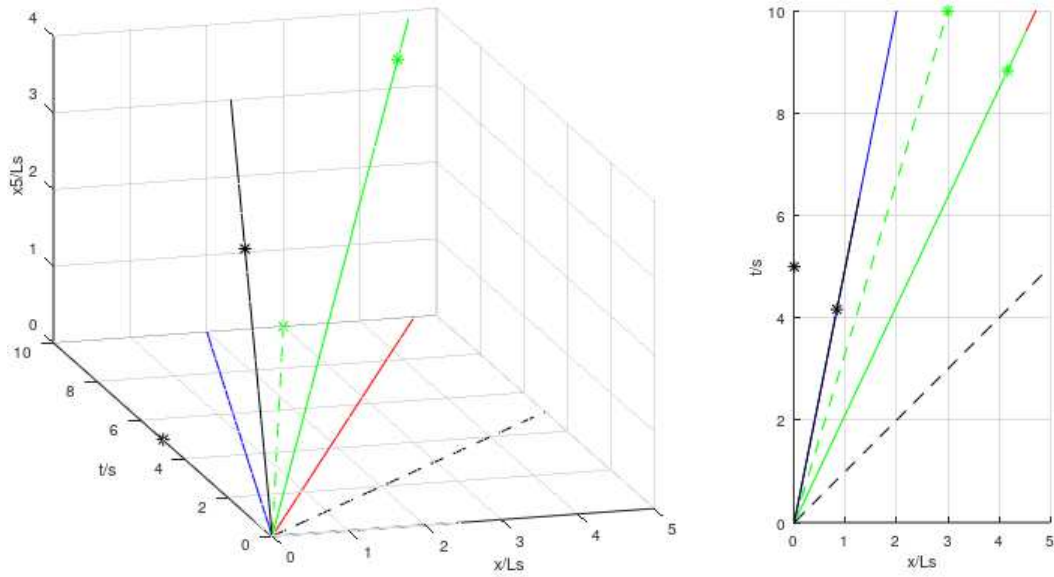


Fig. 7: The example of fig. 3 and 5, left side the spatial representation with the axes x , t and x_5 , the blue worldline for $v = 0.2 c$, the dotted green worldline for $u' = 0.3 c$, the red worldline of the resulting velocity $u = 0.4717 c$, the turned black t -axis and the turned green worldline. Right side the vertical projection shows that the blue worldline is the exact projection of the t -axis and the red worldline is the exact projection of the turned green worldline.

By clicking the icon with the double arrow (rotate) in one of the figures 7 the figure can be rotated respectively the point of view can be changed. This way the vertical projection can be produced.

The computer program "Video_turning_to_plan.mp4" (see attachment after fig. 9) shows this rotation from an oblique view to the plan.

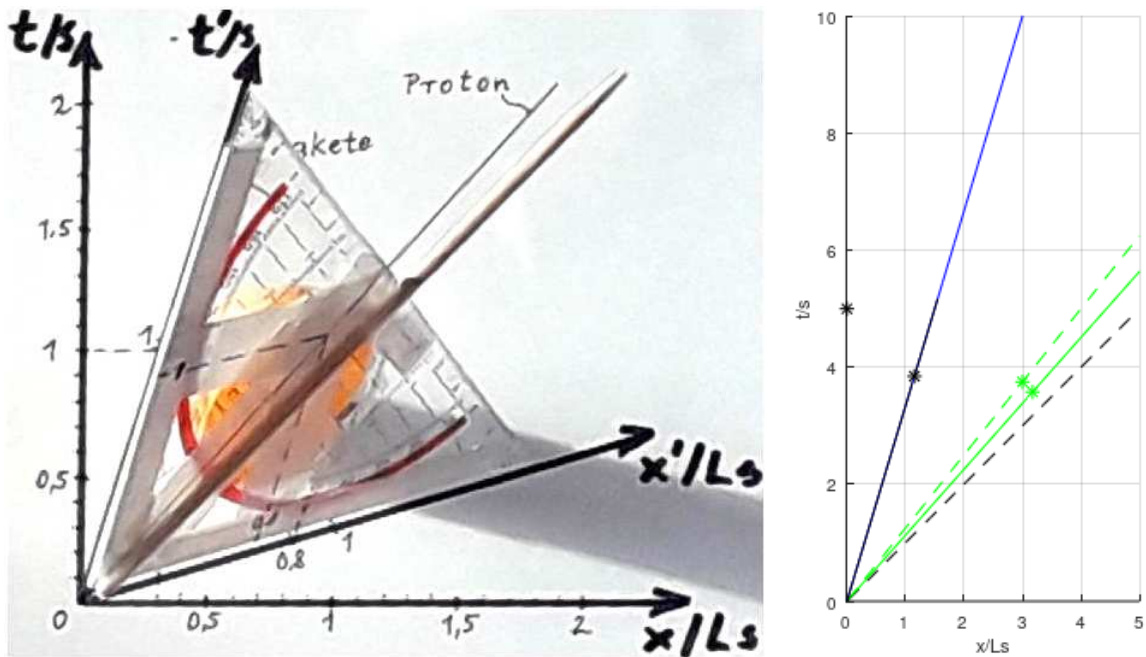


Fig. 8: Comparison of the central projection (left side) and the parallel projection (right side) for $v = 0.3 c$ and $u' = 0.8 c$ ($u = 0.8871 c$). The red worldline is covered by the

green worldline. Comparable angles are equal in the left and right picture; they are independent of the scales. See also attachment fig. 9.

The resulting velocity u can also be calculated by means of analytic geometry using the blue lines g , d and e in fig. 3. And

u can as well be calculated from the angles φ , α and β' (see fig. 9).

5. Resolution of the paradox " $c + u' = c$ "

In this five-dimensional model of the world resolves the paradox " $c + u' = c$ " which was confirmed by an experiment at CERN in 1964, when pi-mesons with $v = 0.00075 c$ emitted gamma-rays in the direction of movement, the velocity of which was measured as c . In the case of

$v = c$ the worldline of the first object is the angle bisector and the turning angle results in $\varphi = 90^\circ$. Then the orthogonal projection of the worldline of the second object of any velocity in S' coincides with the angle bisector in S , see fig. 9.

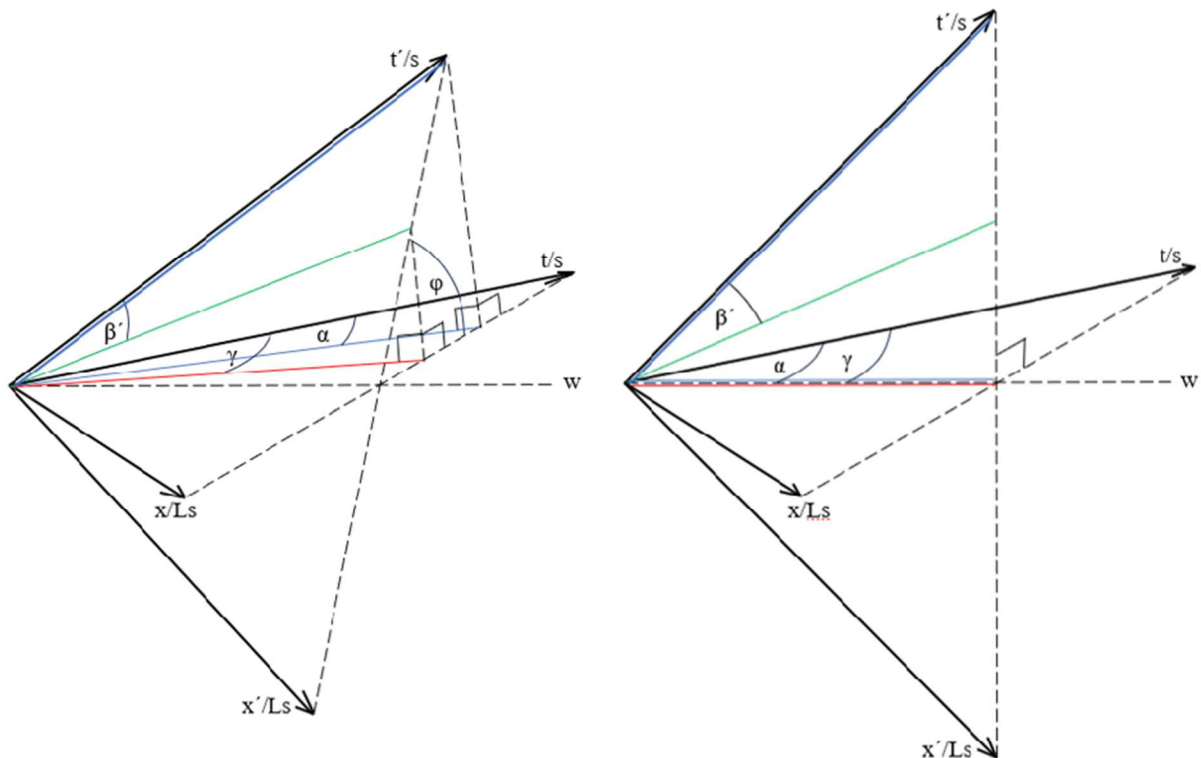


Fig. 9: Left side applies $0 \leq \varphi < 90^\circ$, right side applies $\varphi = 90^\circ$. If $\varphi = 90^\circ$ the orthogonal projection of the green worldline of the second object coincides with the angle bisector w for any angle β' ($0 < \beta' \leq 45^\circ$). So the resulting velocity in S always is c . Paradoxically the first and the second object move together in S although the first object emits the second one. But in the fifth dimension they move away from each other at the speed u' .

6. Summary

The shown method of adding relativistic velocities together with the derivation of the addition theorem of velocities supports the ingenious idea of Theodor Kaluza, who introduced a fifth dimension as the first one in 1921 [3], as well as the

Kaluza-Klein-theory [4]. Furthermore a five-dimensional spacetime may give the possibility to explain quantum entanglement and the orbitals of atoms being standing waves of the three-dimensional space.

6. Acknowledgement

Thanks to the author at the science forum matheplanet.de with the pseudonym MontyPythagoras, who doubted in the existence of a fifth dimension which I found in a preceding manuscript for matheplanet.de in 2022. He particularly gave the example of the relativistic addition of velocities as contradiction to a fifth dimension. That challenged me to deal with this subject and show that it is

working within a five-dimensional spacetime. Thereby I found an even stronger indication for the existence of a fifth dimension than in the previous manuscript.

Further thanks to Jonas Kaiser who gave me the idea to produce perfect projections of worldlines with GNU Octave by doing that with freeCAD.

References

- [1] Metzler Physik, J.B. Metzlersche Verlagsbuchhandlung, Stuttgart, 1988, p. 343-349
- [2] <https://de.wikipedia.org/wiki/Drehmatrix>
- [3] T. Kaluza: *Zum Unitätsproblem der Physik*. In: *Sitzungsberichte Preußische Akademie der Wissenschaften*, 1921, S. 966–972, archive.org.
- [4] Klein, O. Quantentheorie und fünfdimensionale Relativitätstheorie. *Z. Physik* **37**, 895–906 (1926). <https://doi.org/10.1007/BF01397481>

Attachment

GNU-Octave-Program for fig. 7:

```

1 # Relativistic_addition_of_velocities_in_RS
2 # Minkowski-diagram with rotation by the angle bisector
3
4 # t = (c/v)*x = (1/a) * x ; c/u' = 1/b
5 a=0.2 ; disp('v/c = '); a
6 b=0.3 ; disp('u'/c = '); b
7 x=linspace(0,5,50);
8 t1=(1/a)*x; t2=(1/b)*x; # worldlines of a 1st and a 2nd body in S
9 z=linspace(0,5,50);
10 clf; figure(1);
11 plot3(x,t1,0*z,'b'); # worldline of a 1st body in S
12 axis equal; grid on; hold on;
13 plot3(x,t2,0*z,'--g'); # worldline of the 2nd body (dotted green) before rotation
14 plot3(x,x,0*z,'--k'); # angle bisector (dotted black) in S
15 p=0:pi/2; # p = phi # rotation angle
16 disp('rotation angle phi := p')
17 p=acos(tan((pi/4)-atan(a)))
18 phi=(p/pi)*180
19 q=sqrt(2);
20 A=[0;5;0]; plot3(A(1),A(2),A(3),'*k')
21 B=[2;2/b;0]; plot3(B(1),B(2),B(3),'*g')
22 R=[(1-cos(p))/2+cos(p), (1-cos(p))/2, sin(p)/q; (1-cos(p))/2, (1-cos(p))/2+cos(p), -sin(p)/q; -sin(p)/q, -sin(p)/q, sin(p)/q, cos(p)];
23 # R = rotation matrix for rotation around the angle bisector
24 disp('rotation matrix R'); R
25 E=R*A; plot3(E(1),E(2),E(3),'*k')
26 F=R*B; plot3(F(1),F(2),F(3),'*g')
27 disp('location vector of point F'); F
28 plot3(E(1)*x,E(2)*x,E(3)*x,'k'); # t'-axis = turned worldline of the 1st body in S' (black)
29 plot3(F(1)*x,F(2)*x,F(3)*x,'g'); # turned worldline of the 2nd body in S' (green)
30 # worldline of the 2nd body in S:
31 plot3(F(1)*x,F(2)*x,0*x,'r'); # projection of the green worldline from S' to S (red)
32 xlim([-0.01 5+0.01])
33 ylim([0 10])
34 zlim([0 4])
35 xlabel('x/Ls'); ylabel('t/s'); zlabel('x5/Ls')
36 title('Relativistic addition of velocities in RS')
37 disp('u/c =')
38 d=F(1)/F(2) # wanted resulting velocity u/c of the 2nd body in S

39
40 # for comparison calculation with the addition theorem
41 # uc=(u'+v)/(1+u'*v/c^2) = [(a + b)/(1 + a*b)] c
42 hold off; disp(' ')
43 disp('comparison with the addition theorem:')
44 d=(a+b)/(1+a*b)
45
46 # the same diagram in vertical projection:
47 figure(2); clf; hold on; grid on; axis equal;
48 plot3(x,t1,0*z,'b'); # worldline of the 1st body (blue) in S
49 plot3(x,t2,0*z,'--g'); # worldline of the 2nd body (dotted green) before rotation
50 plot3(x,x,0*z,'--k'); # angle bisector (dotted black) in S
51 A=[0;5;0]; plot3(A(1),A(2),A(3),'*k')
52 B=[2;2/b;0]; plot3(B(1),B(2),B(3),'*g')
53 E=R*A; plot3(E(1),E(2),E(3),'*k')
54 F=R*B; plot3(F(1),F(2),F(3),'*g')
55 plot3(E(1)*x,E(2)*x,E(3)*x,'k'); # t'-axis = turned worldline of the 1st body in S' (black)
56 plot3(F(1)*x,F(2)*x,F(3)*x,'g'); # turned worldline of the 2nd body in S' (green)
57 plot3(F(1)*x,F(2)*x,0*x,'r'); # projection of the green worldline from S' to S (red)
58 xlim([-0.01 5])
59 ylim([0 10])
60 zlim([0 4])
61 xlabel('x/Ls'); ylabel('t/s'); zlabel('x5/Ls')
62 title('Fig. 1 in vertical projection')
63 hold off;

```

First type this code into the editor window (as shown here), because it can be corrected only there. Store it with a name followed by .m ; then copy it to the command window and start it with that name, but without .m

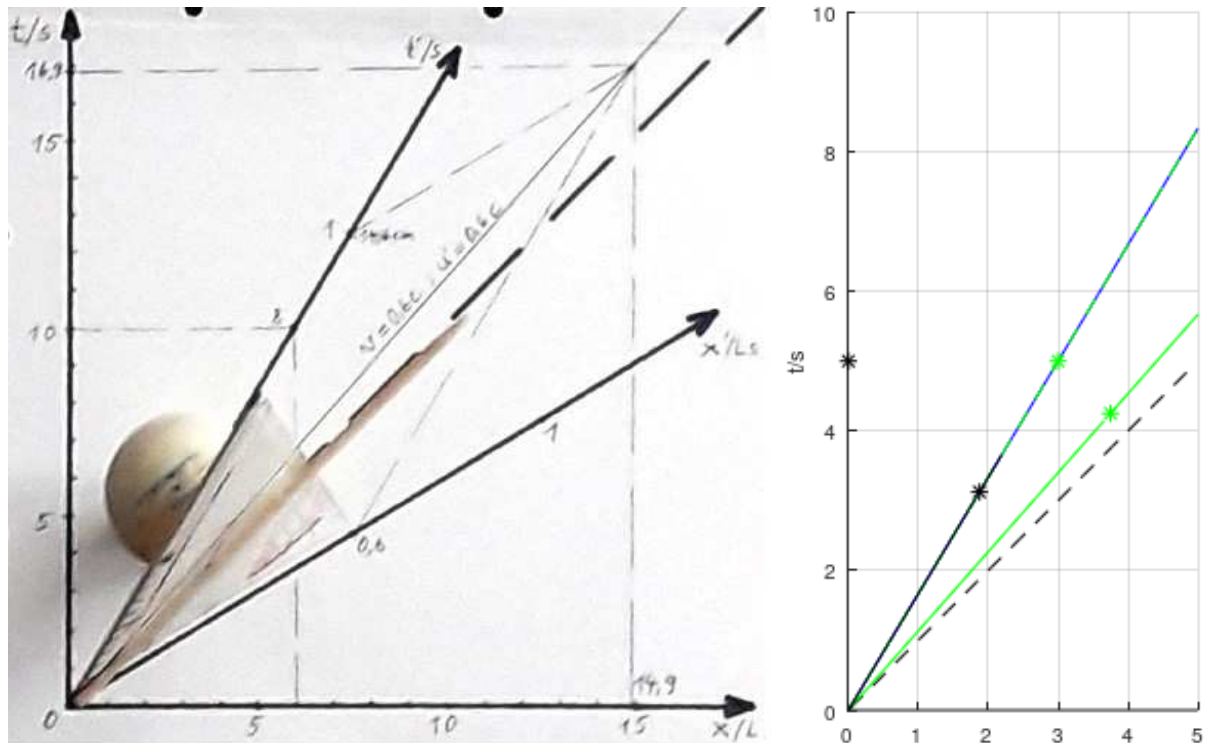


Fig. 9: : Comparison of the central projection (left side) and the parallel projection (right side) for $v = 0.6 c$ and $u' = 0.6 c$ ($u = 0.8824 c$).

GNU-Octave-program for the “Video_turning_to_plan.mp4”:

```

1 fn='Video_turning_to_plan.mp4';
2 w=VideoWriter(fn);
3 a=0.7; b=0.3;
4 x=linspace(0,5,50);
5 z=linspace(0,5,50);
6 p=0:pi/2;
7 p=acos(tan((pi/4)-atan(a)));
8 q=sqrt(2);
9 R=[(1-cos(p))/2+cos(p), (1-cos(p))/2, sin(p)/q; (1-cos(p))/2, (1-cos(p))/2+cos(p), -sin(p)/q; -sin(p)/q, sin(p)/q, cos(p)];
10
11 figure(301); clf; grid on; axis equal;
12 xlim([-0.01 5+0.01])
13 ylim([0 8])
14 zlim([0 4])
15 hold on; view(3)
16 h=plot3(5.01,8,4,'.');
17 title('Turning to plan')
18 xlabel('x/Ls'); ylabel('t/s'); zlabel('x5')
19
20 for n=1:301
21 t1=(1/a)*x;
22 t2=(1/b)*x;
23 plot3(x,t1,0*z,'b');
24 view([-5+(5/300)*(n-1), -20+(20/300)*(n-1), 4+(30/300)*(n-1)]);
25 plot3(x,t2,0*z,'--g');
26 plot3(x,x,0*z,'--k');
27 A=[0;5;0]; plot3(A(1),A(2),A(3),'*k')
28 B=[2;2/b;0]; plot3(B(1),B(2),B(3),'*g')
29 E=R*A; plot3(E(1),E(2),E(3),'*k')
30 F=R*B; plot3(F(1),F(2),F(3),'*g')
31 plot3(E(1)*x,E(2)*x,E(3)*x,'k');
32 plot3(F(1)*x,F(2)*x,F(3)*x,'g');
33 plot3(F(1)*x,F(2)*x,0*x,'r');
34 drawnow;
35 writeVideo(w,getframe(gcf));
36 endfor
37 close(w)

```

Recommendation: First type in the command window 'pkg load video', copy the code to the command window and start it with 'open Video_turnig_to_plan.mp4'.

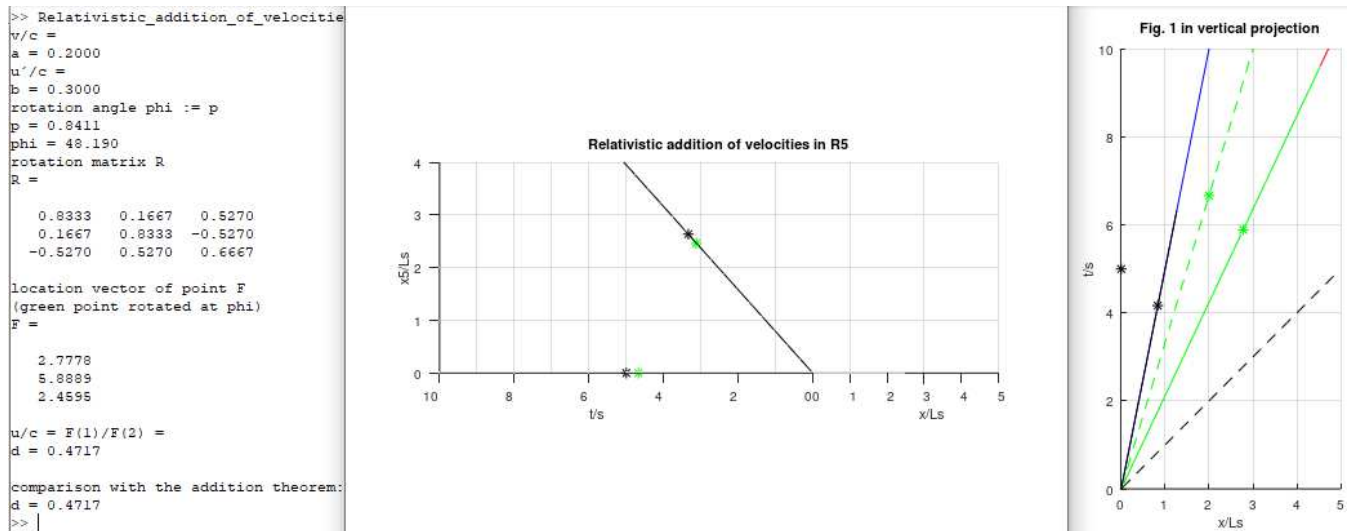


Fig. 10: Parallel projection of example 2 with $v = 0.2 c$, $u' = 0.3 c$ and $\varphi = 48,19^\circ$. The measurement of φ in the turned left diagram gives the angle 48.2° .