Theodor Kaluza's Theory of Everything: revisited

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Abstract

Using a Kaluza-type of model, describing the laws of electromagnetism within the formalism of differential geometry, provides a coherent, comprehensive and quantitative description of phenomena related to particles, including a convergent series of quantized particle energies with limits given by the energy values of the electron and the Higgs vacuum expectation value as well as the values for electroweak coupling constants. The geometry of the solutions for spin 1/2 defines 6 lepton-like and 6 quark-like objects and allows to calculate the fractional electric charges as well as the magnetic moments of baryons.

Electromagnetic and gravitational terms will be linked by a series expansion, the corresponding relation suggests the existence of a cosmological constant in the correct order of magnitude.

The model can be expressed *ab initio*, necessary input parameters are the electromagnetic constants.

1 Introduction

Theodor Kaluza in 1919 developed a unified field theory of gravitation and electromagnetism that produced the formalism for the field equations of the general theory of relativity (GR) and Maxwell's equations of electromagnetism (EM) thus unifying the major forces known at his time. His 5-dimensional model [1] is not suited to give properties related to particles, a problem addressed by Oskar Klein [2] who introduced the idea of compactification and attempted to join the model with the emerging principles of quantum mechanics. Therefore the theory is mainly known as Kaluza-Klein theory today. This version became a progenitor of string theory. The original Kaluza model was developed further as well [3], Wesson and coworkers elaborated a general non-compactified version to describe phenomena extending from particles to cosmological problems. The equations of 5D space-time may be separated in a 4D Einstein tensor and metric terms representing mass and the cosmological constant, Λ. Particles may be described as photon-like in 5D, traveling on time-like paths in 4D. This version is known as space-time-matter theory [4]. Both successor theories give general relationships rather than providing quantitative results for specific phenomena such as particle energy.

The model described in the following does not attempt to give a complete solution for a 5D theory but to demonstrate that Kaluza's ansatz provides very simple, parameter-free and in particular quantitative solutions for a wide range of phenomena. Basic equations from the existing literature may be used, with one significant *simplification*:

Kaluza discovered that Maxwell's equations may be described within the formalism of GR. To get both these *and* the Einstein field equations (EFE) he needed an additional dimension and had to insert the constant of gravitation in his metric. He chose the gravitational term to keep the electromagnetic potential terms in the metric dimensionless, a rather unfitting combination[1]. If one settles for electromagnetic phenomena as first approximation there is no need for the gravitational constant. This does not give a unification of EM and GR, however, it is a suitable ansatz to "unify" EM and particle physics. Gravitational terms can be recovered via a series expansion of the electromagnetic equations and such a proceeding may actually reflect the huge difference in order of magnitude of both phenomena better than the more linear original approach.

Curvature of space-time based on an electromagnetic version of the field equations of GR will be strong enough to localize a photon in a self-trapping kind of mechanism, yielding energy states in the range of the particle zoo. Circular polarized light is part of conventional electromagnetic theory, in the following this feature will be treated equivalently with the terms angular momentum or spin as intrinsic property of a photon and will be a necessary boundary condition in the equations used. In particular, unless noted otherwise, it is assumed that particles posses spin 1/2 or are composed of spin 1/2 components (e.g. mesons).

The basic proceeding will be as follows:

Kaluza's equations for flat 5D-space-time may be arranged to give [4, chapter 6.6]

1) Einstein-like equations for space-time curved by electromagnetic and scalar fields (equ. (5)),

1 In the closing remarks of [1] Kaluza suggests to reconsider „die etwas fragwürdige Gravitationskonstante" – „the somewhat questionable constant of gravitation".

2) Maxwell equations where the source depends on the scalar field,
3) a wave-like equation connecting the scalar Φ with the electromagnetic tensor (equ. (6)).

Solutions of 3) for Φ in a flat 5D-metric will be used as general ansatz in a 4D-metric. This is considered to be a proof of concept only, a more thorough ansatz has to be expected to incorporate angular momentum/spin into the field equations appropriately.

The solution for Φ gives $\Phi \sim \exp(-(\rho/r)^3)$ and may be seen as representing curvature of 4D space-time. Due to the derivation from a Kaluza ansatz coefficient ρ is a function of the electromagnetic potential, A, in the static approximation of this work the electric potential. The only other parameter entering ρ will be a function of the fine-structure constant [2], α, which enters the equations through the boundary condition spin 1/2, requiring a relationship between the values of the electric potential, elementary charge and electric constant, and ħ/2, see chpt. 2.4 [3]. Since a geometric interpretation allows to give α in terms of Γ-functions ρ may be given in terms of elementary charge, electric constant and mathematical constants only.

Based on this the model yields absolute particle energies in the range expected for a neutrino and as a set of converging series with limits given by the energy of the electron and the Higgs vacuum expectation value. Assuming that a 2nd term in a series expansion of EM-terms represents gravitation and should not exceed the EM-term, some of the α-terms included in ρ can be identified with the ratio of electron and Planck energy, see chpt. 2.6f, 4.1 [4]. With this ansatz additional minor terms in the field equations will be in the correct order of magnitude for the cosmological constant, Λ.

Focusing on the angular momentum aspects of the model, in chpt. 3 the rotation of a set of orthogonal E, B, C-vectors, attributed to the electromagnetic fields and the propagation with the speed of light, C, will be modeled via quaternions. This gives 3 possible solutions for spin 1/2 defining 6 distinct geometric objects that can be matched with the properties of the 6 leptons and the 6 quarks of the standard model of particle physics (SM). Using the results of this model for energy, values of magnetic moments for J = 1/ 2 baryons of the uds-octet may be calculated *ab initio*.

Typical accuracy of the calculations is in the order of 0.0001 [5]. The deviation of calculated results from the experimental values is typically in the range 0.01 - 0.001, consistent with a variation of input parameters related to elementary charge in an order of magnitude of QED corrections, which are not included in this model.

To focus on the more fundamental relationships some minor aspects of the model are exiled to an appendix, related topics will be marked as [A].

2 Calculation

2.1 System of natural units

The approach sketched in the introduction requires the use of an electromagnetic unit system appropriate for the general formalism of GR. It is common to define natural electromagnetic units by referring them to the value of the speed of light. The same will be done here, thus subscript c will be used. Retaining SI units for length, time and energy the electromagnetic constants may be defined as:

$$c_0^2 = (\varepsilon_c \, \mu_c)^{-1} \tag{1}$$

with $\varepsilon_c = (2.998\text{E+8 [m}^2\text{/Jm]})^{-1} = (2.998\text{E+8})^{-1}$ [J/m]
$\mu_c = (2.998\text{E+8 [Jm/s}^2])^{-1} = (2.998\text{E+8})^{-1}$ [s²/Jm] .

From the Coulomb term $b_0 = e^2/(4\pi\varepsilon_0) = e_c^2/(4\pi\varepsilon_c) = 2.307\text{E-28}$ [Jm] follows for the square of the elementary charge: $e_c^2 = 9.671\text{E-36}$ [J²]. In the following $e_c = 3.110\text{E-18}$ [J] and $e_c/(4\pi\varepsilon_c) = 7.419\text{E-11}$ [m] may be used as natural unit of energy and length.

With the unit system above the T_{00}-component of the electromagnetic stress-energy-tensor in the field equation in an electrostatic approximation will simply be $T_{00} = E^2/2$ [m⁻²]. In the case of T_{00} referring to energy density the constant G/c_0^4 [m/J] in the Einstein field equations (EFE) will be replaced by:

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2 The relation of the masses e, μ, π with α was noted first in 1952 by Nambu [5]. MacGregor calculated particle mass and constituent quark mass as *multiples* of α and related parameters [6].

3 The coefficient of angular moment may be interpreted as either σ, which will in general indicate the integration limit, $(r/\rho)^3$ for calculating the incomplete gamma functions, or its main component $\alpha_{lim} \approx 1.5/\alpha$, see chpt. 2.4, 2.5.

4 Giving $\rho^3 \approx \sigma\, \alpha_{Pl}\, \rho_0^3$;

5 Including e.g. errors due to the numerical approximation of incomplete Γ-functions.

$$\left(8\pi\right)G/c_0^4 \quad => \quad \approx \quad 1/\varepsilon_c \tag{2}$$

in an accordingly modified field equation:

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = -\frac{1}{\varepsilon_c}T_{\alpha\beta} \tag{3}$$

2.2 Kaluza theory

Kaluza theory is an extension of general relativity to 5D-space-time with a metric given as [4, equ. 2.2]:

$$g_{AB} = \begin{bmatrix} \left(g_{\alpha\beta} - \kappa^2\Phi^2 A_\alpha A_\beta\right) & -\kappa\Phi^2 A_\alpha \\ -\kappa\Phi^2 A_\beta & -\Phi^2 \end{bmatrix} \tag{4}$$

In (4) roman letters correspond to 5D [6], Greek letters to 4D. κ corresponds to the constant in the field equation (2), A is the electromagnetic potential. In the context of the electrostatic approximation of this model A will be assumed to be represented by the electric potential, $A_{el} = e_c/(4\pi\varepsilon_c r) = \rho_0/r$ [-]. Assuming 5D space-time to be flat, i.e. $R_{AB} = 0$, gives for the 4D-part of the field equations [4, equ. 2.3]:

$$G_{\alpha\beta} = \frac{\kappa^2\Phi^2}{2}T_{\alpha\beta}^{EM} - \frac{1}{\Phi}\left(\nabla_\alpha\left(\partial_\alpha\Phi\right) - g_{\alpha\beta}\Box\Phi\right) \tag{5}$$

From $R_{44} = 0$ follows:

$$\Box\Phi = -\frac{\kappa^2\Phi^3}{4}F_{\alpha\beta}F^{\alpha\beta} \tag{6}$$

In the following only derivatives with respect to r of a spherical symmetric coordinate system will be considered. Equation (6) will be used to obtain an ansatz for a metric to get a solution of the 00-component in (3). A function Φ_N

$$\Phi_N \approx \left(\frac{\rho}{r}\right)^{N-1} e^{\nu/2} = \left(\frac{\rho}{r}\right)^{N-1}\exp\left(-\left(\frac{\rho}{r}\right)^N/2\right) \tag{7}$$

yields solutions for an equation of general type of (6), where the term of highest order of exponential N, given by $\Phi'' \sim \rho^{3N-1}/r^{3N+1}$, may be interpreted to provide the terms for $A'(r) \sim e_c/(4\pi\varepsilon_c r^2) \sim \rho_0/r^2$, see [A1]:

$$\Phi_N'' \sim \left(\frac{\rho^{3N-1}}{r^{3N+1}}\right)e^{\nu/2} \sim \boldsymbol{\Phi_N^3}\ e^{-\nu}\left(\boldsymbol{A_0}'\right)^2 \approx \left[\left(\frac{\boldsymbol{\rho}}{\boldsymbol{r}}\right)^{\boldsymbol{N-1}}\boldsymbol{e^{\nu/2}}\right]^{\boldsymbol{3}} e^{-\nu}\left(\frac{\boldsymbol{\rho}}{\boldsymbol{r^2}}\right)^{\boldsymbol{2}} = \left(\frac{\rho}{r}\right)^{3N-3}e^{\nu/2}\left(\frac{\rho}{r^2}\right)^2 \tag{8}$$

The significance of (7)f lies in providing the relationship of exponential and pre-exponential terms and first of all in the requirement to contain powers of $A_{el} \sim (\rho_0/r)$ in the exponent of Φ_N, to be used in the following.

2.3 Example for metric, point charge energy

The following 4D-metric for N = 3 in (7)f, using only diagonal components will be used as a general ansatz:

$$g_{\mu\mu} = \left(\frac{\rho_0}{r}\right)^2\exp\left(-\left(\frac{\rho}{\boldsymbol{r}}\right)^3\right),\ -\left(\frac{\rho_0}{r}\right)^2\exp\left(\left(\frac{\rho}{\boldsymbol{r}}\right)^3\right),\ -r^2,\ -r^2\sin^2\vartheta \tag{9}$$

This is based on the following considerations (cf. [A2]):

1) flat 5-D-space-time;

2) the limit in absence of electromagnetic fields will not be given by a component $g_{\alpha\beta}$ related to gravitational effects as given in equ. (4), gravitational terms (and flat space-time limits) may be recovered by a series expansion of the exponential terms of (9) [7], see chpt. 4.1;

3) coefficient ρ_0 in the pre-exponential terms ensures Coulomb terms as limit cases;

4) Equation (9) is an approximation not only in neglecting contributions of the magnetic potentials but also in not considering spin, a necessary boundary condition for particles which is not represented in Kaluza's equations either. Thus some modification in the metric of (4) has to be expected. A metric according to (9) will give correct quantitative particle related results. However, only the exponential part of Φ_3, $e^{\nu/2}$, will be squared in the metric terms, giving $e^\nu A_\alpha A_\beta$ instead of $e^\nu(A_\alpha A_\beta)^2$. This is somewhat ad hoc and considered a

6 ct, x1, x2, x3, x4

7 Or other terms according to [A2.2]

proof of concept only [8].

Equation (3) gives a generic example that may be modified significantly in 4D or 5D, the dominant term for particle energy will originate from the angular terms [9], see [A2.1].

The Einstein tensor component G_{00} will be:

$$G_{00} = - \rho_0^2/r^4 e^v \qquad (10)$$

and using equ. (3) will give:

$$\frac{\rho_0^2}{r^4} e^v \approx \frac{w}{\varepsilon_c} \quad => \quad \frac{\varepsilon_c \rho_0^2}{r^4} e^v \approx w \qquad (11)$$

The volume integral over (11) gives the energy of particle n according to:

$$W_n = \varepsilon_c \rho_0^2 \int_0^{r_n} \frac{e^{v(n)}}{r^4} d^3 r = 4\pi \varepsilon_c \rho_0^2 \int_0^{r_n} \frac{e^{v(n)}}{r^2} dr \qquad (12)$$

Solutions for integrals over e^v, with v according to (7), times some function of r can be given by:

$$\int_0^{r_n} \exp(-(\rho_n/r)^N) r^{-(m+1)} dr = \Gamma(m/N, (\rho_n/r_n)^3) \frac{\rho_n^{-m}}{N} = \int_{(\rho_n/r_n)^3}^{\infty} t^{\frac{m}{N}-1} e^{-t} dt \frac{\rho_n^{-m}}{N} \qquad (13)$$

in this work used for N = {3; 4}, m = {-2; -1; 0; +1;+2}. The term $\Gamma(m/N, (\rho_n/r_n)^3)$ denotes the upper incomplete gamma function, given by the Euler integral of the second kind [10]. In the range of values relevant in this work, for $m/N \geq 1$ the complete gamma function $\Gamma_{m/N}$ is a sufficient approximation, for $m/N \leq 0$ the integrals have to be calculated numerically, requiring an integration limit, see 2.4.

Equation (12) will give as energy for a particle n:

$$W_{n,elstat} = 4\pi \varepsilon_c \rho_0^2 \int_0^{r_n} \frac{e^{v(n)}}{r^2} dr = b_0 \Gamma(+1/3, (\rho_n/r_n)^3) \rho_n^{-1}/3 \approx b_0 \Gamma_{+1/3} \rho_n^{-1}/3 \qquad (14)$$

including the integral for the energy of a point charge term modified by e^v. Particles are supposed to be electromagnetic objects possessing photon-like properties, thus it will be assumed that particle energy has equal contributions of electric and magnetic energy, i.e.:

$$W_n = W_{n,elstat} + W_{n,mag} = 2W_{n,elstat} \approx 2 b_0 \Gamma_{+1/3} \rho_n^{-1}/3 \qquad (15)$$

Except for the Coulomb-term, ρ_0, entering via the Kaluza ansatz, the only other terms in ρ may be given as function of the fine-structure constant, α [11], which is a consequence of the boundary condition of spin 1/2 ħ.

2.4 Angular momentum, coefficient σ

The integral limits required for Euler integrals of (13) with $m/N \leq 0$ are r_n („particle radius" of state n; with respect to J_Z; $r_n \neq \lambda_C$, see (63)) in integrals over e^v and $(\rho_n/r_n)^3$ in the Euler integrals. The latter will be expressed via a constant defined as 8/σ [12]:

$$(\rho_n/r_n)^3 = 8/\sigma \qquad (16)$$

whose value may be derived from the condition for angular momentum J_z = 1/2 [ħ]. A simple relation with angular momentum J_z for spherical symmetric states will be given by applying a semi-classical approach [13]:

$$J_z = r_2 x\, p(r_1) = r_2 W_n(r_1)/c_0 \equiv 1/2\ [\hbar] \qquad (17)$$

Using term $2b_0$ of equ. (15) as constant factor and integrating over a circular path of radius $|r_2| = |r_1|$, equation

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8 A solution of (7) with N = 1 could be inserted directly in Kaluza's term $\Phi^2 A_\alpha A_\beta$ of (4). Using the coefficients of this work this would yield the energy of the electron, however, not the energy relation (24)ff, etc. A more general solution of (7) with N = {1;2;3} is discussed in [A2.2].

9 Consistent with curvature being due to the lateral extension of the E-vector in the quaternion / dreibein model;

10 Euler integrals yield positive values, the sign convention of Γ-functions gives negative values for negative arguments. The abbreviation $\Gamma_{-1/3}$ will be used for $|\Gamma(-1/3)|$;

11 Or the related Γ functions, see below;

12 Chosen to give coefficient σ in the exponent of e^v, see [A3.2].

13 In 1st approximation: using the term for energy (15) and length (31) requires $\sigma^{1/3}$ to be of order of the inverse fine-structure constant α^{-1}: $1/c_0 \int w(r)\, dr * \int dr \approx b_0/\rho_n * \sigma^{1/3} \rho_n / c_0 \equiv \hbar/2 \;=>\; \sigma^{1/3} \approx \alpha^{-1}$.

(13) will give for m = 0:

$$J_z = \int_0^{r_n}\int_0^{2\pi} J_z(r,\varphi)\,d\varphi\,dr = 4\pi\frac{b_0}{c_0}\int_0^{r_n} e^{v} r^{-1} dr = 4\pi\alpha\hbar\int_0^{r_n} e^{v} r^{-1} dr = \frac{4\pi}{3}\alpha\hbar\int_{8/\sigma}^{\infty} t^{-1} e^{-t} dt \equiv 1/2\ [\hbar] \qquad (18)$$

To obtain $J_z = 1/2$ [ħ] the integral over $e^v r^{-1}$ dr of (18), *has to* yield $\alpha^{-1}/8\pi$.

$$\int_0^{r_n} e^{v} r^{-1} dr = 1/3\int_{8/\sigma}^{\infty} t^{-1} e^{-t} dt \equiv \frac{\alpha^{-1}}{8\pi} \approx 5.45 \qquad (19)$$

Relation (19) may be used for a numerical calculation of the integration limit, $8/\sigma$, giving a value of σ_0 for spherical symmetry, σ_0 = 1.810E+8 [-]. Assuming the coefficient $\Gamma_{-1/3}/3$ according to (13) has to be part of the expression for σ_0 [14] this results in $\sigma_0 \approx 8(1.5\alpha^{-1}\Gamma_{-1/3}/3)^3$. This value may be interpreted as a coefficient representing geometry, with a value close to the numerical one:

$$\sigma_0 \approx 8\ (1.5\ \alpha^{-1}\ \Gamma_{-1/3}\ /3)^3 \approx 8\left(\frac{4\pi\Gamma_{-1/3}{}^{3}}{3}\right)^3 = 1.772\text{E+8 [-]} \qquad (20)$$

As a consequence a dimensionless volume-like term appears in the denominator of the energy expression (15) for spherical symmetry. Expression (20) is closely related to the value of α and will be used in this context in chpt. 2.10.

In [A3] some additional aspects of the terms supposed to comprise σ will be discussed, giving an alternate expression for (20) and demonstrating that coefficient σ has to be part of the exponent of e^v: $\rho_n^3 \sim \sigma$,

$$\Phi_\sigma \sim \exp(-\sigma A_{el}^3) \qquad (21)$$

Calculating energy according to Φ_σ and (5) will give ≈ 0.1 eV, a value in the estimated energy range of a neutrino.

2.5 Lower limit of σ

The minimal possible value for σ is defined by the Γ-term in the integral expression for length, $\Gamma_{-1/3}/3$, (13), and the integers in (55) to be:

$$\sigma_{min} = (2\Gamma_{-1/3}/3)^3 \quad {}^{15} \qquad (22)$$

leaving a term

$$\alpha_{\lim} \approx 1.5\alpha^{-1} \approx 4\pi\Gamma_{-1/3}{}^{2} \qquad (23)$$

as variable part in σ to consider non-spherical symmetric states (see 2.7, [A3]) [16]).

2.6 Quantization with powers of $1/3^n$ over α

Most relations given here are valid for any particle energy which should be expected as there is a continuous spectrum of energies according to special relativity. However, a particular set of energies may be identified by relaxing the condition of orthogonality of different states according to quantum mechanics to requiring different states to be expressible in simple terms of a ground state coefficient, α_0, in the exponent of e^v and not to exhibit any dependence on intermediary states [17].

There are 2 lines of thought for an estimation of α_0.

2.6.1 Condition a)

The condition that energy/length of a charged particle has to be higher/smaller than the value given by a pure electrostatic term.

Since r_l according to (13), (16), (20)f will be proportional to α^{-2} the term in the exponential has to be: $\alpha_0 < \alpha^6$. The relationship between a photon-like object and a point charge object of elementary charge is based on the coefficient α, suggesting a photon-like state to differ by a factor of α from a pure point charge state and to use a ground state coefficient $\alpha_0 \approx \alpha^9$. This fits the relationship of a set of fundamental particle energies with

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14 Since according to (16) $\sigma^{1/3}$ is proportional to a length parameter, r_n, which according to (13) includes $\Gamma_{-1/3}/3$.

15 If the term of (20) is interpreted as a (cube of a) volume parameter, a term of the kind of (22) would approximately represent the (cube of a) 1D parameter.

16 $\sigma_0 \approx (\alpha_{\lim}\, 2\Gamma_{-1/3}/3)^3$

17 cf. $W_n^2 \sim \left(\alpha_0^{1/3}\alpha_0^{1/9}\ldots.\alpha_0^{1/(3\wedge(n-1))}\boldsymbol{\alpha_0^{1/(3\wedge n)}}\right) / \left(\boldsymbol{\alpha_0^{1}}\alpha_0^{1/3}\alpha_0^{1/9}\ldots.\alpha_0^{1/(3\wedge(n-1))}\right) = \boldsymbol{\alpha_0^{1/(3\wedge n)}/\alpha_0}$

the charged particle of lowest energy, the electron, as a ground state quite well, however, requiring an ad hoc factor ≈ 3/2 for the electron itself. With W_e as ground state, W_n would be given by (24)ff *relative* to the electron state as:

$$W_n/W_e \approx 3/2\frac{\alpha\wedge(1.5/3^n)}{\alpha^{1.5}} \approx 3/2\Pi_{k=1}^{n}\alpha\wedge(-3/3^k) \qquad n = \{1;2;..\} \qquad (24)$$

see table 1.

However, to calculate absolute values of energy requires another factor in addition to α_0.

2.6.2 Condition b)

In a series expansion of the exponential in terms of force, potential, etc., such as given below, particles beyond the electron enter the terms according to their coefficients from (24)ff. In order for the 2nd order term not to exceed the 1st order term the energy of spherical symmetric particles - including relativistic effects - should not exceed $\alpha_0^{-1} = \alpha^{-9}$. However, this restriction should apply for non-spherical symmetric particles as well, requiring $\alpha_{lim} \approx 1.5/\alpha$ as additional factor. With some minor assumptions one will get (cf. chpt. 4):

$$\frac{1.5^3\alpha_0}{2\alpha_{\lim}} = 1.5^2\alpha^{10}/2 = 4.8E\text{-}22 \approx \frac{W_e}{W_{Pl}} = \alpha_{Pl} \qquad (25)$$

The additional factor of ≈ 2/3 of the electron might be related to the difference in α_0 and α_{Pl}. The electron coefficient in the exponential of e^v and the energy term equ. (15) would be given as: $\alpha_e \approx (3/2)^3\alpha^9$. ρ_n may be given as (δ = 1 for electron, = 0 otherwise; n = {0;1;2;..}):

$$\rho_n^3 \approx (1.5)^\delta \sigma_0 \boldsymbol{\alpha_{lim}^{-1}/2\ 1.5^3\alpha^9}\ \alpha^{4.5}/\alpha\wedge(4.5/3^n)\ (e_c/(4\pi\varepsilon_c))^3 \approx (1.5)^\delta\sigma_0\boldsymbol{\alpha_{Pl}}\ \alpha^{4.5}/\alpha\wedge(4.5/3^n)\ (e_c/(4\pi\varepsilon_c))^3 \qquad (26)$$

2.7 Non-spherical symmetric states

Assuming the angular part to be related to spherical harmonics and exhibiting the corresponding nodes would give the analog of an atomic p-state for the 1st angular state, y_1^0. With the additional assumption that $W_{n,l} \sim 1/r_{n,l} \sim 1/V_{n,l}^{1/3} \sim (2l+1)^{1/3}$ ($V_{n,l}$ = volume) is applicable for non-spherically symmetric states as well, this would give $W_1^0/W_0^0 = 3^{1/3} = 1.44$ and $\sigma^{1/3} = 3^{-1/3}\sigma_0^{1/3}$. The considerations of chpt. 3.1 support that a y_1^0-like symmetry for particle states has to be considered and a second partial product series of energies in addition to (24) corresponding to these values approximately fits the data, see tab. 1.

A change in angular momentum has to be expected for a transition from spherical symmetric states, y_0^0, to y_1^0 which is actually observed with ΔJ = ± 1 except for the pair μ/π with Δ J = 1/2.

With $\sigma^{1/3}/2 = \{4\pi\Gamma_{-1/3}{}^3/3\ (y_0^0);\ 3^{-1/3}4\pi\Gamma_{-1/3}{}^3/3\ (y_1^0);\ \Gamma_{-1/3}/3\ (\max)\}$ energy relative to the electron state may be given as:

$$W_n/W_e \approx 3/2\frac{\alpha\wedge(1.5/3^n)}{\alpha^{1.5}}\frac{\sigma_0^{1/3}}{\sigma^{1/3}} = 3/2\Pi_{k=0}^{n}\alpha\wedge(-3/3^k)\ \frac{\sigma_0^{1/3}}{\sigma^{1/3}} \qquad n = \{1;2;..\} \qquad (27)$$

According to the variable part in σ, (23), the maximum additional contribution to W_{max} with respect to a spherical symmetric state would be:

$$\Delta W_{max} \approx 3/2\ \alpha^{-1} \qquad (28)$$

With (24) and (28) the total maximum energy will be $W_{max} \approx W_e\ 9/4\ \alpha^{-2.5}$ = 4.05E-8 [J] (= 1.03 Higgs vacuum expectation value, VEV = 246GeV = 3.941E-8 [J] [7]) [18].

2.8 Results of energy calculation

Table 1 presents the results of the energy calculation according to (15), (26) for y_0^0 (bold), y_1^0. Only states given in [7] as 4-star, characterized as „*Existence certain, properties at least fairly well explored*", are included, up to Σ'^0 all states given in [7] are listed. Coefficients given in col. 4 refer to (24), (26), starting with the electron coefficient in W_e, including its extra term of 2/3. Exponents of -9/2 for Δ and tau are equal to the limit of the partial product of α(n), including the electron coefficient. The term [$3/2\alpha^{-1}$] represents (28).

In col. 5 equ. (15) and (26) are used to calculate energy with σ_0 according to the value of the *fit for J_Z =1/2 and α_{Pl} given by W_e/W_{Pl} according to the experimental value* of the electron and definition (25) for Planck energy.

18 For the Higgs boson see [A6.3].

$$W_n = 2b_0 \int_0^{r_n} \exp\left(-\left(1.5^{3\delta} \sigma_0 \alpha_{Pl} \frac{\boldsymbol{\alpha}^{\mathbf{4.5}}}{\boldsymbol{\alpha}^{(\mathbf{4.5/3^n})}} \left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right)\right) r^{-2} dr \quad => \quad W_\mu = \frac{2}{3} \frac{\Gamma_{+1/3} \boldsymbol{\alpha}^{\mathbf{-1}}}{(\sigma_0 \alpha_{Pl})^{1/3}} e_c \tag{29}$$

(n = {0;1;2;..}; 1.5^δ = extra coefficient for the electron only, δ = δ(0,n); bold: particle coefficient; muon given as example[19]). In col. 6 an alternate version for calculating σ_0 according to (61)f of [A3.3] is given for comparison.

Additional particle states and blanks in the table are discussed in [A6]. The values of physical constants are taken from [7].

To illustrate possible QED-effects and the non-linearity of the Γ-functions, a calculation of σ_0 with values of (18)f varying within +/-1.00116 gives a range of energy values of +/-1.006, varying within +/-1.00116^2 gives a range of energy values of +/-1.013 compared to the values given in table 1. Additional effects due to e.g. different charge in the nucleons have to be expected.

The accuracy of ~1% of the values calculated for leptons, mesons and baryons is comparable to that of LQCD calculations for baryons [10].

	n, l	$W_{n,Lit}$ [MeV]	α-coefficient in W_n $\alpha(n)^{-1/3}$ [f(l)]	Wcalc/ Wlit Equ.(29)	Wcalc/ WLit Equ.(62)	J	uds
ν	**-1 ***	**~ E-7 (calc)**	**0**			**1/2**	
e^{+-}	**0, 0**	**0.51**	**2/3 α^{-3}**	**1.014**	**1.002**	**1/2**	O
μ^{+-}	**1, 0**	**105.66**	**$\alpha^{-3}\alpha^{-1}$**	**1.007**	**0.996**	**1/2**	O
π^{+-}	1, 1	139.57	$\alpha^{-3}\alpha^{-1}$ [$3^{1/3}$]	1.101	1.088	0	uds
K		495	see I 3.8			0	uds
η^0	**2, 0**	**547.86**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}$**	**1.002**	**0.990**	**0**	LC
ρ^0	2, 1	775.26	($\alpha^{-3}\alpha^{-1}\alpha^{-1/3}$) [$3^{1/3}$]	1.022	1.009	1	LC
ω^0	2, 1	782.65	($\alpha^{-3}\alpha^{-1}\alpha^{-1/3}$) [$3^{1/3}$]	1.012	1.000	1	LC
K*		894	see I 3.8			1	uds
p^{+-}	**3, 0**	**938.27**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.011**	**0.999**	**1/2**	uds
n	**3, 0**	**939.57**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.010**	**0.998**	**1/2**	uds
η'		958	see I 3.8			0	LC
Φ^0		1019	see I 3.8			1	uds
Λ^0	**4, 0**	**1115.68**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$**	**1.020**	**1.008**	**1/2**	uds
Σ^0	**5, 0**	**1192.62**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$**	**1.014**	**1.002**	**1/2**	uds
Δ	**∞, 0**	**1232.00**	**$\alpha^{-9/2}$**	**1.012**	**1.000**	**3/2**	uds
Ξ		1318				1/2	uds
Σ^{*0}	3, 1	1383.70	($\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$) [$3^{1/3}$]	0.989	0.977	3/2	uds
Ω^-	4, 1	1672.45	($\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$) [$3^{1/3}$]	0.982	0.970	3/2	uds
N(1720)	5, 1	1720.00	($\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$) [$3^{1/3}$]	1.014	1.002	3/2	
tau^{+-}	∞, 1	1776.82	($\alpha^{-9/2}$) [$3^{1/3}$]	1.012	1.000	1/2	O
Higgs	∞,∞ **	1.25 E+5	($\alpha^{-9/2}$) [3/2 α^{-1}] /2	1.042	1.066	0	
VEV	∞,∞***	2.46 E+5	($\alpha^{-9/2}$) [3/2 α^{-1}]	1.059	1.083		

Table 1: Particle energies; col.2: radial, angular quantum number; col.4: α-coefficient in W_n according to (29), n = {0;1;2;..}; col.5,6: ratio of calculated energy, W_{calc} according to (29), (62) and literature value [7]; col.7: angular momentum J_z [ħ];

2.9 Photon energy

In the following a term for length expressed via the Euler integral of (13) will be introduced for $\lambda_{C,n}$:

$$r_x = \int_0^{r_x} e^v dr \quad = \quad \rho_n/3 \int_{(\rho_n/r_x)^3}^{\infty} t^{-4/3} e^{-t} dt \quad \approx \Gamma(-1/3, (\rho_n/r_x)^3)\ \rho_n/3 \tag{30}$$

19 The term for the muon is given as reference to avoid ambiguities due to extra term ≈ 3/2 of the electron.

In the limit $(\rho_x/r_x)^N$ -> 0

$$\Gamma(-1/N, (\rho_x/r_x)^N) = \int_{(\rho_x/r_x)^N}^{\infty} t^{-(1/N+1)} e^{-t}\, dt \approx N\, (\rho_x/r_x)^{-1} = N\, \sigma^{1/3}/2 \tag{31}$$

holds. Equation (31) inserted in the right side of (30) gives back r_x, however, (30)f may be seen as expressing r_x in terms useful for this model, i.e. ρ_n, σ_0 and Γ-functions. Using equ. (31) for the incomplete Γ-function and multiplying r_x in the integration limit $(\rho_n/r_x)^3$ by √3, the ratio of total angular momentum and its z-component (see [A4, (63)]), gives in good approximation (using (20)):

$$\lambda_{C,n} \approx 3^{1.5}\, \sigma_0^{1/3}/2\; \rho_n/3 \approx 3^{0.5}\, 4\pi\, \Gamma_{-1/3}^{3}/3\, \rho_n \tag{32}$$

With (32) energy of a photon may be expressed as:

$$W_{Phot,n} = hc_0/\lambda_{C,n} = hc_0 \Big/ \int^{\lambda_{C,n}} e^{v} dr = \frac{2\, hc_0}{3^{0.5} \rho_n \sigma_0^{1/3}} \approx \frac{3\, hc_0}{3^{0.5}\, 4\pi\, \Gamma_{-1/3}^{3} \rho_n} \tag{33}$$

2.10 Fine-structure constant, α

The energy of a particle is assumed to be the same in both photon and point charge description. Equating (15) with (33) gives:

$$W_{pc,n} = W_{Phot,n} = 2b_0\, \Gamma_{+1/3}\, \rho_n^{-1}/3 \approx \frac{2\, hc_0}{3^{0.5} \rho_n \sigma_0^{1/3}} \approx \frac{3\, hc_0}{3^{0.5}\, 4\pi\, \Gamma_{-1/3}^{3} \rho_n} \tag{34}$$

Solving equ. (34) for α involves a term of two Γ-functions with an argument of same value and opposite sign for which the relation Γ(+x)Γ(-x) = π /(x sin(πx) holds [11], giving for the Γ-functions of (15) and (33):

$$\Gamma_{+1/3}\, \Gamma_{-1/3} = 3^{0.5}\, 2\pi \tag{35}$$

Using equation (34) with (35) will give (note: h => ħ):

$$\alpha^{-1} = \frac{hc_0}{2\pi\, b_0} \approx \left(\frac{2\Gamma_{+1/3}}{3^{0.5}\, 2\pi}\right) \left(\frac{4\pi}{3} \Gamma_{-1/3}^{3}\right) \approx \frac{2}{3} \frac{\Gamma_{-1/3}}{\Gamma_{+1/3}}\, 4\pi\, \Gamma_{+1/3} \Gamma_{-1/3} \approx \boldsymbol{4\pi\, \Gamma_{+1/3} \Gamma_{-1/3}} \tag{36}$$

The last expression is emphasized since it has a simple interpretation in terms of the coefficients of the integrals over $\exp(-(\rho/r)^N)$. Equations (34)ff are based on the integral over a 3-dimensional point charge term modified by the exponential term according to (7) with N = 3, and a complementary integral - in 3D for length, λ_C - to yield a dimensionless constant. This may be generalized to N dimensions (N ={3; 4}), to give a point charge term (S_N = geometric factor for N-dimensional surface, in case of 3D: 4π; 4D: $2\pi^2$):

$$\int_0^r e^{v(N)} r^{-2(N-1)}\, d^N r = S_N \int_0^r e^{v(N)} r^{-(N-1)} dr \tag{37}$$

that has to be multiplied by a complementary integral

$$\int_0^r e^{v(N)} r^{(N-3)} dr \tag{38}$$

The exact result depends on the integration limit of the second integral, cf. [A4]. However, in terms of the Γ-functions both electroweak coupling constants can be given in 1st approximation as

$$\alpha_N^{-1} = S_N \frac{\Gamma(+m/N)\, \Gamma(-m/N)}{m^2} = S_N \frac{\Gamma(+(N-2)/N)\, \Gamma(-(N-2)/N)}{(N-2)^2} \qquad \text{(m = N-2, cf. (13))} \tag{39}$$

Dimension – space	coupling constant	Value of *inverse* of coupling constant, α_N^{-1}	
4D	$\alpha_4 = \alpha_{weak}$	$2\pi^2\, \Gamma_{+1/2}\, \Gamma_{-1/2}/4 = \pi^3 =$	31.0
3D	$\alpha_3 = \alpha$	$4\pi\, \Gamma_{+1/3} \Gamma_{-1/3} = 4\pi\, \Gamma_{+1/3} \Gamma_{-1/3} =$	136.8

Table 2: Values of electroweak coupling constants [20]

20 Values of coupling constants refer to a rest frame and the related charges (i.e. α_{weak} is not defined via lifetime).

The ratio of α and α_{weak} represents the weak mixing angle, θ_W, and may be expressed as:

$$\sin^2\theta_W = \frac{\alpha}{\alpha_{weak}} = \frac{\pi^3}{4\pi\,\Gamma_{+1/3}\,\Gamma_{-1/3}} = 0.227 \tag{40}$$

(Experimental values: PDG [7]: $\sin^2\theta_W = 0.231$, CODATA [8]: $\sin^2\theta_W = 0.222$). The mass ratio of the W- and Z-bosons will be given by $\cos\theta_{W,calc} = (m_W/m_Z)_{calc} = 0.879 = 0.998(m_W/m_Z)_{exp}$ [7].

3 Quaternion ansatz

3.1 Basic approach

The model as described above emphasizes a Kaluza-like ansatz with spin as boundary condition. Reversing the main focus, emphasizing angular momentum and implicitly assuming curvature of space as necessary boundary condition for localization is a straight forward alternate way to get additional information about the states of this model [9], details are given in [A5].

A circular polarized photon with its intrinsic angular momentum interpreted as having its E- and B-vectors rotating around a central axis of propagation, C [21], will be transformed into an object of SO(3)-type symmetry where the center of rotation is the origin of an EBC-dreibein, supposed to be locally orthogonal and subject to standard Maxwell equations. This has the following qualitative consequences:

1) Such a rotation is related to the group *SO(3)* and *SU(2)* as important special case. In the following a quaternion ansatz will be used for modeling the respective rotations.
2) E-vector constantly oriented to a fixed point implies *charge*. As implicitly assumed above, neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of reversed E-vector orientation and opposite polarity.
3) A local coordinate system = rest system implies *mass*.
4) In case of any lateral extension of the E-field, for r -> 0 the overlap of a rotating E-vector implies rising energy density, resulting in *rising curvature of space-time* according to GR or its modification as of equ. (3).
5) The EBC-dreibein can be given in 2 different *chiral* states (left- right-handed).
6) As essentially electromagnetic waves such states are consistent with a “point-like” structure function on the other hand imply a spatial distribution of energy density and angular momentum / spin.
7) Antiparticles may be constructed by reversing orientation of the fields.

For quantitative results 3 orthonormal vectors E, B, C, each described as imaginary part of a quaternion with real part 0, will be subject to alternate, incremental rotations around the axes E, B and C. In the following only solutions where one of the incremental angles of rotation has half the value of the other two will be considered. This may serve as a primitive model for spin $J_Z = 1/2$. There are 3 possible solutions corresponding to half the angular frequency for each of the components E, B, or C. The trajectory of the E-vector encloses a spherical cone, the spherical cap of the cone encompasses a fraction of the area of a hemisphere of 2/3, 1/3 and 1/3, respectively. Mirroring at the center of rotation gives the equivalent double cone (dark grey in fig.1), the fractions of both caps in relation to the surface of the total sphere may be interpreted to give partial charges of 2/3, 1/3 and 1/3 according to Gauss’ law. It is suggestive to identify such components with uds-quarks. In the following the assignment (half-frequency-E-rotation, charge +2/3, U), (half-B, charge -1/3, D), (half-C, charge -1/3, S) will be used.

The E-vector might as well be interpreted to enclose the complement of the double cone of a 3D-ball (white in fig.1), to be called a spherical wedge in the following. This gives the objects complement-U, complement-D, complement-S with charges 1/3, 2/3, 2/3. These objects may be attributed to c, b, t-quarks [22].

Such UDSCBT-entities of (single) spherical cones and toroidal wedges may be used as elementary building blocks to be combined to form more complex objects pending on fitting charge / phase / angular momentum and chirality as well as interference of the fields itself. A mismatch in charge /phase / chirality may result in nodal planes and higher energy states. In the following it will be left undecided if more complex compositions of such objects might be interpreted to represent time averages of propagating EBC-vectors or a standing wave.

21 In the limit r -> λ_C => |C| -> c_0;
22 There exist no data to compare calculated properties with experiments. However, it may be expected that their more extended geometry might be less favourable in a combination for hadrons, leading to higher energy states.

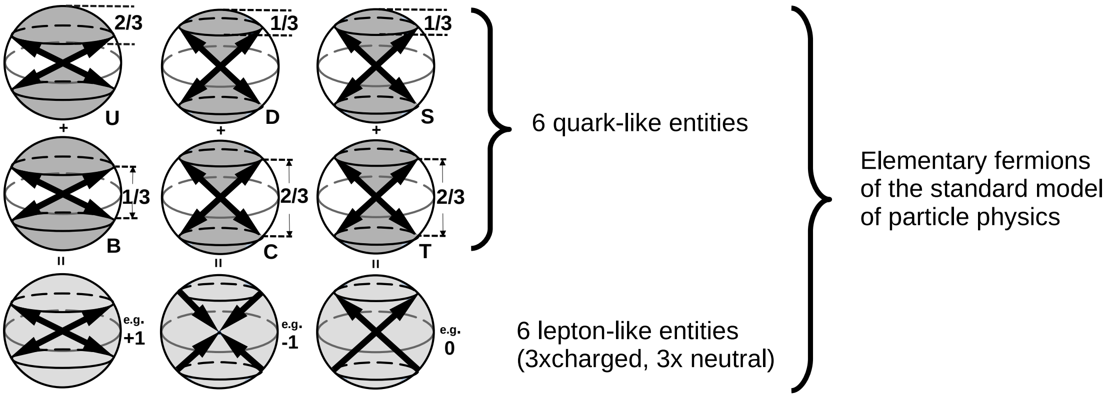

Fig.1: Trajectories of the E-vector, enclosing spherical cones and spherical wedges

A combination of two cones to give a double cone will always give a valid solution with any spin or chirality and is considered to correspond to the y_1^0 solutions of chpt. 2.7 [23].
The simplest combination will consist of 2 complementary segments of same charge etc., to recover a simple sphere with no nodal planes (last row of fig. 1). Such particles should represent the lowest possible energy state, $J_Z = 1/2$ should still be valid and their charge could have values of +/-1 or 0. An electron might be considered e.g. as an (anti-U + (U-Complement = B)) particle, however, unlike a B-meson with spin 1/2. While this is not possible with quarks, i.e. objects with particle character, it is the simplest solution for such a type of an electromagnetic wave.
The neutral configuration will have to be distinct from all other particles by representing a state where the center of rotation is not at the “tip” of an E-vector, but at its “middle”, see last row right in figure 1. This will be an “intrinsically” neutral particle unlike particles consisting of components of opposite charge, such as the neutron and a unique solution that for spatial reasons is not suited as component to build other particles. It will not be subject to the conditions of 2.6.1, 2.6.2 and α_{Pl}, which are related to “charge”.

3.2 Magnetic moments of baryons

There is a crucial test for the applicability of such a quaternion ansatz: calculation of magnetic moments of uds-baryons. Though it is possible to give values for all combinations of the uds-octet of spin 1/2 that match the experiment within a few percent they have to be selected from a large set of solutions. Unique solutions require additional boundary conditions. For nucleons this will be isospin. Exchanging U- and D-components results in switching the values for magnetic moment of p and n [25].

In the quaternion/dreibein model both E- and B-fields are oriented to the center (magnetic monopole character on particle level) and will feature average fields of 1/3 and 2/3 for quark-like objects. The B-field for u- and d-entities will have Cartesian components of ± 2/9, ± 2/9, ± 1/9 (d) and ± 4/9, ± 4/9, ± 2/9 (u). Permutations of these values give a large set of solutions, isospin will serve as a restricting boundary condition for the nucleons. Unique solutions (except for arbitrary orientation in space) for B-field components of nucleons will be e.g. ($B_{avg}=((\sum x_i)^2+(\sum y_i)^2+(\sum z_i)^2)^{0.5}/3$):
proton - uud -4/9, -4/9, -2/9 / -2/9, -4/9, -4/9 / +2/9, -2/9, +1/9 $B_{avg} = 141^{0.5}/27 \approx 0.440$
neutron - ddu -2/9, -2/9, -1/9 / -1/9, -2/9, -2/9 / +4/9, -4/9, +2/9 $B_{avg} = 66^{0.5}/27 \approx 0.301$
To get absolute values one has to multiply by $ec_0\lambda_C/2 = 2\pi\mu_B$ (λ_C = Compton wavelength, μ_B = Bohr magneton), see tab. 3. The ratio of both values is $(141/66)^{0.5} = 1.461631$, which compared to the ratio from experiments [7] gives 1.461631 / 1.459898 = 1.001187.

		λ_C	$e\, c_0\, \lambda_C/2$	B_Avg	$ec_0\lambda_C\, B_{avg}/2$	$\lvert M\rvert Exp[Am^2]$	$\lvert M\rvert Calc/\lvert M\rvert Exp$
p^{+-}	UUD	1.32E-15	3.17E-26	0.440	1.39E-26	1.41E-26	0.988
n	DDU	1.32E-15	3.17E-26	0.301	9.55E-27	9.66E-27	0.988

Table 3: Magnetic moments for proton and neutron; For greater accuracy values of λ_C according to [7] are used;

23 Composite objects - in particular if composed of 3 UDS-components – may feature sufficient spherical symmetry to conform to the respective energy equation (29). The spherical symmetry of nucleons as assumed in chpt. 2 may be given by suitable linear combinations of the states discussed in [A5], [A6.2, η].
25 U and D are symmetric in their E and B-fields while in S-components E- and B-fields are symmetric to each other.

These solutions are distinguished by one U and one D-component being collinear [26], indicating a particular stable configuration involving oppositely charged components (see [A7]).
Table 4 compares some ratios of particle pairs for calculations with the average of the B-field as calculated above, i.e. geometry only, and calculations of the actual moment, using the experimental value of the Compton wavelength. A simple analysis for particles with S-components is not possible since the U- and S-components of the B-fields are identical (cf. tab. 5 in [A5]).

	U,D,S-components	|M|Calc (λ_C exp)	B_avg
M(p/n)_Calc/M(p/n)_Exp	UUD/DDU	0.999809	1.001187
M(Σ^+/Σ^-)_Calc/M(Σ^+/Σ^-)_Exp	UUS/DDS	1.007813	1.001111
M(Ξ^0/Ξ^-)_Calc/M(Ξ^0/Ξ^-)_Exp	USS/DSS	0.974652	0.969601

Table 4: Ratio of particle magnetic moments of baryon pairs compared for calculated and experimental values [7] (col.3: geometry only, B_avg; col.2 inc. exp. particle energy);

3.3 Chirality / Color

The orthonormal EBC-vectors feature two possible chiral configurations, right-handed "R" and left-handed "L", suggesting to be a possible source for a factor 3 frequently appearing in the quantitative interpretation of processes involving a quark-antiquark-pair, such as in the decay, e.g. of the W- or Z-boson, or in the coefficient R of electron-positron-annihilation. While this is attributed to the 3 "colors" of quarks in the SM, the same factor would result for any UDS-pair having the possibility to exist in triplet-like states, "LL", "RR" and 1/√2 (LR+RL) [27] (referring to an axial vector representing the EBC-configuration).

4 Recovering terms of the original Einstein field equation

4.1 Planck scale

In this work the expression

$$b_0 = G\, m_{Pl}^2 = G\, W_{Pl}^2 / c_0^4 \tag{41}$$

is used as definition for Planck terms, giving for the Planck energy, W_{Pl}:

$$W_{Pl} = c_0^2\, (b_0 / G)^{0.5} = c_0^2\, (\alpha\hbar c_0/G)^{0.5} = 1.671\ \mathrm{E{+}8}\ [\mathrm{J}] \tag{42}$$

The value of W_{Pl} according to definition (42) allows to identify the ratio of W_e and W_{Pl} with the α-terms given in (25), i.e. the relation between W_e and W_{Pl} is given by $\alpha_e \approx (3/2)^3\ \alpha^9$, the electron coefficient in the exponent of e^v divided by two times the limit factor, α_{lim}, according to (23) [28]. The constant G may be given as:

$$G \approx \frac{\alpha_{Pl}^2 c_0^4 b_0}{W_e^2} \tag{43}$$

Since W_e may be expressed as function of π, $\Gamma_{+1/3}$, $\Gamma_{-1/3}$ and e_c only, (62), G may be expressed as a coefficient based on electromagnetic constants only, $G \approx 2/3\ c_0^4\alpha^{24}/(4\pi\varepsilon_c) \approx 2/3\ c_0^4\, (4\pi\Gamma_{+1/3}\Gamma_{-1/3})^{24}/(4\pi\varepsilon_c)$.

4.2 Gravitation

Terms for gravitation may be recovered via a series expansion of either $\Gamma(+1/3, (\rho_n/r_n)^3)$ of (14) [15] or the exponential e^v in any suitable expression, e.g. potential ρ_0/r, resulting in a general term such as:

$$\frac{\rho_0}{r}\left[1\ -\ \sigma\alpha_{Pl}\left(\frac{e_c}{4\,\pi\varepsilon_c r}\right)^3\right] \approx \text{Coulomb-term}\left[1\ -\ \sigma\alpha_{Pl}\left(\frac{e_c}{4\,\pi\varepsilon_c r}\right)^3\right] \tag{44}$$

which is a very good approximation for $r > \alpha\lambda_C$. The 1st term is the classical Coulomb term, the 2nd term contains by definition the ratio between Coulomb and gravitational terms for *one* electron, α_{Pl}. To turn this into the exact Coulomb / gravitation relationship requires

1) coefficient σ to approach unity, which may be approximately justified by considering the limit of chpt. 2.5 or [A3.2],

26 Time average! All E,B-components involved are orthogonal at any given point in time.
27 With a singlet state corresponding to destructive interference; alternatively: 3 simple combinations RR, LL, RL;
28 A factor 2 might correspond to relate only the electrostatic contributions of (15) for the electron with the electrostatically defined value of a Planck state.

2) parameter r in e_c /($4\pi\varepsilon_c$r) to turn into a constant,
3) parameter r to approach the value $e_c/(4\pi\varepsilon_c)$.

For condition 2) one has to consider that r *in the exponential* may not be considered to be a free parameter for r > λ_C, the limit of a real solution for an equation such as (55). Using the limit of σ_{min} of (22) and inserting the Compton wavelength of the electron in (44) would give a value two orders of magnitude off to yield the expected value for the electrostatic / gravitation ratio. Since σ is essentially related to spin of a particle and it has to be assumed that spin does not play a role for r > λ_C, one might omit the related coefficient in (44) as well as in the term for λ_C [29] and thus by definition of α_{Pl} recover the exact gravitational term.
The general expression for the series expansion would be:

Coulomb-term (1 - α_{Pl}).

Particle interaction would be given by the square of the α_{Pl} term multiplied by appropriate coefficients from the α-series according to (24) for particles of spherical symmetry in a rest system. Since the 2nd term of such a series expansion should not exceed the 1st, electromagnetic one, the maximum relativistic mass for such particles would be defined by $\alpha_e^{-1} \approx \alpha^9$, while the inverse of the maximum non-spherical symmetry term, i.e. α_{lim}^{-1} as given in (25) secures that particles that are not spherical symmetric in a rest system can not exceed the Planck limit either.
The approach using assumptions 1) - 3), is supported by the considerations of chpt. 4.3, yielding a term for the cosmological constant in the correct order of magnitude.

4.3 Cosmological constant Λ

The full 5D equation (5), including $\sim 1/\Phi\left(\nabla_\alpha(\partial_\alpha \Phi) - g_{\alpha\beta}\Box\Phi\right)$, offers the possibility to produce additional terms that might be considered as a natural candidate for the cosmological constant term, $g_{\alpha\beta}\Lambda$. Its exact expression will depend on the complete metric used. Nevertheless G_{00} may in general contain terms such as $\rho_n^3/\mathbf{r}^5$ or $\rho_n^6/\mathbf{r}^8$ with all r originating from derivatives of the exponential only, see [A2] [30]. Using $\mathbf{r} = e_c/(4\pi\varepsilon_c)$ as upper bound of r, as suggested in 4.2 will yield approximate values in the order of magnitude of critical, vacuum density, ρ_c, ρ_{vac}:

$$\frac{\Phi''}{\Phi} \approx \frac{\rho^3}{\mathbf{r}^5} \approx \frac{\alpha_{Pl}}{\left(e_c/(4\pi\varepsilon_c)\right)^5}\left(\frac{e_c}{4\pi\varepsilon_c}\right)^3 = \alpha_{Pl}\left(\frac{4\pi\varepsilon_c}{e_c}\right)^2 = 0.089\ [\mathrm{m}^{-2}] \tag{45}$$

Multiplied by ε_c this gives an energy density of 2.97E-10 [J/m³].
Multiplied by the conversion factor for the electromagnetic and gravitational equations, equ. (2), $8\pi\varepsilon_c G/c_0^4$ (45) gives as estimate for the cosmological constant, Λ:

$$\alpha_{Pl}\frac{(4\pi)^2\varepsilon_c^3}{e_c^2}\frac{8\pi G}{c_0^4} \approx \quad 6.17\text{E-}53\ [\mathrm{m}^{-2}] \quad ^{31} \tag{46}$$

5 Discussion

Theory of everything is a somewhat ironic and pompous term and maybe an unachievable goal. At the time Theodor Kaluza’s unification of general relativity (GR) and electromagnetism was conceived, it came pretty close, yet the emerging theory of quantum mechanics (QM) moved the finish line. It is a common thought ever since that the theory of GR somehow has to be unified with QM. The model presented here suggests that the ansatz of Kaluza is sufficient to give an excellent model for particles, in particular in combination with the boundary condition spin 1/2, bypassing QM in 1st approximation. The major deviation from conventional GR is dropping the constant of gravitation in the field equations, a minor thing from a mathematical point of view. The resulting objects of interest are waves only, which naturally fits basic concepts of QM. Other general features of quantum mechanics that emerge from such an ansatz include quantization of energy or the pivotal constant of quantum mechanics, Planck’s constant, h, that may be derived from the electromagnetic constants and geometry as expressed in the derivation of α.

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29 I.e. condition 2.6.2 would not have to apply to wavelength, while the more general condition 2.6.1, 1st term ≥ 2nd term, would still require α_{Pl} in the series expansion;

30 Such as $\rho^3/\mathbf{r}^5$ in [A2.1] though this term cancels in the specific example for G_{00}; The 1st term of (54), representing field-free space, i.e. vacuum, might be a suitable starting point as well.

31 $\Lambda \approx 1.11\text{E-}52\ [\mathrm{m}^{-2}]$ with Hubble constant H_0 = 67.66 [km/s/Mpc] [12]

The results of the quaternion ansatz of chpt. 3 reproduce the set of elementary fermions of the standard model of particle physics (SM). The number of 6 basic building blocks of matter can be traced back to the 3 possibilities to single out one of the orthogonal EBC-vectors and in a broad sense is a consequence of the 3 space dimensions in 4D space-time. While the properties of quarks, such as partial charges, are deduced from experimental particle data, in particular symmetry, they can be *derived* in the quaternion ansatz. Leptons are an integral part of the particle classification scheme.

Though the ansatz of this model yields features that match those of quarks, there seems to be no deeper connection with the concepts of QCD, such as color or gluons. Properties such as confinement or the need for adhering to the Pauli principle in e.g. the Δ^{++} are obsolete from the outset for an object that is basically a (5D-) electromagnetic wave. The development of the SM from constituent quarks towards QCD, based on valence and sea quarks plus gluons, was in part required by the limitations in explaining some scattering experiments with 3 point-like objects only. The waves of this model are consistent with a point like structure function and still feature spatial extension from the outset.

On the other hand, regarding electroweak theory, there are several features of the model that indicate a close relationship in addition to the obvious common root in EM: SO(3), SU(2) symmetry, the energy of the Higgs boson /VEV as upper limit for particle energy [32] and the possibility to calculate the IR-limits of the electroweak coupling constants. As for chirality the inherent chiral character of a circular polarized EM-wave is transferred via the quaternion model to particles. All of this strongly suggests a deeper connection of both concepts, worth to be examined more closely in the future.

As far as other aspects of QM are concerned, QED terms are considered to be a necessary correction for the results of this model.

Comparing the computational power of the Kaluza ansatz with the SM, for e.g. calculating particle energy and magnetic moments, in the SM calculations of relative energy/mass of hadrons by LQCD-calculations use *current*-quark masses as input parameters, see e.g. [10], while the currently best results for magnetic moments of uds-baryons originate from a fit to p, n and Λ^0-magnetons related to *constituent*-quark mass [7]. The model presented here achieves a precision for energy comparable to the LQCD-calculations, however, the energy range is not limited to hadrons but essentially covers the complete range of energy relevant for particles (ignoring the sub-particle t-quark). Magnetic moments can be given with a precision an order of magnitude better than that of the values given in [7]. Both calculations do not rely on any input parameter but are *ab initio.*

Not all details of the SM are reproduced by the particle model presented here, several SM assumptions seem to become obsolete. Anyway the relevant benchmark is the agreement with experiments and the small set of assumptions of a Kaluza ansatz, essentially based on established physical theories, should hardly offer any points for a fundamental refutation. For potential minor discrepancies this model should be flexible enough to adapt. The range of particle properties covered is quite comprehensive and in some aspects the capabilities of the SM are exceeded considerably.

The strongest point of this model is the root in Kaluza’s combination of GR and electromagnetism. Kaluza’s ansatz was able to produce correct expressions for both theories but failed to reconcile the huge difference in order of magnitude of both effects with the properties of particles. The remedy proposed here is an almost trivial one, series expansion. That the coefficients used in this approach have some significance is attested by both the results for particles as well as the possibility to produce a reasonable term for the cosmological constant. The ansatz for a metric still needs considerable improvement, but there seems to be an opportunity to gain a new perspective to address problems at the scale of cosmology as well.

32 Some additional relation with electroweak bosons might be given by [A6.3].

Conclusion

A formalism based on 5D-differential geometry and electromagnetic concepts, with spin 1/2 as boundary condition, provides a simple, coherent, comprehensive and first of all quantitative description of phenomena related to particles, such as

- a convergent series of particle energies quantized as a function of the fine-structure constant, α, with electron and the Higgs VEV energy as lower and upper limit, equ. (27),
- an energy for a distinct, neutral particle class at ca. 0.1 eV,
- a single expression for the values of electroweak coupling constants, equ. (39),
- 3D-space and spin 1/2 define a set of 6 lepton-like and 6 quark-like objects with the associated charges,
- magnetic moments of the nucleons.

A series expansion links electromagnetic and gravitational terms with a cosmological constant in the correct order of magnitude.

The model works *ab initio* without free parameter and allows to remove some values from the set of fundamental constants:
electromagnetic constants, h, G, α, α_{weak}, energies of elementary particles => electromagnetic constants.

References

[1] Kaluza, T., "Zum Unitätsproblem in der Physik". Sitzungsber. Preuss. Akad. Wiss. Berlin. 966–972 (1921)
[2] Klein, O., "Quantentheorie und fünfdimensionale Relativitätstheorie", Zeitschrift für Physik A. 37 (12), 895–906 (1926); doi:10.1007/BF01397481
[3] Wesson, P.S., Overduin, J.M., arxiv.org/abs/gr-qc/9805018v1 (1998)
[4] Wesson, P.S., Overduin, J.M., "Principles of Space-Time-Matter", Singapore, World Scientific (2018)
[5] Nambu,Y. Progress of theoretical physics 7, 595-596; 1952
[6] MacGregor, M., "The power of alpha", Singapore, World Scientific (2007)
[7] Workman, R.L. et al., Particle Data Group, “REVIEW OF PARTICLE PHYSICS”, Prog. Theor. Exp. Phys. 2022, 083C01 (2022); https://doi.org/10.1093/ptep/ptac097; https://pdg.lbl.gov/
[8] Mohr, P.J., Newell, D.B.,Taylor, B.N., "CODATA Recommended Values of the Fundamental Physical Constants: 2014", arxiv.org 1507.07956; RevModPhys. 88.035009 (2016)
[9] Schindelbeck, T., Raets. Phaen. Vol. 2, 22-24 (2022)
[10] Dürr, S. et al., “Ab Initio Determination of Light Hadron Masses”, Science 322, 1224 (2008); arXiv:0906.3599
[11] Paris, R. B. in Olver, F.W.J. et al. "NIST Handbook of Mathematical Functions", Cambridge University Press (2010); http://dlmf.nist.gov/8.7.E3
[12] Planck Collaboration; Aghanim, N. et al., "Planck 2018 results. VI. Cosmological parameters". ArXiv:1807.06209 (2018)
[13] Aubert, J.J. et al., "The ratio of the nucleon structure functions F2N for iron and deuterium", Phys. Lett. B. 123B (3–4): 275–278 (1983)
[14] Abrams, D. et al., “Measurement of the Nucleon Fn2/Fp2 Structure Function Ratio by the Jefferson Lab MARATHON Tritium/Helium-3 Deep Inelastic Scattering Experiment”, arXiv:2104.05850v2 [hep-ex] (2021)
[15] Schindelbeck, T., http://doi.org/10.5281/zenodo.832957 (2023)

Appendix
[A1] Scalar potential Φ
The solutions for the scalar Φ depend on the complete metric used. The easiest method to get a solution of order N is to use spherical coordinates of dimension N+1. Using e.g. the line element for a 4D metric of [4, equ. 6.76]

$$ds^2 = e^{v} dt^2 - e^{\lambda} dr^2 - e^{\mu} r^2 (d\vartheta^2 + \sin^2\vartheta\, d\varphi^2) \tag{47}$$

and $A_\alpha = (A_{el}, 0, 0, 0)$ gives as solution for equ.(6) (cf. [4, equ. 6.77], prime corresponds to derivatives with respect to r):

$$\Phi'' + \left(\frac{v' - \lambda' + 2\mu'}{2} + \frac{2}{r}\right)\Phi' - \frac{1}{2}\Phi^3 e^{-v} (A_{el}')^2 = 0 \tag{48}$$

This can be solved with a function of type (7) for N = 2:

$$\Phi_2' = \left[-\left(\frac{\rho}{r^2}\right) + 2\left(\frac{\rho^3}{r^4}\right)\right] e^{v} \tag{49}$$

and

$$\Phi_2'' = \left[2\left(\frac{\rho}{r^3}\right) - 10\left(\frac{\rho^3}{r^5}\right) + 4\left(\frac{\rho^5}{r^7}\right)\right] e^{v} \tag{50}$$

The ρ^1 terms cancel in (48), the ρ^3 terms can be eliminated by appropriate choice of v', λ' and μ', a remaining factor in the ρ^5 term could be compensated by assuming a corresponding factor in A_{el}. For N = 3 hyperspherical coordinates with the line element

$$ds^2 = e^{v} dt^2 - e^{\lambda} dr^2 - e^{\mu} r^2 (d\psi^2 + \sin^2\psi (d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)) \tag{51}$$

may be used. A more complex metric of the kind given in [A2] may be used as well to solve equation (8).

[A2.1] Metric / point charge
A general metric using solutions for Φ according to [A1] will be:

$$g_{\alpha\alpha} = \left(\frac{\rho_0}{r}\right)^{N-1} \exp\left(-\left(\frac{\rho}{r}\right)^{N}\right),\ -\left(\frac{\rho_0}{r}\right)^{N-1} \exp\left(\left(\frac{\rho}{r}\right)^{N}\right),\ -r^2,\ -r^2\sin^2\vartheta \tag{52}$$

The following uses the metric of (52) with N = 3. The exponential part represents $\Phi^2 \sim e^v$ in the metric. The variable r is marked bold if originating from the exponential term to facilitate a discussion of the implications of its restricted range of validity.

$\Gamma_{01}^{0} = \Gamma_{10}^{0} = -1/r^1 + 3/2\, \rho^3/\mathbf{r}^4$ $\quad \Gamma_{00}^{1} = -1/r^1 e^{-2v} + 3/2\, \rho^3/\mathbf{r}^4 e^{-2v}$
$\Gamma_{11}^{1} = -1/r^1 - 3/2\, \rho^3/\mathbf{r}^4$
$\Gamma_{12}^{2} = \Gamma_{21}^{2} = \Gamma_{13}^{3} = \Gamma_{31}^{3} = +1/r^1$ $\quad \Gamma_{22}^{1} = -r^3/\rho_0^2\, e^{-v} = \Gamma_{33}^{1}/\sin^2\vartheta$
$\Gamma_{23}^{3} = \Gamma_{32}^{3} = \cot\vartheta$ $\quad \Gamma_{33}^{2} = -\sin\vartheta\cos\vartheta$
$R_{00} = e^{2v} [+1/r^2 + 6\,\rho^3/\mathbf{r}^5 - 9/2\,\rho^6/\mathbf{r}^8]$
$R_{11} = +3/r^2 - 6\,\rho^3/\mathbf{r}^5 + 9/2\rho^6/\mathbf{r}^8$
$R_{22} = -1 + e^{+v} [+ r^2/\rho_0^2 + 3\rho^3 r^3/(\rho_0^2 \mathbf{r}^4)]$
$R = +2/r^2 + e^{v} [(-4/\rho_0^2 - 6\rho^3 r/(\rho_0^2 \mathbf{r}^4) + 12\, a\, \rho^3 r^2/(\rho_0^2 \mathbf{r}^5) - 9\, a^2 \rho^6 r^2/(\rho_0^2 \mathbf{r}^8)]$
G_{00} will be:
$G_{00} = e^{2v} [+1/r^2 + 6\rho^3/\mathbf{r}^5 - 9/2\rho^6/\mathbf{r}^8)] - e^{v} \rho_0^2/r^4 + e^{2v} [2/r^2 + 3\rho^3/(r\mathbf{r}^4) - 6\, a\, \rho^3/\mathbf{r}^5 + 9/2\, \rho^6/(\rho_0^2\, \mathbf{r}^8)] =$
$- e^{v} \rho_0^2/r^4 + e^{2v} [3/r^2 + 3\rho^3/(r\mathbf{r}^4)]$

Volume integrals over any ρ^n/r^{n+2} terms will yield energy results $\varepsilon_c \int e^v\, \rho^n/r^{n+2}\, d^3r \approx \varepsilon_c\, \rho \approx$ 1E-22 [J] compared to the term $\varepsilon_c \int e^v\, \rho_0^2/r^4\, d^3r \approx \varepsilon_c\, \rho_0^2\, \rho^{-1} \approx$ 1E-13 [J] (both with coefficients for the electron, $\sigma_0\alpha_{Pl}$) giving negligible contributions to particle energy within the parameter range discussed here. This leaves the first term as leading order:
$G_{00} = -e^{v} \rho_0^2/r^4$

[A2.2] General solution N = {1; 2; 3}
This article has a focus on a solution of (7) with N = 3. However, all solutions in a 5D space-time according to [A1], i.e. up to using hyperspherical coordinates, N = {1; 2; 3}, might be used for the ansatz of a metric such as

$$g_{00} = \sum_{N=1}^{3} \left(\frac{\rho_0}{r}\right)^{N-1} \exp\left(-\left(\frac{\rho}{r}\right)^{N}\right) \tag{53}$$

With the approximation σ ≈ 1 this gives for g_{00}:

$$g_{00} = \exp\left(-\alpha_{Pl}\left(\frac{\rho_0}{\mathbf{r}}\right)\right) + \left(\frac{\rho_0}{r}\right)\exp\left(-\alpha_{Pl}\left(\frac{\rho_0}{\mathbf{r}}\right)^2\right) + \left(\frac{\rho_0}{r}\right)^2 \exp\left(-\alpha_{Pl}\left(\frac{\rho_0}{\mathbf{r}}\right)^3\right) \tag{54}$$

Each term might be expanded and split in EM and gravitational part as shown in chpt. 4.2.
The 3rd term corresponds to the case discussed above, resulting in terms giving the square of the E-field in G_{00} and eventually particle energy as well as an equivalent term for gravitation from the series expansion. The second term is the linear version and might be used to construct a Schwarzschild-like solution.
The first term might represent a general vacuum solution, i.e. without presence of any field ρ_0/r. A series expansion would give the 1 for flat space-time, while the minor terms of G_{00} could give Λ-like orders of magnitude equivalent to the reasoning of chpt. 4.3.

[A3] Model coefficients

[A3.1] Coefficient σ as component in ρ

The exponential term, $\exp(-\rho^3/r^3)$, together with the r^{-2} dependence of the field of a point charge define a maximum of particle energy near $r_{W(max)} \approx \rho$, rapidly approaching 0 for $r_{W(max)} > \rho$, effectively allowing to calculate energy terms without using a specific upper integration limit, r_n [33]. On the other hand the weaker r-dependence of angular momentum, ~1/r results in the calculated values being completely dominated by an integration limit. The limit of the Euler integral is given by ρ_n^3/r_n^3, a constant which will be denoted 8/σ in this work.
A general exponential function of radius featuring a limit radius, assumed to correspond to a damped oscillator-like solution, may be given in 1st approximation as:

$$e^{\nu'} = \exp\left(-\left(\frac{ß\rho'^3}{2r^3} + \left[\left(\frac{ß\rho'^3}{2r^3}\right)^2 - 4\frac{\rho'^3}{2r^3}\right]^{0.5}\right)\right) \tag{55}$$

ß being some general coefficient. At the limit r_n of the real solution (55)

$$\left(ß\rho'^3/r_n^3\right)^2 = 8\rho'^3/r_n^3 \quad => \quad ß = 8\left(\frac{r}{\rho}'\right)^3 = \sigma \tag{56}$$

holds, reproducing the definition of σ (16). Within the parameter range of this work the function $e^{\nu'} \approx \exp(-(ß\rho'^3/r^3))$ is a very good approximation of an equation of the kind of (55) and consequently coefficient σ will be part of the exponential.

[A3.2] Coefficient σ, coefficient 1.5x

The basic relation of α(n) and σ with the fine-structure constant α and coefficient $\Gamma_{-1/3}/3$ is due to the considerations of chpt. 2.4 To get a more detailed description in a range of 1 percent precision is difficult since there are several options conceivable and in this range of accuracy, QED and other minor effects may be expected which might be amplified due to the non-linear nature of the Γ-functions involved. A factor ≈ 3/2 appears in several terms such as $\sigma_0 \sim 1.5\alpha^{-1}$ of (20), the ratio of electron and muon energy =1.5088, $\Gamma_{-1/3}/\Gamma_{+1/3}$ =1.516, π/2 = 1.5707 and the irregular electron coefficient in the power series that is part of α_{Pl} as well. The following discusses some relevant aspects with a focus on identifying possible underlying relationships while minimizing assumptions about the term ≈ 3/2 in particular.
In this model elementary charge may be given as $b_0\int\exp(-(e_c/(4\pi\varepsilon_c r))^3)r^{-2}dr \approx e_c$, the corresponding radial distribution of energy has its maximum at $r_c \approx e_c/(4\pi\varepsilon_c)$. To get the exact value of e_c coefficient Γ(+1/3)/3 is required to appear as a term in $W(e_c)$ due to the Euler integral, thus a counter term *must* be part of ρ in (14)f:

$$W(e_c) = \frac{e_c^2}{4\pi\varepsilon_c}\int \exp\left(\frac{-\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\varepsilon_c}\right)^3 r^{-2}dr = \frac{e_c^2}{4\pi\varepsilon_c}\frac{\Gamma_{+1/3}}{3}\left(\frac{\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\varepsilon_c}\right)^{-1} = e_c \tag{57}$$

For r_c follows, considering the basic coefficients only, using (32), (35)

$$\lambda_C \sim 3^{0.5}\int \exp-\left(\frac{\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\varepsilon_c}\right)^3 dr \sim \frac{\Gamma_{-1/3}\Gamma_{+1/3}}{3^{0.5}}\ \frac{e_c}{4\pi\varepsilon_c} = \frac{e_c}{2\varepsilon_c} \tag{58}$$

again removing all coefficients that are not part of a Coulomb-expression and suggesting an additional term of 2π in the denominator of ρ (note: for elementary charge σ = 1 has to be assumed; otherwise one gets (59)).
Looking only at the basic mathematical coefficients entering the equation (30)ff (i.e. σ -> $2\Gamma_{-1/3}/3$) an additional term $((2\pi)^{-1}\Gamma_{+1/3}/\Gamma_{-1/3})^3$ (bold in (59)) in ρ would cancel redundant $\Gamma_{-1/3}/3$ terms in the length expression as well:

$$\lambda_C \sim 3^{0.5}\frac{\Gamma_{-1/3}}{3}\frac{\sigma^{1/3}}{2}\rho \sim 3^{0.5}\Gamma_{-1/3}\frac{\Gamma_{-1/3}}{3}\frac{2\Gamma_{-1/3}}{3}\ \frac{\boldsymbol{\Gamma_{+1/3}}}{\boldsymbol{2\pi\Gamma_{-1/3}}} = \frac{2\Gamma_{-1/3}}{3} \tag{59}$$

The term $((2\pi)^{-1}\Gamma_{+1/3}/\Gamma_{-1/3})^3$ consists of components related to angular momentum and (with an additional factor 2) seems to be a suitable replacement for $1/(2\alpha_{lim})$ e.g. in (25) and may thus be used in expressions such as (60)ff [34].

33 For an upper limit $r_n \geq 10\rho$ other limitations supersede the attainable precision.
34 The need of $\Gamma_{+1/3}/\Gamma_{-1/3}$ to appear in (57)ff and its more pronounced relationship with angular terms is the reason to prefer $(2\alpha_{lim})^{-1} \approx 2\,((2\pi)^{-1}\,\Gamma_{+1/3}/\Gamma_{-1/3})^3$ over $(2\alpha_{lim})^{-1} \approx 2((2\pi)^{-1}\,2/3)^3$ which would give σ_0=1.821E+8[-], i.e. a term very

Using these coefficients considered essential for yielding basic quantities such as e_c, including the typical term 2π associated with angular momentum and corresponding to the 3rd power structure of the equations best would give for σ_0:

$$\sigma_0 = \left[\frac{1}{4}\left(\frac{\Gamma_{-1/3}2\pi}{\Gamma_{+1/3}}\right)^3 \frac{2\Gamma_{-1/3}}{3}\right]^3 = \left[\left(\frac{\Gamma_{-1/3}\pi}{\Gamma_{+1/3}}\right)^3 \frac{4\Gamma_{-1/3}}{3}\right]^3 = 2.008\text{E}+8\ [-] \qquad (60)$$

[A3.4] Model calculations for e^v

In col. 6 of tab. 1 equ. (15) and (26) are used with σ_0 according to (60), α_{Pl} will be replaced by $\alpha_{lim}^{-1}/2\ (3/2\ \alpha^9)$ with α_{lim} being recalculated from $\alpha_{lim}^{-1} = \sigma_0^{-1/3}\, 2\Gamma_{-1/3}/3$. This gives the following expression for e^v [35]:

$$\exp\left(-\left[(\rho_n/r)^3\right]\right) \approx \exp\left(-\left[1.5^{3\delta}\sigma_0\alpha_{Pl}\,\alpha(n+1)\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right]\right) \approx \exp\left(-\left[1.5^{3\delta}\left[\left(\frac{\Gamma_{-1/3}\pi}{\Gamma_{+1/3}}\right)^3\frac{4\Gamma_{-1/3}}{3}\right]^3\frac{\alpha(n)}{2\alpha_{\lim}}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right]\right)$$

$$\approx \exp\left(-\left[1.5^{3\delta}\left[\left(\frac{\Gamma_{-1/3}\pi}{\Gamma_{+1/3}}\right)^3\frac{4\Gamma_{-1/3}}{3}\right]^3 2\left(\frac{\Gamma_{+1/3}}{\Gamma_{-1/3}2\pi}\right)^3\left(\frac{3}{2}\right)^3 \boldsymbol{\Pi_{k=0}^{n}\alpha\wedge(9/3^k)}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right]\right) \approx \qquad (61)$$

$$\left(\exp\left(-\left[1.5^{3\delta}\frac{\pi^2\Gamma_{-1/3}{}^3}{\Gamma_{+1/3}{}^2}\boldsymbol{\Pi_{k=0}^{n}\alpha\wedge(3/3^k)}\frac{e_c}{4\pi\varepsilon_c r}\right]^3\right)\right)^2 \qquad n = \{0;1;2;..\}$$

Inserted in the equation for energy, (14)f, gives

$$W_n = 2b_0\int_0^{r_n}\left(\exp\left(-\left[1.5^{3\delta}\frac{\pi^2\Gamma_{-1/3}{}^3}{\Gamma_{+1/3}{}^2}\boldsymbol{\Pi_{k=0}^{n}\alpha\wedge(3/3^k)}\frac{e_c}{4\pi\varepsilon_c r}\right]^3\right)\right)^2 r^{-2}\,dr \quad => \qquad n = \{0;1;2;..\} \qquad (62)$$

$$W_\mu = 2e_c\frac{\Gamma_{+1/3}}{3}\ 2^{-1/3}\left[\frac{\Gamma_{+1/3}{}^2}{\pi^2\Gamma_{-1/3}{}^3}\boldsymbol{\alpha^{-4}}\right] = \frac{2^{2/3}}{3\pi^2}\left(\frac{\Gamma_{+1/3}}{\Gamma_{-1/3}}\right)^3\boldsymbol{\alpha^{-4}}\ e_c$$

(1.5^δ = extra coefficient for the electron only, $\delta = \delta(0,n)$; bold: particle coefficient; muon given as example)

[A4] Coupling constant in N dimensions

The integration limits for calculating angular momentum in z-direction, r_n of J_z, (17)ff, and (Compton-)wavelength, λ_C, supposed to represent the rotating E-vector and in turn total angular momentum J should be related by the factor $\sqrt{3}$ of the ratio J/J_z:

$$\lambda_C / r_n = (1/2(1/2+1))^{0.5} / (1/2) = \sqrt{3} \quad [36] \qquad (63)$$

The 3D case of the coupling constant is easy to interpret, for the 4D-case some assumptions have to be made concerning the integration limit. The following gives an alternative, more detailed interpretation than 2.10 ($\varphi_N = \exp(-(\rho/r)^N)$).

3D case:

The exact value of the product of the integrals (37)f, depends on the integration limit relevant for the second integral, i.e. the lower integration limit of the Euler integrals, which can be expressed as 3D volume with $\Gamma_{-1/3}$ as radius (20):

$$\rho_n^3/\lambda_{C,n}^3 = 8/(3^{1.5}\sigma_0) = \left(3^{0.5}\frac{4\pi}{3}\ \Gamma_{-1/3}{}^3\right)^{-3} \qquad (64)$$

The additional factor $3^{0.5}$ may be interpreted as the ratio between r_n of equ. (16) and $\lambda_{C,n}$ as required in the expression for photon *energy*. This gives $\Gamma(-1/3, 1/\sigma_0) \approx 36\pi^2\Gamma_{-1/3}$ and

$$2\int_0^r \varphi_3 r^{-2} dr \int_0^r \varphi_3 dr \approx 2\left[\frac{\Gamma_{1/3}}{3}\right]\left[2\pi\, 2\pi\, 9\frac{\Gamma_{-1/3}}{3}\right] = 4\pi\,\Gamma_{1/3}\Gamma_{-1/3}\ \mathbf{2\pi} = \mathbf{2\pi}\ \alpha^{-1} \quad [37] \qquad (65)$$

The result of (65) yields a dimensionless constant $\alpha' = h\, c_0\, 4\pi\, \varepsilon/e^2$ and it is a matter of choice to include 2π in the dimensionless coupling constant. Factor 9 cancels the corresponding factors from the Euler integrals. The remaining factor of 4π is needed to yield the correct value of α.

A general N-dimensional version of (64) may be given as:

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close to the value of σ_0 fitted to J_z.

35 Expression intended to emphasize 3rd power relationship, a remaining factor of 2 is attributed to $e^{v/2}$ being squared.

36 Alternatively: $\lambda_{C,n} = 3\rho h c_0/(2b_0\Gamma_{+1/3}) = 3\pi\alpha^{-1}\rho/\Gamma_{+1/3}$; $r_n = 3/2\ \alpha^{-1}\rho\ \Gamma_{-1/3}/3$ => $\lambda_{C,n}/r_n = 6\pi/(\Gamma_{+1/3}\Gamma_{-1/3}) = 6\pi/(2\pi\sqrt{3}) = 3^{0.5}$

37 Factor 2 from adding electric and magnetic contributions to energy;

$$8/\sigma_N = \left(3^{0.5\delta} V_N \ \left(\Gamma(-1/\mathrm{N})\right)^N\right)^{-N/(N-2)} \tag{66}$$

V_N is the coefficient for volume in N-D, coefficient $3^{0.5}$ will be omitted in 4D where coordinate r is considered to be directly related to energy via $r_n \sim 1/W_n$ and r_n might be directly identified with $\lambda_{C,n}$; subscript in σ_N corresponds to dimension in the following.

4D case:

Using φ_4 according to the definition (7) and (66) for 4D:

$$\rho_n^4/r_n^4 = 8/\sigma_4 = \left(\frac{\pi^2}{2} \ \left(\Gamma_{-1/4}\right)^4\right)^{-2} = 1.232\text{E-}7 \tag{67}$$

as integration limit, with (13) the non-point-charge integral in 4D will be given by:

$$\int_0^r \varphi_4 r\, dr \sim \Gamma(-1/2, 8/\sigma_4) = \int_{8/\sigma_4}^{\infty} t^{-1.5} e^{-t}\, dt = 5687 \approx 16\pi^4 \Gamma_{-1/2} \tag{68}$$

The 4D equivalent of (65) will be:

$$2\int_0^r \varphi_4 r^{-3} dr \int_0^r \varphi_4 r\, dr \approx 2\left[\frac{\Gamma_{1/2}}{4}\right]\left[16\pi^4 \frac{\Gamma_{-1/2}}{4}\right] = \frac{\pi^2}{2}\Gamma_{1/2}\Gamma_{-1/2} \ \mathbf{4\pi^2} = \pi^3 \mathbf{4\pi^2} = \alpha_{weak}^{-1} \mathbf{4\pi^2} \tag{69}$$

The interpretation is the same as in the 3D-case:

A $4\pi^2$ term originating from the second integral of equation (69) is required for turning h^2 into $\hbar^2$ since the integral refers to ρ_n^2 and thus to the square of energy and h, ħ. Factor 16 cancels the corresponding factors from the Euler integrals. The remaining factor of $\pi^2/2$ is needed to yield the correct value of α_{weak}.

2D case:

the 2D case is not as straightforward as the 4D case. The integral over the 1D point charge

$$\int_0^r \varphi_2 r^{-1} dr = \Gamma(0, \rho_n^2/r_2^2) \ /2 \tag{70}$$

features Γ(0, x), with Γ(0, x) -> ∞ for x -> 0 and m = N-2 = 0 in the equations above. Setting nevertheless m=1 in the 2D equivalent of the integration limit

$$\rho_n^2/\lambda_{C,n}^2 = 8/(\sigma_2) = \left(3^{0.5}\pi \ \ \Gamma_{-1/2}{}^2\right)^{-2} \approx 1/4676 \tag{71}$$

and calculating $\Gamma(0, \rho_2^2/r_2^2)$ numerically gives $\int\varphi_2 r^{-1}\, dr \approx \Gamma(0, \rho_2^2/r_2^2)/2 = 7.872/2$. In the 2D case the complementary integral would be identical to the point charge integral, giving $2(\int\varphi_2 r^{-1}\, dr)^2 \approx 4\pi^3/4 = \pi^3$, i.e. the same value as 4D, maybe giving an alternate candidate for α_{weak}.

[A5] Quaternion-based quark-like model

[A5.1] Quaternion UDS-components

In the following the model described in chpt. 3 will be explained in some more detail. A standard algorithm for rotation with quaternions will be used.

Three orthonormal vectors E, B, C described as imaginary part of a quaternion with real parts set to 0, will be subject to alternate, incremental rotations around the axes E, B and C. For each E, B and C the following variables will be defined:

- de, db, dc: incremental step for rotation angle,
- de_sum, db_sum, dc_sum: total rotation angle,
- ex, ey, ez, bx, by, bz, cx, cy, cz: Cartesian components of the respective vectors,
- eex, eey, eez, bbx, bby, bbz, ccx, ccy, ccz: Cartesian components of the respective vectors to be buffered until rotation around the axes E, B and C is complete,
- sih, qw, qx, qy, qz: internal variables for quaternion-rotation calculation.

The following part of the algorithm gives the rotation of B around the E axis for an incremental step de:

```
de_sum = de_sum + de;   sih = Sin(de / 2);   qw = Cos(de / 2);    qx = ex * sih    qy = ey * sih;    qz = ez * sih;
bx = bbx;    by = bby;    bz = bbz;
bxx = bx * (qx * qx + qw * qw - qy * qy - qz * qz)  + by * (2 * qx * qy - 2 * qw * qz) + bz * (2 * qx * qz + 2 * qw * qy);
byy = bx * (2 * qw * qz + 2 * qx * qy)   + by * (qw * qw - qx * qx + qy * qy - qz * qz) + bz * (-2 * qw * qx + 2 * qy * qz);
bzz = bx * (-2 * qw * qy + 2 * qx * qz)  + by * (2 * qw * qx + 2 * qy * qz) + bz * (qw * qw - qx * qx - qy * qy + qz * qz);
bx = bxx;  by = byy;  bz = bzz;
```

This has to be followed by rotation of C around the E axis; and equivalent routines for the rotation of E, B around the C axis and the rotation of E, C around the B axis. After each incremental step for de, db, dc the Cartesian components of the E, B, C vectors may be stored in a list, tab. 6 gives an example for the results. A rotation is considered complete if all vectors regain there starting values, see flowchart, fig.2.

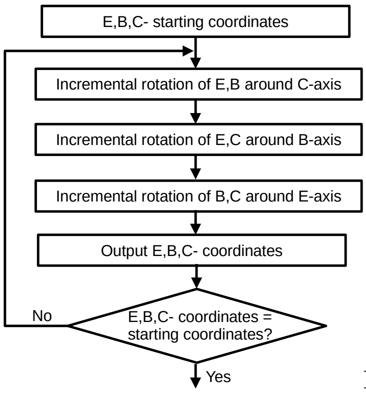

Fig. 2: Flowchart quaternion calculation

The vectors are thought to indicate spatial orientation only, *polarity of E and B has to be considered in the analysis of the results*. Orientation of angular momentum remains a free parameter.
In the following only solutions where one of the incremental angles of rotation has half the value of the other two will be considered. This may serve as a primitive model for spin J = 1/2.
There are 6 possible solutions for de, db and dv, respectively, to be called U, D, S, C, B, T:

	de = 0.5 db = 0.5 dc				de = 0.5 db = 0.5 dc				de = 0.5 db = 0.5 dc			
	E-comp	E-avg	B-comp	B-avg	E-comp	E-avg	B-comp	B-avg	E-comp	E-avg	B-comp	B-avg
Spherical	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	4/9, 4/9, 2/9	2/3
cone	U				D				S			
Toroidal	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	2/9, 2/9, 1/9	1/3
wedge	C				B				T			

Tab.5: Inc. average of x,y,z-components (E,B-comp) and total average (E,B-avg) of E-and B-field for complete rotation;

The average of the x, y, z-components of the fields are multiples of 1/9th of the original vector length, the average total sum of E- and B-fields is 1/3 or 2/3, respectively. Surface area / fractional charge of 1/3 and 2/3 correspond to an average of the E-field of 2/3 and 1/3.
The diagram for the E,B, C-components as function of the angle dc_sum is given in fig. 3a.
From a coordinate transformation to a representation with one Cartesian coordinate as axis of rotation (in fig. 3b transformation of z-axis +26,6°, x-axis -41,8°, to give y-axis as axis of rotation) one can infer that the E-vector circumvents a spherical cap of area 2πr 2/3r. Mirroring at the center of rotation gives a value of 2/3 of the surface of a sphere, which according to Gauss' law may represent 2/3 of a full point charge. The analogue procedure yields a value of 1/3 of a point charge for D and S-rotations.

Fig. 3.: a) E-components for Cartesian starting values b) E-components after coordinate transformation

[A5.2] Magnetic moments of baryons from U, D, S-components

To calculate magnetic moments of uds-baryons three components of U,D,S will be combined that represent orthonormal starting conditions for E, B. Spin/angular moment of the 3 components has to add up to $J_Z = 1/2$. Within this model this is not an assumption but may be calculated in principle in detail. In the following it will be sufficient to have two components sharing the same orientation of the axis of rotation, i.e. both can be transformed according to fig. 3 above with the same set of rotation angles, or - in a trivial case – to have 2 identical components. Together with the freedom in choosing direction of rotation, allowing for canceling or adding up spin as needed, this will be sufficient to model $J_Z = 1/2$ baryons. Table 6 gives an example for UUD and DDU.

In D_inv and U_inv the sign of E- and B-components is inverted. The D and U for calculation of the effective B-field include the appropriate sign from their charge while U_inv, D_inv components represent the actual geometric orientation of the E, B-vector only, which is needed for calculation of the angular momentum J from the square of the electromagnetic fields. In table 6 "Rot_X_axis" and "Rot_Z_axis" give the angle of rotation needed to transform to a representation with y-coordinate as axis of rotation for the B-field. For U_1 and D_inv of the proton as well as for D_2 and U_inv of the neutron the angles of transformation are identical, so is their transformed y-axis, i.e. they posses identical orientation of spin (average of B) while still maintaining their orthonormal relationship (B(t)). Since orientation of rotation is a free parameter opposite spin will cancel both contributions, leaving the 3rd component's spin of $J_Z = 1/2$ as total spin of the nucleon.

The U and D components of proton / neutron are complementary with respect to the sign and relative value of the components of the E- and B- fields (given in tab. 6 only for the Bx, By, Bz-components (bold) relevant for calculating a geometry-based average value of B, B_Avg). The starting values of E, B, C are given for reference only, each pair represents the same rotation.

	UUD	Proton			DDU	Neutron	
	U_1				D_1		
Start value	-Ez	-Bx	Cy		-Ex	-Bz	Cy
Bx, By, Bz	**-0.444444**	**0.444444**	**-0.222222**		**-0.222222**	**0.222222**	**-0.111111**
	E	B			E	B	
Rot_Z_axis	-45	135			-45	135	
Rot_X_axis	19.47	19.47			19.47	19.47	
	U_2				D_2		
Start value	-Ex	By	-Cz		Ey	-Bx	-Cz
Bx, By, Bz	**-0.222222**	**0.444444**	**-0.444444**		**-0.111111**	**0.222222**	**-0.222222**
	E	B			E	B	
Rot_Z_axis	-26.57	116.56			-26.57	116.56	
Rot_X_axis	41.82	41.81			41.82	41.81	
E, B inverted	D_inv				U_inv		
Start value	-Ey	-Bz	Cx		-Ez	-By	Cx
	E	B			E	B	
Rot_Z_axis	-45	135			-26.57	116.56	
Rot_X_axis	19.47	19.47			41.82	41.82	
	D				U		
Start value	Ey	Bz	Cx		Ez	By	Cx
Bx, By, Bz	**0.222222**	**0.222222**	**0.111111**		**0.444444**	**0.444444**	**0.222222**
Bx, By, Bz Avg(UUD)	-0.148148	0.37037	-0.185185		0.037037	0.296296	-0.037037
B_Avg			**0.439790**				**0.300890**

Table 6: Example for appropriate combinations of U- and D-components for proton and neutron;

The results for U and D are exceptional in regard to the exchangeability of U and D-components. Exchangeability of components for other particle pairs is difficult to asses due to identical B-field components of U and S, different internal symmetry of S-components may play a role as well.

In the case of the solutions examined, compliance with condition $J_Z = 1/2$ for the lambda-particle (UDS) can be maintained by using a spin-cancelling UD-solution in combination with an S-component, for UUS, DDS, USS-combinations trivial solutions with two identical components exist, in the case of DSS, Xi^-, one can resort to the method used for the nucleons to find a $J_Z = 1/2$ solution. Results for the best fitting appropriate UDS-combinations are shown in tab. 7.

	USD	Lambda		UUS	Sigma +		DDS	Sigma -		USS	Xi 0		DSS	Xi -	
	U			U			D			S			S		
Bx, By, Bz	-0.444	0.444	-0.222	-0.222	0.4444	-0.444	-0.111	-0.222	0.222	-0.222	-0.444	-0.444	0.444	-0.222	0.444
	S			U			D			S			S		
Bx, By, Bz	0.444	-0.444	0.222	-0.222	0.4444	-0.444	-0.111	-0.222	0.222	-0.222	-0.444	-0.444	-0.444	0.444	-0.222
	D			S			S			U			D		
Bx, By, Bz	0.222	0.222	0.111	0.4444	0.4444	0.222	0.444	0.444	0.222	0.444	0.444	0.222	0.222	-0.222	0.111
Bx, By, Bz Avg(UUD)	0.074	0.074	0.037	0.000	0.444	-0.222	0.074	0.000	0.222	0.000	-0.148	-0.222	0.074	0.000	0.111
B_Avg			**0.111**			**0.497**			**0.234**			**0.267**			**0.134**

Table 7: Combinations of UDS-components for calculating magnetic moments of baryons.

To calculate magnetic moments, above factors of B_avg, derived from the purely geometric quaternion model, have to be multiplied by a factor considering the absolute strength of fields. Using the simple model of a current loop, M = I*A, gives for magnetic moments of baryons with $J_Z = 1/2$:

$$\boldsymbol{M_n} \approx e c_0 \lambda_C / 2 \; * \; \text{B\_avg} \quad (= 2\pi \mu_{Bohr} * \text{B\_avg}) \tag{72}$$

see tab. 8. Factor 2π of the Bohr magneton, μ_{Bohr}, applicable for the electron and muon, is considered to represent a degree of rotational freedom of simple particles that more complex structures composed of several U, D, S-components do not exhibit.

		λ_C	$e\, c_0 * \lambda_C /2$	B_Avg	|M|Calc = $e c_0 \lambda_C\, B_{avg}/2$	|M|Exp[Am^2]	|M|Calc/ |M|Exp	|M|Calc/|M|Exp Const. quark
p^{+-}	UUD	1.32E-15	3.17E-26	0.440	1.39E-26	1.41E-26	0.988	-
n	DDU	1.32E-15	3.17E-26	0.301	9.55E-27	9.66E-27	0.988	0.973*
Λ^0	UDS	1.10E-15	2.64E-26	0.111	2.94E-27	3.10E-27	0.949	-
Σ^+	UUS	1.04E-15	2.50E-26	0.497	1.24E-26	1.24E-26	1.002	1.090
Σ^-	DDS	1.04E-15	2.50E-26	0.234	5.83E-27	5.86E-27	0.994	0.897
Ξ^0	USS	9.43E-16	2.26E-26	0.267	6.05E-27	6.31E-27	0.958	1.152
Ξ^-	DSS	9.38E-16	2.25E-26	0.134	3.01E-27	3.06E-27	0.983	0.784

Table 8: Magnetic moments for UDS-Baryons; col.3: Compton wavelength [7]; col.4: magnetic moment for current loop; col.5: average B-component from quaternion calc.; col.6: calculated magnetic moments; col.7: values from experiment [7]; col.8: ratio calculated / experiment value; col.9: ratio (calculated constituent quark model, [7]) / experiment [7]), *calc. via Clebsch-Gordan coefficients relative to p; Σ, Ξ via fit based on p, n, Λ^0.

[A6] Additional particle states

Assignment of more particle states will not be obvious. The following gives some possible approaches.

[A6.1] Partial products

One more partial product might be inferred from considering the next spherical harmonic, y_2^0 with a factor of $(2l+1)^{1/3}$ = $5^{1/3}$ as energy ratio relative to η, giving the start of an additional partial product series at $5^{1/3}$ W(η) = 937MeV i.e. close to energy values of the first particles available as starting point, η', Φ^0. However, in general it is not expected that partial products can explain all values of particle energies.

[A6.2] Linear combinations

Though the model reproduces basic properties of the quarks the fundamental differences might offer some alternate interpretations based on extended, non-point-like objects.

The linear combination state of the kaons, the first particle family that does not fit to the partial product series scheme, and the η-particle might be an example for such an interpretation:

The kaons are designated to the linear combination of $(d\bar{s}$ +/- $\bar{d}s)/\sqrt{2}$ in the SM. They might be considered to be a linear combination of 2 extended y_1^0 states (double cones of $s|\bar{d}$, $\bar{s}|d$, etc., composition with 1 angular node) similar to the linear combination of 2 atomic p-orbitals, assumed to exhibit 2 angular nodes. A linear combination which would yield the basic symmetry properties of the 2 neutral kaons would be a planar structure such as:

$$K_S^o \quad \begin{matrix} & \bar{s} & \\ d & & s \\ & \bar{d} & \end{matrix} \qquad K_L^o \quad \begin{matrix} & d & \\ s & & \bar{s} \\ & \bar{d} & \end{matrix}$$

providing two neutral kaons of different structure and parity (considering either flavour or chirality), implying a decay with different parity and MLT values.

A linear combination of 3 such states would result in a linear combination of 3 orthogonal y_1^0 states implying an essentially spherical symmetric object which might be attributable to the η-particle $((u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6})$.

[A6.3] Electroweak bosons

The considerations of chpt. 2.4, 2.5 suggest to interpret the Higgs VEV as 1D object and the Higgs boson with half its energy value might be interpreted correspondingly if both objects are considered to be in the relationship of a double cone/cone with opening angle approximating 0°. The use of the maximum term for angular contributions implies a minimum lateral extension of the E-vector and essentially no space left for rotation, i.e. Spin -> 0 [38].

Using the alternate definition of the Higgs VEV as 〈Φ〉 = VEV/√2 [7] would relate a Higgs boson to 〈Φ〉 through the "1D"-term, $\Gamma_{-1/3}/3$. Moreover, the Z boson would correspond to a 2D-object, the W bosons to a 3D-object (if the inverse of the coefficient of the integral for energy, $3/\Gamma_{+1/3}$, is considered to represent a length parameter attributed to λ_C). Except for a factor of 2 the volume term of (20) would give the Δ-particle as starting point of the energy series from the high energy side. This seems to be another hint that some aspects of this model might be expressible in terms of Euclidean geometry.

Electroweak bosons + VEV/√2	W [GeV]	Γ-coefficient relative to VEV/√2	VEV/√2 divided by Γ-coeff. [GeV]	W(calc)/ W(Lit.)
VEV/√2	174.1			
Higgs	125.4	$\Gamma_{-1/3}/3$	128.6	1.026
Z^0	91.2	$(\Gamma_{-1/3}/3)^2$	95.0	1.041
$W^{+/-}$	80.4	$(\Gamma_{-1/3})^2\ /(3\Gamma_{+1/3})$	84.8	1.055
Δ	1232	$4\pi/3\ (\Gamma_{-1/3})^3$	1.24/2	1.006

Tab. 9: Electroweak bosons and Δ-particle relative to the Higgs VEV/√2

[A7] Nucleons – stability, bonding in nuclei, scattering

Apart from the quantitative results for partial charges and magnetic moments some qualitative trends for nucleon properties may be inferred from the quaternion-based model.

The spin-cancelling of a UD-unit involves 2 collinear components with opposite charges occupying approximately the same spatial area, which is energetically favorable. This suggests among other things:

1) Comparatively lower energy for particles with UD-component;
2) High stability / life time of the nucleons;
3) A possible contribution to bonding in nuclei via UD-U—D-UD, a direct U-D-bond even without meson intermediate;
4) If such an inter-nucleon UD-bond plays a role in bonding in nuclei this would suggest a significant change in UD-structure between isolated and bound nucleons, which might play a role in the "EMC-effect" [13];
5) In DIS-experiments the ratio of the structure functions of neutron and proton, $F_2^n(x)/F_2^p(x)$ approaches 1 for x -> 0 (x = Bjorken-scale) which would be in agreement with a supposed identical field distribution of E and B-fields in the nucleons. For x -> 1 this model predicts the ratio $F_2^n(x)/F_2^p(x)$ to approach

$(z(UD)^2 + Z(D)^2)/(z(UD)^2 + Z(U)^2) = ((+1/3)^2 + (-1/3)^2)/((+1/3)^2 + (+2/3)^2) = 2/5$

which is in good agreement with high precision scattering experiments which yield values in the range 0.4 – 0.5 [14].

38 Assuming that an extremal, not rotating E-vector state is not accompanied by a B-field gives factor 1/2 as well.