ISSN- 2350-0530(O), ISSN- 2394-3629(P)
DOI: 10.5281/zenodo.1095439

INTERNATIONAL JOURNAL OF RESEARCH – GRANTHAALAYAH
A knowledge Repository

Social

ASSESSING STUDENTS' UNDERSTANDING OF PRE-CALCULUS CONCEPTS

Dr. Jyoti Sharma [*1]
[*1] Associate Professor, Cluster Innovation Centre, University of Delhi, India

Abstract

Calculus is one of the most momentous achievements of the human intellect (Boyer, 1949). It has given a new direction to the work of mathematicians and scientists. Calculus has exponentially expanded the scope and use of mathematics in other fields. Learning calculus is important to pursue career in applied mathematics.

Keywords: Pre-Calculus; Concepts; Assessing Students.

Cite This Article: Dr. Jyoti Sharma. (2017). "ASSESSING STUDENTS' UNDERSTANDING OF PRE-CALCULUS CONCEPTS." *International Journal of Research - Granthaalayah*, 5(11), 217-221. https://doi.org/10.5281/zenodo.1095439.

1. Introduction

Calculus comprises significant part of senior secondary mathematics in Indian school mathematics curriculum (Position paper on Teaching of Mathematics- NCF-2005, NCERT). Students are expected to learn differential and integral calculus along with concept of functions and limits. Learning to differentiate and integrate various functions in specific interval is followed by some standard applications of differentiation and integration.

Students continue doing advanced calculus in their Mathematics Major course and other Science & Technology based professional courses at undergraduate level. Sound foundations of calculus is important for three main reasons: first, understanding of operational calculus is important for pursuing career in applied mathematics; second, conceptual understanding of calculus is central to develop structural foundations of mathematics and third, learning about origin of calculus is significant to appreciate human efforts in demystifying the language of nature in the most efficient way.

Objectives of introducing calculus at school level shall aim to all three reasons mentioned above. The roots of calculus should start from the most fundamental concept of mathematics, 'measurement'.

ISSN- 2350-0530(O), ISSN- 2394-3629(P)
DOI: 10.5281/zenodo.1095439

Mathematics is always a dreadful subject for majority of students. Only few students enjoy learning mathematics for sheer excitement. For majority of students, mathematics is a subject of formulae, methods and standard procedures. The way mathematics is being taught since the beginning, it kills the spirit of 'learning mathematics for understanding'. For the understanding of mathematics, it is important that students are taught to understand language of mathematics; meaning what a mathematical statement communicates, expresses or represents. It shall also include re-stating a mathematical statement in multiple ways or framing a mathematical argument; writing a mathematical justification or producing an elegant mathematical solution.

The journey to learn mathematics shall start from talking about mathematical concepts, inter-relation of mathematical concepts and boundaries of a give mathematical concept. These talks shall be initially non-formal and later it shall develop as a more formal mathematical language.

The present paper is based on an empirical study about students' understanding of 'limit and Continuity of Functions', which is a pre-required concept to start with calculus (as per Indian school mathematics curriculum prescribed by CBSE)[1].

2. Objectives of the Study

Objectives of the study were as follows:

- To assess the students' understanding of defining and representing idea of limit in different contexts.
- To assess students' understanding of defining and applying concept of continuous functions in different contexts.
- To develop sequence of learning for introducing the concept of calculus.

3. Methodology

It was an exploratory study. The study was qualitative in nature though qualitative analysis was also done to analyse the findings.

Sample consisted of 35 students who had studied calculus as part of their high school mathematics curriculum and who were pursuing under graduate degree course either with math major or math as an applied or a minor subject. All students had studied mathematics till grade XII and as per CBSE school mathematics curriculum, they had extensively studied limit of functions, continuous functions and differential and integral calculus as part of their grade XI & XII mathematics syllabus.

Study consisted of set of three questions on the concept of 'limit and continuity. Questions aimed to assess the conceptual understanding of students about the concept of 'limit and continuity'.

[1] National Council of Educational Research and Training is the national council responsible to provide national curriculum framework for school education and write textbooks for all subjects and for all grades at school level in India whereas Central Board of Secondary Education (CBSE) is the national board of school education mainly responsible for monitoring the implementation of norms and standards of school education).

 ISSN- 2350-0530(O), ISSN- 2394-3629(P)
DOI: 10.5281/zenodo.1095439

The set of questions consisted of following items:

Q1) $For\ any\ function\ g(x),\ \ Explian\ the\ following\ statement\ in\ your\ own\ words:$
$$Function\ is\ continuous\ at\ x = c$$
$$Function\ is\ approaching\ to\ a\ limit\ at\ x = c$$
Q2) $Represent\ the\ concept\ of\ limit:$
$$graphically$$
$$symbolically$$
$Q3)\ Accept\ or\ refute\ the\ following\ statements. Support\ your\ choice\ with\ suitable\ example.$
$$If\ a\ function\ has\ a\ limit\ at\ a\ point\ p, then\ it\ is\ a\ continuous\ function.$$
$$Only\ a\ continuous\ function\ can\ have\ limit.$$
$$If\ a\ function\ is\ undefined\ at\ a\ certain\ point\ then\ it\ does\ not\ have\ limit\ at\ that\ point.$$
$$All\ polynomial\ functions\ have\ limits.$$

4. Data Collection

Questions were prepared after looking into the syllabus of calculus of high school mathematics. Brief discussion sessions on introduction to calculus were held before students were given the question set. Students were given forty five minutes to answer all the questions. Students were encouraged to answer questions in more than one way.

5. Data Analysis

Out of 35 students, eight students could not answer even one questions correctly. Only two students answered all, except one questions correctly.

Question 1 (a & b part) was attempted by more than 85% of students but only 43% of students answered part (a) correctly and only 23% of students answered part (b) correctly.

Question 2 (a & b part) was attempted by only 67% of students and 9% of total students answered part (a) correctly whereas 29% of total students answered part (b) correctly.

Question 3 consisted of four parts: (a, b, c & d). This question was attempted by 91% of students. 34% of total students attempted part (a) correctly; 4% of total students answered part (b) correctly; 35% of students answered part (c) correctly and 16% of students answered part (d) correctly.

Since, answers to all questions could be expressed in many different ways, modes of expressions used by students were also a point of consideration. Choice of examples in justifying their argument also helped to analyze their responses. Many students could write textbook definition but were unable to express it in other contextual form. For example, only three students could represent concept of limit graphically though among three of them only one could represent the idea of limit with appropriate symbols and indicators. Similarly, for Question 1, majority of students who attempted the question, could give formal definition of limit but not many could explain what did it mean when it is said that *'a function is approaching to a limit at* $x = c'$.

ISSN- 2350-0530(O), ISSN- 2394-3629(P)
DOI: 10.5281/zenodo.1095439

Even for Question 3, majority of students could answer part (a) but only 4% could think about the existence of limit without (independent of) continuity. Similarly, majority of students who responded to part (d) were unable to connect the concept of limit to the definition of polynomial functions.

The responses of the students highlighted four major issues:

1) Students could not express their own conceptual understanding of concepts of limit and continuity. They could only provide text book definition.
2) Students could not analysis the definition or salient characteristics of the concepts.
3) Students could not locate the concepts in specific contexts.
4) Students could not relate the concept of 'limit' and 'continuity' holistically, how these two concepts are related to each other or dependent on each other.

6. Discussion

Calculus is one of the central concepts in advanced mathematics. It builds up foundations for applied mathematics and also for pure mathematics. Sound understanding of calculus is essential for mathematical modeling and for application of mathematics in other domains. Deeper understanding of calculus is important to appreciate the scope, beauty and coherence of mathematical concepts. Pre-calculus concepts such as functions, nature of functions and describing a function, are all fundamental to develop sound understanding of calculus. The present study highlighted the misconceptions among students who had already learned foundations of calculus and were now pursuing advanced courses either in pure calculus or applied calculus. This shall be an area of concern because if conceptual foundations are weak then any development on incorrect foundations would give unproductive results in future.

Introduction to calculus shall begin with measurement of continuous quantities, called variables when two or more quantities (variables) are related to each other. Special relations among two or more than two quantities when expressed algebraically are called 'functions'. Each function possess special characteristics which are unique to that function. Nature of a function changes as values of variables changes from one value to another value. This change can be studied numerically, algebraically or graphically. Every function is defined under certain conditions or boundaries. Sound understanding of nature of a function and describing a function numerically, algebraically and graphically is essential to start the idea of calculus. Describing a function will help students to explore the function at length such as values of function at various points, nature of change in the values, progression of function and trajectory followed by the function. It is important to highlight students' attention at those crucial points where there is a significant shift in the nature of function. These informal but cautiously planned discussions can help students to build up sound understanding of a function and now they will be better prepared to understand limit of á function; limit of a function at a point, continuity of the function and how limit and continuity of a function are related to each other with respect to a specific function and for various other functions in general. Below is the suggestive sequence of learning that may be followed to teach calculus:

1) Concept of measurement
2) Discrete and continuous measurement
3) Related variables

ISSN- 2350-0530(O), ISSN- 2394-3629(P)
DOI: 10.5281/zenodo.1095439

4) Relation among related variables
5) Relation and function
6) Describing a function (domain, co-domain and range)
7) Plotting a function
8) Characteristic of a given function & parts of function (limit, continuity, maximum and minimum values)
9) Calculating (analysing) the rate of change of variables of a function (differentiation)
10) Summing (synthesising) the dynamic parts of a function (integration)
11) Visualizing differentiation and integration
12) Application of calculus

The twelve point sequence mentioned above can be a guiding line to gradually develop the idea of calculus. It is based on already learned concepts of measurement and quantities. It also requires to recall students' understanding of basic algebra. When calculus is introduced by connecting already learned concepts, students feel more comfortable and confident.

All concepts related to calculus shall be taught through exploration, discussion and concretization. Gradually, students shall be encouraged to generalize the concepts of limits and continuity. Exploring functions shall be the first step in teaching calculus. Mechanical procedures to teach calculus not only force students to develop misconceptions but also deprive them to appreciate inherent beauty of the concepts. Calculus is an interesting as well as important concept. Teaching of calculus shall have broader outlook with scope of discussing historical development of one of the most significant achievement of human minds.

Acknowledgment

The researcher and author acknowledge the support of University of Delhi to conduct the research on topic titled, "Concept Map as Pedagogy Tool for Teaching Calculus" under Research & Development Grant (2015-16). The author acknowledges the Cluster Innovation Centre, University of Delhi, for all infrastructural support to carry out the research work.

References

[1] Boyer, B.C. (1959). The History of Calculus and Its Conceptual Development. United States: Dover Publications.
[2] Mathematics for Class XI by NCERT. India: New Delhi.
[3] Mathematics for Class XII by NCERT. India: New Delhi.
[4] Position Paper on Teaching of Mathematics: National Curriculum Framework-2005. NCERT. India; New Delhi.

*Corresponding author.
E-mail address: jyotisharmacic@ gmail.com