The strong interaction as a nonlinear near-field component of the electromagnetic field

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This model presents a view of the strong nuclear force from the perspective of differential geometry, allowing the effective residual interaction acting between hadrons to be attributed to a nonlinear behavior of the electromagnetic force. The strength of the residual interaction has a fixed relationship to Sommerfeld's fine structure constant. The binding energy of the simplest proton-proton bond can in principle be derived.

Introduction

Even before the introduction of the quark model, the nuclear force between hadrons could be described by an effective Yukawa potential. This is still used as a model today and enables many simplified calculations. But when scattering experiments detected substructures in hadrons, the question arose how they hold together and whether the interaction between hadrons can be described as a residual interaction of the new strong interaction. The answer is yes, but there are limitations.

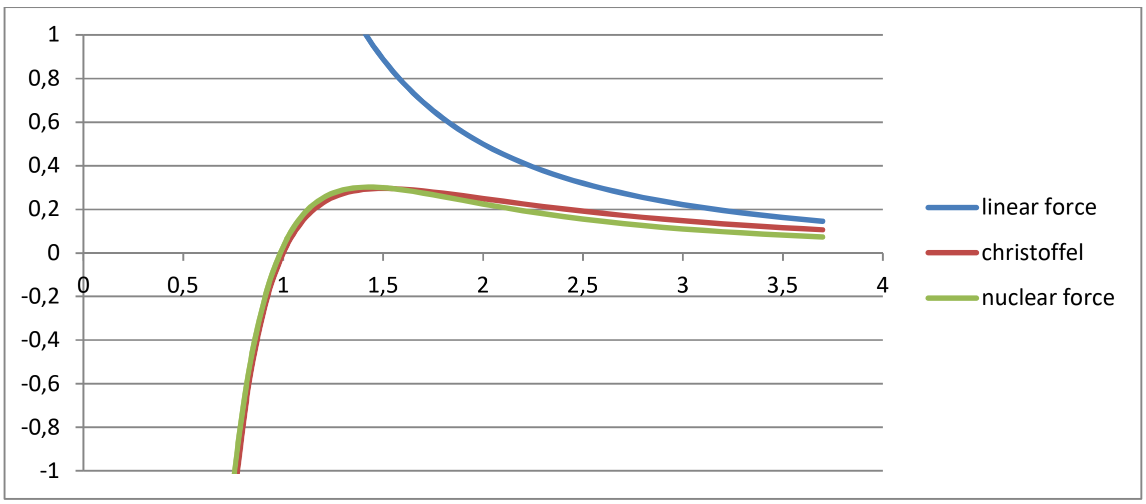
The strong interaction between the quarks can only be imprecisely captured using perturbation calculations. Even grid calculations with supercomputers only provide rough estimates. The strength of the quark force causes non-linear relationships. Models often need to be adjusted to the results of precise measurements. This is currently the case, for example, through newer measurements of the charge radius of protons.

Last but not least, the question of all questions in physics arises! How can the four known interactions be combined? This was successful for two interactions. The electroweak interaction describes quantum electrodynamics and weak nuclear force uniformly, with the help of spontaneous symmetry breaking in the Higgs mechanism. Both forces are relatively weak and can therefore be calculated precisely to many decimal places using linear operators and perturbation calculations. The strong interaction breaks out of the scheme for the reasons mentioned. Gravity, on the other hand, is a property of the geometry of space-time and is described using the means of differential geometry. It is not treated as a gauge field and cannot yet be quantized.

A simple comparison of field gradients now provides a completely new point of view. The course of the effective nuclear force - i.e. the nuclear-residual interaction and the electric field - of a proton appears to be incredibly similar to the behavior of gravity near a black hole. In other words, similar to the behavior of a field that is also highly nonlinear. As a first approximation, this results in an elementary connection between the nuclear force and the electrostatic field of the proton - the external Schwarzschild metric is the metric of a point mass.

I The model

The course of the effective field strength of a proton is strongly reminiscent of the course of the radial Christoffel-Symbol in the Schwarzschild model, the simplest solution in general relativity for masses. A calculation of this type leads to a nonlinear acceleration field.



(Diagram 1: Field gradients)

$$\Gamma^r_{00} = \frac{1}{2} \cdot g^{rr} \cdot g_{00;r}$$

$$\Gamma^r_{00} = \frac{1}{2} \cdot \left(1 - \frac{R_S}{r}\right) \cdot \frac{R_S}{r^2} = \frac{1}{2} \cdot \left(\frac{R_S}{r^2} - \frac{R_S{}^2}{r^3}\right)$$

There is a point at which the field strength effectively becomes zero and this is identical to the Schwarzschild radius of a mass.

If it is now assumed that the distance at which nuclear force and electrical force cancel each other is analogous, a *dual* Schwarzschild radius Rs* can be defined. If Rs* follows from the zero crossing of the effective force, this value must correspond to approximately 2.5fm[1].

If one compare this value to the reduced Compton-wavelength of the proton, the relation follows

$$\frac{R_S^*}{\lambda_C'} \approx 11{,}924$$

The value for this radius is only approximately known. If a fixed connection to the fine structure constant can now be concluded, then the dual radius clearly depends on it and the proton mass.

$$R_S^* = {\alpha_{em}}^{-1/2} \cdot \frac{\hbar}{m \cdot c}$$

The maximum effective force is located at a radius 1.5 times this dual radius.

The force acting on a second proton can be calculated from the Christoffel symbol

$$F = \frac{1}{2} \cdot \left(\frac{{\alpha_{em}}^{-1/2} * \frac{\hbar}{m \cdot c}}{r^2} - \frac{{\alpha_{em}}^{-1} * \frac{\hbar^2}{m^2 \cdot c^2}}{r^3} \right) \cdot c^2 \cdot m$$

$$F = \frac{1}{2} \cdot \left(\frac{{\alpha_{em}}^{-1/2} * \hbar c}{r^2} - \frac{{\alpha_{em}}^{-1} * \frac{\hbar^2}{m}}{r^3} \right)$$

In order for the first term to accurately determine the electrostatic force, the entire expression must be rescaled:

$$2 \cdot {\alpha_{em}}^{3/2} \cdot F = \left(\frac{\alpha_{em} \cdot \hbar c}{r^2} - \frac{{\alpha_{em}}^{1/2} \cdot \frac{\hbar^2}{m}}{r^3} \right)$$

The rescaling does not change the relationship between the force components and the zero point does not shift.

As a force, the Christoffel symbol breaks down into a purely electromagnetic term and a nonlinear term, which appears as a mass-dependent nuclear force. This is actually a nonlinear continuation of the electromagnetic force. The nonlinearity now follows from the assumption that the structure of Euclidean space is influenced in the sense of a Riemann geometry. The direct interaction constant for the nuclear force is then exactly 11.7 times higher than the electromagnetic fine structure constant.

The core potential can then be represented as:

$$V_i = \frac{1}{2} \cdot \frac{{\alpha_{em}}^{1/2} \cdot \hbar \cdot c \cdot \frac{\hbar}{mc}}{r^2}$$

II Results

For the most accurate comparison with the Yukawa potential, the exponential function is approximated using its Taylor series:

$$V_Y = f^2 \cdot \hbar c \cdot \frac{e^{-\mu r}}{r} \approx \frac{f^2 \cdot \hbar c}{r \cdot (1 + \mu \cdot r + \cdots)} = \frac{f^2 \cdot \hbar c}{\mu \cdot r^2}$$

$$\frac{f^2 \cdot \hbar c}{\mu \cdot r^2} = \frac{1}{2} \cdot \frac{{\alpha_{em}}^{1/2} \cdot \hbar \cdot c \cdot \frac{\hbar}{mc}}{r^2}$$

$$f^2 = {\alpha_{em}}^{1/2}$$

$$\frac{1}{\mu} = \frac{1}{2} \cdot \frac{\hbar}{mc}$$

The coefficient comparison is based on this approximation. In fact, the comparison could only be made because

$$\frac{f^2}{\alpha_{em}} > 1$$

The public literature gives different values for the nucleon-pion interaction. A work determines an order of magnitude with great uncertainty[2]:

$$\frac{1}{f^2} = 12{,}1 \ \pm 2{,}7 \approx 11{,}706$$

The value derived from the fine structure constant can be used very well here.

Other works initially speak against this or require borderline behavior[3]:

$$\frac{1}{f^2} = 11{,}904 \ldots 14{,}285$$

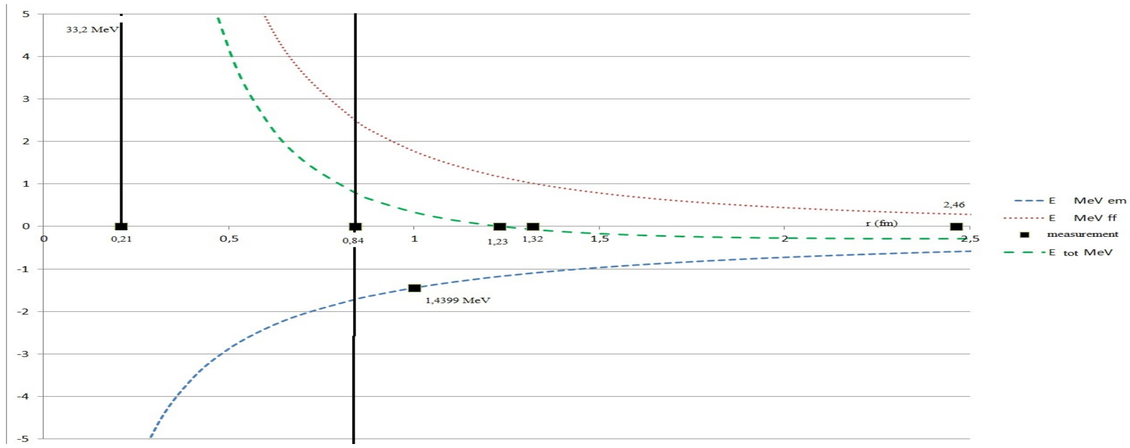
There can be various reasons for this. Exact models cannot be derived from the Quark model precisely, because their strength requires non-linear relationships. The usual methods are always only approximations. Or the measured values are caused by the general running of the coupling strength, by its energy dependence.

Assuming a conservative force field, a running interaction strength can be derived[T2].

$$f^2(r) = \alpha_{em} \quad - \frac{{\alpha_{em}}^{1/2} \cdot \frac{\hbar}{mc}}{r}$$

This results in a correction depending on the radius and the effective coupling, based on the reduced Compton-wavelength, is -0.07812 = -1/12.8. This makes the calculations comparable with the measurements.

The electromagnetic component of the potential exactly matches the binding energy of the simplest proton-proton bond[T1] at the expected radius.



(diagram 2: resulting potentials and excellent points[T1].)

The mass to be used is now the mass of the hadron that is to be described, not the mass of an exchange particle, and the exponential function can be understood as the charge density distribution of the hadron.

The model created is a point charge model. Extensions with a true charge density distribution will be discussed in later work. In particular, the local minimum of the core potential needs to be determined, but this requires a more complex consideration.

III Below the nuclear potential – the quark field

One conclusion from the approach is that nuclear power appears not to be an independent force. But what can this mean for the quark-model?

Below one femtometer there is still room for many degrees of freedom and so far only static fields have been considered. But in quantum mechanics, real solutions are always wave solutions or at least wave-like solutions.

Then it is not impossible to reinterpret the existence of quarks.

They could be viewed as quasi-corpuscular internal field maxima, i.e. zones of maximum field strength of a single coherent structure - the electromagnetic wave field. They would be measurable because, f.e. electrons would be scattered at such locations, but the principle of confinement would no longer be necessary. The hadrons, i.e. mesons and baryons, are already elementary in this picture, although they have an internal structure.

IV The duality principle for a Kaluza-Klein theory

In order to be able to represent the nuclear force as an inner field in the sense of general relativity, the concept of the dual Schwarzschild radius was introduced. But its size is proportional to a mass that is actually far below the Planck scale.

This is exactly the reason that led to the introduction of this term. The Schwarzschild radii of all known elementary masses are far too small to define a difference between an inner field and an outer field in the sense of GR. This is a particularly glaring problem if spacetime should be quantized based on the Planck length (this will be worked out in detail in a subsequent work). Schwarzschild radii can also be derived from the interaction strengths, but these are also too small.

The concept of duality is taken from string theory[5]. Different string theories are dual to each other. They produce similar results via mathematical approaches of varying difficulty, and the more accessible approach is usually preferred.

Here quantities are now called dual, which represent the same physical quantity differently. These are primarily the reduced Compton-wavelength and the gravitational radius of a given mass. In principle, dual quantities can be determined for every Planck unit and the Planck quantities themselves are *dual invariants*.

V Conclusion

The model for the strong Yukawa interaction is based on the use of a dual Schwarzschild radius. Previously, the size and internal structure of hadrons remained completely incomprehensible within the framework of a Kaluza-Klein theory. Leptons sometimes have longer compton wavelengths, but do not show any internal structure.

Why this type of "stretched distribution" only occurs with hadrons and not with leptons remains an open question. The present model is only a working model and is not derived from a higher principle.

A Kaluza-Klein theory must be established which, in addition to quantization, also includes the duality principle. In this respect, the derived metric can provide an indication of the necessary dimensionality and mathematical structure of such a theory.

The rescaling performed determines the (external) metric for hadrons with charge over

$$\Gamma_{hd}^{k} = \frac{1}{2}\left(\sqrt{2}{\alpha_{em}}^{3/4} - \sqrt{2}{\alpha_{em}}^{3/4}\frac{{\alpha_{em}}^{-1/2}\lambda_c'}{r}\right)\sqrt{2}{\alpha_{em}}^{3/4}\frac{{\alpha_{em}}^{-1/2}\lambda_c'}{r^2}$$

$$g_{hd} = \sqrt{2}{\alpha_{em}}^{3/4}\left(1 - \frac{{\alpha_{em}}^{-1/2}\lambda_c'}{r}\right) \approx \frac{1}{28{,}321}\left(1 - \frac{{\alpha_{em}}^{-1/2}\lambda_c'}{r}\right)$$

The metric goes against a different limit at infinity compared to the typical Minkowski metric, but as usual goes against the coordinate singularity at the dual radius.

The theory we are looking for would be simpler in one point than existing approaches including string theories: the near fields appear as abstractions of the long-range fields. These are just gravity and electromagnetism. Expanding space-time by a single dimension would be sufficient, as Theodor Kaluza proved in 1921[4].
The mathematics of differential geometry could open up a new way to describe strong forces, which can only be approximately calculated using QFT.

VI References

[1] Starke Wechselwirkung – Wikipedia

[2] Enrique Ruiz Arriola, Johannes Gutenberg-Universität Mainz, Germany, *The Pion-Nucleon-Nucleon Coupling Constants*, page 2

[3] T.E.O Ericson, B. Loiseau, A.W. Thomas, Cern, 1211 Geneva 23, Switzerland, *Determination of the pion–nucleon coupling constant and scattering length*, page 2

[4] Theodor Kaluza, *On the Unification Problem in Physics*

[5] Edward Witten, School of Natural Sciences, Institute for Advanced Study Olden Lane, Princeton, N.J. 08540, Joseph Polchinski, Institute for Theoretical Physics University of California, Santa Barbara, CA 93106, *Evidence for Heterotic - Type I String Duality*

VII Tables

table 1: excellent points in diagram 2

description	unit	value
reduced Compton-wavelength	Fm	0,2105
proton-size measured	Fm	0,84094
radius for binding-energy He^2	fm	0,99999557
binding-energy He^2	MeV	1,439972098
total potential zeropoint	fm	1,2309514
Compton-wavelenght	fm	1,3214109
dual radius	fm	2,461928

table 2: values of the Yukawa coupling constant using the model

proton radii	fm	Yukawa-coupling f^2
reduced Compton-wavelength	0,210309	-0,07812
charge radius	0,84094	-0,01407
dual radius	2,461928	0
expectationvalue r_1 for measured value of f^2	0,2305	-0,07
expectationvalue r_2 for measured value of f^2	0,1966	-0,084
expectationvalue r_3 strong coupling	0,017966	-0,9927