World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:4, No:8, 2010

Effect of thermal radiation on temperature variation in 2-D stagnation-point flow

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Abstract—Non-isothermal stagnation-point flow with consideration of thermal radiation is studied numerically. A set of partial differential equations that governing the fluid flow and energy is converted into a set of ordinary differential equations which is solved by Runge-Kutta method with shooting algorithm. Dimensionless wall temperature gradient and temperature boundary layer thickness for different combinaton of values of Prandtl number Pr and radiation parameter N_R are presented graphically. Analyses of results show that the presence of thermal radiation in the stagnation-point flow is to increase the temperature boundary layer thickness and decrease the dimensionless wall temperature gradient.

Keywords—Stagnation-point flow, Similarity solution, Thermal radiation.

I. Introduction

Stagnation-point flow is a flow in which the component of velocity normal to a wall is toward the wall everywhere in the region concerned, so that the vorticity created at the wall will be convected toward the wall, in opposition to viscous diffusion away from it. Stagnation-point flow has been found in numerous applications in engineering and technology[1]. Hiemenz[2] discovered that stagnation-point flow can be analyzed by the Navier-Stokes (NS) equations through similarity solution in which the number of variables can be reduced by on or more by a coordinate transformation. Chiam[3] studied heat transfer in stagnation-point flow with variable conductivity. Wang[4] found a similarity solution of stagnation slip flow and convective heat transfer on a moving plate.

Stagnation-point flow with consideration of thermal radiation is rarely and this is important for flow with high temperature environment[5][6]. Recently Bataller[7] consider the radiation effects in the Blasius flow over a flat plate with different Prandtle number and radiation parameter.

The present paper extends the results of [7] by considering the effect of radiation effect in non-isothermal stagnation-point flow. The Navier-Stokes and energy equations are transferred to two uncoupled ordinary different equations which are solved numerically by the shooting method given in [8]. The outline of the paper is as follows. In Section 2, we introduce governing equations for non-isothemal stagnation-point flow with thermal radiation and the process of finding similarity solution. Numerical results of the similarity solution are given and discussed in Section 3. Finally conclusions are drawn in Section 4.

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Manuscript received August 19, 2010; revised October 11, 2010.

II. Governing equations and similarity solution

The Governing equations for 2-dimensional non-isothermal stagnation-point flow with thermal radiation can be written as [6][9]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right] \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right] \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}. \tag{4}$$

where u and v are velocity along the wall direction (x-direction) and normal to the wall (y direction), p is pressure, ν is kinematic viscosity, k is thermal conductivity, c_p is specific heat at constant pressure, ρ is density, T is temperature, and q_r is radiative hear flux. The boundary conditions are

$$\begin{array}{llll} u=0, & v=0, & T=T_w & \text{at} \quad y=0 \\ u=0, & v=-V, & T=T_\infty & \text{at} \quad y\to\infty. \end{array} \tag{5}$$

Where V is ambient fluid velocity approaching to the wall normally, T_w is wall temperature, T_∞ is ambient fluid temperature. It is noted that viscous dissipation is neglected and constant T_w and T_∞ are assumed, so that $T \approx T(y)$ only.

Following the approach of [6][7], the radiative heat flux is given as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}. \tag{6}$$

Where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. It is further assumed that the term T^4 due to radiation within the flow can be expressed as a linear function of temperature itself. Hence T^4 can be expanded as a Taylor series about T_∞ and can be approximated as[7]

$$T^4 \approx 4T_\infty^3 T-3T_\infty^4 \tag{7}$$

after neglecting the higher-order terms. Using Eqs. (6) and (7), we can rewrite Eq. (3) as

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left[1+\frac{4}{3}\frac{4\sigma^* T_\infty^3}{kk^*}\right]\frac{\partial^2 T}{\partial y^2}. \tag{8}$$

Where $\alpha = \frac{k}{\rho c_p}$ is the molecular thermal diffusivity. Eq. (8) indicates that the presence of thermal radiation can be interpreted as an increase in the molecular thermal diffusivity. Let $N_R = \frac{kk^*}{4\sigma^* T_\infty^3}$ be the radiation parameter, Eq. (8) can be rewritten as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{\text{eff}}\frac{\partial^2 T}{\partial y^2}, \tag{9}$$

where $\alpha_{\text{eff}} = \frac{\alpha}{k_0}$ and $k_0 = \frac{3N_R}{3N_R+4}$. It can be seen that the effect of thermal radiation is to replace the original molecular diffusivity, α, by an an effective molecular diffusivity, α_{eff}, in the energy equation at which no thermal radiation is considered. It should be noted that $k_0 \leq 1$ and $\alpha_{\text{eff}} \geq \alpha$. The condition that $k_0 = 1$ or $N_R \to \infty$ represents the stagnation-point flow without thermal radiation.

Following [9] by introducing a similarity variable η, a dimensionless stream function $F(\eta)$, a dimensionless temperature θ, and the Prandtl number Pr, such that

$$\eta = y\sqrt{\frac{B}{\nu}}, \qquad u = BxF'(\eta), \qquad v = -\sqrt{B\nu}F(\eta), \tag{10}$$

and

$$\theta = \frac{T - T_w}{T_\infty - T_w}, \qquad Pr = \frac{\nu}{\alpha}, \tag{11}$$

where B is a constant. Eq. (1) is satisfied automatically. Eqs. (2), (3), and (9) can be reduced to two uncoupled ordinary differential equations

$$F''' + FF' + (1 - F'^2) = 0 \tag{12}$$

$$\theta'' + Prk_0F\theta' = 0. \tag{13}$$

The boundary conditions are

$$F(0) = F'(0) = F'(\infty) - 1 = 0 \tag{14}$$

$$\theta(0) = \theta(\infty) - 1 = 0. \tag{15}$$

The two-point boundary value problem of Eqs. (12)-(15) is solved numerically with Runge-Kutta method of fourth-order[8].

III. Results and discussion

In present study of non-isothermal stagnation-point flow, the fluid flow problem governing by Eq. (12) is uncoupled with the thermal problem of Eq. (13). Hence change of the values of Pr and N_R will have no effect on the fluid flow velocity. Eqs. (12)-(15) are solved with $\Delta\eta = 0.005$. Three quantities of interest in this study are wall heat flux which is related to dimensionless wall temperature gradient $\theta'(0)$, dimensionless temperature boundary layer thickness η_δ, and dimensionless parameter η_∞. The dimensionless temperature boundary layer thickness η_δ is defined as the distance (η) away from the wall such that $\theta(\eta_\delta) = 0.99$. The dimensionless parameter η_∞ is defined as the value of η at which $\theta'(\eta_\infty) = 10^{-4}$.

Numerical results of variation of $\theta'(0)$, η_δ, and η_∞ with Pr without thermal radiation (i.e., k_0 = 1) are given in Table 1. It is noted that the dimensionless wall temperature gradient

TABLE I
Numerical results of $\theta'(0)$, η_δ, and η_∞ for different values of Prandtl number (Pr) without thermal radiation($k_0 = 1$)

k_0	Pr	$\theta'(0)$	η_δ	η_∞
1	0.5	0.4334	4.1224	6.4111
	1	0.5705	3.0517	4.7726
	5	1.0434	1.5925	2.4942
	10	1.3388	1.2244	1.9180
	50	2.3527	0.6849	1.0750
	100	2.9864	0.5362	0.8456
	1000	6.5293	0.2427	0.3895

TABLE II
Numerical results of $\theta'(0)$, η_δ, and η_∞ for different values of radiation parameter (N_R) with thermal radiation at $Pr = 5$

Pr	N_R	$\theta'(0)$	η_δ	η_∞
5	0.5	0.6431	2.6789	4.1961
	1	0.7633	2.2247	3.4865
	5	0.9572	1.7446	2.7342
	10	0.9970	1.6732	2.6182
	50	1.0335	1.6085	2.5208
	100	1.0384	1.6005	2.5075
	1000	1.0429	1.5952	2.4968

TABLE III
Numerical results of $\theta'(0)$, η_δ, and η_∞ for different values of Prandtl number (Pr) with thermal radiation ($N_R = 5$)

N_R	Pr	$\theta'(0)$	η_δ	η_∞
5	0.5	0.3936	4.5819	7.1047
	1	0.5200	3.3771	5.2736
	5	0.9572	1.7446	2.7342
	10	1.2304	1.3394	2.0980
	50	2.1678	0.7442	1.1684
	100	2.7536	0.5842	0.9176
	1000	6.0286	0.2644	0.4205

$\theta'(0)$ increases with increase of Prandtl number, but both the temperature boundary layer thickness η_δ and the dimensionless parameter η_∞ decrease with increase of Prandtl number Pr. It can be explained by the definition of Prandtl number that Prandtl number is inversely proportional to the molecular thermal diffusivity α. Table 2 gives variation of results with N_R for $0.5 \leq N_R \leq 1000$ at $Pr = 5$. It is observed that values of the dimensionless wall temperature gradient $\theta'(0)$ increases with increasing N_R, but both the temperature boundary layer thickness η_δ and dimensionless parameter η_∞ decrease with N_R. It should be noted that all values of $\theta'(0)$ obtained in Table 2 with the presence of thermal radiation at $Pr = 5$ are less than 1.0434, which is the value of $\theta'(0)$ calculated in Table 1 without thermal radiation at $Pr = 5$. Effect of variation of Pr for $0.5 \leq Pr \leq 1000$ at $N_R = 5$ on the numerical results are shown in Table 3. Again increase of $\theta'(0)$, as well as decrease of both η_δ and η_∞ with increasing Pr are found.

Temperature profiles for three different values of radiation parameter (N_R = 0.5, 1, 10) and three different values of Prandtl number (Pr = 0.5, 5, 10) are given in Figs. 1-3. Results obtained for the case without thermal radiation (i.e., k_0 = 1) are also shown in these figures for comparison. When we compare all these figures, it is found that in Figs. 1 and 2 which are for small value of N_R, or large effect of thermal

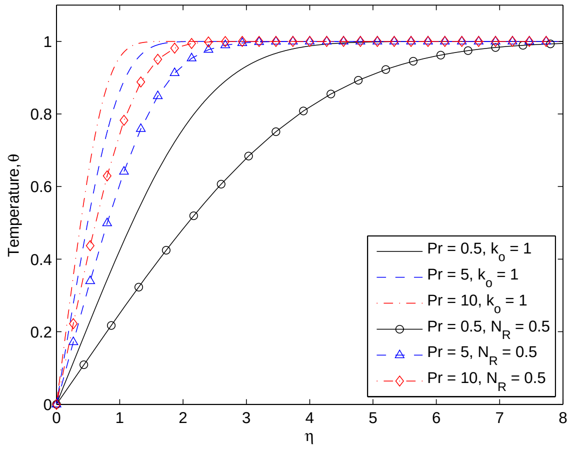

Fig. 1. Dimensionless temperature profiles for three different values of Pr with thermal radiation ($N_R = 0.5$) and without thermal radiatin ($k_0 = 1$).

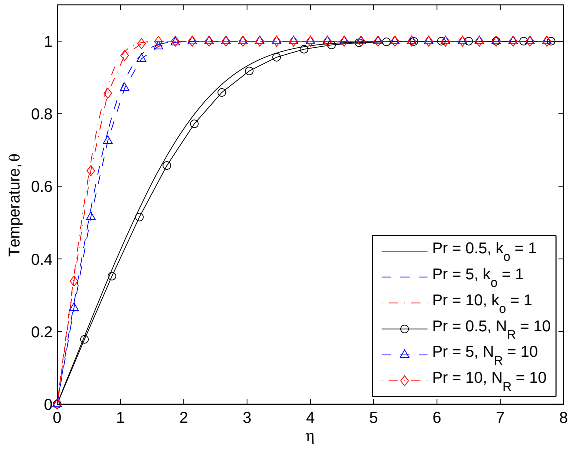

Fig. 3. Dimensionless temperature profiles for three different values of Pr with thermal radiation ($N_R = 10$) and without thermal radiatin ($k_0 = 1$).

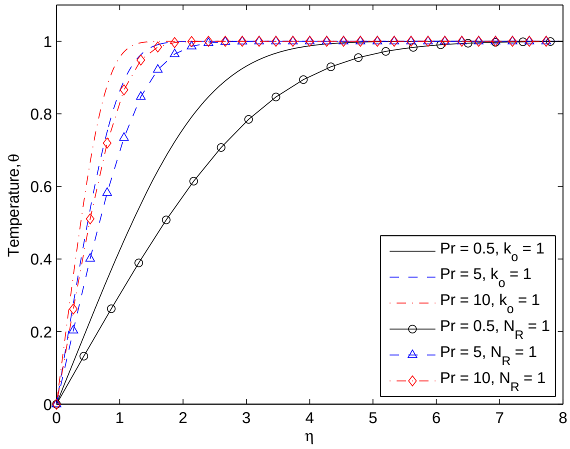

Fig. 2. Dimensionless temperature profiles for three different values of Pr with thermal radiation ($N_R = 1$) and without thermal radiatin ($k_0 = 1$).

radiation, the difference in temperature distribution for the cases with and without thermal radiation is quite large for all three values of Pr. When the value of N_R is large, or the effect of thermal radiation is small, as shown in Fig. 3, it is found that the temperature distribution with or without thermal radiation are close each other for all values of Pr. This is physically consistent because of the fact that k_0 equals to one, or N_R tends to infinity, represents the case without thermal radiation. On the other hand the effect of variation of Pr on the thermal boundary layer is more pronounced for small value of N_R, especially in the case where Pr is less than one. This is consistent with the founding given in Table 3. It is attributed to the fact that decreasing either the Pr or N_R will increase α or α_{eff}, which in turn, will increase the temperature boundary layer thickness.

It is also seen from Figs. 1-3 that the effect of thermal radiation is to increase the temperature boundary layer thickness for a fixed value of Prandtl number. This effect is diminishing when N_R is increasing as expected because a large value of N_R represent a decreasing of thermal radiation effect. All these findings in Figs. 1-3 are consistent with the results shown in Table 2.

IV. Concluding Remark

Numerical calculation of non-isothermal stagnation-point flow with thermal radiation has been investigated by similarity solution of solving the ordinary different equations. This two-point boundary value problem is solved by the Runge-Kutta method and results are presented as both tables and figures. Analyses of the results find that the effect of thermal radiation is to increase the temperature boundary layer thickness and to decrease the wall temperature gradient. But this effect become negligible when radiation parameter N_R is large. On the other hand, increasing the Prandtl number tends to reduce the temperature boundary layer thickness and enlarge the wall temperature gradient for both of the cases with and without thermal radiation.

Acknowledgment

The research is partially supported by Research and Development Administration Office (R&DAO) of University of Macau through the grants RG060/08-09S/SVK/FST & RG079/09-10S/SVK/FST, and Science and Technology Development Fund (FDCT) of Macao SAR through the grant 034/2009/A.

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