A Macroscopic Ray-based Model for Reflective Metasurfaces

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***Abstract*— Reconfigurable Intelligent Surfaces (RISs) have gained significant attention in research studies focused on their technology and/or potential applications. While several modeling approaches have been proposed - ranging from analytical integral formulations to simplified approaches based on scattering matrix theory – there is still a great need for efficient and electromagnetically-consistent macroscopic models that can simulate scattering from RISs, especially for the purpose of large-scale simulations purposes. In the present paper we propose a fully ray-based approach, based on the characterization of the RIS through a "spatial modulation" function that can be easily embedded in efficient, forward ray tracing models. We validate the proposed model by comparison to well established methods available in the literature and show that, although the considered method is completely different and more efficient, results are as accurate if not indistinguishable in a typical benchmark case.**

Keywords— macroscopic modeling, ray model, reconfigurable intelligent surface (RIS), metasurface.

I. INTRODUCTION

In the last years, Reconfigurable Intelligent Surface (RIS) technology has been proposed as a powerful and flexible tool to realize passive, relay-like reflectors for mm-wave and THz applications, to improve channel capacity or to perform basic operations on the signal "at the speed of light", thus limiting the use of digital signal processing devices and power consumption [1].

Several approaches have been proposed in the literature to efficiently model scattering from RISs for design and simulation purposes, while retaining good accuracy and electromagnetic consistency. Some Authors propose hybrid approaches where electromagnetic simulation is used to derive a far-field radar cross section of the RIS to be inserted in ray tracing simulation [2]. Far-field approaches, however, cannot be used to model near-field effects such as focusing, which represents one of the most important RIS applications. Promising *macroscopic modeling* approaches have been proposed that overlooks the RIS microscopic structure in order to directly address the specific wave transformation the RIS realizes [3]-[6]. These approaches assume that the metasurface can be described in terms of an effective surface function - e.g. a surface impedance or a spatial modulation function - that determines such a wave transformation based on Maxwell's equations. In particular, in [6], a realistic macroscopic model for evaluating multi-mode reradiation from generic, finite-size, reflective RISs is introduced.

While in the foregoing paper reradiation is modelled using a Huygens-based formulation, in the present work we build on that macroscopic approach to develop a fully ray-based, efficient formulation for anomalous reradiation, that can be used for large-scale simulation of RIS-based radio network solutions. Differently from [7], we model reradiation with a forward ray tracing approach, therefore avoiding a complex *critical point* search step.

The model is briefly described in the following and then validated by comparison with some reference measurements and models available in the literature.

II. RAY MODEL BASICS

In the following, according to a forward ray-tracing perspective, we assume that the surface of an impedance modulated RIS is divided into small surface elements (or "tiles") and we exploit the locally-plane wave assumption, to define the local incident and (anomalously) reflected/diffracted ray at each tile, in accordance with the classical Geometrical Optics (GO) theory. At the same time, the actual curvature of the incident and reflected wavefronts is taken into account to compute the wave *spreading factor*, that gives the actual attenuation-trend of field with distance.

The final goal is to compute the total field reradiated by the RIS in the far-field and radiative near-field as a set of rays. To this extent, the procedure is composed of the following steps:

i. computation of the anomalous ray direction
ii. computation of the reradiated field at the RIS surface
iii. computation of the spreading factor

If the RIS has multiple reradiation modes (e.g. Floquet's modes of a locally periodic structure), the procedure above must be iterated for each reradiation mode. Moreover, the same procedure is applied also to diffracted rays from the surface edges, following the well-known approach of the Uniform geometrical theory of diffraction (UTD) [8],[9].

Anomalous ray reflection

When a ray impinges on the RIS with propagation direction $\hat{\mathbf{s}}^i$, the field acquires an incidence phase gradient on the tile surface due to the inclination of the locally-plane wavefront of the ray with respect to the tile. Such phase gradient is:

$$\nabla\chi^i = -k_0 \sin\theta_i \, \hat{\mathbf{s}}_\tau^i \tag{1}$$

where θ_i is the incidence angle with respect to the RIS normal, $\hat{\mathbf{s}}_\tau^i$ is a unit vector that defines the orientation of the incidence plane with respect to the surface, and k_0 is the free-space wavenumber.

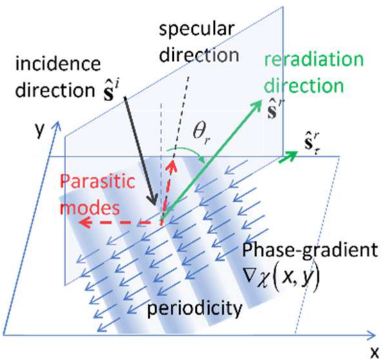

Fig. 1 – Incident and reradiated ray on a point of the RIS surface.

Then, according to a macroscopic approach, the RIS applies the additional phase gradient $\nabla\chi^m$ of the considered reradiation mode so that the *total phase gradient* at the considered tile becomes:

$$\nabla\chi = \nabla\chi^i + \nabla\chi^m \tag{2}$$

Anomalous reflection direction takes place according to total phase gradient (2). In particular, the reflection plane is parallel to the phase gradient direction (see Fig. 1): however, as surface points with a greater phase will reradiate before those with a phase lag, the resulting locally-plane wavefront will have opposite orientation with respect to the total phase gradient $\nabla\chi$. Therefore, the reradiation direction can be easily computed by observing that:

$$\theta_r = \arcsin\left(\frac{|\nabla\chi|}{k_0}\right) \qquad \hat{\mathbf{s}}_\tau^r = -\frac{\nabla\chi}{|\nabla\chi|} \tag{3}$$

with θ_r being the reradiation angle with respect to the surface normal, and $\hat{\mathbf{s}}_\tau^r$ the projection of the reradiation direction on the RIS plane. The reradiation direction $\hat{\mathbf{s}}^r$ is then given by:

$$\hat{\mathbf{s}}^r = \sin\theta_r \hat{\mathbf{s}}_\tau^r + \cos\theta_r \hat{\mathbf{n}} = -\frac{\nabla\chi}{k_0} + \cos\theta_r \hat{\mathbf{n}} \tag{4}$$

with $\hat{\mathbf{n}}$ being the normal unit vector to the RIS surface.

As in [6], [10], the reradiated field can be computed using a proper Spatial Modulation Coefficient (SMC), sometimes called "reflection coefficient". More generally, in order to take into account the polarimetric effect of the RIS we can make use of the *Spatial Modulation Dyadic* (SMD) coefficient:

$$\underline{\mathbf{\Gamma}}(\mathbf{r}') = A^m(\mathbf{r}') e^{j\chi^m(\mathbf{r}')} \cdot \underline{\mathbf{R}}^m \tag{5}$$

where $\mathbf{r}'$ is the position vector of the generic element on the RIS surface, A^m and χ^m are the amplitude and phase modulation of the considered reradiation mode, while the matrix $\underline{\mathbf{R}}^m$ is used to account for the polarization transformation realized by the metasurface.

In addition to the SMD, a proper spreading factor must be also applied to the incident field, in order to compute the reradiated field at a given point along the reflected ray.

The spreading factor for the general case of an astigmatic wave is expressed by [11]:

$$A(s) = \sqrt{\frac{\rho_1\rho_2}{(\rho_1+s)(\rho_2+s)}} \tag{6}$$

where s is a local coordinate along the ray, and ρ_1, ρ_2, are the principal curvature radii of the wavefront at the reference point s=0. The reciprocals of the principal curvature radii (i.e. the principal wave curvatures), are the eigenvalues of the so-called *curvature matrix* $\underline{\mathbf{Q}}$ [12],[13].

The surface impedance modulation of the RIS modifies the local curvature of the incident wave. Then, by following an approach similar to the one adopted in [13], i.e. imposing the phase matching for the incident and reflected wave at the RIS surface, the curvature matrix of the reflected ray can be expressed as a function of the incident curvature matrix $\underline{\mathbf{Q}}^i$ and of the RIS modulation as:

$$\underline{\mathbf{Q}}^r = \left(\underline{\mathbf{1}} - \frac{\hat{\mathbf{n}}\hat{\mathbf{s}}^r}{\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}^r}\right)\cdot\left[\underline{\mathbf{Q}}^i - \frac{1}{k_0}\nabla\nabla\chi^m(\mathbf{r}'_0)\right]\cdot\left(\underline{\mathbf{1}} - \frac{\hat{\mathbf{s}}^r\hat{\mathbf{n}}}{\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}^r}\right) \tag{7}$$

with $\nabla\nabla\chi^m$ being the Hessian matrix of the phase modulation function computed at the reference point $\mathbf{r}'_0$, and $\underline{\mathbf{1}}$ the identity matrix, while the notation $\mathbf{ab} \equiv \mathbf{ab}^T$ stands for the dyadic vector product, which is equivalent in linear algebra to the product of a column vector by a row vector, and $\mathbf{a}\cdot\mathbf{b} \equiv \mathbf{a}^T\mathbf{b}$ is the dot scalar product.

Finally, the (anomalous) reflected field is expressed as:

$$\mathbf{E}^r(\mathbf{r}) = \underline{\mathbf{\Gamma}}(\mathbf{r}')\mathbf{E}^i(\mathbf{r}')\sqrt{\frac{\rho_1^r\rho_2^r}{(\rho_1^r+s)(\rho_2^r+s)}}\, e^{-jk_0 s} \tag{8}$$

with **r'** being the position vector of the considered tile on the RIS surface, **r** the position vector of the observation point, s = |**r-r'**| the local coordinate along the reflected ray, and ρ_1^r, ρ_2^r the reciprocals of the non-zero eigenvalues of the reflection curvature matrix $\underline{\mathbf{Q}}^r$, computed with (7).

A. Anomalous ray diffraction

Beside the GO contributions for the RIS scattered field, edge diffracted ray-fields are also included in the model. This type of contribution is important to smooth out the abrupt field discontinuity predicted by GO when crossing the Reflection Shadow Boundary (RSB) and to predict a nonzero field in the GO shadow region [8],[9].

Since the total phase progression along the RIS edges results from the combination of both the incident wave illumination and the surface impedance modulation, edge diffracted rays are launched toward anomalous directions, similarly to what happens for GO reflected rays.

Namely, according to a *generalized law of diffraction*, the diffracted ray direction $\hat{\mathbf{s}}^d$ must obey to:

$$\cos\beta = \hat{\mathbf{s}}^d \cdot \hat{\mathbf{e}} = \left(\hat{\mathbf{s}}^i - \frac{\nabla\chi^m}{k_0}\right)\cdot\hat{\mathbf{e}} \tag{9}$$

where β is the aperture angle of the *anomalous* Keller's diffraction cone, β' is the incidence angle with respect to the edge and $\hat{\mathbf{e}}$ is the unit vector along the edge, as shown in Figure 2.

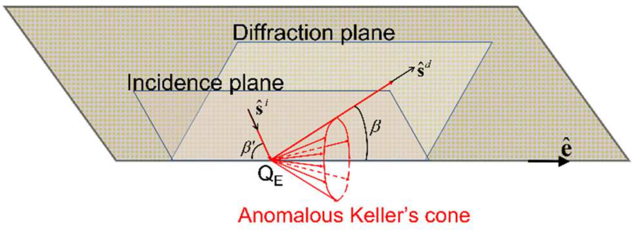

Fig. 2 – Incident and (anomalously) diffracted ray on a RIS edge.

Therefore, one can proceed similarly to the standard UTD case, by recalling that the diffracted wave is astigmatic with one caustic on the edge, and that the diffracted field is computed as:

$$\mathbf{E}^d(s) = \underline{\mathbf{D}}\cdot\mathbf{E}^i(Q_E)\sqrt{\frac{\rho^d}{s(\rho^d+s)}}\,e^{-jk_0 s} \tag{10}$$

In (10), $\underline{\mathbf{D}}$ is the dyadic diffraction coefficient, and ρ^d is the edge-caustic distance, i.e. the distance between the caustic at the edge and the second caustic of the diffracted ray.

In order to extend the UTD theory to the case of a RIS, the diffraction coefficient introduced in [9] for a perfectly conducting wedge is here heuristically modified by multipling it by the spatial modulation dyadic $\underline{\boldsymbol{\Gamma}}$, similarly to the approach adopted in [14],[15] for a non-perfectly conducting surface.

III. Preliminary Results

As a simple benchmark case, we consider a "perfect" anomalous reflector, illuminated with a plane wave at normal incidence. The RIS has size 7x7 m^2, is centered in the origin of an orthogonal reference system *Oxyz*, and lays on the *xy* plane. Furthermore, the RIS is designed for an anomalous reflection angle $\theta_r = 60°$, and a normal incident wave with perpendicular (TE) polarization with respect to the *xz* plane, at the frequency of 3.5 GHz. This can be accomplished by setting the following expressions in the SMD coefficient:

$$\begin{gathered}\chi^m = -k_0(\sin\theta_i - \sin\theta_r)x\\ A^m = \sqrt{\cos\theta_i/\cos\theta_r}\\ \underline{\mathbf{R}}^m = \hat{\mathbf{y}}\hat{\mathbf{y}}\end{gathered} \tag{11}$$

this means that the RIS imposes a constant phase gradient $\nabla\chi^m = -k_0(\sin\theta_i - \sin\theta_r)\hat{\mathbf{x}}$ along the x axis, the wave polarization is perpendicular to the reradiation plane and is not altered by the RIS, while the term $A^m = \sqrt{\cos\theta_i/\cos\theta_r}$ accounts for global power conservation [16].

In order to show the effectiveness of the proposed approach, the reradiated field is computed with the ray model along the Rx segment $x = 10,\ y = 0,\ 0 \le z \le 20$ [m] (crossing the reflection cone of the RIS and with spacing of 0.03 m between receivers), and then compared with the field predicted using the Physical Optics approach.

The PO field is computed through the following radiation integral:

$$\mathbf{E}^s_{PO}(\mathbf{r}) = -\frac{jk_0}{4\pi}\int_S \frac{e^{-jk_0|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}[\eta\hat{\mathbf{r}}\times\mathbf{J}_S(\mathbf{r}')\times\hat{\mathbf{r}} + \mathbf{M}_S(\mathbf{r}')\times\hat{\mathbf{r}}]dS \tag{12}$$

where the equivalent surface currents for an impenetrable metasurface are approximated as [3]:

$$\begin{aligned}\mathbf{J}_S &= \hat{\mathbf{n}}\times(\mathbf{H}^i + \mathbf{H}^r)\\ \mathbf{M}_S &= -\hat{\mathbf{n}}\times(\mathbf{E}^i + \mathbf{E}^r) = -\hat{\mathbf{n}}\times(\mathbf{E}^i + \underline{\boldsymbol{\Gamma}}\cdot\mathbf{E}^i)\end{aligned} \tag{13}$$

with

$$\mathbf{H}^{i,r} = \frac{1}{\eta}\hat{\mathbf{s}}^{i,r}\times\mathbf{E}^{i,r}$$

Figure 3 shows the comparison of the ray model (blue dashed line) with the PO model (red line) along the Rx route. It is evident that the 2 curves are nearly coincident, thus confirming the validity of the adopted approach.

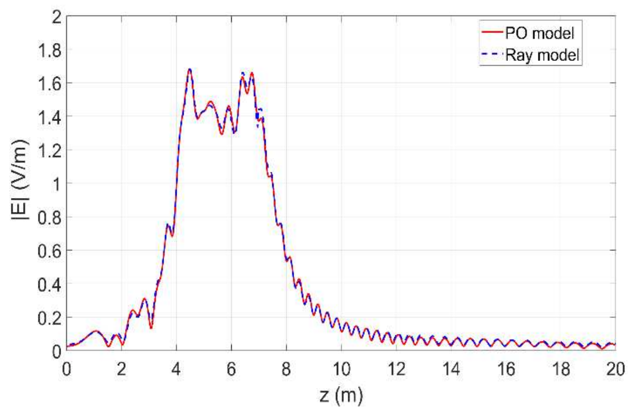

Fig. 3 – Comparison of the ray model with the PO model along a Rx route crossing the reflection cone of the RIS.

IV. Conclusions

Based on the characterization of the RIS through a surface impedance (or "spatial modulation") function and a few parameters, in the present paper we propose a fully ray-based approach for the computation of the field reradiated from a finite-size, reflective RIS that can be easily embedded in efficient, forward ray tracing models. The model is based on the computation of the anomalous direction of the reflected or diffracted ray based on the phase gradient of the spatial modulation function, and on the computation of its spreading factor using the curvature matrix of the local wavefront.

Preliminary results show that the proposed ray model, besides being intrinsically more efficient in terms of computation time, is as accurate as other popular models based on the Physical Optics approach.

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