New Feed-Forward/Feedback Generalized Minimum Variance Self-tuning Pole-placement Controller

S. A. Mohamed, A. S. Zayed and O. A. Abolaeha

Abstract—A new Feed-Forward/Feedback Generalized Minimum Variance Pole-placement Controller to incorporate the robustness of classical pole-placement into the flexibility of generalized minimum variance self-tuning controller for Single-Input Single-Output (SISO) has been proposed in this paper. The design, which provides the user with an adaptive mechanism, which ensures that the closed loop poles are, located at their pre-specified positions. In addition, the controller design which has a feed-forward/feedback structure overcomes the certain limitations existing in similar pole-placement control designs whilst retaining the simplicity of adaptation mechanisms used in other designs. It tracks set-point changes with the desired speed of response, penalizes excessive control action, and can be applied to non-minimum phase systems. Besides, at steady state, the controller has the ability to regulate the constant load disturbance to zero. Example simulation results using both simulated and real plant models demonstrate the effectiveness of the proposed controller.

Keywords—Pole-placement, Minimum variance control, self-tuning control and feedforward control.

I. INTRODUCTION

THE generalized minimum variance controller [8] was developed by Clarck and Gawthrop from Astrom and Wittenmark's minimum variance controller.

It has several useful properties. First, in its performance criteria, the control signal is weighted. Consequently, excessive actuator movements are avoided and at the same time the control of a certain class of non-minimum phase processes is made possible.

Secondly through user-chosen transfer functions $P(z^{-1})$ and $Q(z^{-1})$, the controller can acquire different characteristics such as model following, detuned model following and optimal Smith prediction. The controller can thus be tailored to solve different control problems. To choose the appropriate

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$P(z^{-1})$ and $Q(z^{-1})$, a trail and error procedure may be used. Alternatively, by assigning the closed-loop poles to pre-specified locations, Allidina and Hughes [10] have proposed a pole-placement procedure to determine the polynomials $P(z^{-1})$ and $Q(z^{-1})$. However, the design of [10] has considerable limitation, that are the controller has not the ability to eliminate steady state error to zero if the system to be controlled is subjected to constant load disturbances. This drawback can be overcome by introducing integral action into the design. However, this method may complicate solving Diophantine equation [1]. In this paper a new generalized minimum variance controller which is combined with both pole-placement and feed-forward control designs is proposed in order to overcome all limitations of other designs. The idea behind of this design is based on the ability of feedforward controller to eliminate the effect of load disturbance.

II. FEED-FORWARD CONTROL

Combined feed-forward plus feedback control can significantly improve the performance over simple feedback control whenever there is a major disturbance that can be measured before it affects the process output. In the most ideal situation, feed-forward control can entirely eliminate the effect of the measured disturbance on the process output. Feed-forward/feedback control is always used along with feedback control because a feedback control system is required to track set point changes and minimizes unmeasured disturbance that always present in any real process [4]. Simplified block diagram of the feed-forward/feedback control is shown in the Figure (1).
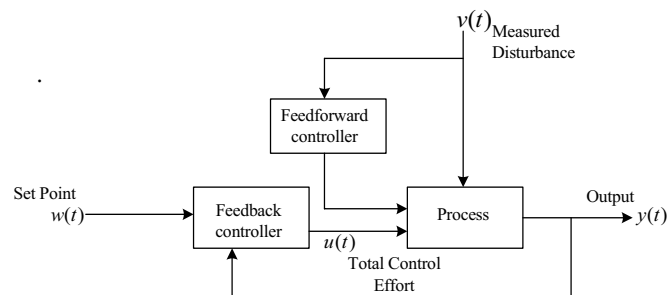

Fig. 1 Simplified block diagram of feed-forward/feedback control.

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III. DERIVATION OF CONTROL LAW

To derive the control law of Feed-forward/Feedback Generalized Minimum Variance Pole-Placement Controller (FF/FB-GMVPPC), it is assumed that the plant can be described by the following model:

$$A(z^{-1})y(t) = B(z^{-1})u(t-k) + C(z^{-1})\xi(t) + D(z^{-1})v(t-k) \qquad (1)$$

where $y(t)$, $u(t)$ and $\xi(t)$ are respectively the measured output, the control input and an uncorrelated sequence of random variables with zero mean at the instant $t = 1, 2, \ldots$, and k is the time delay of the process. The term $v(t)$ in the above equation (1) represent the measured disturbance. The resulting model is a combination of CARMA model plus measured disturbance part. The polynomials $A(z^{-1})$, $B(z^{-1})$, $C(z^{-1})$ and $D(z^{-1})$ are respectively of orders n_a, n_b, n_c and n_d are expressed in terms of the backward shift operators, z^{-1} as:

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots, \ldots + a_{n_a} z^{-n_a} \qquad (2)$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots, \ldots + b_{n_b} z^{-n_b} \qquad (3)$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2} + \ldots, \ldots + c_{n_c} z^{-n_c} \qquad (4)$$

$$D(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2} + \ldots, \ldots + d_{n_d} z^{-n_d} \qquad (5)$$

The main objective of the new Feed-forward/Feedback Generalized Minimum Variance Pole-Placement Controller is to minimize the following cost function:

$$J_N = \mathrm{E}[\phi_2{}^2(t+k)] \qquad (6)$$

Where

$$\phi_2(t+k) = P(z^{-1})y(t+k) + Q(z^{-1})u(t) - R(z^{-1})w(t) - H_v(z^{-1})v(t) \qquad (7)$$

Where $w(t)$ is the set point and $P(z^{-1}), Q(z^{-1}), R(z^{-1})$ and $H_v(z^{-1})$ are user-defined transfer function in the backward shift operator z^{-1} and $\mathrm{E}\{.\}$ is the expectation operator.

Now, we can introduce the following identity:

$$C(z^{-1})P(z^{-1}) = A(z^{-1})E(z^{-1}) + z^{-k}F(z^{-1}) \qquad (8)$$

where $E(z^{-1}), F(z^{-1}), Q(z^{-1})$ and $P(z^{-1})$ are polynomials in z^{-1} that may be expressed as:

$$F(z^{-1}) = f_0 + f_1 z^{-1} + f_2 z^{-2} + \ldots, \ldots + f_{n_f} z^{-n_f} \qquad (9)$$

$$E(z^{-1}) = 1 + e_1 z^{-1} + e_2 z^{-2} + \ldots, \ldots + e_{n_c} z^{-n_e} \qquad (10)$$

$$P(z^{-1}) = 1 + p_1 z^{-1} + p_2 z^{-2} + \ldots, \ldots + p_{np} z^{-np} \qquad (11)$$

$$Q(z^{-1}) = q_0 + q_1 z^{-1} + q_2 z^{-2} + \ldots, \ldots + q_{nq} z^{-nq} \qquad (12)$$

The order of the polynomials $E(z^{-1}), F(z^{-1}), Q(z^{-1})$ and $P(z^{-1})$ in equations (9)-(12) are specified as follows [2,6]:

$$\left. \begin{array}{l} n_f = n_a - 1 \\ n_e = k - 1 \\ n_p = n_a \\ n_Q = n_b \end{array} \right\} \qquad (13)$$

The controller polynomials are calculated according to equation (8). By using equations (1), (7) and (8) we get:

$$Py(t+k) = \frac{F}{C}y(t) + \frac{BE}{C}u(t) + E\xi(t+k) + \frac{DE}{C}v(t) \qquad (14)$$

Making use of equations (7) and (14), we get the following predictive model:

$$\phi_2(t+k) = \frac{F}{C}y(t) + [\frac{BE}{C} + Q]u(t) - Rw(t) + E\xi(t+k) + (\frac{DE}{C} - H_v)v(t) \qquad (15)$$

we define the $\phi_2{}^*(t+k|t)$ as the optimum prediction of $\phi_2(t+k)$ based on the measurement up to and including time (t) and $\widetilde{\phi}_2(t+k)$ as the prediction error as follows:

$$\phi_2{}^*(t+y|t) = \frac{F}{C}y(t) + [\frac{EB}{C} + Q]u(t) - Rw(t) + (\frac{DE}{C} - H_v)v(t) \qquad (16)$$

$$\widetilde{\phi}_2(t+k) = E\xi(t+k) \qquad (17)$$

The minimum variance of $\phi_2(t+k)$ is obtained by selecting the control input $u(t)$ such that $\phi_2{}^*(t+k|t)$ in equation (16) is set to zero. If we set $\phi_2{}^*(t+k|t) = 0$ in equation (16), then the generalized minimum variance control law is obtained as:

$$u(t) = \frac{[CRw(t) - Fy(t) + (H_v C - DE)v(t)]}{[BE + CQ]} \qquad (18)$$

Combining equations (18), (8) and (1), the closed loop system is obtained as:

$$(PBC + QAC)y(t+k) = BRCw(t) + (BCH_v + CDQ)v(t) + C(BE + CQ)\xi(t+k) \qquad (19)$$

Making use of equations (8) and (19) and rearranging, we obtain:

$$y(t) = \frac{z^{-k}RB}{PB + QA}w(t) + \frac{BE + CQ}{PB + QA}\xi(t) + \frac{z^{-k}((BE + CQ)D + H'_v B)}{PB + QA}v(t) \qquad (20)$$

The pole-placement control can be achieved if the closed loop poles defined by the zeros of a chosen polynomial are used to fix P and Q through the identity:

$$PB + AQ = T \qquad (21)$$

where T is a desired closed loop poles which can be expressed as:

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$$T = 1 + t_1 z^{-1} + \ldots , \ldots + t_{n_t} z^{-n_t} \qquad (22)$$

The order of the polynomials T can be selected as [1,5]:

$$n_t \leq n_a + n_b + k - n_c - 1 \qquad (23)$$

By using equations (8) and (21), we get:

$$y(t) = \frac{z^{-k} RB}{T} w(t) + \frac{(BE + CQ)}{T} \xi(t) \qquad (24)$$

Where $\qquad H'_v = H_v C - DE \qquad (25)$

From equations (18) and (19), we can clearly note that:

1) In order to obtain a zero steady state tracking error, equation (19) and (20) gives the following condition:

$$\left. \frac{z^{-k} RB}{BP + QA} \right|_{z=1} = 1 \qquad (26)$$

This can be achieved by setting R as:

$$R = \frac{T(1)}{B(1)} \qquad (27)$$

2) To eliminate the effect of the measured disturbance in the steady state, the user transfer H_v may be chosen as:

$$H_v = -\frac{D(1)Q(1)}{B(1)} \qquad (28)$$

The proposed pole-placement algorithm can be summarized as:

Step 1. Select the desired closed-loop system poles polynomial $T(z^{-1})$.

Step 2. Read the new values of $y(t)$ and $u(t)$.

Step 3. Estimate the process parameters $\hat{A}, \hat{B}, \hat{C}$ and $\hat{D}$ using the least square algorithm

Step 4. Compute $\hat{F}, \hat{P}, \hat{E}$ and $\hat{Q}$ using equations (8) and (21)

Step 5. Set $R = \dfrac{T(1)}{\hat{B}(1)}$ and $H_v = -\dfrac{\hat{D}(1)\hat{Q}(1)}{\hat{B}(1)}$.

Step 6. Apply the control input using equation (18).
Steps 2 to 6 are repeated for every sampling instant.

IV. SIMULATION RESULTS

The objective of this section is to study the ability of the proposed placement controller in controlling a process under set point changes. Two simulation examples will be carried out in order to observe the ability of the proposed algorithm to locate the closed loop poles at the pre-specified locations. The simulation study also includes an investigation of the influence of the constant load disturbances and random disturbances on the systems. Both simulation examples were performed over 600 samples with set point change every 100 sampling instants.

A. Case Study 1: Agitated Heating Tank

The algorithm is tested agitated heating tank system treated previously by Yusof et al. [9] and Zayed et al. [1,2] and described by the following transfer function:

$$(1 + a_1 z^{-1}) y(t) = b_0 u(t-1) + \xi(t)$$

where $a_1 = -0.411$, $b_0 = 0.492$ and the sampled time $50^\circ C$ sec. The simulations were performed over 600 samples (300 minutes) under set point changes from $25^\circ C$ to $50^\circ C$ and from $50^\circ C$ to $25^\circ C$ every 100 sampling instants.

In this example step load disturbance of value 6 was added to the output of closed loop system between 450th and 600th sampling instant. The desired closed loop poles polynomial T was chosen as: $T = 1 - 0.5 z^{-1}$. The output and control input are shown in the Figures (2a) and (2b), respectively.
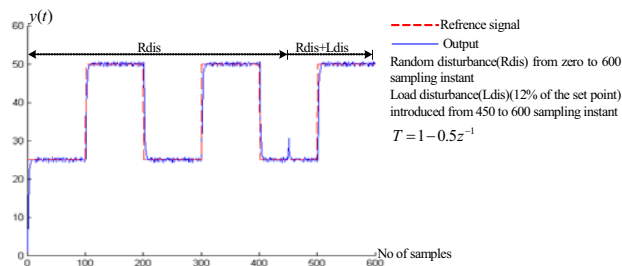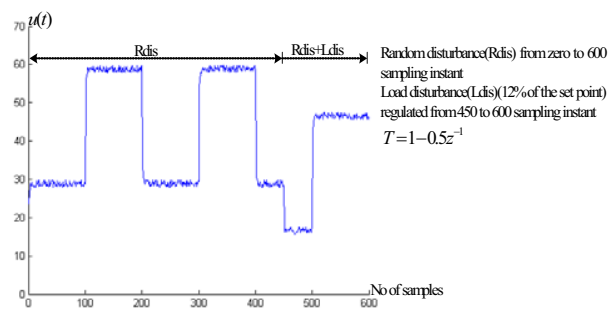

Fig. 2a The output

Fig. 2b The control input

It can clearly be seen from Figure (2a) and (2b) that no excessive control input , transient response is shaped by the choice of the polynomial T and at steady state the controller has the ability to effectively reject the constant load disturbance to zero.

B. Case Study 2: Non-minimum phase System

In the second example, the new proposed controller in section III is also applied to the non-minimum phase double water tank system. The process can be described by the following transfer function [1,7] as:

$$G(z) = \frac{-0.3178 z^{-1} + 1.0375 z^{-2}}{1 - 1.1514 z^{-1} + 0.3248 z^{-2}}$$

where the sample time $= 50 \sec$.

The simulations were performed over 600 samples (300 minutes) under set point changes from 25 to 50 and from 50 to 25 every 100 sampling instants.

The desired closed loop poles polynomial T was chosen

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as: $T = 1 - 1.3z^{-1} + 0.42z^{-2}$. Artificial load disturbance of value 5 was added to the output of closed loop system from 450th to 600th sampling time instant. The output and the control input are shown in the Figures (3a) and (3b), respectively.
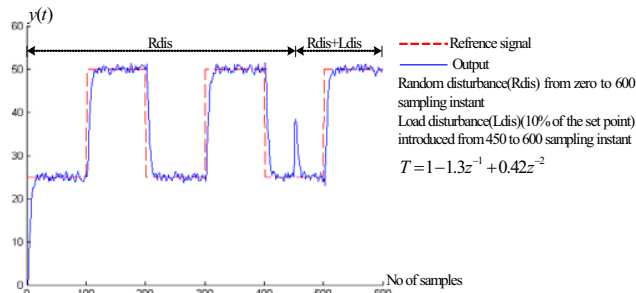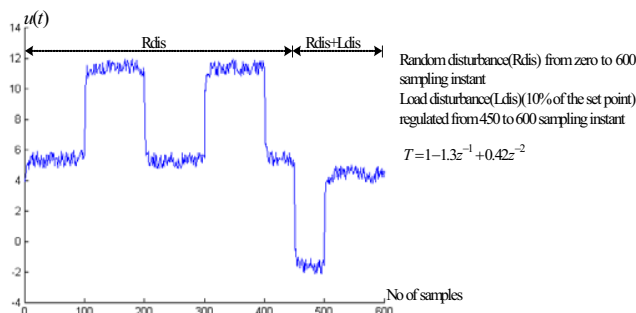

Fig. 3a The output

Fig. 3b The control input

It is clear from the Figure (3a) and (3b) that at steady state the proposed pole-placement controller has the ability to regulate constant load disturbances to zero without producing the excessive control input.

V. CONCLUSION

In this paper generalized minimum variance pole-placement control has been extended to a new feed-forward/feedback generalized minimum variance pole-placement control. The resulting self-tuning controller provides an adaptive mechanism, which ensures that the closed loop poles are located at their pre-specified positions. The design was successfully tested on both simulated and real plant models. The results presented here indicate that the controller tracks set point changes with the desired speed of response, penalizes the excessive control action and can deal with non-minimum phase systems. In addition, the controller has the ability to ensure zero steady state error if the system is subjected to constant load disturbances.

ACKNOWLEDGMENT

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