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Existence and globally exponential stability of equilibrium for BAM neural networks with mixed delays and impulses

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Abstract—In this paper, a class of generalized bi-directional associative memory (BAM) neural networks with mixed delays is investigated. On the basis of Lyapunov stability theory and contraction mapping theorem, some new sufficient conditions are established for the existence and uniqueness and globally exponential stability of equilibrium, which generalize and improve the previously known results. One example is given to show the feasibility and effectiveness of our results.

Keywords—Bi-directional associative memory (BAM) neural networks, mixed delays, Lyapunov stability theory, contraction mapping theorem, existence, equilibrium, globally exponential stability.

I. Introduction

RECENTLY, a class of two-layer heteroassociative networks called bi-directional associative memory (BAM) networks [1-4] with or without axonal signal transmission delays has been proposed and used in many fields, such as pattern recognition and automatic control. They were first proposed by Kosko [1-3], Cao [5-8,16-17,19-20], Liao and Yu [10], Gopalsamy and He [11], Liang [12], Wei and Ruan [13], Jin [14] and Xia and Cao [15]. Though the non-impulsive systems have been well studied in theory and in practice (for example see [1-20] and references cited therein), the theory of impulsive differential equations is now being recognized to be not only richer than the corresponding theory of differential equations without impulse, but also represents a more natural framework for mathematical modeling of many real-world phenomena, such as population dynamic and the neural networks. In recent years, the impulsive differential equations have been extensively studied (see the monographs and the works [21-25]). This class of neural networks has been showed to be a useful network model for applications in pattern recognition, solving optimization problems and automatic control engineering. Hence, they have been the object of intensive analysis by numerous authors in recent years [16-17,19-20,24-25]. In particular, there are extensive results on the problem of the stability and other dynamical behaviors of impulsive. In the present paper, we investigate the following more general BAM neural networks with impulses

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$$\begin{cases} x'_i(t) = -a_i x_i(t) + \sum_{j=1}^{p} c_{ji} f_j(y_j(t-\sigma_{ji})) \\ \quad + \sum_{j=1}^{p} \int_0^{\tau} p_{ji} f_j(y_j(t-s))ds + r_i, \quad t \geq 0, t \neq t_k, \\ \Delta x_i(t) = (\alpha_{ik} - 1)x_i(t), \\ \quad i = 1,2,\cdots,n, k = 1,2,\cdots, t = t_k, \\ y'_j(t) = -b_j y_j(t) + \sum_{i=1}^{n} d_{ij} g_i(x_i(t-\delta_{ij}) \\ \quad + \sum_{i=1}^{n} \int_0^{\tau} q_{ij} g_j(x_i(t-s))ds + s_j, \quad t \geq 0, t \neq t_k, \\ \Delta y_j(t) = (\beta_{jk} - 1)y_j(t), \\ \quad j = 1,2,\cdots,p, k = 1,2,\cdots, t = t_k. \end{cases} \tag{1}$$

where $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$ and $\Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k^-)$ are the impulses at moments t_k then there are $x_i(t_k^-) = x_i(t_k)$ and $y_j(t_k^-) = y_j(t_k)$ $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$. n and p correspond to the number of neurons in X-layer and Y-layer. x_i and y_j are the activations of the ith neurons and the jth neurons, respectively. $c_{ji}, d_{ij}, p_{ji}, q_{ij}$ are the connection weight. $\sigma_{ji} > 0, \delta_{ij} > 0$ are the transmission delay, τ is distributed time-varying delay r_i and s_j denote the external inputs $a_i > 0$ and $b_j > 0$ represent the rate with which the ith neuron and jth neuron will rest its potential to the resting state in isolation when disconnected from the network and external inputs, respectively. Then system (1) is supplemented with initial values given by $x_i(t) = \varphi_i(t)$, $t \in [-\sigma, 0]$, $\sigma = \max\{\max_{\substack{1\leq i\leq n \\ 1\leq j\leq p}} \sigma_{ji}, \tau\}$, $i = 1,2,\cdots,n$. $y_j(t) = \psi_j(t)$, $t \in [-\delta, 0]$, , $\delta = \max\{\max_{\substack{1\leq i\leq n \\ 1\leq j\leq p}} \delta_{ij}, \tau\}, j = 1,2,\cdots,p$ where $\varphi_i(t)$ and $\psi_i(t)$ denote real-valued continuous functions defined on $[-\sigma, 0]$ and $[-\delta, 0]$. $z(t) = (x_1(t),\cdots,x_n(t),y_1(t),\cdots,y_p(t))^T \in R^n$ in which $x_i(\cdot),\cdots,y_j(\cdot)$ are piecewise continuous on $(0,\beta)$ for some $\beta > 0$ such that $z(t_k^+)$ and $z(t_k^+)$ exist and $z(\cdot)$ is differentiable on intervals of the form $(t_{k-1}, t_k) \subset (0,\beta)$ and satisfies(1); We assume that $z(t)$ is left continuous with $z(t_k^-) = z(t_k)$, and $\alpha_{ik}, \beta_{jk}, r_i$ s_j are real numbers. we assume that:

(H1) $a_i, b_j \in (0,\infty)$, $c_{ji}, d_{ij}, p_{ji}, q_{ij}, r_i, s_j \in R, \sigma_{ji}, \delta_{ij} \in (0,\infty), i = 1,\cdots,n, j = 1,\cdots.p$.

(H2) f_j and g_i are Lipschitz-continuous on R with Lipschitz constant $L_j^f, (j = 1, \cdots, p)$ and $L_i^g (i = 1, \cdots, n)$, that is,

$$|f_j(x) - f_j(y)| \le L_j^f |x - y|, \forall x, y \in R.$$
$$|g_i(x) - g_i(y)| \le L_i^g |x - y|, \forall x, y \in R.$$

The aim of this paper is to derive some criteria for the existence and globally exponential stability of a unique equilibrium of system (1). The rest of this paper is organized as follows. In Section II, we shall establish some sufficient conditions for the existence and uniqueness of equilibrium. In Section III, we will investigate the globally exponential stability of the unique equilibrium. Finally, one example is presented to illustrate that our results are feasible and more general.

II. Existence and uniqueness of equilibrium

When neural networks are used for the solution of optimization problems, one of the fundamental issues in the design of a network is concerned with the existence and uniqueness and globally exponentially stable equilibrium state of network without requiring the bounded ness, differentiability or monotonicity, we establish a easily verifiable sufficient conditions for the existence of a unique equilibrium state in this section. An equilibrium solution of (1) is a constant vector

$$z^* = (x_1^*, \cdots, x_n^*, y_1^*, \cdots, y_p^*)^T \in R^{n+p},$$

which satisfies the system

$$\begin{cases} a_i x_i^* = \sum_{j=1}^{p} c_{ji} f_j(y_j^*) + \tau \sum_{j=1}^{p} p_{ji} f_j(y_j^*) + r_i, i = 1, \cdots, n \\ b_j y_j^* = \sum_{i=1}^{n} d_{ij} g_i(x_i^*) + \tau \sum_{i=1}^{n} q_{ij} g_i(x_j^*) + s_j, j = 1, \cdots, p. \end{cases} \quad (2)$$

Or

$$\begin{cases} x_i^* = \sum_{j=1}^{p} \frac{1}{a_i}(c_{ji} + \tau p_{ji}) f_j(y_j{}^*) + \frac{r_i}{a_i}, i = 1, 2, \cdots, n \\ y_j^* = \sum_{i=1}^{n} \frac{1}{b_j}(d_{ij} + \tau q_{ij}) g_i(x_i{}^*) + \frac{s_j}{b_i}, j = 1, 2, \cdots, p. \end{cases} \quad (3)$$

when the impulsive jumps as assumed to satisfy

$$\alpha_{ik}(x_i{}^*) = \beta_{jk}(y_j^*) = 0.$$

We denote the spectral radius of the matrix F by $\rho(F)$.

Lemma 1. ([15]). Let N be a positive integer and B be an Banach space. If the mapping $\phi^N : B \to B$ is a contraction mapping, then $\phi : B \to B$ has unique fixed point in B, where $\phi^N = \phi(\phi^{N-1})$.

Theorem 1. In addition to (H1)-(H2), assume further that $\rho(K) < 1$, where $K = (K_{ij})_{(n+p)\times(n+p)}$, $M_{ji} = \frac{1}{a_i}(|c_{ji}| + |\tau p_{ji}|)$, and $N_{ij} = \frac{1}{b_j}(|d_{ij}| + |\tau q_{ij}|)$,

$$k_{ij} = \begin{cases} M_{j-n,i} L_{j-n}^f, & 1 \le i \le n, n+1 \le j \le n+p, \\ N_{j,i-p} L_i^g, & p+1 \le i \le n+p, 1 \le j \le p, \\ 0, & 1 \le i, j \le n, n+1 \le i, j \le n+p. \end{cases}$$

Then, there exists unique equilibrium of system (1).

Proof: In order to show that (3) has a unique solution. Now consider a mapping: $\phi : \phi(z) = (\phi_1(z), \cdots, \phi_n(z), \phi_{n+1}(z), \cdots, \phi_{n+p}(z))^T, R^{n+p} \to R^{n+p}$

$$\phi(z) = \phi(x_1, x_2, \cdots, x_n, y_1, y_2, \cdots, y_p)^T,$$
$$z = (x_1, x_2, \cdots, x_n, y_1, y_2, \cdots, y_p)^T$$
$$\overline{z} = (\overline{x}_1, \overline{x}_2, \cdots, \overline{x}_n, \overline{y}_1, \overline{y}_2, \cdots, \overline{y}_p)^T. \quad (4)$$

We have

$$\begin{aligned} |\phi(z) - \phi(\overline{z})| &= (|(\phi(z) - \phi(\overline{z}))_1|, \cdots, |(\phi(z) - \phi(\overline{z}))_n|, \\ &\quad |(\phi(z) - \phi(\overline{z}))_{n+1}|, \cdots, |(\phi(z) - \phi(\overline{z}))_{n+p}|)^T \\ &\le [\sum_{j=1}^{p} (M_{j1} |f_j(y_j) - f_j(\overline{y}_j)|), \\ &\quad \cdots, \sum_{j=1}^{p} (M_{jn} |f_j(y_j) - f_j(\overline{y}_j)|), \\ &\quad \cdots, \sum_{i=1}^{n} (N_{i1} |g_i(x_i) - g_i(\overline{x}_i)|), \\ &\quad \cdots, \sum_{i=1}^{n} (N_{ip} |g_i(x_i) - g_i(\overline{x}_i)| \ )]^T \\ &\le [\sum_{j=1}^{p} (M_{j1} L_j^f |y_j - \overline{y}_j|), \\ &\quad \cdots, \sum_{j=1}^{p} (M_{jn} L_j^f |y_j - \overline{y}_j|) \cdots, \sum_{i=1}^{n} (N_{i1} L_i^g |x_i - \overline{x}_i|), \\ &\quad \cdots, \sum_{i=1}^{n} (N_{ip} L_i^g |x_i - \overline{x}_i|)]^T \\ &= \begin{pmatrix} 0, & \cdots, & 0, & M_{11}L_1^f, & \cdots, & M_{1p}L_p^f \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0, & \cdots, & 0, & M_{n1}L_1^f & \cdots & M_{np}L_p^f \\ N_{11}L_1^g & \cdots & N_{1n}L_n^g & 0, & \cdots, & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ N_{p1}L_1^g & & N_{pn}L_n^g & 0, & \cdots, & 0 \end{pmatrix} \\ &\quad \times \begin{pmatrix} |x_1 - \overline{x}_1| \\ \vdots \\ |x_n - \overline{x}_n| \\ |y_1 - \overline{y}_1| \\ \vdots \\ |y_p - \overline{y}_p| \end{pmatrix} \\ &= K(|x_1 - \overline{x}_1|, \cdots, |x_n - \overline{x}_n|, \\ &\quad \cdots |y_1 - \overline{y}_1|, \cdots, |y_p - \overline{y}_p|)^T \\ &= K(|(z - \overline{z})_1|, \cdots, |(z - \overline{z})_n|, \\ &\quad \cdots |(z - \overline{z})_{n+1}|, \cdots, |(z - \overline{z})_{n+p}|)^T. \end{aligned} \quad (5)$$

Let m be a positive integer. Then from (5), we get

$$\begin{aligned} &(|(\phi^m(z) - \phi^m(\overline{z}))_1|, \cdots, |(\phi^m(z) - \phi^m(\overline{z}))_n|, \\ &|(\phi^m(z) - \phi^m(\overline{z}))_{n+1}|, \cdots, |(\phi^m(z) - \phi^m(\overline{z}))_{n+p}|)^T \\ &\le K |(\phi^{m-1}(z) - \phi^{m-1}(\overline{z}))_1|, \\ &\cdots, |(\phi^{m-1}(z) - \phi^{m-1}(\overline{z}))_n|, |(\phi^{m-1}(z) - \phi^{m-1}(\overline{z}))_{n+1}|, \\ &\cdots, |(\phi^{m-1}(z) - \phi^{m-1}(\overline{z}))_{n+p}|)^T \\ &\le K^m(|(z - \overline{z})_1|, \cdots, |(z - \overline{z})_n|, \\ &\cdots |(z - \overline{z})_{n+1}|, \cdots, |(z - \overline{z})_{n+p}|)^T. \end{aligned} \quad (6)$$

From the assumption $\rho(K) < 1$, we obtain

$$\lim_{m\to+\infty} K^m = 0,$$

which implies that there exists a positive integer N and positive constant $\eta < 1$ such that

$$K^N = (h_{kl})_{(n+p)\times(n+p)} \quad and \quad \sum_{l=1}^{n+p} h_{kl} \le \eta, k = 1, \cdots, n+p. \quad (7)$$

In view of (6) and (7), we have

$$\left|(\phi^N(z) - \phi^N(\overline{z}))_k\right| \le \sum_{l=1}^{n+p} h_{kl} \max_{1\le l\le n+p} |z_l - \overline{z}_l| \le \eta \|z - \overline{z}\| \quad (8)$$

for all $k = 1, 2, \cdots, n+p$. Therefore, it follows from (8) that

$$\left|(\phi^N(z) - \phi^N(\overline{z}))\right| \le \eta \|z - \overline{z}\|_B. \quad (9)$$

This implies that the mapping

$$\phi^N : R^{n+p} \to R^{n+p}$$

is a contraction mapping. By Lemma 1, ϕ has unique fixed point $z^* \in R^{n+p}$ such that $\phi(z^*) = z^*$. Thus, system (1) has unique equilibrium. This completes the proof of Theorem 1.

III. Exponential stability of equilibrium

In this section, we will show that the conditions in Theorem 2 also guarantee globally exponentially stability the unique equilibrium of the impulsive system (1). For convenience, we introduce some definitions.

Definition 1. Let functions

$$\varphi(s) = (\varphi_1(s), \cdots, \varphi_n(s)), s \in [-\sigma, 0], R^{n+p} \to R^{n+p}$$

and

$$\psi(s) = (\psi_1(s), \cdots, \psi_n(s)), s \in [-\delta, 0], R^{n+p} \to R^{n+p}$$

The function $\phi(s) = (\varphi(s), \psi(s))$ is said to be a continuous function if the following two conditions are satisfied:

(a) ϕ is piecewise continuous with first kind discontinuity at the points $\{t_k\}$,and ϕ is left-continuous at each discontinuity points.

(b)

$$\varphi_i(t^+) = \alpha_{ik}\varphi_i(t), t \in t_k \cap [-\sigma, 0],$$

$$\psi_j(t^+) = \beta_{jk}\psi_j(t), \ \ t \in t_k \cap [-\delta, 0],$$

$$i = 1, 2, \cdots, n, k = 1, 2, \cdots$$

We define the norms $\|(z(t)\|$ as

$$\|(z(t)\| = \sum_{i=1}^{n} |x_i(t)| + \sum_{j=1}^{p} |y_j(t)|,$$

and the norm $\|\phi\|$ by

$$\|\phi\| = \sup\left\{\sum_{i=1}^{n} |\varphi_i(s)| + \sum_{j=1}^{p} |\psi_j(s)|\right\}.$$

Definition 2. The unique equilibrium $z^* = (x_1^*, \cdots, x_n^*, y_1^*, \cdots, y_p^*)^T$ of (1) is said to be globally exponentially stable, if there exist constant $\varepsilon > 0$ and $M(\varepsilon) > 0$ such that

$$\|z - z^*\| \le M(\varepsilon)e^{-\varepsilon t} \|\phi - z^*\|, \ \ \forall \ \ t \ge 0.$$

Definition 3. The Dini right upper derivative of a continuous function, which is defined by

$$D^+f(t) = \lim_{h\to 0^+} \sup_{0\le\Delta t\le h} \left\{\frac{f(t+\Delta t) - f(t)}{\Delta t}\right\},$$

and

$$D^+|f(t)| = \begin{cases} f'(t), & if\ f(t) > 0\ and\, f'(t) > 0 \\ -f'(t), & if\ f(t) < 0\ and\, f'(t) < 0. \\ 0 & if\ f(t) = 0\ and\, f'(t) = 0 \end{cases}$$

Theorem 2. Under assumptions (H1) and (H2), system (1) is globally exponentially stable if the following conditions are satisfied:

(1) $|\alpha_{ik}| \le 1$, $|\beta_{jk}| \le 1, i = 1, 2, \cdots, n, j = 1, 2, \cdots, p, k = 1, 2, \cdots$.

(2) There exist positive constants λ_i , μ_j such that

$$\begin{cases} \sum\limits_{j=1}^{p} \mu_j L_i^g(d_{ij} + q_{ij}\tau) - \lambda_i a_i < 0, \ j = 1, 2, \cdots, p. \\ \sum\limits_{i=1}^{n} \lambda_i L_j^f(c_{ji} + p_{ji}\tau) - \mu_j b_j < 0, \ i = 1, 2, \cdots, n. \end{cases}$$

Proof. From condition (2) of Theorem 2 for continuation, there is a small positive constant ε such that

$$\begin{cases} \sum\limits_{j=1}^{p} \mu_j L_i^g(d_{ij}e^{\varepsilon\delta_{ij}} + q_{ij}\int_0^\tau e^{\varepsilon s}ds) + \lambda_i(\varepsilon - a_i) < 0 \\ \sum\limits_{i=1}^{n} \lambda_i L_j^f(c_{ji}e^{\varepsilon\sigma_{ji}} + p_{ji}\int_0^\tau e^{\varepsilon s}ds + \mu_j(\varepsilon - b_j) < 0 \end{cases} \quad (10)$$

We construct Lyapunov functional as follows:

$$V(t) = V_1(t) + V_2(t)$$

$$V_1(t) = \sum_{i=1}^{n} \lambda_i[\sum_{j=1}^{p} L_j^f |c_{ji}| \int_{t-\sigma_{ji}}^{t} |y_j(r) - y_j{}^*| e^{\varepsilon(r+\sigma_{ji})}dr]$$
$$+ \sum_{i=1}^{n} \lambda_i[|x_i(t) - x_i{}^*| e^{\varepsilon t}$$
$$+ \sum_{j=1}^{p} L_j^f |p_{ji}| \int_0^\tau \int_{t-s}^{t} |y_j(r) - y_j{}^*| e^{\varepsilon(r+s)}drds].$$

$$V_2(t) = \sum_{j=1}^{p} \mu_j[|y_j(t) - y_j{}^*| e^{\varepsilon t}$$
$$+ \sum_{i=1}^{n} L_i^g |q_{ij}| \int_0^\tau \int_{t-s}^{t} |x_i(r) - x_i{}^*| e^{\varepsilon(r+s)}drds]$$
$$+ \sum_{j=1}^{p} \mu_j(\sum_{i=1}^{n} L_i^g |d_{ij}| \int_{t-\delta_{ij}}^{t} |x_i(r) - x_i{}^*| e^{\varepsilon(r+\delta_{ij})}dr).$$

$$D^+V_1(t) = \sum_{i=1}^{n} \lambda_i e^{\varepsilon t}[(\sum_{j=1}^{p} L_j^f |c_{ji}| e^{\varepsilon\sigma_{ji}} |y_j(t) - y_j{}^*|$$
$$-(\sum_{j=1}^{p} L_j^f |c_{ji}| |y_j(t-\sigma_{ji}) - y_j{}^*|)]$$
$$+ \sum_{i=1}^{n} \lambda_i e^{\varepsilon t} \{D^+ |x_i(t) - x_i{}^*| + \varepsilon |x_i(t) - x_i{}^*|$$
$$+ \sum_{j=1}^{p} L_j^f |p_{ji}| |y_j(t) - y_j{}^*| \int_0^\tau e^{\varepsilon s}ds$$
$$- \sum_{j=1}^{p} L_j^f |p_{ji}| \int_0^\tau |y_j(t-s) - y_j{}^*|ds\}.$$

$$D^+V_2(t)=\sum_{j=1}^{p}\mu_j e^{\varepsilon t}[D^+\left|y_j(t)-y_j{}^*\right|+\varepsilon\left|y_j(t)-y_j{}^*\right|$$
$$+\sum_{i=1}^{n}L_i^g\left|q_{ij}\right|\left|x_i(t)-x_i^*\right|\int_0^\tau e^{\varepsilon s}ds$$
$$-\sum_{i=1}^{n}L_i^g\left|q_{ij}\right|\int_0^\tau\left|x_i(t-s)-x_i^*\right|ds]$$
$$+\sum_{j=1}^{p}\mu_j e^{\varepsilon t}[(\sum_{j=1}^{p}L_i^g\left|d_{ij}\right|e^{\varepsilon\delta_{ij}}\left|x_i(t)-x_i^*\right|)$$
$$-(\sum_{j=1}^{p}L_i^g\left|d_{ij}\right|\left|x_i(t-\delta_{ij})-x_i^*\right|)].$$

$$D^+\left|x_i(t)-x_i^*\right|\le -a_i\left|x_i(t)-x_i^*\right|$$
$$+\sum_{j=1}^{p}\left|c_{ji}\right|\left|f_j(y_j(t-\sigma_{ji})-f_j(y_j^*))\right|$$
$$+\sum_{j=1}^{p}\left|p_{ji}\right|\int_0^\tau\left|f_j(y_j(t-s)-f_j(y_j^*)\right|ds$$
$$\le -a_i\left|x_i(t)-x_i^*\right|+\sum_{j=1}^{p}L_j^f\left|c_{ji}\right|\left|y_j(t-\sigma_{ji})-y_j^*\right|$$
$$+\sum_{j=1}^{p}L_j^f\left|p_{ji}\right|\int_0^\tau\left|y_j(t-s)-y_j^*\right|ds.$$
$$D^+\left|y_j(t)-y_j^*\right|\le -b_j\left|y_j(t)-y_j^*\right|$$
$$+\sum_{i=1}^{n}\left|d_{ij}\right|\left|g_i(x_i(t-\delta_{ij}))-g_i(x_i^*)\right|$$
$$+\sum_{i=1}^{n}L_i^g\int_0^\tau\left|q_{ij}\right|\left|x_i(t-s)-x_i^*\right|ds \qquad (11)$$
$$\le -b_j\left|y_j(t)-y_j^*\right|+\sum_{i=1}^{n}L_i^g\left|d_{ij}\right|\left|x_i(t-\delta_{ij})-x_i^*\right|$$
$$+\sum_{i=1}^{n}L_i^g\int_0^\tau\left|q_{ij}\right|\left|x_i(t-s)-x_i^*\right|ds.$$

From inequalities (10) and (11), we have

$$D^+V_1(t)\le\sum_{i=1}^{n}\lambda_i e^{\varepsilon t}[(\varepsilon-a_i)\left\|x_i(t)-x_i^*\right\|$$
$$+\sum_{j=1}^{m}L_j^f\left|c_{ji}\right|e^{\varepsilon\sigma_{ji}}\left|y_j(t)-y_j^*\right|$$
$$+\sum_{j=1}^{m}L_j^f\left|p_{ji}\right|\left|y_j(t)-y_j^*\right|\int_0^\tau e^{\varepsilon s}ds].$$
$$D^+V_2(t)\le\sum_{j=1}^{m}\mu_j e^{\varepsilon t}[(\varepsilon-b_j)\left|y_j(t)-y_j^*\right|$$
$$+\sum_{i=1}^{n}L_i^g\left|d_{ij}\right|e^{\varepsilon\delta_{ij}}\left|x_i(t)-x_i^*\right|$$
$$+\sum_{i=1}^{n}L_i^g\left|q_{ij}\right|\left|x_i(t)-x_i^*\right|\int_0^\tau e^{\varepsilon s}ds].$$

$$D^+V(t)\le e^{\varepsilon t}[\sum_{i=1}^{n}\lambda_i(\varepsilon-a_i)$$
$$+\sum_{j=1}^{p}\mu_j\sum_{i=1}^{n}L_i^g(d_{ij}e^{\varepsilon\delta_{ij}}+\left|q_{ij}\right|\int_0^\tau e^{\varepsilon s}ds)]\left|x_i(t)-x_i^*\right|$$
$$+e^{\varepsilon t}[\sum_{j=1}^{p}\mu_j(\varepsilon-b_j)+\sum_{i=1}^{n}\lambda_i\sum_{j=1}^{p}L_j^f(\left|c_{ji}\right|e^{\varepsilon\sigma_{ji}}$$
$$+\left|p_{ji}\right|\int_0^\tau e^{\varepsilon s}ds)]\left|y_j(t)-y_j^*\right|$$
$$=e^{\varepsilon t}\sum_{i=1}^{n}[\lambda_i(\varepsilon-a_i)+\sum_{j=1}^{p}\mu_j L_i^g(\left|d_{ij}\right|e^{\varepsilon\delta_{ij}}$$
$$+\left|q_{ij}\right|\int_0^\tau e^{\varepsilon s}ds)]\left|x_i(t)-x_i^*\right|$$
$$+e^{\varepsilon t}\sum_{j=1}^{p}[\mu_j(\varepsilon-b_j)+\sum_{i=1}^{n}\lambda_i L_j^f(\left|c_{ji}\right|e^{\varepsilon\sigma_{ji}}$$
$$+\left|p_{ji}\right|\int_0^\tau e^{\varepsilon s}ds)]\left|y_j(t)-y_j^*\right|\le 0.$$

Case

$$\left|x_i(t_k{}^+)-x_i^*\right|=\left|\alpha_{ik}\right|\left|x_i(t_k)-x_i^*\right|\le\left|x_i(t_k)-x_i^*\right|,$$
$$\left|y_j(t_k{}^+)-y_j^*\right|=\left|\beta_{ik}\right|\left|y_j(t_k)-y_j^*\right|\le\left|y_j(t_k)-y_j^*\right|.$$

Hence,

$$V(t_k{}^+)=\sum_{i=1}^{n}\lambda_i(\sum_{j=1}^{p}L_j^f\left|c_{ji}\right|\int_{t_k-\sigma_{ji}}^{t_k}\left|y_j(r)-y_j{}^*\right|),$$
$$\times e^{\varepsilon(r+\sigma_{ji})}dr+\sum_{i=1}^{n}\lambda_i[\left|x_i(t_k^+)-x_i^*\right|e^{\varepsilon t_k^+}$$
$$+\sum_{j=1}^{p}L_j^f\left|p_{ji}\right|\int_0^\tau\int_{t_k-s}^{t_k}\left|y_j(r)-y_j^*\right|e^{\varepsilon(r+s)}drds]$$
$$+\sum_{j=1}^{p}\mu_j[\left|y_j(t_k^+)-y_j^*\right|e^{\varepsilon t_k^+}$$
$$+\sum_{i=1}^{n}L_i^g\left|q_{ij}\right|\int_0^\tau\int_{t_k-s}^{t_k}\left|x_i(r)-x_i^*\right|]e^{\varepsilon(r+s)}drds$$
$$+\sum_{j=1}^{p}\mu_j\sum_{i=1}^{n}L_i^g\left|d_{ij}\right|\int_{t_k-\delta_{ij}}^{t_k}\left|x_i(r)-x_i{}^*\right|e^{\varepsilon(r+\delta_{ij})}dr\le V(t_k).$$

Combining with the above discussion, for any $t\ge 0$

$$V(t)\le V(t_k{}^+)\le V(t_k)\le V(t_{k-1}{}^+)\le\cdots\le V(0).$$

On the other hand, from the expression of $V(t)$, we have

$$V(t)\ge e^{\varepsilon t}\sum_{i=1}^{n}\lambda_i\left|x_i(t)-x_i{}^*\right|+e^{\varepsilon t}\sum_{j=1}^{p}\mu_j\left|y_j(t)-y_j{}^*\right|$$
$$\ge e^{\varepsilon t}\gamma\left[\sum_{i=1}^{n}\left|x_i(t)-x_i{}^*\right|+\sum_{j=1}^{p}\left|y_j(t)-y_j{}^*\right|\right]$$
$$=e^{\varepsilon t}\gamma\left\|(x(t),y(t))-(x^*,y^*)\right\|$$
$$=e^{\varepsilon t}\gamma\left\|z(t)-z^*\right\|, \qquad (12)$$

where $\gamma=\min\{\min_{1\le i\le n}\{\lambda_i\},\min_{1\le j\le p}\{\mu_j\}\}$

$$V(0)=\sum_{i=1}^{n}\lambda_i(\sum_{j=1}^{p}L_j^f\left|c_{ji}\right|\int_{-\sigma_{ji}}^0\left|y_j(r)-y_j^*\right|e^{\varepsilon(r+\sigma_{ji})}dr)$$
$$+\sum_{i=1}^{n}\lambda_i[\left|x_i(0)-x_i^*\right|$$
$$+\sum_{j=1}^{p}L_j^f\left|p_{ji}\right|\int_0^\tau\int_{-s}^0\left|y_j(r)-y_j^*\right|e^{\varepsilon(r+s)}drds]$$
$$+\sum_{j=1}^{p}\mu_j[\left|y_j(0)-y_j^*\right|$$
$$+\sum_{i=1}^{n}L_i^g\left|q_{ij}\right|\int_0^\tau\int_{-s}^0\left|x_i(r)-x_i^*\right|e^{\varepsilon(r+s)}drds]$$
$$+\sum_{j=1}^{p}\mu_j(\sum_{i=1}^{n}L_i^g\left|d_{ij}\right|\int_{-\delta_{ij}}^0\left|x_i(r)-x_i{}^*\right|e^{\varepsilon(r+\delta_{ij})}dr)$$
$$=\sum_{i=1}^{n}\lambda_i(\sum_{j=1}^{p}L_j^f\left|c_{ji}\right|\int_{-\sigma_{ji}}^0\left|\psi_j(r)-y_j{}^*\right|e^{\varepsilon(r+\sigma_{ji})}dr)$$
$$+\sum_{i=1}^{n}\lambda_i[\left|\varphi_i(0)-x_i^*\right|$$
$$+\sum_{j=1}^{p}L_j^f\left|p_{ji}\right|\int_0^\tau\int_{-s}^0\left|\psi_j(r)-y_j^*\right|e^{\varepsilon(r+s)}drds]$$
$$+\sum_{j=1}^{p}\mu_j[\left|\psi_j(0)-y_j^*\right|$$
$$+\sum_{i=1}^{n}L_i^g\left|q_{ij}\right|\int_0^\tau\int_{-s}^0\left|\varphi_i(r)-x_i^*\right|e^{\varepsilon(r+s)}drds]$$
$$+\sum_{j=1}^{p}\mu_j(\sum_{i=1}^{n}L_i^g\left|d_{ij}\right|\int_{-\delta_{ij}}^0\left|\varphi_i(r)-x_i^*\right|e^{\varepsilon(r+\delta_{ij})}dr)$$
$$\le\sum_{i=1}^{n}[\lambda_i+\sum_{j=1}^{p}\mu_j L_i^g(q_i j\int_0^\tau\int_{-s}^0 e^{\varepsilon(r+s)}drds$$

$$+d_{ij}\int_{-\delta_{ij}}^{0} e^{\varepsilon(r+\delta_{ij})}dr)) \times \sup_{-\delta\le s\le 0}\sum_{i=1}^{n}|\varphi_i(s)-x_i{}^*|$$

$$+\sum_{j=1}^{p}[\mu_j+\sum_{j=1}^{p}\lambda_i L_j^f(c_{ji}\int_{-\sigma_{ji}}^{0} e^{\varepsilon(r+\sigma_{ji})}dr$$

$$+p_{ji}\int_0^\tau\int_{-s}^{0} e^{\varepsilon(r+s)}drds)]\times \sup_{-\sigma\le s\le 0}\sum_{j=1}^{p}\left|\psi_j(s)-y_j^*\right|.(13)$$

From (12) and (13), we obtain

$$e^{\varepsilon t}\gamma\left\|z(t)-z^*\right\| \le V(t)\le V(0)\le M(\varepsilon)\gamma\left\|\phi-z^*\right\|,$$

so that

$$\left\|z(t)-z^*\right\| \le M(\varepsilon)e^{-\varepsilon t}\left\|\phi-z^*\right\| \quad for\ all\ \ t\ge 0. \quad (14)$$

where $M(\varepsilon) = \gamma^{-1}\max\{\sum_{i=1}^{n}\{\lambda_i + \sum_{j=1}^{p}\mu_j L_i^g[|q_{ij}|\int_0^\tau\int_{-s}^0 e^{\varepsilon(r+s)}drds + |d_{ij}|\int_{-\delta_{ij}}^0 e^{\varepsilon(r+\delta_{ij})dr}]\}, \sum_{j=1}^{p}\mu_j + \sum_{j=1}^{p}\lambda_i L_j^f \times[|c_{ji}|\int_{-\sigma_{ji}}^0 e^{\varepsilon(r+\sigma_{ji}}]dr + |p_{ji}|\int_0^\tau\int_{-s}^0 e^{\varepsilon(r+s)}drds]\}\}$. Thus, from (14) we directly obtain that system (1) is globally exponentially stable, the proof is completed.

Corollary 1. Under assumptions (H1) and (H2), system (1) is globally exponentially stable, if the following conditions are satisfied:

(1) $|\alpha_{ik}|\le 1$, $|\beta_{jk}|\le 1$, $i=1,2,\cdots,n$, $j=1,2,\cdots,p$, $k=1,2,\cdots$.

(2) $$\begin{cases}\sum_{j=1}^{p}L_i^g(d_{ij}+q_{ij}\tau)-a_i<0,\ j=1,2,\cdots,p.\\ \sum_{i=1}^{n}L_j^f(c_{ji}+p_{ji}\tau)-b_j<0,\ i=1,2,\cdots,n.\end{cases}$$

Remark 1. Obviously, when there is no impulse in system (1), it reduces to the following model:

$$\begin{cases} x'_i(t) = -a_ix_i(t)+\sum_{j=1}^{p}c_{ji}f_j(y_j(t-\sigma_{ji}))\\ +\sum_{j=1}^{p}\int_0^\tau p_{ji}f_j(y_j(t-s))ds+r_i, t\ge 0,\\ y'_j(t) = -b_jy_j(t)+\sum_{i=1}^{n}d_{ij}g_i(x_i(t-\delta_{ij})\\ +\sum_{i=1}^{n}\int_0^\tau q_{ij}g_j(x_i(t-s))ds+s_j, t\ge 0,\end{cases} \quad (1^*)$$

by the process of proof of Theorems 1. Theorems 1 can guarantee that the system(1^*) has a unique equilibrium point. It is easy to show that the following corollaries hold.

Corollary 2. Under assumptions (H1) and (H2), system (1^*) is globally exponentially stable if there exist positive constants such that

$$\begin{cases}\sum_{j=1}^{p}\mu_jL_i^g(d_{ij}+q_{ij}\tau)-\lambda_ia_i<0,\ j=1,2,\cdots,p.\\ \sum_{i=1}^{n}\lambda_iL_j^f(c_{ji}+p_{ji}\tau)-\mu_jb_j<0,\ i=1,2,\cdots,n.\end{cases}$$

IV. An illustrative example

Consider the following impulsive BAM neural network with mixed delays:

$$\begin{cases} x'_i(t) = -a_ix_i(t)+\sum_{j=1}^{3}c_{ji}f_j(y_j(t-\sigma_{ji}))\\ +\sum_{j=1}^{3}\int_0^\tau p_{ji}f_j(y_j(t-s))ds+r_i, t\ge 0, t\ne t_k,\\ \Delta x_i(t) = (\alpha_{ik}-1)x_i(t), i=1,\cdots,n, k=1,\cdots, t=t_k\\ y'_j(t) = -b_jy_j(t)+\sum_{i=1}^{3}d_{ij}g_i(x_i(t-\delta_{ij})\\ +\sum_{i=1}^{3}\int_0^\tau q_{ij}g_j(x_i(t-s))ds+s_j, t\ge 0, t\ne t_k, \qquad (15)\\ \Delta y_j(t) = (\beta_{jk}-1)y_j(t), j=1,\cdots,p, k=1,\cdots,\ t=t_k.\end{cases}$$

Where $(a_1,a_2,a_3)=(1,1,1)^T, (b_1,b_2,b_3)=(1,1,1)^T$, σ_{ji}, $\delta_{ij}\in(0,\infty), \mu_j=\lambda_i, L_j^f=L_i^g=\tau=1, i,j=1,2,3.$ $\alpha_{ik}=\frac{1}{2}\sin(1+k), \beta_{jk}=\frac{2}{3}\cos 2k,\ f_j(x)=g_i(x)=\frac{1}{2}(|x+1|-|x-1|)$, $c_{ji}, d_{ij}, p_{ji}, q_{ij}$ are positive and

$$M_{ji}=\frac{1}{a_i}(|c_{ji}|+|\tau p_{ji}|)=c_{ji}+p_{ji}$$

$$N_{ij}=\frac{1}{b_j}(|d_{ij}|+|\tau q_{ij}|)=d_{ij}+q_{ij}$$

$$\begin{pmatrix} M_{11} & M_{12} & M_{13}\\ M_{21} & M_{22} & M_{23}\\ M_{31} & M_{32} & M_{33}\end{pmatrix}=\begin{pmatrix} 1/2 & 1/18 & 0\\ 5/2 & 1/2 & 0\\ 0 & 0 & 1/2\end{pmatrix},$$

$$\begin{pmatrix} N_{11} & N_{12} & N_{13}\\ N_{21} & N_{22} & N_{23}\\ N_{31} & N_{32} & N_{33}\end{pmatrix}=\begin{pmatrix} 1/3 & 0 & 0\\ 0 & 1/3 & 1/32\\ 0 & 2 & 1/3\end{pmatrix}$$

$$\begin{pmatrix} r_1\\ r_2\\ r_3\end{pmatrix}=\begin{pmatrix} -2\\ 4/9\\ 1/2\end{pmatrix}, \quad \begin{pmatrix} s_1\\ s_2\\ s_3\end{pmatrix}=\begin{pmatrix} 2/3\\ -4/3\\ 61/96\end{pmatrix}$$

Simple computation shows that

$$K=(k_{ij})_{6\times 6}=\begin{pmatrix} 0 & 0 & 0 & 1/2 & 1/18 & 0\\ 0 & 0 & 0 & 5/2 & 1/2 & 0\\ 0 & 0 & 0 & 0 & 0 & 1/2\\ 1/3 & 0 & 0 & 0 & 0 & 0\\ 0 & 1/3 & 1/32 & 0 & 0 & 0\\ 0 & 2 & 1/3 & 0 & 0 & 0\end{pmatrix}$$

Hence, by using matlab, it shows

$$\rho(K)=\text{maxeigenvalues}(K)=0.602262<1.$$

Therefore, it follows that Theorem 1 and Theorem 2 that system (15) has a unique equilibrium

$$z^*=(x_1^*,x_2^*,x_3^*,y_1^*,y_2^*,y_p^*)^T=(1,1,1,1,1,1)^T$$

from the given date, we obtain

$$\begin{cases}\sum_{j=1}^{p}\mu_jL_i^g(d_{ij}+q_{ij}\tau)-\lambda_ia_i=0,\ j=1,2,\cdots,p.\\ \sum_{i=1}^{n}\lambda_iL_j^f(c_{ji}+p_{ji}\tau)-\mu_jb_j<0,\ i=1,2,\cdots,n.\end{cases}$$

So that system (15) is globally exponentially stable.

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