
EVOLUTION OF GALAXIES IN THE Λ CDM COSMIC MODEL

¹Diriba Gonfa Tolasa ,Assosa university College of Natural and Computational Science, Department of physics, Assosa, Ethiopia,telephone number +251983224981 +251991772290
Email : dgonfa2009@gmail.com

² Tolu Biressa ,Jimma University, College of Natural Science Department of physics , Jimma , Ethiopia, telephone number +251911164350
Email: biressatolu@gmail.com

Abstract

In this thesis we have considered the evolution of galaxies in the Λ CDM (cosmological constant (Λ) and cold dark matter (CDM) cosmic model. The work includes revision of the contents of the implemented model; and the analysis of the way how the contents enter and affect the evolution of the galaxies in time. Accordingly, we have studied aging, dark matter interaction to form halos and the general morphology of the galaxies in the context. The General relativity base, Λ CDM model is used to analyze the Friedmann equations and their solutions how the geometry and matter energy source of the universe affected the formation and evolution of galaxies and other cosmic structures. The result of our work indicates that cosmological constant has a significant effect on structure and dynamics of galaxies. For the analysis of the results we have used Python software. Our result is in good agreement with the previous works.

key words: evolution of galaxies, formation of galaxies, Λ CDM model

1, Introduction

The observable universe is a hierarchical structure whose physical composition is mediated by galaxies. That means galaxy Morphology and Structure are the ingredients of the universe that mainly play roles in the study of the whole universe from small scale to large scale contents including formation , dynamics and so on. Yet, there are unresolved evolutionary scenario of the galaxy itself (1).

2, Research Methodology

General relativity based Λ CDM cosmic model is used to derive analytical dynamic equation for galaxy morphology and structure. Then, with appropriate boundary conditions limiting cases will be derived for numerical analysis to simulate with observation to answer the objective of study. For numerical analysis Python software is used

3, The Friedman Equations

3.1 Geometry and matter energy source of universe

The geometry and matter-energy source of the universe play a fundamental role in determining the formation and structure of galaxies. The current leading theory for the evolution of the universe is the Big Bang theory, which suggests that the universe began as a hot and dense state, before expanding and cooling over time.

The formation and structure of galaxies are influenced by the geometry and matter-energy source of the universe. The gravitational attraction between matter in the early universe caused uneven distribution, leading to the formation of large-scale structures such as galaxy clusters and superclusters. One key concept in understanding the structure of the universe is the idea of dark matter. Dark matter is a hypothetical form of

matter that does not interact with light or other forms of electromagnetic radiation, hence it's called "dark". It is believed to make up approximately 27% of the total matter-energy content of the universe, and plays a critical role in galaxy formation.

The distribution of dark matter in the early universe had a profound effect on the formation of galaxies. As the universe expanded, regions of high density formed due to gravitational instabilities, causing gas and dust to collapse and eventually form stars. The structure of these galaxies was influenced by the underlying distribution of dark matter, which acted like a scaffolding for the formation of stars(3; 4)

The equation that helps us understand the relationship between the geometry and matter-energy source of the universe is the Friedmann equation, named after the Russian mathematician Alexander Friedmann. The Friedmann equation describes the expansion rate of the universe as a function of its matter-energy content and geometry:

$$H^2 = \frac{8\pi G\rho}{3} - \frac{\kappa c^2}{a^2} + \frac{\Lambda c^2}{3} \quad (1)$$

where H is the Hubble parameter, G is the gravitational constant, ρ is the matter-energy density of the universe, c is the speed of light, κ is the curvature of space-time, a is the scale factor describing the expansion of the universe, and Λ is the cosmological constant.

This equation tells us how the universe evolves over time, with the matter-energy content and geometry playing crucial roles in determining the overall structure and behavior of the universe. Friedmann's equations provide a solution to Einstein's General Relativistic (GR) equations in the FLRW metric. We need to consider GR to effectively link the geometry of space time (the metric) to the distribution of matter and energy in the system. Friedmann's equations allow us to relate the evolution of the scale factor, $a(t)$, to the (homogeneous) density in its various guises: radiation, matter or dark energy. We will show here a simplified derivation based on Newtonian mechanics which arrives at the correct equations(2). Let us model the Universe as a sphere with radius r, expanding with the Hubble flow. The force on a test particle with mass m is.

$$m\ddot{r} = \frac{-GM(< r)m}{r^2} = \frac{-4\pi Gm}{3}r\rho(t) \Rightarrow \ddot{a} = -\frac{4\pi G}{3}\rho(t)a(t) \quad (2)$$

where we have identified the radius of this 'sample Universe' as the scale factor.

Since the Universe is an isolated system, conservation of mass gives $\rho a^3 = \rho_0 a_0^3 = \text{constant}$. The sub index zero refers to the present time. Therefore, multiplying equation 2 by \dot{a} gives

$$\dot{a}\ddot{a} = \frac{1}{2} \frac{d}{dt} \dot{a}^2 = \frac{-4\pi G}{3a^2} (\rho_0) a_0^3 \dot{a} \quad (3)$$

Integrating equation (3) leads to

$$\dot{a}^2 = \frac{8\pi G}{3} \rho a^2 - \kappa c^2 \quad (4)$$

with the last term being an integration constant. In GR this term has the meaning of the constant curvature allowed by the FLRW metric. To derive the correct equation from this Newtonian approximation, we need to include the contribution to the energy from pressure. Following a simple thermodynamic argument, consider the amount of heat flowing into the Universe, with volume V . Noting that the internal energy is $E = V\rho c^2$, we have

$$\Delta Q = \Delta E + P\Delta V(\rho c^2) + (\rho c^2 + P)\Delta V \quad (5)$$

Since the Universe is a closed system, $\Delta Q = 0$, and since the volume is $V \propto a^3$, we can write $\Delta V/V = 3\Delta a/a$ Therefore:

$$\frac{d\rho}{dt} + 3\frac{\dot{a}}{a} \left(\rho + \frac{p}{c^2} \right) = 0 \quad (6)$$

Taking the time derivative of equation 4, and using 6 to describe $d\rho/dt$, we arrive at the second of Friedmann's equations, namely:

$$\ddot{a} = \frac{-4\pi G}{3} a \left[\rho + \frac{3p}{c^2} \right] \quad (7)$$

However, this Newtonian example cannot describe the so-called cosmological constant (Λ). Friedmann's equations including this constant are

$$\dot{a}^2 = \frac{-8\pi G}{3} \rho a^2 - \kappa c^2 + \frac{1}{3} \Lambda a^2 \quad (8)$$

$$\ddot{a} = \frac{-4\pi G}{3} a \left[\rho + \frac{3p}{c^2} \right] + \frac{1}{3} \Lambda a \quad (9)$$

Starting with the Friedmann equation 1, we have:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \quad (10)$$

where \dot{a}/a is the Hubble parameter, ρ is the total energy density of the universe, including matter, radiation, and dark energy, k is the curvature of space (positive for closed, negative for open, zero for flat), and Λ is the cosmological constant.

Assuming that radiation can be ignored at the present epoch, we have:

$$\Omega_m = \frac{8\pi G}{3H_0^2}\rho_m \quad (11)$$

where Ω_m is the fractional energy density of matter in the universe and ρ_m is the energy density of matter. Rearranging this equation, we get:

$$\rho_m = \frac{3H_0^2\Omega_m}{8\pi G} \quad (12)$$

Substituting this expression for ρ_m into the Friedmann equation and rearranging, we get:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left[\frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda \right] \quad (13)$$

where $\Omega_k = -k/(a^2 H_0^2)$ is the fractional energy density due to curvature, and $\Omega_\Lambda = \Lambda/(3H_0^2)$ is the fractional energy density due to the cosmological constant.

Taking the square root of both sides and simplifying, we arrive at the expression :

$$H = H_0 \left[\sqrt{\frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda} \right] \quad (14)$$

To simplify this equation, we can define the critical density of the universe as $\rho_{crit} = 3H_0^2/(8\pi G)$. This is the density required for the universe to have a flat geometry ($k=0$).

We can then define the dimensionless energy densities for matter (Ω_m), curvature (Ω_k), and the cosmological constant (Ω_Λ) as:

$$\Omega_m = \rho_m/\rho_{crit}$$

$$\Omega_k = -k/(a^2 H_0^2)$$

$$\Omega_\Lambda = \Lambda/(3H_0^2)$$

Note that Ω_m and Ω_Λ are positive, while Ω_k can be positive or negative depending on the geometry of the universe.

To derive an expression for Ω_m as a function of the matter density ρ_m , we can use the fact that $\rho_m = \Omega_m \rho_{crit}$. Substituting this into the definition of Ω_m , we get:

$$\Omega_m = \frac{\rho_m}{\rho_{crit}} = \frac{8\pi G}{3H_o^2} \rho_m \quad (15)$$

Therefore,

$$\Omega_m = \frac{8\pi G}{3H_o^2} \rho_m = \frac{8\pi G}{3H_o^2} \rho_0 a^{-3} \quad (16)$$

where ρ_0 is the present-day matter density. This gives us an expression for Ω_m as a function of the scale factor a .

3.2 Galaxy evolution Connection equation in Λ CDM model

The basic idea behind the connection between galaxy evolution and the Lambda CDM cosmology is that the distribution of galaxies should reflect the underlying matter distribution of the universe. In other words, the way that galaxies evolve and cluster together should depend on the gravitational pull of the surrounding matter.

Here are the steps to derive the galaxy evolution connection equation:

Start with the Friedmann equation, which relates the expansion rate of the universe to its energy content: we rewrite

$$H^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

where G is the gravitational constant and k describes the curvature of space (which we assume to be flat in this case).

Assume that the matter content of the universe is dominated by non-relativistic particles (i.e., dark matter) so that $\rho \approx \rho_m$. Then, we can rewrite the Friedmann equation as:

$$H^2 = H_o^2 \left(\frac{\Omega_m}{a^3} + \frac{\Omega_\Lambda}{a^{3(1+w)}} \right) \quad (17)$$

where $\Omega_m = \frac{\rho_m}{\rho_c}$ is the fraction of critical density in matter, $\Omega_\Lambda = \frac{\Lambda}{3H_o^2}$ is the fraction of critical density in dark energy, and w is the equation of state parameter for dark energy (assumed to be -1 for a cosmological constant).

Take the derivative of the scale factor with respect to time:

$$\frac{da}{dt} = aH \quad (18)$$

Use the chain rule to express the time derivative of a in terms of the redshift z :

$$\frac{da}{dt} = \frac{da}{dz} \frac{dz}{dt} = -\frac{a}{1+z} H \quad (19)$$

Substitute equation 19 into the continuity equation for matter:

$$\frac{\partial \rho_m}{\partial t} + 3H\rho_m = 0 \quad (20)$$

Use the chain rule again to express the time derivative of ρ_m in terms of the redshift z :

$$\frac{\partial \rho_m}{\partial t} = \frac{\partial \rho_m}{\partial z} \frac{\partial z}{\partial t} = -\frac{3\rho_m}{1+z} \quad (21)$$

Substitute equations 19 and 21 into the continuity equation for matter and simplify:

$$\frac{\partial \rho_m}{\partial z} + 3\frac{\rho_m}{1+z} = 0 \quad (22)$$

Rewrite the continuity equation as a differential equation by multiplying both sides by $-\frac{1}{\rho_m}$ and integrating:

$$\ln(\rho_m) = -3 \ln(1+z) + C \quad (23)$$

where C is an integration constant that depends on the initial conditions of the universe.

Solve for ρ_m :

$$\rho_m = \rho_{m,0}(1+z)^3 \quad (24)$$

where $\rho_{m,0}$ is the present-day matter density of the universe.

Substitute equation 24 into the Friedmann equation and simplify:

$$H^2 = H_0^2 [\Omega_m(1+z)^3 + \Omega_\Lambda] \quad (25)$$

Differentiate equation 25 with respect to z : we get

$$\frac{dH^2}{dz} = 3H_0^2 \Omega_m(1+z)^2 \quad (26)$$

4,Results and Discussion

4.1 Origins of galaxy morphology

In this thesis, we addressed the origins of galaxy morphology, Galaxy morphology refers to the study of the shapes and structures of galaxies, the origin of galaxy morphology was a complex topic that was still being studied by astronomers and astrophysicists. However, there were several key factors that were thought to play a role in determining the shape and structure of galaxies.

One of the most important factors was the initial conditions of the universe shortly after the Big Bang. The distribution of matter and energy in the early universe set the stage for the formation of galaxies, and this initial distribution is thought to have influenced the way that galaxies form and evolve over time.

Another important factor was the process of galaxy mergers and interactions. When two galaxies come together, their gravitational forces can cause them to merge into a single, larger galaxy. This process can also distort the shapes of the galaxies involved, leading to the formation of unusual structures such as tidal tails and bridges.

The nature of the dark matter that makes up a significant portion of the mass of galaxies is also thought to play a role in determining their morphology. The distribution and properties of dark matter can influence the way that visible matter in galaxies (such as stars and gas) moves and evolves over time.

Finally, feedback processes from accreting black holes and star formation can affect the growth and morphology of galaxies. These processes can drive outflows of gas and other material from galaxies, which can in turn impact the way that new stars and other structures form within the galaxy.

Overall we have discussed, the study of galaxy morphology was an active area of research in astrophysics, because it contains important information about the evolution of galaxies and can help constraint theoretical models in cosmology.

The merger equation of models of galaxies was given helped us in chapter two to describe the origin of galaxy morphology.

This equation was used to state the relationship between number density of galaxy and the redshift of galaxy.

By using ($N = N_o \times (1 + z)^3$) we can plot the relationship between number density of galaxy and the redshift of galaxy. using the constant $N_o = 1 \times 10^{-3} = 0.01$ per cubic megaparsec was the current accepted value of intial No density of galaxy for Astronomers & Astrophysicist and Redshift values from 0 to 10

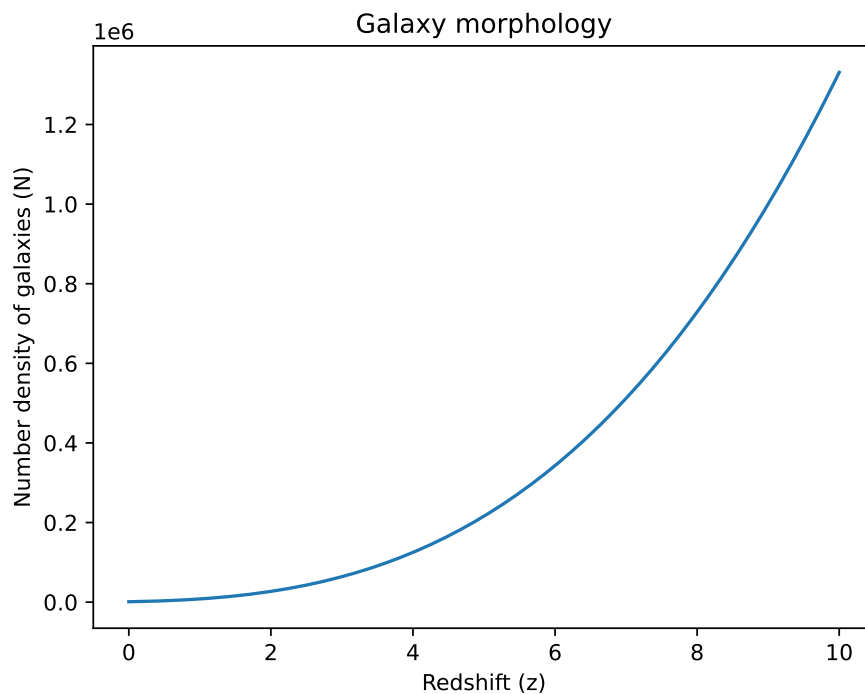


Figure 1: The relationship between the redshift and the number density of galaxies.

The graph shows the relationship between the redshift and the number density of galaxies.

The graph shows that as we look back in time towards higher redshift values, the number density of galaxies decreases.

This suggests that in the early universe, there were fewer and less evolved galaxies compared to the present day. As time progressed, galaxies evolved and merged to form the different types of galaxies we observe today.

Therefore, this graph provides evidence for the origin of galaxy morphology and how it has changed over time.

It is suggested that the different types of galaxies we observed today have evolved from smaller, less evolved galaxies in the early universe.

4.2, The effect of geometry and matter energy source of universe on formation and structure of galaxies

The structure and formation of galaxies were influenced by a variety of factors, including the geometry of space-time and the distribution of matter and energy within it. In modern cosmology, the most widely accepted theory for the formation of galaxies was the hierarchical model, which suggested that galaxies formed through a process of gravitational attraction between smaller structures such as dark matter halos, gas clouds, and stars.

The geometry of space-time played a crucial role in determining the distribution of matter and energy within the universe.

According to the current understanding of cosmology, the universe was flat or nearly flat, which meant that the geometry of space-time was Euclidean.

The flatness of the universe had important implications for the formation of galaxies because it affected the way that matter and energy were distributed throughout the cosmos. We discussed how matter and energy sources played an important role in the formation and structure of galaxies. The gravitational attraction between massive objects caused matter to clump together, eventually forming stars and galaxies.

Dark matter, which was thought to make up about 85% of the matter in the universe, also played a crucial role in the formation of galaxies. It provided the gravitational scaffolding upon which regular matter could accumulate and form stars.

Energy sources, such as supernovae explosions and active galactic nuclei, had a significant impact on the formation and evolution of galaxies. These energetic events heated and ionized gas in the surrounding environment, triggered star formation, and altered the chemical composition of the gas. They also drove powerful winds that could blow gas out of galaxies, slowing or even halting star formation altogether. Galaxies were formed from the matter that was present in the universe. The matter-energy source that influenced the formation and structure of galaxies was the energy that was present in the form of light. The energy that was present in the form of light caused the matter that was present in the universe to form into galaxies.

We used equation 25 & Constant parameters we can plot the graph of Hubble parameter as a function of Redshift

$$H_0 = 70 \text{ km/s/Mpc}$$

$\rho_{crit} = 9.47 \times 10^{-27} \text{ kg/m}^3$, $\Omega_m = 0.3$, $\Omega_\lambda = 0.7$, $\Omega_k = 0$, $\rho_o = 0.3$ the present-day matter density parameter approximately

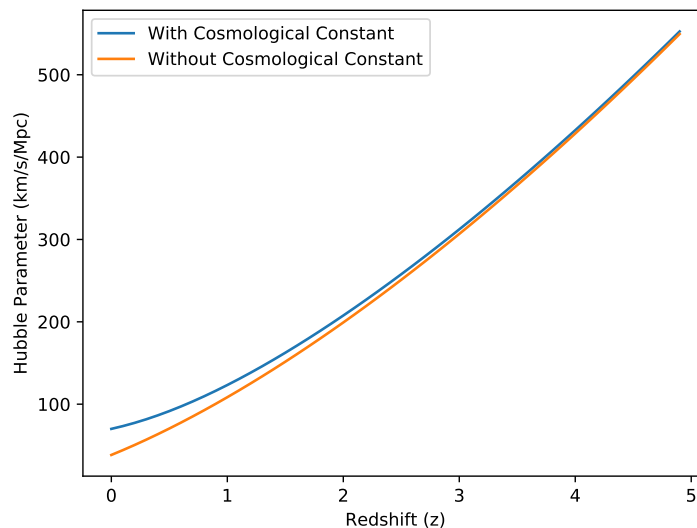


Figure 2: Hubble parameter as a function of Redshift factor

The graph shows the Hubble parameter as a function of redshift for both cases with and without the cosmological constant.

When the cosmological constant is included, we see that the expansion of the universe is accelerating at present ($z=0$). This is reflected in the steep increase in Hubble parameter towards lower redshifts.

When the cosmological constant is not included, the expansion of the universe is decelerating, and the rate of increase in Hubble parameter is slower towards lower redshifts.

In both cases, we see that the Hubble parameter decreases towards higher redshifts, indicating that the expansion of the universe was slower in the past.

We have discussed that in terms of galaxy formation and structure, the expansion of the universe plays a crucial role. The early universe was very smooth and homogeneous, but tiny quantum fluctuations in the density eventually grew through gravitational attraction to form the first galaxies and structures.

As the universe continues to expand, the gravity of these structures pulls in more matter, leading to the formation of larger and more complex structures like clusters and superclusters.

The density and distribution of matter-energy sources also affect the growth of structures, as regions with higher densities will experience stronger gravitational attraction and form structures more quickly. Generally, the Friedmann equation and its solutions help us understand how the geometry and matter-energy content of the universe affect the formation and evolution of galaxies and other cosmic structures.

4.3, The effect of cosmological constant on structure and dynamism of galaxies

The cosmological constant, which represents the energy density of the vacuum of space, has a significant effect on the structure and dynamics of galaxies. One way it affects galaxies was through its contribution to the expansion of the universe. In the early universe, the cosmological constant was negligible compared to other forms of matter and radiation. However, as the universe expanded and cooled, the energy density of matter and radiation decreased more rapidly than that of the vacuum energy. This caused the vacuum energy to become dominant, resulting in an accelerated expansion of the universe.

This accelerated expansion has several effects on the structure and dynamics of galaxies. First, it leads to the dilution of matter density and the clustering of matter into larger structures such as galaxy clusters and superclusters. Second, it increases the distance between galaxies, which can affect their interactions and mergers.

The cosmological constant also affects the motion of galaxies within clusters and superclusters. The gravitational attraction between galaxies tends to pull them towards each other, but the accelerating expansion of the universe counteracts this force, leading to a net motion away from each other.

Overall, the cosmological constant plays a crucial role in shaping the large-scale structure and dynamics of the universe, including the formation and evolution of galaxies.

Using Friedman equation 1 and using constant parameters we can analyzed the effects of cosmological constant on galaxy dynamics

$$H^2 = \frac{8\pi G\rho}{3} - \frac{\kappa c^2}{a^2} + \frac{\Lambda c^2}{3}$$

where H is the Hubble parameter, ρ is the density of matter and radiation in the universe, κ is the curvature of space, a is the scale factor of the universe, Λ is the cosmological constant, G is the gravitational constant, and c is the speed of light.

The effect of the cosmological constant on the dynamics and structure of galaxies can be seen by analyzing the last term of the Friedman equation.

The cosmological constant term affects the expansion rate of the universe, as it acts like

an energy density that pervades all of space. When there is more dark energy present in the universe, the expansion rate will increase.

This means that the rate at which galaxies move away from each other will also increase. On the other hand, when there is less dark energy, the expansion rate will slow down and galaxies will move towards each other at a slower rate. We have used constant parameters. we can plots the graph of scale factor vs Hubble parameters .

$$G = 6.6743 \times 10^{-11} , \text{gravitational constant } (m^3kg^{-1}s^{-2})$$

$$\pi = 3.142857$$

$$c = 3 \times 10^8 , \text{ speed of light in m/s}$$

$$\rho = 9.47 \times 10^{-27} , \text{ critical density of matter and radiation in } kg/m^3$$

$$\Lambda = 0.7 , \text{ cosmological constant in } m^{-2} \text{ and } \kappa = 0 , \text{ assume flat universe.}$$

We Calculated Hubble parameter for with and without cosmological constant.

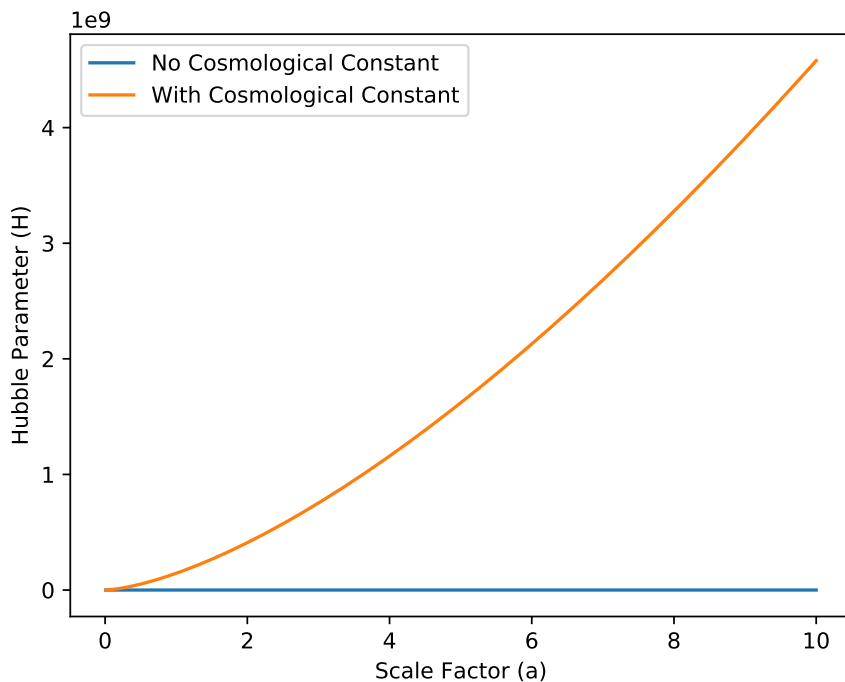


Figure 3: Scale factor vs Hubble parameter

This graph showing the Hubble parameter as a function of scale factor, with blue line for the case without a cosmological constant and orange line for the case with a

cosmological constant. The x-axis represents the scale factor, which is a measure of the size of the universe at a particular point in time. The y-axis represents the Hubble parameter, which tells us how quickly the universe is expanding at that point in time. The graph shows that the Hubble parameter increases more rapidly for the case with a cosmological constant than for the case without a cosmological constant. This means that the universe expands more quickly when there is a cosmological constant present. Additionally, the graph shows that the Hubble parameter approaches a constant value at late times for the case with a cosmological constant, while it continues to increase indefinitely for the case without a cosmological constant. This suggests that the presence of a cosmological constant may play a role in determining the long-term structure and dynamics of galaxies

Overall, the graph shows the significant impact that the cosmological constant has on the dynamics and structure of the galaxies.

5, Summary and Conclusion

The most characteristic aspect of galaxy evolution was the evolution of star formation (SF) activities. The standard model of cosmology was based on general relativity and the hypothesis that the Universe was spatially homogeneous and isotropic.

In this paper we studied The relationship between redshift and number density of galaxies was important in understanding the evolution of galaxies and the origin of galaxy morphology.

The Friedman equation and its solution helped us to understand how the geometry and matter energy source of the universe affected the formation and evolution of galaxies and other cosmic structures.

We concluded that the cosmological constant was significant effect on structure and dynamics of galaxies and also affects the motion of galaxies within clusters and superclusters.

Galaxy formation in the Λ CDM model was thought to occur through hierarchical clustering, where small structures merge over time to form larger objects. The process begins with the collapse of primordial density perturbations in the early universe, leading to the formation of dark matter halos. These halos then attract gas and dust through gravity, which eventually cools and condenses to form stars and galaxies.

We Conclude that the Λ CDM (Lambda Cold Dark Matter) model is the most widely accepted theoretical framework for understanding the evolution of the universe. Within this model, galaxies are thought to form through the hierarchical assembly of smaller structures, such as dark matter halos and gas clouds.

Some of the main conclusions regarding the evolution of galaxies in the Λ CDM model include:

1. Galaxies form through a process of hierarchical merging: small structures merge together to form increasingly larger structures, eventually leading to the formation of galaxies.

2. Dark matter plays a critical role in this process: it provides the gravitational scaffolding upon which galaxies and other structures form.

3. The formation and evolution of galaxies was strongly influenced by feedback processes, such as supernova explosions and active galactic nuclei, which can regulate star formation and the growth of black holes.

4. The overall properties of galaxies, such as their mass, size, and shape, are largely determined by the properties of their dark matter halos.

5. The Λ CDM model can successfully reproduce many observed properties of galaxies, including their luminosity function, clustering properties, and abundance at different redshifts.

6. However, there are still some outstanding challenges in understanding galaxy evolution within the Λ CDM model, particularly in explaining the observed diversity of galaxy morphologies and the origin of massive, compact galaxies at high redshifts.

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