Ranking Fuzzy Numbers based on Epsilon-Deviation Degree

Vincent F. Yu, Ha Thi Xuan Chi

Abstract—Nejad and Mashinchi (2011) proposed a revision for ranking fuzzy numbers based on the areas of the left and the right sides of a fuzzy number. However, this method still has some shortcomings such as lack of discriminative power to rank similar fuzzy numbers and no guarantee the consistency between the ranking of fuzzy numbers and the ranking of their images. To overcome these drawbacks, we propose an epsilon-deviation degree method based on the left area and the right area of a fuzzy number, and the concept of the centroid point. The main advantage of the new approach is the development of an innovative index value which can be used to consistently evaluate and rank fuzzy numbers. Numerical examples are presented to illustrate the efficiency and superiority of the proposed method.

Keywords—Ranking fuzzy numbers, Centroid, Deviation degree.

I. INTRODUCTION

RANKING and comparing of fuzzy numbers is one of the main research problems in fuzzy set theory, and plays a key role in the process of decision making, data analysis and artificial intelligence. In uncertain environments, making the right choice among different alternatives is a difficult task. Selecting the optimum or the best alternative is completely based on the ranking of fuzzy numbers. An efficient ranking method for fuzzy numbers can be used to evaluate alternatives and order them consistently so that a correct decision can be made.

Since Jain [1] proposed the first fuzzy number ranking method, many researchers have investigated various approaches for ranking fuzzy numbers [1]-[11]. However, some of these ranking methods suffered from some shortcomings [12]. A number of studies used the concept of centroid point in fuzzy number ranking methods [11], [13]-[17]. Yager [11] first proposed incorporating the concept of centroid point into the ranking method for fuzzy numbers. Along this line of thinking, Mabuchi [16] suggested a centroid index ranking approach with weighting function. Lee and Li [15] investigated the concept of probability measure and its use in ranking fuzzy numbers. Later, Cheng [13] suggested using the coefficient of variation to improve Lee and Li's method [15]. Nevertheless, Cheng's method still has some drawbacks. Therefore, Chu and Tsao [14] proposed a ranking method based of the area between the centroid point and the original point to overcome the shortcomings of Cheng's method. Ramli and Mohamad [17] provided a detailed review of several ranking methods based on centroid or ranking index, and compared the effectiveness of these approaches.

In recent years, synthesizing the centroid point and the left and right (L-R) deviation degree to form an index value has attracted the attention of many researchers.

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This index value can be employed as a measurement for ranking fuzzy numbers. Wang et al. [18] proposed a novel method based on L-R deviation degree. The ranking of fuzzy numbers is determined by their ranking index values, which are determined by the relationship between transfer coefficient and the L-R deviation degree. However, Nejad and Mashinchi [19] pointed out that Wang et al.'s method is inconsistent in ranking for some cases. Thus, they suggested an improved approach based on the area of the left and the right side of fuzzy number and the concept of centroid point to overcome the shortcomings of Wang et al.'s method. Asady [20] pointed out that the inconsistency between the ranking of fuzzy numbers and their images is another shortcoming of Wang et al.'s method and proposed a new method to resolve this issue. However, several studies, Hajjari and Abbasbandy [21], and Hajjari [22] pointed out that some of these ranking methods [18]-[20] still cannot always provide consistent ranking results for fuzzy numbers and their images. To overcome these shortcomings, we propose a new epsilon-deviation degree approach which is capable of ranking generalized and symmetric fuzzy numbers consistently.

The rest of this paper is organized as follows. Section 2 briefly reviews the concept of fuzzy numbers. In Section 3, several existing deviation degree based fuzzy number ranking methods are reviewed. Their shortcomings are analyzed and demonstrated through counter-examples. In Section 4, we propose a new ranking approach based on epsilon-deviation degree and define an innovative index for ranking fuzzy numbers. The advantages of the proposed approach are verified by numerical examples in Section 5. Finally, conclusions are drawn in Section 6.

II. CONCEPT OF FUZZY NUMBER

In this section, the concept of fuzzy numbers is defined [7].

Definition 1.

A real fuzzy number $A = (a,b,c,d,\varpi)$ is described as any fuzzy subset of the real line R with membership function f_A which is given by

$$f_A(x) = \begin{cases} f_A^L(x), & a \le x \le b, \\ \varpi, & b \le x \le c, \\ f_A^R(x), & c \le x \le d, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where $0 \le \varpi \le 1$ is a constant.

$f_A^L(x) : [a,b] \to [0,\varpi]$ and $f_A^R(x) : [c,d] \to [0,\varpi]$ are two strictly monotonic and continuous, mapping from R to closed interval $[0,\varpi]$. Since $f_A^L(x)$ and $f_A^R(x)$ are strictly monotonic and continuous, their inverse functions exist.

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Let $g_A^L : [0, \varpi] \to [a,b]$ and $g_A^R : [0, \varpi] \to [c,d]$ denote the inverse functions of $f_A^L(x)$ and $f_A^R(x)$, respectively. Since $f_A^L(x) : [a,b] \to [0, \varpi]$ is continuous and strictly increasing, $g_A^L(x) : [0, \varpi] \to [a,b]$ is continuous and strictly increasing. Similarly, $f_A^R : [c,d] \to [0, \varpi]$ is continuous and strictly decreasing. Thus, $g_A^R : [0, \varpi] \to [c,d]$ is continuous and strictly decreasing. Both $g_A^L(x):$ and $g_A^R :$ should be integrable on the close interval $[0, \varpi]$. In other words, both $\int_0^\varpi g_A^L(y)dy$ and $\int_0^\varpi g_A^R(y)dy$ should exist.

Definition 2.

Let E be the set of all fuzzy numbers. A trapezoid fuzzy number $A = (a,b,c,d; \varpi)$ is described as any fuzzy subset of the real line R, with the membership function $f_A(x)$ expressed as [23]

$$f_A(x) = \begin{cases} \dfrac{\varpi(x-a)}{b-a}, & a \leq x \leq b, \\ \varpi, & b \leq x \leq c, \\ \dfrac{\varpi(d-x)}{d-c}, & c \leq x \leq d, \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

If $\varpi = 1$, then A is a normal fuzzy number; otherwise, it is said to be a non-normal fuzzy number. If $b \neq c$, A is referred to as a fuzzy interval [24] or a flat fuzzy number [25]. If $L_A(x)$ and $R_A(x)$ are both linear, then A is referred to as a trapezoidal fuzzy number and is usually denoted by $A = (a,b,c,d; \varpi)$ or simply $A = (a,b,c,d)$ if $\varpi = 1$. In particular, when $b = c$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number, and can be denoted by $A = (a,b,d; \varpi)$ or $A = (a,b,d)$ if $\varpi = 1$. Thus triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers.

Definition 3.

For a fuzzy set A, the support set of A is defined as follows[26]

$$S(A) = \{x \in R : f_A(x) > 0\}$$

The set of all fuzzy numbers is denoted by E. The image of fuzzy number $A = (a,b,c,d)$ is denoted by $-A = (-d,-c,-b,-a)$ [27], [28].

III. REVIEW OF EXISTING APPROACHES BASED ON DEVIATION DEGREE

In this section, the methods of Wang et al.'s [18], and Nejad and Mashinchi [19] are briefly reviewed. Several counter-examples are provided to indicate the shortcomings of these approaches.

A. Wang et al.'s Method [18].

Definition 4.

For any group of fuzzy numbers, $A_1, A_2, ..., A_n$ of E with support sets $S(A_i)$, $i = 1, 2, ..., n$. Let $S = \bigcup_{i=1}^n S(A_i)$. and $x_{\min} = \inf S$ and $x_{\max} = \sup S$. The minimal and maximal reference sets, $A_{\min}$ and $A_{\max}$, are defined by Chen [23] as

$$g_{A_{\min}}(x) = \begin{cases} \dfrac{x_{\max} - x}{x_{\max} - x_{\min}}, & \text{if } x \in S, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

$$g_{A_{\max}}(x) = \begin{cases} \dfrac{x - x_{\max}}{x_{\max} - x_{\min}}, & \text{if } x \in S, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

where $S = \bigcup_{i=1}^n S(A_i)$.

Definition 5.

For any group of fuzzy numbers, $A_1, A_2, ..., A_n$ in E, let $A_{\max}$ and $A_{\min}$ be the maximal and the minimal reference sets of these fuzzy numbers, respectively. The left and right deviation degrees of A_i, $i = 1, 2, ..., n$ are defined as

$$d_{A_i}^L = \int_{x_{\min}}^{\alpha_i} (f_{A_{\min}}(x) - f_{A_i}^L(x))dx, \tag{5}$$

$$d_{A_i}^R = \int_{\beta_i}^{x_{\max}} (f_{A_{\max}}(x) - f_{A_i}^R(x))dx, \tag{6}$$

where α_i and β_i, $i = 1, 2, ..., n$ are the abscissas of the crossover points of $g_{A_i}^L$, $g_{A_i}^R$, and $u_{A_{\min}}$, $u_{A_{\max}}$, respectively.

Definition 6.

For a L-R fuzzy number $A_i = (m_i, n_i, \alpha_i, \beta_i)_{LR}$, the expectation value of centroid is defined as [29].

$$M_i = \dfrac{\int_{m_i - \alpha_i}^{n_i + \beta_i} x f_{A_i}(x)dx}{\int_{m_i - \alpha_i}^{n_i + \beta_i} f_{A_i}(x)dx}. \tag{7}$$

For fuzzy numbers $A_1, A_2, ..., A_n$, the transfer coefficient of A_i $(i = 1, 2, ..., n)$ is given by [30].

$$\lambda_{A_i} = \dfrac{M_i - M_{\min}}{M_{\max} - M_{\min}} \tag{8}$$

where $M_{\max} = \max\{M_1, M_2, ... M_n\}$, $M_{\min} = \min\{M_1, M_2, ... M_n\}$ and $M_{\max} \neq M_{\min}$. Based on (7) and (8) the ranking index value of fuzzy number A_i, $i = 1, 2, ..., n$ is given by

$$d_{A_i} = \begin{cases} \dfrac{d_{A_i}^L \lambda_{A_i}}{1 + d_{A_i}^R(1 - \lambda_{A_i})}, & M_{\max} \neq M_{\min}, i = 1, 2, ..., n. \\ \dfrac{d_{A_i}^L}{1 + d_{A_i}^R}, & M_{\max} \neq M_{\min}, i = 1, 2, ..., n. \end{cases} \tag{9}$$

Using (9), the ranking order is defined as

1) $A_i \succ A_j$, if and only if $d_{A_i} > d_{A_j}$,

2) $A_i \prec A_j$, if and only if $d_{A_i} < d_{A_j}$,

3) $A_i \sim A_j$, if and only if $d_{A_i} = d_{A_j}$.

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B. Nejad and Mashinchi's Method [19].

Definition 7.

Let $A_i = (a_i, b_i, c_i, d_i, \varpi)$, $i = 1, 2, ..., n$ be fuzzy numbers and $a_{min} = \min\{a_1, a_2, ..., a_n\}$, $d_{max} = \max\{d_1, d_2, ..., d_n\}$. The left area $(s_{A_i}^L)$ and right area $(s_{A_i}^R)$ of A_i are defined as

$$s_{A_i}^L = \int_0^\varpi (g_{A_i}^L(y) - a_{min})dy, \qquad (10)$$

$$s_{A_i}^R = \int_0^\varpi (d_{max} - g_{A_i}^R(y))dy. \qquad (11)$$

Addition of two triangular fuzzy numbers $A_0 = (a_0, b_0, d_0)$ and $A_{n+1} = (a_{n+1}, b_{n+1}, d_{n+1})$ is used to prevent the value of d_i^L, d_i^R, λ_i, and $(1 - \lambda_i)$ being equal to zero, where

$a_0 = 2b_0 - d_0$, $b_0 = \min\{a_i, i = 1, 2, ..., n\}$,

$d_0 = (d + b_0)/2$, $d = \min\{d_i, i = 1, 2, ..., n\}$,

$a_{n+1} = (a + b_{n+1})/2$, $b_{n+1} = \max\{d_i, i = 1, 2, ..., n\}$,

$d_{n+1} = 2b_{n+1} - a_{n+1}$, $a = \max\{a_i, i = 1, 2, ..., n\}$.

For any fuzzy number, $A_i = (a_i, b_i, c_i, d_i; \varpi)$, the transfer coefficient of A_i, $i = 1, 2, ..., n$, is given by

$$\lambda_{A_i} = \frac{M_i - M_{min}}{M_{max} - M_{min}}, \qquad (12)$$

where $M_0 = \min\{M_0, M_1, ..., M_n\}$, $M_{n+1} = \max\{M_0, M_1, ..., M_n\}$. Based on (4) ~ (6), the ranking index value of A_i is given by

$$S_{A_i} = \frac{s_{A_i}^L \lambda_{A_i}}{1 + s_{A_i}^R (1 - \lambda_{A_i})}, \qquad (13)$$

Using (13), the ranking order is defined as

1) $A_i \succ A_j$, if and only if $S_{A_i} > S_{A_j}$,

2) $A_i \prec A_j$, if and only if $S_{A_i} < S_{A_j}$,

3) $A_i \sim A_j$, if and only if $S_{A_i} = S_{A_j}$.

C. Counter-examples

The drawbacks of these methods are analyzed and discussed through three counter-examples as follows.

The first weakness to be considered in Wang et al. [18] and Asady's method [20] is the construction of transfer coefficient. In these methods, transfer coefficient is constructed by the expectation value of centroid to present the relative variation of left and right deviation degree of a fuzzy number. However, when either transfer coefficient indicates that the relative variation of left or right deviation degree is equal to zero, the left deviation degree and the right deviation degree respectively, are worthless, as shown in Example 1.

Example 1.

Consider the following set of fuzzy numbers, $A_1 = (2, 4, 6)$, $A_2 = (1, 5, 6)$, $A_3 = (3, 5, 6)$ adopted from [19], as shown in Fig.1-3.

According to Wang et al.'s method [18], in set 1, $\lambda_1 = \lambda_2 = 0 \Rightarrow d_1 = d_2 = 0$. it is concluded that transfer

coefficients (λ_1, λ_2) are equal to zero, rendering worthless the left deviation degrees (d_1^L, d_2^L). Similarly, in set 2, $\lambda_1 = 0$, $\lambda_2 = \lambda_3 = 1 \Rightarrow 1 - \lambda_2 = 1 - \lambda_3 = 0 \Rightarrow d_1 = d_2 = 0$. Transfer coefficients (λ_1, λ_2) are equal to one and thus the right deviation degrees (d_1^R, d_2^R) become worthless.
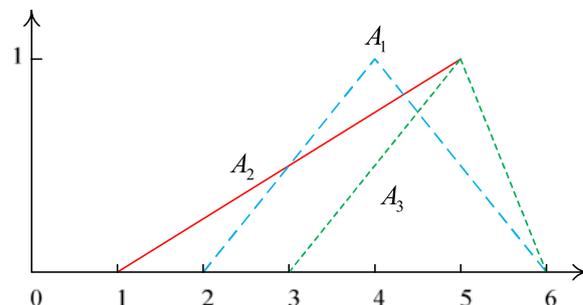

Fig. 1 Fuzzy numbers A_1, A_2 and A_3 in Set 1 of Example 1

Example 2.

Consider the following sets of fuzzy numbers adopted from [22], as shown in Fig. 4-5.

Set 1: $A_1 = (1, 2, 6)$, $A_2 = (2.5, 2.75, 3)$ and $A_3 = (2, 3, 4)$,

Set 2:

$$f_{A_1}(x) = \begin{cases} x - 1, & \text{if } x \in [1, 2], \\ 3 - x, & \text{if } x \in [2, 3], \\ 0, & \text{otherwise.} \end{cases}$$

$$f_{A_2}(x) = \begin{cases} \left[1 - (x-2)^2\right]^{1/2}, & \text{if } x \in [1, 2], \\ \left[1 - \frac{1}{4}(x-2)^2\right]^{1/2}, & \text{if } x \in [2, 4], \\ 0, & \text{otherwise.} \end{cases}$$

In set 1:

Applying Wang et al.'s method [18], the ranking order of the three fuzzy numbers is $A_2 \prec A_1 \prec A_3$ and the ranking order of their images is $-A_1 \sqcup -A_3 \prec -A_2$, which are not logical. Using the method of Nejad and Mashinchi[19], the ranking results are also unreasonable because the ranking order of fuzzy numbers and their images obtained are $A_1 \prec A_2 \prec A_3$, $-A_3 \prec -A_1 \prec -A_2$, respectively.

In set 2:

Using Asady's method [20], the ranking order obtained is $A_1 \prec A_2$. However, the ranking order of their images is $-A_1 \prec -A_2$.

Finally, these methods may not obtain correct ranking order when symmetric fuzzy numbers are involved. Wang et al. [18] and Asady's method [19] obtain the same ranking orders as shown in Example 3.

Example 3.

Consider symmetric triangular fuzzy numbers $A_1 = (0.2, 0.5, 0.8)$, $A_2 = (0.4, 0.5, 0.6)$ adopted from [22], as shown in Fig. 6. It can be verify that the centroid point of A_1 and A_2 are identical. According to Nejad and Mashinchi's

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method, the ranking result of A_1 and A_2 conflicts with the ranking order of their images. The ranking index values obtained $D_{A_1} = 0.148$, $D_{A_2} = 0.183$. Therefore, $A_1 \prec A_2$. However, $D_{-A_1} = 0.148$, $D_{-A_2} = 0.183$ so the order of their images is $-A_1 \prec -A_2$. Consequently, Nejad and Mashinchi's method is incorrect in this case. It can conclude that these examples indicate that the existing approach [19] based on deviation degree cannot rank fuzzy numbers correctly in all situations.
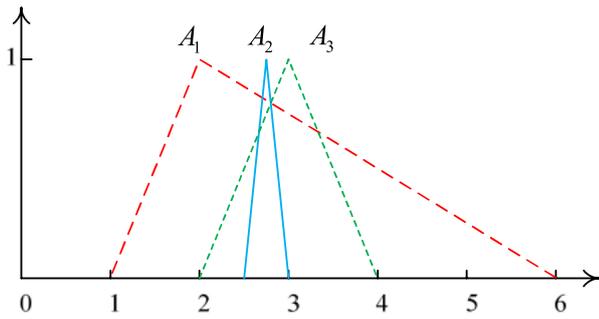

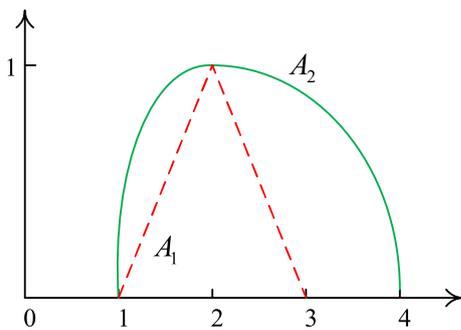Fig. 2 Fuzzy numbers A_1, A_2 and A_3 in Set 1 of Example 2

Fig. 3 Fuzzy numbers A_1, A_2 and A_3 in Set 2 of Example 2
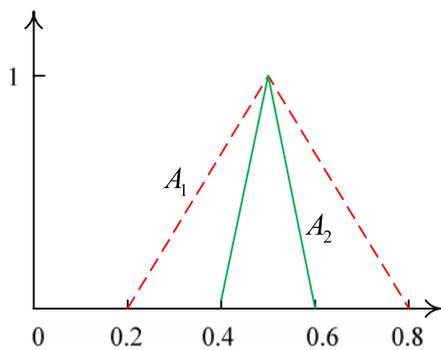

Fig. 4 Fuzzy numbers A_1, A_2 and A_3 in Example 3

IV. THE PROPOSED EPSILON-DEVIATION DEGREE METHOD

To overcome the weaknesses of the existing approaches based on deviation degree, we introduce an epsilon-deviation degree method for ranking fuzzy numbers $A_i = (a_i, b_i, c_i, d_i, \varpi)$, $i = 1, 2, ..., n$. The proposed ranking index combines epsilon-transfer coefficient with L-R

area of fuzzy numbers. This index is reliable for ranking not only generalized fuzzy numbers, but also symmetric fuzzy numbers

Definition 8.

For any fuzzy number, $A_i = (a_i, b_i, c_i, d_i; \varpi)$, the epsilon-transfer coefficient of A_i, $i = 1, 2, ..., n$, based on expectation value of centroid of fuzzy number, is given by

$$\tau_i = \frac{x_{M_i} - x_{M_{\min}} + \dfrac{\varepsilon_1}{2}}{x_{M_{\max}} - x_{M_{\min}} + \varepsilon_1}, \tag{14}$$

Where $0 < \varepsilon_1 \le 1$,

$x_{M_{\min}} = \min\{x_{M_1}, x_{M_2}, ..., x_{M_n}\}$ and

$x_{M_{\max}} = \max\{x_{M_1}, x_{M_2}, ..., x_{M_n}\}$. Adding an approximate epsilon to avoid transfer coefficient becoming zero is an advantage of the proposed method.

Definition 9.

The innovative ranking index value is defined as:

$$D_i = \frac{\varepsilon_2 + d_i^L \tau_i}{\varepsilon_2 + d_i^R (1 - \tau_i)}, \tag{15}$$

where $i = 1, 2, ..., n$, $\rho > 0$, $0 < \varepsilon_2 \le 1$.

The left deviation degree and the right deviation degree [18] based on the minimal and maximal reference sets are not applicable to the calculation of the L-R area of generalized fuzzy numbers with different ϖ. To overcome this limitation, the L-R areas of triangular fuzzy numbers are defined as

$$d_i^L = \int_{x_{\min}}^{b_i} \varpi_i \, dx - \int_{a_i}^{b_i} \varpi_i f_{A_i}^L(x) \, dx$$
$$= \varpi_i(b_i - x_{\min}) - \varpi_i \int_{a_i}^{b_i} f_{A_i}^L(x) \, dx, \tag{16}$$

$$d_i^R = \int_{b_i}^{x_{\max}} \varpi_i \, dx - \int_{b_i}^{c_i} \varpi_i f_{A_i}^R(x) \, dx$$
$$= \varpi_i(x_{\max} - b_i) - \varpi_i \int_{b_i}^{c_i} f_{A_i}^R(x) \, dx. \tag{17}$$

The L-R areas for trapezoidal fuzzy numbers are defined as

$$d_i^L = \int_{x_{\min}}^{b_i} \varpi_i \, dx - \int_{a_i}^{b_i} \varpi_i f_{A_i}^L(x) \, dx$$
$$= \varpi_i \left(\frac{a_i + b_i}{2} - x_{\min} \right),$$

$$d_i^R = \int_{c_i}^{x_{\max}} \varpi_i \, dx - \int_{c_i}^{d_i} \varpi_i f_{A_i}^R(x) \, dx$$
$$= \varpi_i \left(x_{\max} - \frac{c_i + d_i}{2} \right).$$

where $x_{\min} = \min\{x_1, x_2, ..., x_n\}$ and $x_{\max} = \max\{x_1, x_2, ..., x_n\}$.

The ranking order is defined on the basis of the following properties:

1) $A_i \succ A_j$, if and only if $D_{A_i} > D_{A_j}$,

2) $A_i \prec A_j$, if and only if $D_{A_i} < D_{A_j}$,

3) $A_i \sqcup A_j$, if and only if $D_{A_i} = D_{A_j}$.

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Algorithm 1:

Step 1.

Calculate d_i^L and d_i^R of each fuzzy number, according to (16) and (17).

Step 2.

Determine the transfer coefficient τ_{A_i} through (14).

Step 3.

Obtain the ranking index value D_i using (15).

Step 4.

Rank fuzzy numbers $A_1, A_2, ..., A_n$ based on the ranking index value using Definition 9.

V. NUMERICAL EXAMPLES

In this section, the reliability and superiority of the epsilon-deviation degree method are demonstrated through the following examples. These examples address the shortcoming of existing deviation degree based methods by Wang et al. [18], Asady[20], and Nejad and Mashinchi [19]. In addition, the ranking results are compared with previous studies to validate the proposed method.

Consider the three counter examples discussed in Section 3. Let $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-4}$ in all the examples.

Example 1.

Set 1:

Following Algorithm 1, the ranking index values obtained are $D_{A_1} = 2.949$, $D_{A_2} = 5.757$, $D_{A_3} = 15.384$. The ranking order of fuzzy numbers is therefore $A_1 \prec A_2 \prec A_3$. The ranking order of their images is $-A_1 \succ -A_2 \succ -A_3$ since $D_{-A_1} = 0.339$, $D_{-A_2} = 0.174$, $D_{-A_3} = 0.065$.

Set 2:

According to Algorithm 1, we have $D_{A_1} = 0.049$, $D_{A_2} = 0.126$, $D_{A_3} = 0.09$, $D_{-A_1} = 20.494$, $D_{-A_2} = 7.957$, $D_{-A_3} = 11.119$. The ranking order of fuzzy numbers is therefore $A_2 \succ A_3 \succ A_1$ and the ranking order of their images is $-A_1 \succ -A_3 \succ -A_2$. It can be seen that the ranking result is reasonably using the proposed method.

Set 3:

Applying the proposed method, we obtain $D_{A_1} = 0.133$, $D_{A_2} = 0.2.18$, $D_{-A_1} = 7.516$, $D_{-A_2} = 4.589$. The ranking orders of the fuzzy numbers is $A_1 \prec A_2$, consistent with the ranking order of their images $-A_1 \succ -A_2$. The result agrees with most of the existing approaches [3, 19, 20, 31]. It can be concluded that the epsilon-deviation degree overcomes

the shortcoming of L-R deviation degree becoming worthless in existing deviation degree based methods.

Example 2.

Set 1:

By using the proposed method, we have $D_{A_1} = 0.174$, $D_{A_2} = 0.284$, $D_{A_3} = 0.404$. The ranking order of fuzzy numbers is therefore $A_2 \prec A_1 \prec A_3$. Since $D_{-A_1} = 5.757$, $D_{-A_2} = 3.522$, $D_{-A_3} = 2.473$, the order of their images is $-A_2 \succ -A_1 \succ -A_3$. The ranking order of fuzzy numbers is consistent with that of their images. Therefore, the proposed approach resolves the shortcomings of existing deviation degree based methods [18]-[20].

Set 2:

Using the proposed method, we obtain $D_{A_1} = 0.612$, $D_{A_2} = 0.072$, $D_{-A_1} = 1.631$, and $D_{-A_2} = 1.397$. Consequently, the ranking order of the fuzzy numbers is $A_1 \succ A_2$ and the ranking order of their images is $-A_2 \succ -A_1$. The results agree with the results obtained by Asady's method [20]. It can be seen that the ranking results of the proposed method are both consistent and intuitive.

TABLE I
COMPARING THE RANKING RESULTS OF THE PREVIOUS STUDIES OF EXAMPLE 2

Method	Index value of fuzzy numbers			Index value of their images		
Abbasbandy and Hajari (2009)	2.25	2.75	3	-2.25	-2.75	-3
	$A_1 \sqcup A_2 \prec A_3$			$-A_1 \sqcup -A_2 \succ -A_3$		
Wang et al. (2009)	0.417	0	1.167	0	2.119	0
	$A_2 \prec A_3 \prec A_1$			$-A_2 \succ -A_1 \sqcup -A_3$		
Asady (2010)	3.5	0	4	0	6.5	0
	$A_2 \prec A_1 \prec A_3$			$-A_2 \succ -A_1 \sqcup -A_3$		
Nejad and Mashinchi (2011)	0.184	0.220	0.281	1.40	1.65	1.27
	$A_1 \prec A_2 \prec A_3$			$-A_2 \succ -A_1 \succ -A_3$		
Asady (2011)	2.743	2.75	3	-2.743	-2.75	-3
	$A_1 \prec A_2 \prec A_3$			$-A_1 \succ -A_2 \succ -A_3$		
Proposed method	0.174	0.284	0.404	5.757	3.522	2.47
	$A_1 \prec A_2 \prec A_3$			$-A_1 \succ -A_2 \succ -A_3$		

Example 3.

Following Algorithm 1, the ranking indices of the fuzzy numbers are $D_{A_1} = D_{A_2} = 1$ so $A_1 \sim A_2$. The ranking of their image is $-A_1 \sim -A_2$ since $D_{-A_1} = D_{-A_2} = 1$. Apparently, the

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results of this example show that the revised approach can tackle the case where the centroid point of the fuzzy numbers is the same.

The superiority of the epsilon-deviation degree method is shown by employing the attitude of decision maker to rank symmetric fuzzy numbers consistently. In addition, the proposed method ranks similar fuzzy number with strong discrimination, as demonstrated by Examples 4 and 5.

Example 4.

Consider the three fuzzy numbers $A_1 = (1, 1, 1, 15)$, $A_2 = (-2, 4, 10)$, and $A_3 = (0, 2, 6, 8)$, as shown in Fig.8. Using Algorithm 1, we have $D_{A_1} = 0.489$, $D_{A_2} = 0.341$, and $D_{A_3} = 0.341$. In this case, the L-R deviation degree and the expectation of centroid point of fuzzy number of A_2 and A_3 are equal, i.e. $d_{A_2}^l = d_{A_3}^l = 3$, $d_{A_2}^r = d_{A_3}^r = 7$, $M_{A_2} = M_{A_3} = 4$. As a result, the ranking order is $A_1 \succ A_2 \sim A_3$.
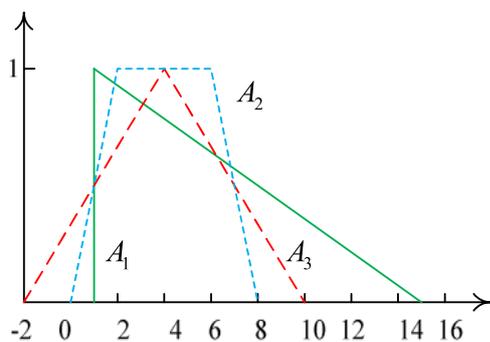

Fig. 5 Fuzzy numbers A_1, A_2 and A_3 in Example 4

Example 5.

Consider the three triangular fuzzy numbers $A_1 = (5, 6, 7)$, $A_2 = (5.9, 6, 7)$, and $A_3 = (6, 6, 6, 7)$, as shown in Fig. 9. The ranking index values obtained are $D_{A_1} = 1$, $D_{A_2} = 1.309$, and $D_{A_3} = 1.350$. The order of the fuzzy numbers is therefore $A_1 \prec A_2 \prec A_3$. The order of their images is $-A_1 \succ -A_2 \succ -A_3$ since $D_{-A_1} = 1$, $D_{-A_2} = 0.7634$, $D_{-A_3} = 0.7407$. As can be seen from Table 2, the ranking result obtained by the proposed approach agrees with most of the previous approaches.

VI. CONCLUSION

This study proposes an epsilon-deviation degree approach based on deviation degree. There are four advantages of this approach which overcome existing deviation degree methods. First, an epsilon-transfer coefficient is proposed to overcome the problem of the L-R deviation degree becoming worthless. Second, a reliable ranking index is suggested to avoid the inconsistency between the ranking order of fuzzy numbers and that of their images, which exists in existing deviation degree

based approaches [8-10]. Third, the proposed approach ranks symmetric fuzzy numbers effectively by considering the optimism index. Lastly, the computation procedure of the proposed approach is simpler than existing approaches. These high-quality characteristics of the proposed method make it a valuable tool for comparing and ranking fuzzy numbers.
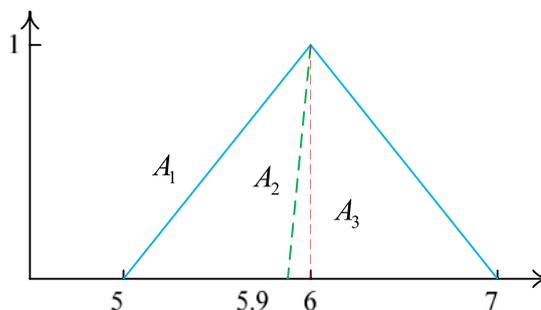

Fig. 6 Fuzzy numbers A_1, A_2 and A_3 in Example 5

TABLE II
COMPARING THE RANKING RESULTS OF THE PREVIOUS STUDIES OF EXAMPLE 5

Method	Index value of fuzzy numbers			Index value of their images		
Abbasbandy and Hajari (2009)	6	6.075	6.083	-6	-6.075	-6.083
	$A_1 \prec A_2 \prec A_3$			$-A_1 \succ -A_2 \succ -A_3$		
Wang et al. (2009)	0.000	0.620	0.750	0.333	0.020	0
	$A_1 \prec A_2 \prec A_3$			$-A_1 \succ -A_2 \succ -A_3$		
Asady (2010)	0.666	0.960	1	0.66	0.449	0.428
	$A_1 \prec A_2 \prec A_3$			$-A_1 \succ -A_2 \succ -A_3$		
Nejad and Mashinchi(2011)	0.5	0.938	1	0.285	0.154	0.142
	$A_1 \prec A_2 \prec A_3$			$-A_1 \succ -A_2 \succ -A_3$		
Asady (2011)	6	6.225	6.25	-6	-6.225	-6.25
	$A_1 \prec A_2 \prec A_3$			$-A_1 \succ -A_2 \succ -A_3$		
Proposed method	1	3.381	3.817	1	0.296	0.262
	$A_1 \prec A_2 \prec A_3$			$-A_1 \succ -A_2 \succ -A_3$		

REFERENCES

[1] R. Jain, Decisionmaking in the Presence of Fuzzy Variables, IEEE Transactions on Systems, Man and Cybernetics, 6 (1976) 698-703.
[2] S. Abbasbandy, B. Asady, Ranking of fuzzy numbers by sign distance, Information Sciences, 176 (2006) 2405-2416.
[3] S. Abbasbandy, T. Hajjari, A new approach for ranking of trapezoidal fuzzy numbers, Computers & Mathematics with Applications, 57 (2009) 413-419.
[4] B. Asady, A. Zendehnam, Ranking fuzzy numbers by distance minimization, Applied Mathematical Modelling, 31 (2007) 2589-2598.
[5] S.M. Baas, H. Kwakernaak, Rating and ranking of multiple-aspect alternatives using fuzzy sets, Automatica, 13 (1977) 47-58.
[6] J.F. Baldwin, N.C.F. Guild, Comparison of fuzzy sets on the same decision space, Fuzzy Sets and Systems, 2 (1979) 213-231.
[7] D. Dubois, H. Prade, Operations on fuzzy numbers, International Journal of Systems Science, 9 (1978) 613-626.
[8] E. Kerre, E., The use of fuzzy set theory in electrocardiological diagnostics, in: MM. Gupta and E. Sanchez, Eds, Approximate

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Reasoning in Decision Analysis (North-Holland, Amsterdam, 1982), (1982) 277-282.

[9] A. Kumar, P. Singh, A. Kaur, Ranking of Generalized Exponential Fuzzy Numbers using Integral Value Approach, International Journal of Advances in Soft Computing and Its Applications, 2 (2010) 209-220.

[10] H. Xiaowei, D. Hepu, An Area-based Approach to Ranking Fuzzy Numbers in Fuzzy Decision Making, Journal of Computational Information Systems, 7 (2011) 3333- 3342

[11] R.R. Yager, Ranking fuzzy subsets over the unit interval, in: Decision and Control including the 17th Symposium on Adaptive Processes, 1978 IEEE Conference on, 1978, pp. 1435-1437.

[12] S.-Y. Chou, L.Q. Dat, V.F. Yu, A revised method for ranking fuzzy numbers using maximizing set and minimizing set, Computers & Industrial Engineering, 61 (2011) 1342-1348.

[13] C.H. Cheng, A new approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Systems, 95 (1998) 307-317.

[14] T.C. Chu, C.T. Tsao, Ranking fuzzy numbers with an area between the centroid point and original point, Computers & Mathematics with Applications, 43 (2002) 111-117.

[15] E.S. Lee, R.J. Li, Comparison of fuzzy numbers based on the probability measure of fuzzy events, Computers & Mathematics with Applications, 15 (1988) 887-896.

[16] S. Mabuchi, An approach to the comparison of fuzzy subsets with an α-cut dependent index, Systems, Man and Cybernetics, IEEE Transactions on, 18 (1988) 264-272.

[17] N. Ramli, D. Mohamad, A comparative analysis of centroid methods in ranking fuzzy numbers, European Journal of Scientific Research, 28 (2009) 492-501.

[18] Z.X. Wang, Y.J. Liu, Z.P. Fan, B. Feng, Ranking L-R fuzzy number based on deviation degree, Information Sciences, 179 (2009) 2070-2077.

[19] A.M. Nejad, M. Mashinchi, Ranking fuzzy numbers based on the areas on the left and the right sides of fuzzy number, Computers & Mathematics with Applications, 61 (2011) 431-442.

[20] B. Asady, The revised method of ranking LR fuzzy number based on deviation degree, Expert Systems with Applications, 37 (2010) 5056-5060.

[21] T. Hajjari, S. Abbasbandy, A note on "The revised method of ranking LR fuzzy number based on deviation degree", Expert Systems with Applications, 38 (2011) 13491-13492.

[22] T. Hajjari, On Deviation Degree Methods for Ranking Fuzzy Numbers, Australian Journal of Basic and Applied Sciences, 5 (2011) 750-758.

[23] S.-H. Chen, Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets and Systems, 17 (1985) 113-129.

[24] D. Dubois, H. Prade, Ranking fuzzy numbers in the setting of possibility theory, Information Sciences, 30 (1983) 183-224.

[25] B. Matarazzo, G. Munda, New approaches for the comparison of L–R fuzzy numbers: a theoretical and operational analysis, Fuzzy Sets and Systems, 118 (2001) 407-418.

[26] C.-H. Cheng, D.-L. Mon, Fuzzy system reliability analysis by interval of confidence, Fuzzy Sets and Systems, 56 (1993) 29-35.

[27] A. Kaufmann, M. Gupta, Introduction to Fuzzy Arithmetic: Theory and Applications 2nd ed ed., Van Nostrand Reinhold, New York, 1991.

[28] L.A. Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.

[29] H.B. Mitchell, P.A. Schaefer, On ordering fuzzy numbers, International Journal of Intelligent Systems, 15 (2000) 981-993.

[30] Z.-X. Wang, Y.-J. Liu, Z.-P. Fan, B. Feng, Ranking L–R fuzzy number based on deviation degree, Information Sciences, 179 (2009) 2070-2077.

[31] B. Asady, Revision of distance minimization method for ranking of fuzzy numbers, Applied Mathematical Modelling, 35 (2011) 1306-1313.