To,

Professor Ralph Cohen

Chief Editor

Communications of the American Mathematical Society

Dear Professor Cohen,

I hope and pray you and your family are all well.

I am herewith submitting my original manuscript "On the Structure and Logic of Conceptual Mind" to be considered for publication in your journal: Communications of the American Mathematical Society. In my manuscript, using category theoretic mathematical methods and constructions, I address two foundational questions of the science of mind:

What is the abstract essence of the mind?

What is the objective logic of the mind?

The genesis of my investigations is the principle guiding the [so-called] new science of mind: "mind is a set of processes carried out by the brain" (https://www.cell.com/action/showPdf?pii=S0896-6273%2813%2900991-4). This is like saying: society is a collection or set of people. Surely, a society is different from the sum of its people. For this reason, conceptualizing society as a set is, at best, a first approximation. The additional structure of a society, above and beyond that of 'set', is in the way its people are related to one another. The same is true of the mind. We can find the structure of human mind by looking at how its contents are related to one another. Upon examining the relations between mental contents, we find that conceptual mind has the mathematical structure of a graph. (This is analogous to saying that language is not merely a collection of words, but also has sentences, which have words as their subject / predicate.) Next, objective logic of the mind is calculated from its structural essences. Particularly noteworthy features of the logic of the mind are degrees of truth, varieties of negation, admission of contradiction, and failure of one of the two de Morgan's laws.

I'd like to note that this is the first time that the mathematics of calculating the objective logic of a universe of discourse from its structural essence is applied to find the logic intrinsic to the mind. I also show how the unity of mind, which has been recognized since antiquity but left unaccounted, follows from its reflexive graph structure. Once again, my manuscript is the first to bring the mathematical definition of cohesion to bear on the long-standing question of the unity of mind. Equally importantly, the mathematics of abstracting the essence of mind and the subsequent calculation of its objective logic is presented in a manner readily accessible to the multidisciplinary investigators of the mind. As such, I am confident that my work will inspire further applications of category theory to elucidate the structural essence and logical form of various notions encountered in the study of mind and matter.

Summing it all, my manuscript, in mathematically answering the age-old questions of the science of human mind, paves way for a useful theoretical understanding of the mental realm on par with that of the indispensable physical theories of the material world.

If I may, I’d like to request you to review my manuscript.

The following external reviewers may also be considered for reviewing my manuscript.

Professor Michael A. Arbib (arbib@usc.edu)

Professor Andrée C. Ehresmann (andree.ehresmann@u-picardie.fr)

Professor Donald Geman (geman@jhu.edu)

Professor Valeria Giardino (Valeria.Giardino@ens.fr)

Professor Piet Hut (piet@ias.edu)

Professor F. William Lawvere (wlawvere@buffalo.edu)

Professor Giuseppe Longo (Giuseppe.Longo@ens.fr)

Professor Richard P. Stanley (rstan@math.mit.edu)

I earnestly hope that you will find my original paper suitable for publication in your journal: Communications of the American Mathematical Society. I sincerely thank you for your kind consideration of my manuscript and eagerly look forward to hearing from you.

Thanking you,

Yours truly,

Posina Venkata Rayudu

ORCID ID: https://orcid.org/0000-0002-3040-9224

Google Scholar: https://scholar.google.co.in/citations?user=cnMxV9MAAAAJ&hl=en&oi=ao

Email: posinavrayudu@gmail.com; Mobile: +91-963-222-4686

Title: On the Structure and Logic of Conceptual Mind

Author: Venkata Rayudu Posina

ORCID ID: https://orcid.org/0000-0002-3040-9224

Running title: Conceptual Mind

Word count: 7691; **Figure count:** 9

2020 Mathematics Subject Classification: 18D99, 00A69.

Affiliation: Independent Scientist

Address for correspondence: Posina Venkata Rayudu, 101-B2 Swathi Heights, A. S. Rao Nagar, Hyderabad - 500062, Telangana, India

Email: posinavrayudu@gmail.com; **Mobile:** +91-963-222-4686

Acknowledgements: I dedicate my paper, with gratitude and admiration, to late Professor Baldev Raj. I thank Dr. Ruadhan O'Flanagan for suggesting the application of category theory to the study of concepts and reasoning, Professor Andrée C. Ehresmann for kindly correcting mistakes in an earlier version, and Professor F. William Lawvere for invaluable help in learning category theory. I am grateful for the NIAS-Mani Bhaumik and NIAS-Consciousness Studies Programme Research Fellowships.

Title: On the Structure and Logic of Conceptual Mind

Abstract

Mind, according to cognitive neuroscience, is a set of brain functions. But, unlike sets, our minds are cohesive. Moreover, unlike the structureless elements of sets, the contents of our minds are structured. Mutual relations between mental contents endow the mind its structure. Here we characterize the structural essence and the logical form of the mind by focusing on thinking. Examination of the relations between concepts, propositions, and syllogisms involved in thinking revealed the reflexive graph structure of the conceptual mind. Objective logic of the conceptual mind is calculated from its structure. Noteworthy features of the logic of conceptual mind are degrees of truth, varieties of negation, admission of contradiction, and failure of a de Morgan's law. Furthermore, cohesion of the conceptual mind follows from its reflexive graph structure. Our characterization of the structure and logic of mind constitutes a substantial refinement of the contemporary cognitive neuroscientific conceptualization of the mind as a set.

Keywords: category theory; cohesion; negation; proposition; reflexive graph; syllogism; truth.

1. Introduction

Mind is useful in making sense of and maneuvering through reality. As such, mind has been an object of serious study since antiquity. Carefully thinking about thinking, which takes place within our minds, led to logic (Lawvere & Rosebrugh, 2003, pp. 193-195, 239-240). Recently, cognitive neuroscience has highlighted the differences between unconscious and conscious thought (Kandel, 2013; Kahneman, 2013). Fascinating as these may be, we still do not have a clear understanding of the nature and workings of the mind (Fodor, 2006). In the present note, as part of scientifically accounting for the effectiveness of mind in the material world (Lawvere, 1980, pp. 377-379; Lawvere, 1994, pp. 43-44; Lawvere & Schanuel, 2009, pp. 84-85; Picado, 2007, p. 25), we address two foundational questions of the science of mind:

What is the structural essence of mind?

What is the objective logic of mind?

We begin with the contemporary cognitive neuroscientific conceptualization of mind: 'mind is a set of processes carried out by the brain' (Kandel, 2013, p. 546; see also Bunge, 1981, p. 68; Kandel et al., 2013, p. 5, 334, 384). In contrast to the structureless elements of a set, the contents of our minds [even when identified with neural processes] are structured. More importantly, since sets have no other property besides the number of elements that they contain i.e. size (Lawvere & Rosebrugh, 2003, p. 1), if minds are sets, then all that we can say about minds: mind A is bigger than mind B; mind X is smaller than mind Y, etc. However, we have many more things to say about minds (e.g. brilliant mind, restless mind, etc.), besides their size. Thus the idea of mind as a set is, at best, a first approximation. In other words, mind is much more structured than a set.

In an effort to refine the current conceptualization of mind as a set, we examine the relations between mental contents, which endow mind its structure. We treat mind as a space where thinking takes place. More explicitly, we limit our consideration to the thinking part of the mind i.e. conceptual mind. Thinking involves concepts, propositions, and combinations of propositions as part of reasoning, i.e. syllogisms. Examination of the relations between concepts and propositions led us to put forth the structure of graph (Lawvere & Schanuel, 2009, pp. 141-142) as an essence of the conceptual mind. In characterizing the essence (theory) of mind, we are using the mathematical method of theorizing about objects, which, in the words of F. William Lawvere, 'consists of taking the main structure [of an object], in the sense that it is mainly responsible for the workings of the object, by itself as a first approximation to a theory of the object, i.e. mentally operating as though all further structure of the object simply did not exist' (Lawvere, 1972, pp. 9-10). Our mathematical characterization of conceptual mind is along the lines of Lawvere's category theoretic characterization of kinship (Lawvere, 1999).

Objective logic of a universe of discourse (e.g. sets, graphs) follows from the structural essence(s) of the universe (Lawvere & Schanuel, 2009, pp. 149-151, 339-347). Using this general method, we calculated the logic of conceptual mind from its structural essence of graph. The logic of conceptual mind, with its degrees of truth and varieties of negation, differs markedly from the Boolean logic of sets. In this context, failure of the de Morgan's law:

not (X *and* Y) = *not* (X) *or not* (Y) is particularly noteworthy (see Lawvere & Rosebrugh, 2003, p. 200). Upon further examination, we find that conceptual mind has the added structure of reflexive graph (Lawvere & Schanuel, 2009, p. 145). We show that the conceptual mind, in light of its reflexive graph structure, is cohesive (Lawvere, 2005, 2007).

In accounting for the combination of propositions as part of reasoning (syllogisms), we further refine our model of conceptual mind as an object consisting of three component sets:

(set of concepts, set of propositions, set of syllogisms)

equipped with eight structural functions specifying the relations between concepts, propositions, and syllogisms. In the following, we provide an intuitively accessible description of structural essences and calculation of the objective logic from structural essences. In our subsequent work, we plan to provide a category theoretic account of abstracting the theoretical essence(s) of minds, and of interpreting the thus abstracted essences to obtain concrete models of the mind in terms of functorial semantics (Lawvere, 2004).

2. Structural Essence of Conceptual Mind

How are we going to find the structural essence(s) of mind? The structure of an object is determined by its contents and their mutual relations. Thus, a first step in characterizing the structure of a given object is to find its contents and their interrelationships. We use this general method to characterize the structure of mind. If we imagine looking into minds, we might find, for example, some concepts such as DOG, GOOD, SKY, etc. in one mind X, and another such lot of concepts LINE, RED, WALK, etc. in another mind Y. If concepts in a mind are all that there are in the mind, then, with concepts as structureless elements, mind can be modeled as a set (Fig. 1a). With minds as sets, the structural essence of minds is a single-element set **1** = {•} (Fig. 2a; Lawvere & Schanuel, 2009, p. 245; Reyes, Reyes, & Zolfaghari, 2004, p. 30). Simply put, having a concept is the essence in which all minds partake, and with which every mind can be constructed. There is, of course, more to a mind than the concepts that it contains. Upon looking further into our minds, we might find, in addition to concepts, a set of propositions {SKY is CLEAR, WATER is CLEAN…} in one mind X, and another set of propositions {BIRD is FLYING, BUS is RED…} in another mind Y. With both concepts and propositions represented as structureless elements, albeit of two different types of sets, mind can be modeled as a pair of sets: (a set of concepts, a set of propositions) (Fig. 1b; Reyes, Reyes, & Zolfaghari, 2004, p. 17).

Concepts and propositions are, however, not unconnected [sets] within our minds. Concepts and propositions in our minds are related to one another in systematic ways. In particular, the subject of a proposition is a concept (e.g. *subject* (SKY is CLEAR) = SKY); so is its predicate (*predicate* (SKY is CLEAR) = CLEAR). Thus, mind can be modeled as a pair of sets:

(a set C of concepts, a set P of propositions)

equipped with a parallel pair of functions:

(*subject*: P –> C, *predicate*: P –> C)

assigning to each proposition in the set P of propositions its subject, predicate concept in the set C of concepts. These relations between concepts and propositions endow mind the structure of irreflexive graph (Lawvere & Schanuel, 2009, pp. 141-142). In modeling minds as irreflexive graphs, concepts and propositions within a mind are represented as dots and arrows, respectively. To each arrow representing a proposition, there is a source and a target dot representing the subject and the predicate concept, respectively, of the proposition (Fig. 1c). With minds as irreflexive graphs, the structural essence of minds is a pair of graph morphisms specifying the inclusion of concept into proposition as its subject, predicate (Fig. 2b; Lawvere & Schanuel, 2009, p. 150). In the next section, we characterize the logic of conceptual mind that follows from these structural essences.

3. Objective Logic of the Conceptual Mind

Objective logic of a universe of discourse (e.g. category of sets) is the logic intrinsic to the universe. Logical operations (*and*, *or*, *not*) can be characterized in terms of the truth value object (totality of truth values) of the universe (Lawvere & Rosebrugh, 2003, pp. 193-201; Lawvere & Schanuel, 2009, pp. 335-357). The totality of parts of the essence (abstract general) of a given universe of discourse constitutes the truth value object of the universe (Reyes, Reyes, & Zolfaghari, 2004, pp. 93-101; see Appendix for the calculation of truth value objects). We now characterize the logic of mind using these methods.

If minds are sets (of concepts, with concepts as structureless elements; Fig. 1a), then the logic of minds is the logic of sets. The truth value object of sets is a two-element set Ω = {false, true} (Fig. 3a). The two-element truth value set can be calculated from the essence of sets, which is a single-element set **1** = {•} (Fig. 2a). The single element set **1** has two parts (**0** = {}, **1** = {•}), which correspond to the two elements (false, true, respectively) of the truth value set **Ω** (Lawvere & Schanuel, 2009, p. 343, 353; Reyes, Reyes, & Zolfaghari, 2004, pp. 95-96). These two elements are the two possible truth values (false, true) a statement (to give an illustration): 'FUNCTOR *is in* X', asserting that a concept FUNCTOR is in a part X (of a mind M), can take.

Once we have the truth value object **Ω**, we can characterize logical operations (*and*, *or*, *not*) as maps to and from the truth value object (Lawvere & Schanuel, 2009, pp. 353-355). The negation operation

$$\textit{not}: \mathbf{\Omega} \rightarrow \mathbf{\Omega}$$

is an endomap on the truth value object **Ω**, while binary operations

$$\textit{and}: \mathbf{\Omega} \times \mathbf{\Omega} \rightarrow \mathbf{\Omega}$$

$$\textit{or}: \mathbf{\Omega} \times \mathbf{\Omega} \rightarrow \mathbf{\Omega}$$

are projection maps from the product **Ω** × **Ω** to **Ω**. A complete specification of these logical operations (construed as maps) is as follows:

not (false) = true, *not* (true) = false

and (false, false) = false, *and* (true, false) = false, *and* (false, true) = false, *and* (true, true) = true

or (false, false) = false, *or* (true, false) = true, *or* (false, true) = true, *or* (true, true) = true.

Note that, in the case of sets, double negation applied to any part A (of a given object) results in the same part, i.e.

not (*not* (A)) = A

Also, note that logical contradiction, by the definition of *not* operation, equals false, i.e.

A *and not* (A) = false

(Lawvere & Schanuel, 2009, p. 355). Furthermore, the two de Morgan's laws:

not (A *and* B) = *not* (A) *or not* (B)

not (A *or* B) = *not* (A) *and not* (B)

which relate the three logical operations (*and*, *or*, *not*), are satisfied in the case of sets. These characteristic features of the logic of sets are not shared by the logic of conceptual mind, which becomes apparent once we recognize the graph structure of conceptual mind (Fig. 2b).

With minds as irreflexive graphs (Fig. 1c), the first thing we notice is the degrees of truth in between false and true (Fig. 3b). Consider a mind M consisting of a proposition P, say, 'SKY is BLUE'. Given a part C (say, conscious part of M), a statement—P *is in* C—can take the truth value: *true*, if P is in C. The statement is *false*, if P is not in C. In addition to these two truth values, there are three more truth values: (i) *tt* if the proposition P is not in C, but its subject and predicate concepts (SKY, BLUE) are in C, (ii) *tf* if the proposition P is not in C, but its subject (SKY) is in C, and (iii) *ft* if the proposition P is not in C, but its predicate (BLUE) is in C. The totality of these five truth values is the truth value object of conceptual minds (see Appendix for the calculation of the truth value object). Note that these five degrees of truth correspond to the five parts of the generic proposition (e.g. SKY is BLUE). The five parts are: 1. entire proposition (SKY is BLUE); 2. subject and predicate concepts (SKY, BLUE); 3. subject (SKY); 4. predicate (BLUE); and 5. empty (Lawvere & Schanuel, 2009, pp. 344-346). In addition to these degrees of truth, which distinguish the logic of conceptual minds from that of sets, conceptual minds (modeled as irreflexive graphs) admit varieties of negation, as discussed below.

A familiar negation is the logical operation *not*, which is defined as: for any part X of an object, *not* (X) is the part of the object that is largest among all parts whose intersection with X is empty (Lawvere & Schanuel, 2009, p. 355). A different negation operation *non* can be defined dually: for any part X of an object, *non* (X) is the part of the given object that is smallest among all parts whose union with X is the entire object (Lawvere, 1986, 1991). Unlike the case of sets, where *non* and *not* are identical operations, in the case of conceptual minds (construed as irreflexive graphs), these two operations give different results (Fig. 4a). In this context, it is fascinating to note that the negation operation *non*, unlike *not*, permits logical contradiction (Fig. 4b; Lawvere, 1991, 1994; Lawvere & Rosebrugh, 2003, p. 201). Also note that, depending on the exact form of negation, double negation can be larger

$$\mathit{not}\ (\mathit{not}\ (\mathrm{A})) > \mathrm{A}$$

or smaller

$$\mathit{non}\ (\mathit{non}\ (\mathrm{A})) < \mathrm{A}$$

than the identity operation (Fig. 4c, d). More importantly, one of the de Morgan's laws:

$$\mathit{not}\ (\mathrm{X}\ \mathit{and}\ \mathrm{Y}) = \mathit{not}\ (\mathrm{X})\ \mathit{or}\ \mathit{not}\ (\mathrm{Y})$$

can fail in the case of conceptual minds (irreflexive graphs; Fig. 5). The other de Morgan's law:

$$\mathit{not}\ (\mathrm{X}\ \mathit{or}\ \mathrm{Y}) = \mathit{not}\ (\mathrm{X})\ \mathit{and}\ \mathit{not}\ (\mathrm{Y})$$

is valid in the case of *not*, while both laws are valid in the case of *non*. All of this logic, which distinguishes conceptual minds (irreflexive graphs) from sets, follows from merely recognizing that there are concepts and propositions within our minds, and that to each proposition there is a concept which is its subject, predicate. This irreflexive graph model of the conceptual mind can be further refined, as shown in the following sections.

4. Cohesive Mind

We have been, up until now, considering the consequences of modeling minds as irreflexive graphs. More specifically, we modeled conceptual mind as a pair of sets:

(a set C of concepts, a set P of propositions)

equipped with a parallel pair of functions:

(*subject*: P –> C, *predicate*: P –> C)

Let us now refine this irreflexive graph model of conceptual mind. If we imagine, again, looking into our minds, then we notice that, for each concept (e.g. ROSE) in a mind, there is a proposition, more specifically, an identity proposition (*identity* (ROSE) = ROSE is ROSE) in the mind. This observation suggests modeling conceptual mind as a pair of sets:

(a set C of concepts, a set P of propositions) equipped with three functions:

(*subject*: P –> C, *predicate*: P –> C, *identity*: C –> P)

with the added third function *identity* assigning to each concept in the set C of concepts its identity proposition in the set P of propositions. These three functions together constitute a reflexive graph (Fig. 1d; Lawvere & Schanuel, 2009, p. 145). One immediate question: what, if any, are the implications of modeling conceptual mind as reflexive graph? An immediate consequence of refining the model of conceptual mind from irreflexive graph to reflexive graph is that it accounts for the unity of mind, as shown in the following.

The cohesiveness of a universe of discourse (such as sets and graphs) can be assessed using the axioms of cohesion (Lawvere, 2005, 2007). One of the necessary conditions for [the objects of] a universe of discourse to be cohesive is that its truth value object is connected, i.e. one piece (Lawvere & Schanuel, 2009, pp. 358-359; Axiom 2 in Lawvere, 2005). Another condition of cohesion is: number of pieces of a product equals the product of pieces of the factors (Lawvere & Schanuel, 2009, pp. 260, 372-373; Axiom 1 in Lawvere, 2005). Let us now examine our models of mind in light of these axioms. Consider our initial model of mind, wherein minds consist of concepts only. With concepts as structureless elements, minds are sets (of concepts). The truth value set {false, true}, consistent with the zero cohesion of discrete sets, is not connected (Fig. 3a). Next, consider minds consisting of propositions and concepts, along with the specification that every proposition has a subject and a predicate concept. With propositions and concepts as arrows and dots, respectively, conceptual minds are irreflexive graphs. The truth value object of irreflexive graphs is connected (Fig. 3b). However, the second condition for cohesion involving products is not satisfied in the case of irreflexive graphs (as shown in Fig.

6a). This additional condition is satisfied in case of conceptual minds, wherein for every concept (e.g. SKY) in a mind, there is an identity proposition (SKY is SKY) in the mind (Fig. 6b). Moreover, since reflexive graphs satisfy additional axioms of cohesion (Lawvere, 2005, 2007), conceptual mind, with its reflexive graph structure, is cohesive.

5. Composing Propositions

In addition to the static aspects of thought (concepts, propositions), which we examined in the above, there are dynamical aspects of thinking. An elementary dynamic of the motion of thought involves combination of given propositions to arrive at novel propositions as conclusions. As part of this reasoning, we compose propositions (such as):

APPLE is FRUIT + FRUIT is EDIBLE = APPLE is EDIBLE

We can represent these syllogisms as commutative triangles (satisfying $f + g = h$, where '+' denotes composition of propositions, which are represented by arrows f: A –> B, g: B –> C, and h: A –> C, while A, B, and C denote concepts; Fig. 7a; Lawvere & Schanuel, 2009, pp. 16-21). This composition of propositions satisfies two identity laws (exemplified by):

FRUIT is FRUIT + FRUIT is EDIBLE = FRUIT is EDIBLE

APPLE is FRUIT + FRUIT is FRUIT = APPLE is FRUIT and as illustrated in (Fig. 7b, c). Based on these observations, we can further refine our model of the conceptual mind as an object consisting of three component sets:

(a set C of concepts, a set P of propositions, a set S of syllogisms)

which are structured by eight functions (Fig. 8).

With a generic syllogism (commutative triangle $f + g = h$) as the essence of conceptual mind, we can calculate the truth value object in terms of the parts of the commutative triangle. The generic syllogism (commutative triangle $f + g = h$) has nineteen parts. They are: 1. $f + g = h$ (entire syllogism); 2. f, g, h (no syllogism, but all three propositions); 3. f, g (two propositions); 4. g, h; 5. h, f; 6. f, C (one proposition and all three concepts); 7. g, A; 8. h, B; 9. f (one proposition); 10. g; 11. h; 12. A, B, C (no proposition, but all three concepts); 13. A, B (two concepts); 14. B, C; 15. C, A; 16. A (one concept); 17. B; 18. C; and 19. empty (no syllogism, no proposition, no concept). These nineteen parts correspond to nineteen degrees of truth ranging from FALSE to TRUE in the truth value triangle (Fig. 9; Lawvere, 1989, pp. 282-283). The truth value triangle is constructed from the incidence relations of triangles, edges, and dots using the same procedure used to calculate the truth value graph (Fig. 3b; Reyes, Reyes, & Zolfaghari, 2004, pp. 93-101; calculation of truth value objects is discussed in detail in the Appendix). The part $f + g = h$ (triangular surface) corresponds to TRUE, which is the truth value of, say, the statement (that a syllogism):

'APPLE is FRUIT + FRUIT is EDIBLE = APPLE is EDIBLE' *is in* X

(where X is a given part of a mind) when the syllogism is in X. The part 'empty' corresponds to FALSE, which is the truth value of the statement when the syllogism is not in X. In between these two extremes, there are seventeen truth values corresponding to various scenarios such as:

the syllogism is not in X, but the three propositions APPLE is FRUIT, FRUIT is EDIBLE, APPLE is EDIBLE are in X, or just one of three concepts FRUIT is in X.

Thus we find that a mere recognition of the all too clearly visible mental contents (concepts, propositions, and syllogisms) and their mutual relations reveals the rich structure and logic of the conceptual mind. The structural essences of a universe of discourse (such as graphs or minds), their extraction and subsequent interpretation to obtain models can all be given a comprehensive mathematical account in terms of functorial semantics (Lawvere, 2004), which we plan to present in a subsequent paper.

6. Concluding remarks

Conceptualizing mind as a set as in 'mind is a set of brain functions' (Bunge, 1981, p. 68; see also Kandel, 2013, p. 546; Kandel et al., 2013, p. 5, 334, 384) is a first approximation. A little more realistic conception of mind would take into account the distinctions between mental contents, say, by way of modeling mind as a pair of sets: (a set of concepts, a set of propositions). A further refinement would take into account the relations between these different sets. This is exactly what we did in the present note. We modeled, via successive refinements, conceptual mind as a structure made up of three component sets: (a set of concepts, a set of propositions, a set of syllogisms) equipped with eight structural functions. These structural functions specify the relations between concepts and propositions (cf. a proposition has a concept as its subject / predicate), and the relations between propositions and syllogisms (cf. a syllogism has a proposition as its minor / major premise / conclusion; Fig. 8). Thus characterized logic of conceptual minds is distinct from that of sets by virtue of its degrees of truth (Fig. 3b, 9). The objective logic of conceptual mind is further distinguished from the Boolean logic of sets in light of the varieties of negation (Fig. 4a). Particularly noteworthy logical features are the admission of contradiction (Fig. 4b) and the failure of one of the de Morgan's laws (Fig. 5).

Summing it all, our characterization of the mathematical structure and the non-Boolean logic of the conceptual mind is a substantial refinement of the contemporary cognitive neuroscientific conceptualization of the mind as a set. Our mathematical characterization of mind can help develop definitive theories of motion of thought on par with that of the mathematical theories of motion of matter. Bringing about this parity between the science of thinking and that of things is a first step towards accounting for the effectiveness of thinking—of thinking about things.

References

Bunge, M. 1981: *Scientific Materialism.* Boston: D. Reidel Publishing Company.

Fodor, J. A. 2006: How the mind works: What we still don't know. *Daedalus,* 135 (3), 86-94.

Kahneman, D. 2013: *Thinking, fast and slow.* New York: Farrar, Straus and Giroux.

Kandel, E. R. 2013: The new science of mind and the future of knowledge. *Neuron,* 80 (3), 546-560.

Kandel, E. R., Schwartz, J. H., Jessell, T. M., Siegelbaum, S. A., and Hudspeth, A. J. 2013: *Principles of Neural Science.* (5th ed.) New York: McGraw Hill Education.

Lawvere, F. W. 1972: *Perugia Notes: Theory of Categories over a base Topos.* Perugia: University of Perugia Lecture Notes.

Lawvere, F. W. 1980: Toward the description in a smooth topos of the dynamically possible motions and deformations of a continuous body. *Cahiers de Topologie et Géométrie Différentielle Catégorique,* 21 (4), 337-392.

Lawvere, F. W. 1986: Introduction. In F. W. Lawvere and S. H. Schanuel (eds.), *Categories in Continuum Physics* (pp. 1-16). New York: Springer-Verlag.

Lawvere, F. W. 1989: Qualitative distinctions between some toposes of generalized graphs. *Contemporary Mathematics,* 92, 261-299.

Lawvere, F. W. 1991: Intrinsic co-Heyting boundaries and the Leibniz rule in certain toposes. In A. Carboni, M. C. Pedicchio, and G. Rosolini (eds.), *Category Theory* (pp. 279-281). New York: Springer-Verlag.

Lawvere, F. W. 1994: Tools for the advancement of objective logic: Closed categories and toposes. In J. Macnamara and G. E. Reyes (eds.), *The Logical Foundations of Cognition* (pp. 43-56). New York: Oxford University Press.

Lawvere, F. W. 1999: Kinship and mathematical categories. In P. Bloom, R. Jackendoff and K. Wynn (eds.), *Language, Logic, and Concepts* (pp. 411-425) Cambridge: MIT Press.

Lawvere, F. W. 2004: Functorial semantics of algebraic theories and some algebraic problems in the context of functorial semantics of algebraic theories. *Reprints in Theory and Applications of Categories,* 5, 1-121.

Lawvere, F. W. 2005: Categories of spaces may not be generalized spaces as exemplified by directed graphs. *Reprints in Theory and Applications of Categories,* 9, 1-7.

Lawvere, F. W. 2007: Axiomatic cohesion. *Theory and Applications of Categories,* 19 (3), 41-49.

Lawvere, F. W. and Rosebrugh, R. 2003: *Sets for Mathematics.* Cambridge: Cambridge University Press.

Lawvere, F. W. and Schanuel, S. H. 2009: *Conceptual Mathematics: A first Introduction to Categories.* (2nd ed.) Cambridge: Cambridge University Press.

Picado, J. 2007: An interview with F. William Lawvere. *CIM Bulletin,* 23, 23-28.

Reyes, M. L. P., Reyes, G. E. and Zolfaghari, H. 2004: *Generic Figures and their Glueings: A Constructive Approach to Functor Categories.* Milano: Polimetrica.

Appendix

In this appendix we will discuss the calculation of truth value objects. The truth value object of a universe of discourse (e.g. category of sets) is an object of the universe (i.e. set). For example, the truth value object of the category of sets is a two-element set **Ω** = {false, true}. Calculating the truth value object of a category requires finding the generic object(s) of the category. The defining property of generic objects is that any two maps in the category are equal if and only if the two maps are equal at every generic object-shaped figure. In the category of sets, a single-element set **1** = {•} is the generic object, since any two functions *f* and *g* are equal if and only if the two functions are equal at every **1**-shaped figure x, i.e., *f* = *g* if and only if *f* (x) = *g* (x), for every x. Once we have the generic object(s), calculation of truth value object involves enumerating parts of the generic object. In the category of sets, the generic object **1** has two parts. They are *0*: **0** –> **1**, *1*: **1** –> **1**, where **0** = {}. The defining property of the truth value object **Ω** of a category is: for any object X of the category, there is a 1-1 correspondence between parts Y –> X of the object X and maps from the object X to the truth value object **Ω**:

Y –> X

X –> **Ω**

Taking X = **1**, we find that, corresponding to the two parts (*0*, *1*) of the generic object **1**, there are two maps from **1** to **Ω**, which means that there are two **1**-shaped figures in **Ω**. In the category of sets, since all that there is to a set is the **1**-shaped figures in it, i.e. points or elements in the set, the truth value object **Ω** has two elements, i.e. **Ω** = {false, true}.

In the category of irreflexive graphs there are two generic objects:

generic dot, D = •

generic arrow, A = • –> •

The generic dot D has two parts. Going by the 1-1 correspondence between parts (Y –> D) of the generic dot D and maps from D to the truth value object **Ω**:

Y –> D

D –> Ω

there are two dot-shaped figures, i.e., there are two dots (F, T) in the truth value object **Ω** of the category of graphs. Next, the generic arrow A has five parts. Going by the 1-1 correspondence between parts (Y –> A) of the generic arrow A and maps from A to the truth value object **Ω**:

Y –> A

A –> Ω

there are five arrow-shaped figures, i.e., there are five arrows (*false*, *ft*, *tf*, *tt*, *true*) in the truth value object **Ω**. Now we determine how these two dots (F, T) and five arrows (*false*, *ft*, *tf*, *tt*, *true*) fit-together into the truth value graph **Ω**. In other words, we have to determine the incidence relations between the dots and arrows of the truth value graph Ω. More explicitly, we have to determine which one of the two dots is the source / target dot of each one of the five arrows. Inverse images of parts of generic objects along structural maps give the incidence relations between the generic object-shaped figures in the truth value graph. There are two structural maps *s*, *t*: D –> A inserting the generic dot D into the generic arrow A as source, target dot, respectively. The inverse images of each one of the five arrows (*false*, *ft*, *tf*, *tt*, *true*; corresponding to the five parts of the generic arrow A) along the source *s*, target *t* structural maps give the source, target dot of the corresponding arrow, as follows:

1. The arrow *false* (of the truth value graph Ω) corresponds to the empty part of the generic arrow A, and its inverse image along the structural map *s*: D –> A is the empty part of the generic dot D, i.e. the dot denoted by F (of Ω). Similarly, its inverse image along the structural map *t*: D –> A is also the empty part of D, i.e. dot F. So, dot F is both the source and the target dot of the arrow *false* of the truth value graph Ω.
2. The arrow *ft* corresponds to the target dot (part) of the generic arrow A, and its inverse image along the structural map *s*: D –> A is the empty part of the generic dot D, i.e. the dot denoted by F. Similarly, its inverse image along the structural map *t*: D –> A is the dot (part) of D, i.e. dot T. So, the source and target dots of the arrow *ft* are the dots F and T, respectively.
3. The arrow *tf* corresponds to the source dot (part) of the generic arrow A, and its inverse image along the structural map *s*: D –> A is the dot (part) of the generic dot D, i.e. the dot denoted by T. Similarly, its inverse image along the structural map *t*: D –> A is the empty part of D, i.e. dot F. So, the source and target dots of the arrow *tf* are the dots T and F, respectively.

4. The arrow *tt* corresponds to the part of the generic arrow A consisting of both the source and the target dots, and its inverse image along the structural map *s*: D –> A is the dot (part) of the generic dot D, i.e. the dot denoted by T. Similarly, its inverse image along the structural map *t*: D –> A is also the dot (part) of D, i.e. dot T. So, dot T is both the source and the target dot of the arrow *tt*.
5. The arrow *true* corresponds to the (entire) arrow part of the generic arrow A, and its inverse image along the structural map *s*: D –> A is the dot (part) of the generic dot D, i.e. the dot denoted by T. Similarly, its inverse image along the structural map *t*: D –> A is also the dot (part) of D, i.e. dot T. So, dot T is both the source and the target dot of the arrow *true*.

Thus we obtain the truth value graph Ω of the category of irreflexive graphs (Fig. 3b). Along similar lines, the truth value triangle (Fig. 9) is calculated.

Figure Legends

Figure 1: Modeling mind. (a) If minds consist of concepts only, then we can model mind as a set of concepts. In this model, concepts are construed as structureless elements. As an illustration, M = {CAT, WE, OF} is a mind consisting of three concepts CAT, WE, and OF (depicted as dots within a circle denoting the mind M). **(b)** A mind M modeled as a pair of sets: (a set M_C of concepts, a set M_P of propositions). Here, both concepts and propositions are construed as structureless elements, albeit of two different types of sets. **(c)** A mind M consisting of a proposition 'SKY is BLUE', and a concept GOOD is modeled as an irreflexive graph. Here, concepts and propositions are displayed as dots and arrows, respectively. Note that the subject, predicate concepts (SKY, BLUE) of the proposition (SKY is BLUE) are depicted as the source, target dots integral to the arrow representing the proposition. **(d)** A mind M consisting of a proposition 'SKY is BLUE' and a concept DOG is modeled as a reflexive graph. In this reflexive graph model, for each concept (e.g. SKY) in a mind, there is an identity proposition (SKY is SKY) in the mind. Note that concepts are displayed as loops (arrows with target dot same as the source dot).

Figure 2: Essence of minds. (a) With mind as a set (of concepts), the structural essence of minds is a set (mind) consisting of one element (concept), i.e. a single-element set **1** = {•}. **(b)** With minds modeled as irreflexive graphs (Fig. 1c), the structural essence of minds consists of two graphs: concept (depicted as dot D) and proposition (depicted as arrow A), along with two graph morphisms *s*: D –> A, *t*: D –> A. These two morphisms specify the inclusion of concept

(dot D) into proposition (arrow A) as its subject, predicate concept (source, target dot; Lawvere & Schanuel, 2009, p. 150).

Figure 3: Degrees of truth. **(a)** With minds as sets, the truth value object of minds is a two-element set Ω = {false, true}. The truth value set Ω is the totality of the two parts **0** (= {}) and **1** (= {•}) of the essence (**1** = {•}) of sets (see Fig. 2a). The two elements of Ω = {false, true} correspond to the two parts (**0**, **1**, respectively) of the single-element set **1**. **(b)** With minds as irreflexive graphs, the truth value object Ω is an irreflexive graph consisting of five arrows (corresponding to the five degrees of truth at the level of propositions, which are represented as arrows) and two dots (corresponding to the two truth values at the level of concepts, which are represented as dots). The five arrows (*false*, *ft*, *tf*, *tt*, *true*) correspond to the five possible truth values a statement—P *is in* C—asserting the inclusion of a proposition P in a part C (of a mind) can take. If P is in C, then the truth value of the statement 'P *is in* C' is *true*; if P is not in C, then the truth value of 'P *is in* C' is *false*. In addition to these two truth values (*false*, *true*), there are three more truth values: (i) *tt* is the truth value of 'P *is in* C', if P is not in C, but both its subject and predicate concepts are in C, (ii) *tf* is the truth value of 'P *is in* C', if P is not in C, but its subject is in C, and (iii) *ft* is the truth value of 'P *is in* C', if P is not in C, but its predicate is in C. The two dots (F, T) in the truth value graph correspond to the two possible truth values (as in the case of sets) a statement asserting the inclusion of a concept (dot) in a part (of a mind) can take. The truth value graph is constructed based on the incidence relations (of dots and arrows) calculated as inverse images, along structural maps, of parts of the generic arrow (Reyes, Reyes, & Zolfaghari, 2004, pp. 93-101; calculation of the truth value graph is discussed in detail in the Appendix).

Figure 4: Varieties of negation. **(a)** Consider a mind consisting of two propositions: 'CAT is ANIMAL' and 'DOG is ANIMAL'. Next, consider a part X = 'CAT is ANIMAL' of the given mind. *not* (X) is the largest part among all parts of the mind whose intersection with the part X is empty, which means *not* (X) = DOG. *non* (X) is the smallest among all parts whose union with X is the entire mind. So, *non* (X) = 'DOG is ANIMAL'. **(b)** Again, let X = 'CAT is ANIMAL'. *non* (X) = 'DOG is ANIMAL'. X *and non* (X) = ANIMAL. Thus, logical contradiction 'X *and non* (X)' extracts from X (from the proposition 'CAT is ANIMAL') its boundary, i.e. the concept ANIMAL (Lawvere, 1991). **(c)** Consider a mind consisting of a proposition 'CAT is ANIMAL'. Let X denote a part (of the mind) consisting of two concepts: CAT, ANIMAL. *not* (X) is the largest among all parts whose intersection with X is empty. So, *not* (X) is empty. Since negating the empty part gives the proposition 'CAT is ANIMAL', double negation of X, i.e., *not* (*not* (CAT, ANIMAL)) is the entire proposition 'CAT is ANIMAL', which is bigger than X (i.e. both the concepts CAT, ANIMAL; Lawvere & Schanuel, 2009, p. 355). **(d)** Consider a mind consisting of two propositions: 'CAT is ANIMAL' and 'DOG is ANIMAL', with the concept ANIMAL as the common predicate of both the propositions. Let X denote a part (of the given mind) consisting of the proposition 'CAT is ANIMAL' and the concept DOG. *non* (X) is the smallest of all parts whose union with X is the entire mind. So, *non* (X) = 'DOG is ANIMAL'. Since *non* (DOG is ANIMAL) = 'CAT is ANIMAL', double negation of X, i.e., *non* (*non* (CAT is ANIMAL, DOG)) = 'CAT is ANIMAL', which is smaller than X (i.e., the proposition 'CAT is ANIMAL', along with the concept DOG).

Figure 5: Failure of de Morgan's law. Consider a mind consisting of one proposition: 'CAT is ANIMAL'. Let X denote the subject concept CAT, and Y denote the predicate concept ANIMAL. X *and* Y is empty. *not* (X *and* Y) = 'CAT is ANIMAL'. *not* (X) = ANIMAL, while *not* (Y) = CAT. *not* (X) *or not* (Y) is both concepts CAT, ANIMAL of the proposition. Since *not* (X *and* Y) ≠ *not* (X) *or not* (Y), the de Morgan's law: *not* (X *and* Y) = *not* (X) *or not* (Y) fails in the case of conceptual minds (irreflexive graphs).

Figure 6: Cohesion of conceptual mind. **(a)** In the irreflexive graph model of conceptual mind (Fig. 1c), a proposition A is an arrow along with its source and target dots representing the subject and predicate concepts of the proposition. Since the subject and predicate concepts of a proposition are integral to the proposition, the arrow A along with its source and target dots constitutes one connected piece (Lawvere & Schanuel, 2009, pp. 358-359). The product $A \times A$ consists of one arrow along with its source and target dots and, in addition to these two dots integral to the arrow, two more disconnected dots. Thus the product consists of three pieces (one arrow plus two disconnected dots). Hence, the number of pieces of the product is not equal to the product of pieces of the factors ($3 \neq 1 \times 1$; Lawvere & Schanuel, 2009, pp. 260, 372-373), which is a required condition for cohesion (Axiom 1 in Lawvere, 2005). **(b)** In the reflexive graph model of conceptual mind (Fig. 1d), for every concept (depicted as a dot), there is an identity proposition (depicted as a loop with a single dot as both source and target dot). Now consider a proposition A (an arrow with loops representing its subject and predicate concepts), which is one piece. The product $A \times A$ is also one piece, as shown. Hence, the number of pieces of the product is equal to the product of pieces of the factors ($1 = 1 \times 1$), thereby satisfying the product condition for cohesion.

Figure 7: Syllogisms as commutative triangles. (a) A pair of successive propositions: (APPLE is FRUIT, FRUIT is EDIBLE), wherein second proposition's subject (FRUIT) is same as the first proposition's predicate (FRUIT), can be composed to obtain a composite proposition: APPLE is EDIBLE. Composition of propositions (as in this syllogism) can be modeled as a commutative triangle, with concepts as dots and propositions as arrows (Lawvere & Schanuel, 2009, p. 201). **(b)** Syllogisms satisfy two identity laws: left and right identity laws. Left identity law: Composing a proposition with the identity proposition of its subject concept results in the proposition (as in): APPLE is APPLE + APPLE is FRUIT = APPLE is FRUIT. **(c)** Right identity law: Composing a proposition with the identity proposition of its predicate concept results in the proposition (as in): APPLE is FRUIT + FRUIT is FRUIT = APPLE is FRUIT.

Figure 8: Model of the conceptual mind. Mind consists of three components sets: 1. a set C of concepts (dots), 2. a set P of propositions (arrows, with a source and a target dot), and 3. a set S of syllogisms (commutative triangles formed of three arrows and three dots). (For the sake of clarity, only one generic element of each one of the three sets C, P, and S is displayed.) These three sets are structured by eight functions. The structural function *identity* from the set C of concepts to the set P of propositions inserts each concept (e.g. FRUIT) in the set of concepts into the set of propositions as an identity proposition (FRUIT is FRUIT). The functions *subject*, *predicate* from the set P of propositions to the set C of concepts assign to each proposition (e.g. 'SKY is CLEAR') its subject, predicate concept (SKY, CLEAR), respectively. The structural functions *lt*, *rt*, and *comp* from the set S of syllogisms to the set P of propositions extract a

proposition from a syllogism (e.g. *lt* (APPLE is FRUIT + FRUIT is EDIBLE = APPLE is EDIBLE) = APPLE is FRUIT). The functions id_L and id_R from the set P of propositions to the set S of syllogisms insert propositions as identity syllogisms (e.g. id_L (APPLE is FRUIT) = (APPLE is APPLE + APPLE is FRUIT = APPLE is FRUIT).

Figure 9: Truth value triangle. Triangulated surface of the truth value triangle is calculated based on the nineteen parts: $\{\{f+g=h\}, \{f, g, h\}, \{f, g\}, \{g, h\}, \{h, f\}, \{f, \mathrm{C}\}, \{g, \mathrm{A}\}, \{h, \mathrm{B}\}, \{f\}, \{g\}, \{h\}, \{\mathrm{A, B, C}\}, \{\mathrm{A, B}\}, \{\mathrm{B, C}\}, \{\mathrm{C, A}\}, \{\mathrm{A}\}, \{\mathrm{B}\}, \{\mathrm{C}\}, \{\}\}$ of the generic syllogism (commutative triangle $f+g=h$). The nineteen degrees of truth corresponding to these nineteen parts are displayed as triangles. The triangular surface TRUE corresponds to the truth value of a statement (that a syllogism): ‘APPLE is FRUIT + FRUIT is EDIBLE = APPLE is EDIBLE’ *is in* X (where X is a given part of a mind) when the syllogism is in X. The triangular surface FALSE is the truth value of the statement when the syllogism is not in X. In between these two extremes, there are seventeen degrees of falsity corresponding to various scenarios such as: the syllogism is not in X, but (i) the three propositions APPLE is FRUIT, FRUIT is EDIBLE, APPLE is EDIBLE are in X (triangle formed by the three arrows labeled *true*), or (ii) just one of three concepts FRUIT is in X (triangle formed by the three arrows labeled *ft*, *tf*, *false*).

Figure 1

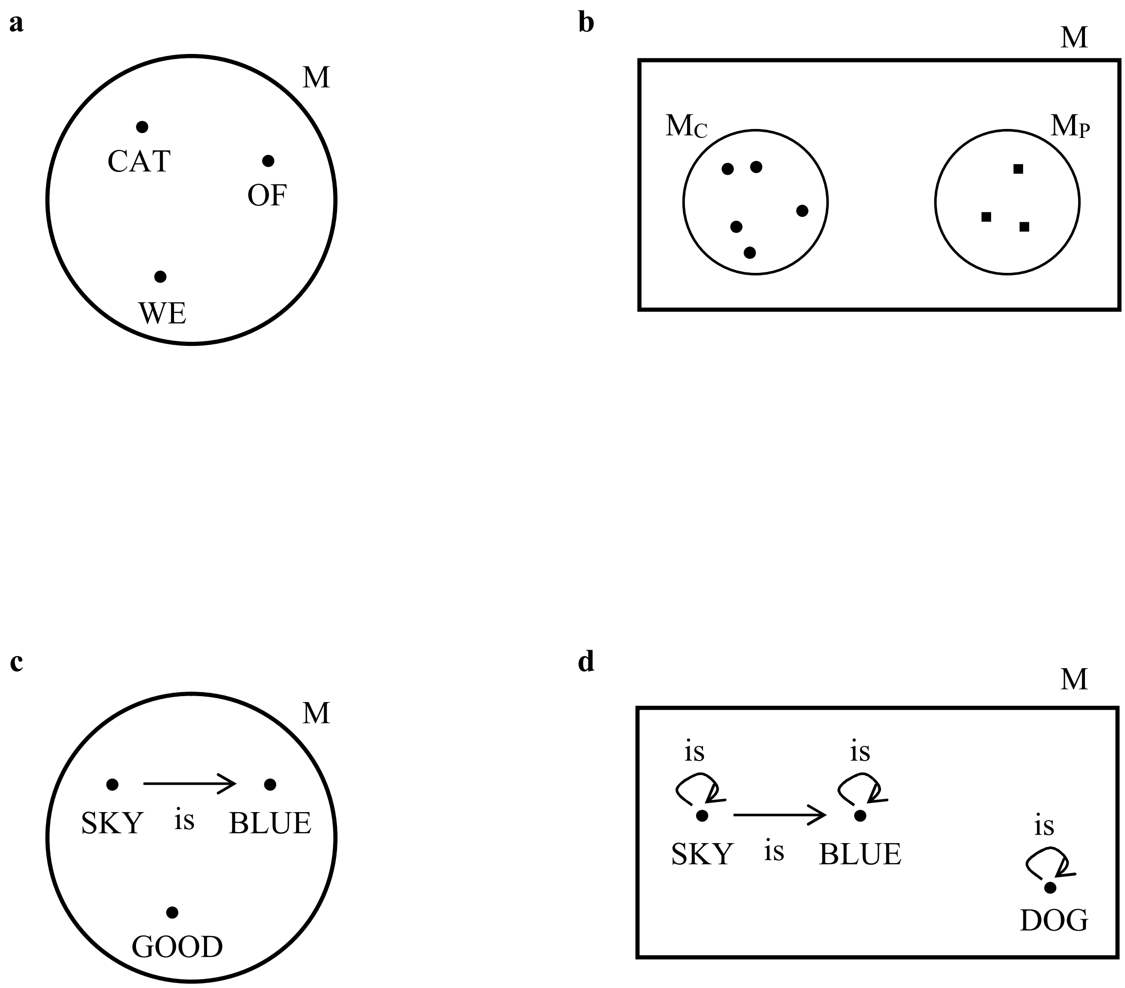

a
M
CAT
OF
WE
b
M
MC
MP
c
M
SKY
is
BLUE
GOOD
d
M
is
is
SKY
is
BLUE
is
DOG

Figure 2

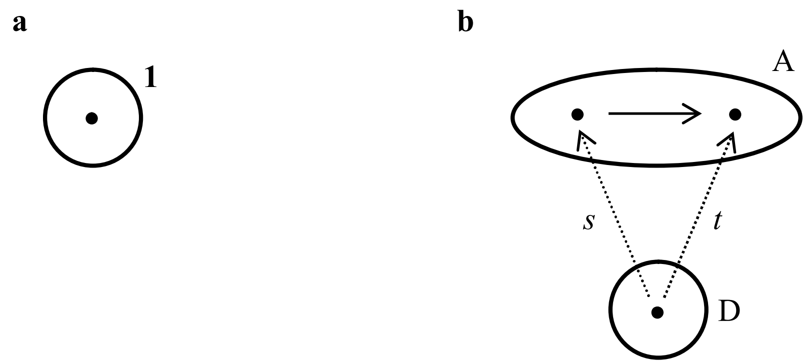


Figure 3

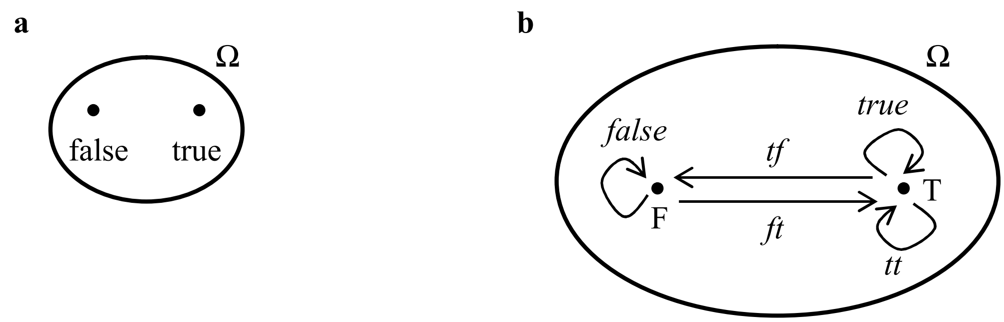


Figure 4

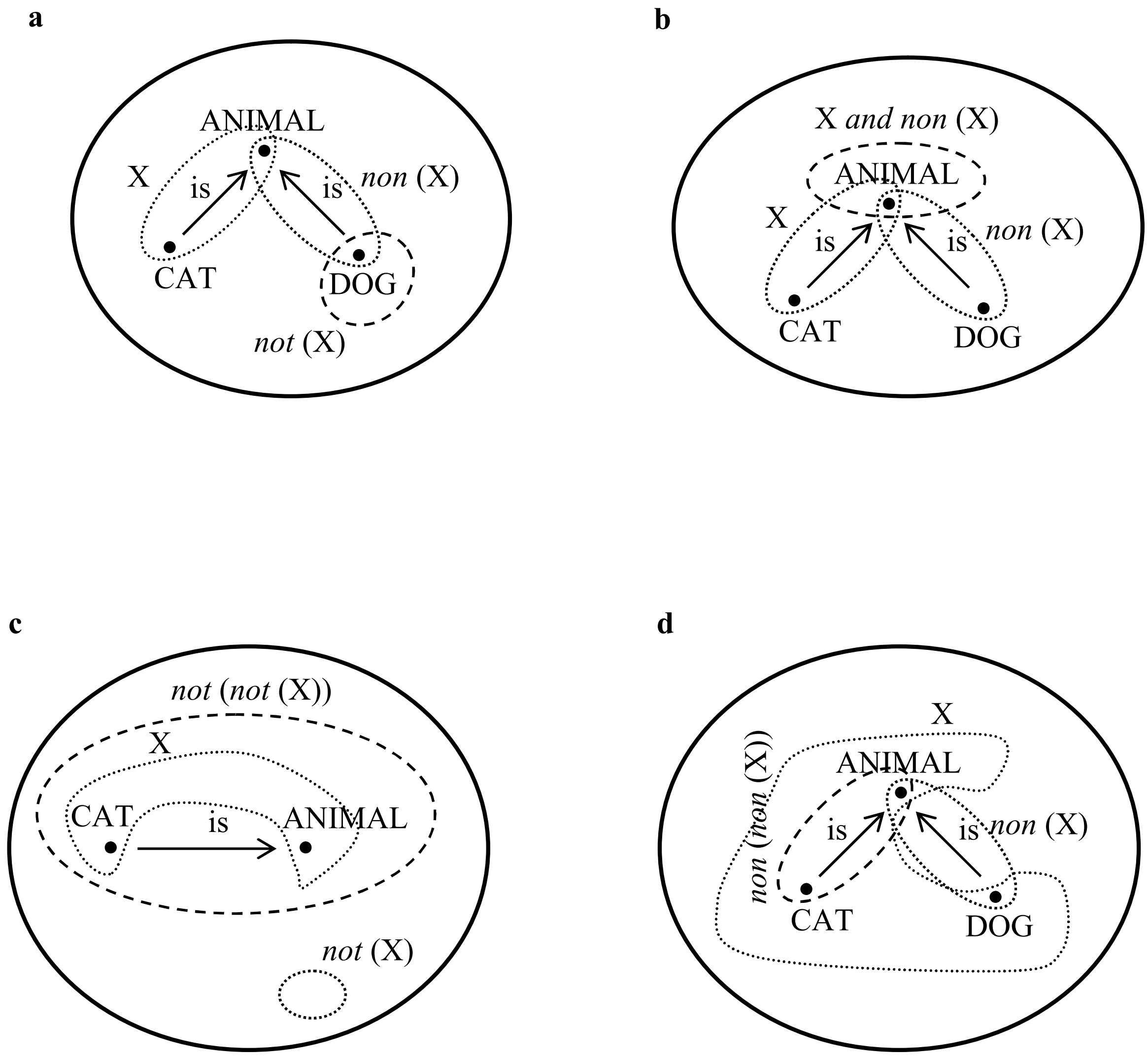

a
ANIMAL
X
is
is
non (X)
CAT
DOG
not (X)
b
X *and non* (X)
ANIMAL
X
is
is
non (X)
CAT
DOG
c
not (*not* (X))
X
CAT
is
ANIMAL
not (X)
d
X
ANIMAL
non (*non* (X))
is
is
non (X)
CAT
DOG

Figure 5

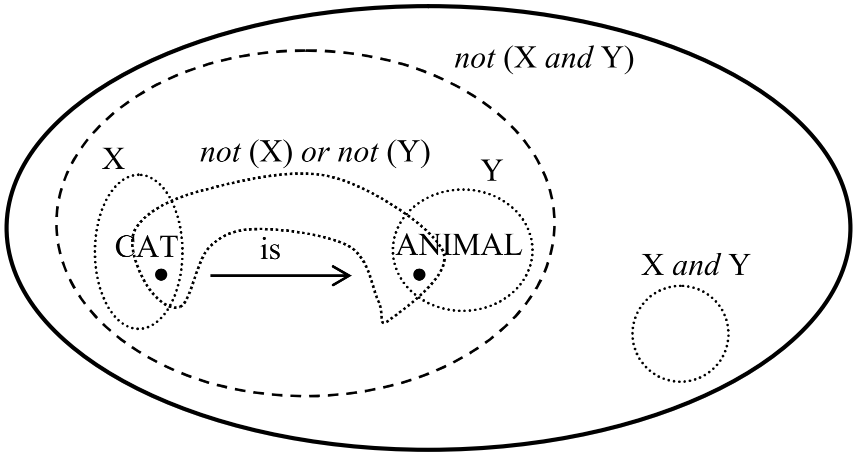


Figure 6

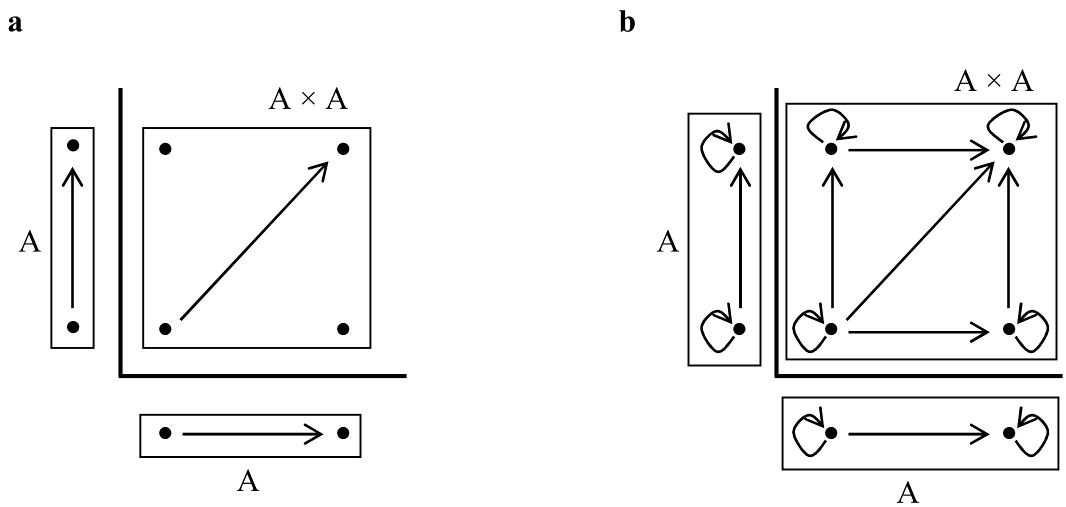


Figure 7

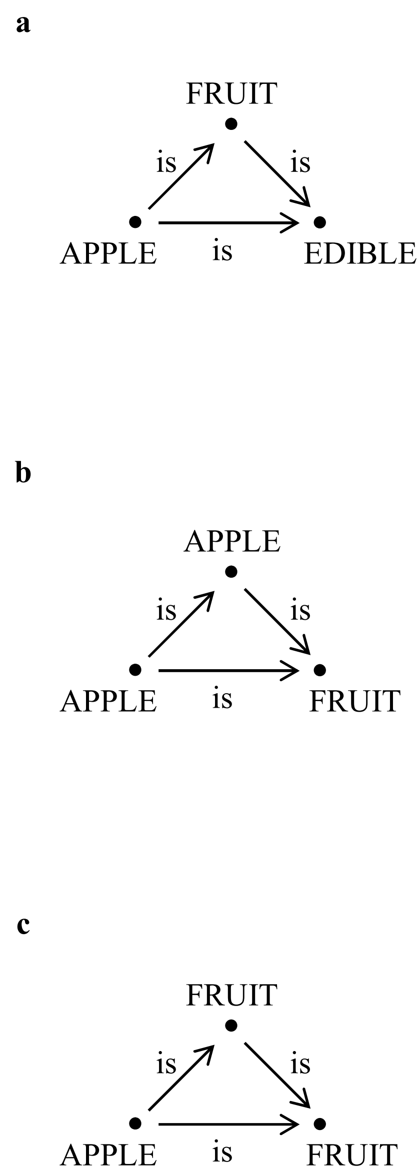

a
FRUIT
is
is
APPLE
is
EDIBLE
b
APPLE
is
is
APPLE
is
FRUIT
c
FRUIT
is
is
APPLE
is
FRUIT

Figure 8

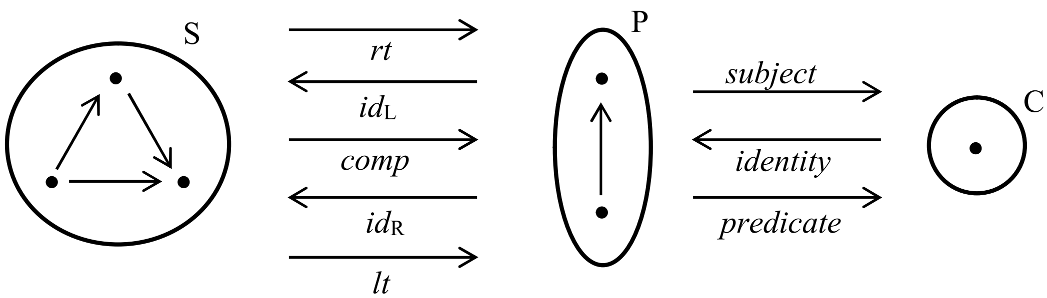
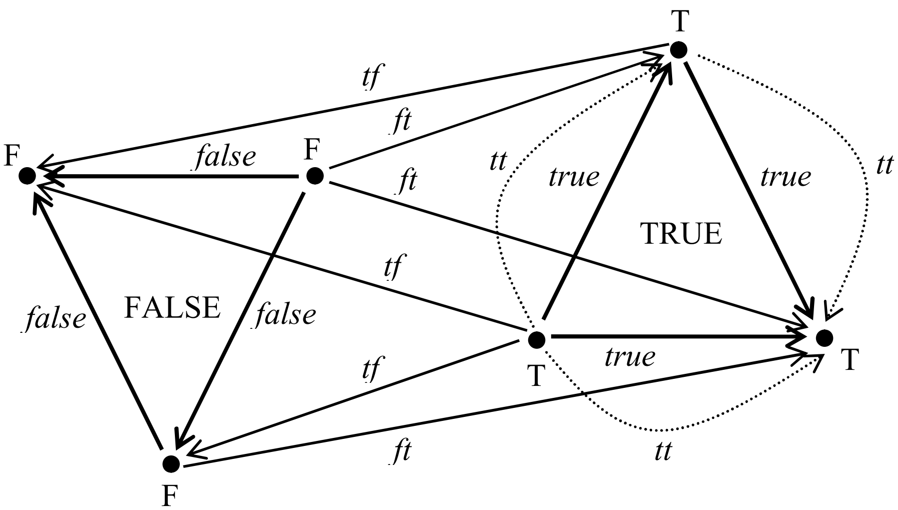


Figure 9


T
tf
ft
F
false
F
ft
tt
true
true
tt
TRUE
tf
false
FALSE
false
tf
T
true
T
ft
tt
F

Gmail

Venkata Rayudu Posina <posinavrayudu@gmail.com>

Review: Structure and Logic of Conceptual Mind

1 message

Editorial Office "Mind and Matter" <editor@mindmatter.de> Sun, May 27, 2018 at 12:52 PM
To: Venkata Rayudu Posina <posinavrayudu@gmail.com>

Dear Venkata,

we now have received two referee reports for your paper "structure and logic of conceptual mind." (comments attached) The reviews were mixed, both positive and negative.

We would thus like to give you the chance to produce a revision and resubmit it for another round of review. However, please note that the revised verson needs substantial revisions, cf. in particular the comments by reviewer 1: You should place your work more in the context of the (already existing and quite vast) literature on the topic, perhaps even add a new chapter on this. Also, we noted that the paper is in many parts quite similar to the your paper co-authered and recently published in Mind and Matter. Of course, the comments by reviewer 2 should also be incorporated or replied to.

If you accept to do the revision, please supply us with the new manuscript and, separatly, a detailed list of changes and replies to the reviewers.

Best wishes,

Robert

--
Dr. Dr. Robert Prentner
Editorial Team "Mind and Matter"
http://www.mindmatter.de/journal

reviewers' comments.docx
16K

Reviewer 1:

The author mentions three noteworthy features of the logic of conceptual mind:

1. degrees of truth,
2. varieties of negation
3. admission of contradiction

4. failure of de Morgan's law.

Concerning the first three points, there is a bulk of research in the relevant fields of logic, semantics, philosophy of mind, and cognitive psychology. Unfortunately, the author decided not even to mention the key references of these enormously rich fields of exploration.

The author is proposing to base his mathematical treatment of conceptual mind on Lawvere's category theoretic characterization. In this context, the author mentions that "The logic of conceptual mind, with its degrees of truth and varieties of negation, differs markedly from the Boolean logic of sets". This is certainly right and the literature of quantum interaction and quantum cognition gives a sound explanation why the mind cannot be based on a Boolean logic (Atmanspacher, Römer, & Walach, 2002; Busemeyer & Bruza, 2012). Unfortunately, the author does not even mention this literature. Instead, he refers to a failure of the de Morgan's law (point 4 on the list above).

However, in the context of quantum cognition, the de Morgan's law are satisfied. Hence, it would be important to give the empirical evidence for the failing of these laws in the context of Human reasoning. Unfortunately, the author fails to provide us with this evidence.

Another potentially interesting topic is the composition of concepts (propositions). Again, the author does not even refer to the most important literature. The given examples are trivial and do not illustrate the envisaged non-Boolean treatment.

Summarizing, the author proposes a mathematical treatment of conceptual mind based on Lawvere's category theory. Unfortunately, he does not give a satisfying motivation of this approach. Further, the presented examples are rather trivial and not interesting.

Atmanspacher, H., Römer, H., & Walach, H. (2002). Weak Quantum Theory: Complementarity and Entanglement in Physics and Beyond. *Foundations of Physics,* 32(3), 379-406.
Busemeyer, J. R., & Bruza, P. D. (2012). *Quantum Cognition and Decision*. Cambridge, UK Cambridge University Press.

Reviewer 2:

I think that the main contribution of this paper is philosophical. From such point of view, this paper is original. However, some historical and current contributions to this subject might bring objections, but the proposal remains acceptable in my opinion. I mention only three possible objections.

1) The contribution is based on the definition of the mind, specifically the conceptual mind, as a set. The paper brings a clear-cut foundation that elaborates on such core definition by means of using the

graphs´ math and some of their properties. However, the philosophy of mind has thoroughly discussed during the last decades the algorithmic nature of the human mind (Minsky, Dennett, Searle, Hofstadter among others mentioned in this paper like Fodor). Furthermore, the most radical conception of the mind states that the human mind can be understood as an axiomatic system, which is more than a set, that is, elements and relations among them within a universe of discourse. However, the initial optimism of artificial intelligence as foundation for cognitive science was gradually replaced by a softer version of the mind-computer analogy proposed by Turing. So, the objection or question is: Why is it better to consider a set instead of an axiomatic system as a tool to account for the conceptual mind? Why such issue is not discussed in this paper?

2) Some current theories of human thinking, reasoning in particular, which is somehow the conceptual mind in movement, use a multivalued-logic approach to the attribution of truth. That is, the true-false polarity was replaced by degrees of truth, probabilities. This approach can be found in the contributions made by the Rational Analysis framework (Mike Oaksford, Nick Chater) and the multivalued logic applied to cognitive modeling by Michiel van Lambalgen and Keith Stenning, among others. Since this paper brings novel perspectives to the same field of research, some discussion concerning the relation between these theories and this paper might be interesting. So, the objection is: Can a multivalued-logic be considered instead of a two-valued function of truth? This might be interesting in particular to account for practical reasoning, which is close related to the theories of concepts and categories.

3) This kind of foundational contributions require consistency, which this paper has in my opinion, but also require simulations to test formal consistency and experimental evidence to achieve predictive capacity. That is, some published experiments are consistent with this paper in my opinion, but others are not. Under some experimental conditions, both DeMorgan´s laws are often correctly applied by many experimental participants. For example, when they are exposed to a prior formal explanation about the laws of compound negation (DeMorgan´s laws in sentential reasoning). Of course, these considerations aim to promote future papers, not to reject the current contribution. Concerning simulation, the PSYCOP model (by Lance Rips)

might be a good example to elaborate this theory of the conceptual mind. Concerning experimental evidence, the Theory of Mental Models (by Phil Johnson-Laird, Sunny Khemlani and others concerning negation in the human mind) might provide interesting strategies to generate empirical evidence. The Dual-Process theories (by Jonathan Evans and many others) might also provide inspiration to generate experiments for this conceptual mind approach.

In sum, I think that this paper brings an interesting and consistent approach to the theory of concepts and mental representation. Some objections might be brought, but the approach is acceptable in my opinion. That is, from a mathematical perspective, since the Bourbaki group made their influential contributions, infinite models can be proposed. This is valid for pure mathematics, but I think that all the branches of science are deeply concerned with pure mathematics including models of the conceptual mind.