Decoupled Search for the Masses: A Novel Task Transformation for Classical Planning – Appendix

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The presented material contains the full proofs of our paper.

Lemma 1. *Let Π be a SAS$^+$ planning task and $\mathcal{F}$ be a factoring for Π. Then $\Pi^{dec}_{\mathcal{F}}$ is a well-formed FDR planning task.*

Proof. The initial state and the goal are consistent by construction, because the original initial and goal states of Π are inherently consistent, and therefore their projection onto the center variables maintains that consistency. All other variables are assigned exactly one value or none.

The preconditions and unconditional effects of operators remain consistent because their projections onto the center variables remain consistent, and all other variables appear only in combination with the 1 value. Looking at the conditional effects in isolation, each condition and effect refers exclusively to a single value for each considered variable – either 0 or 1. When these conditional effects are considered together, they do not assign conflicting values to the same variable, because the conditions for assigning a value of 0 or 1 to a variable are mutually exclusive.

Regarding the axioms $\mathcal{A}^{dec}$, a single layer forms a valid stratification, because no secondary variable appears in any axiom body condition with the default value of 0. □

Lemma 2. *Let $s^{\mathcal{D}} \in S^{\mathcal{F}}$ be a decoupled state and $s^L \in S^{\mathcal{L}}$ a leaf state. Then $s^L \in \mathsf{leaves}^*(s^{\mathcal{D}})$ iff $\mathcal{A}(\varphi(s^{\mathcal{D}}))(d_{s^L}) = 1$.*

Proof.

"⇒": $s^L \in \mathsf{leaves}^*(s^{\mathcal{D}})$ is true iff (a) $s^L \in \mathsf{leaves}(s^{\mathcal{D}})$ or (b) $s^L \notin \mathsf{leaves}(s^{\mathcal{D}})$ but s^L can be reached with leaf-only operators from a leaf state t^L that is reached in $s^{\mathcal{D}}$:

(a) By the definition of φ, it holds that $\varphi(s^{\mathcal{D}})(v_{s^L}) = 1$. Further, with the frame axioms, $\mathcal{A}(\varphi(s^{\mathcal{D}}))(d_{s^L}) = 1$.

(b) By definition of φ, the center variables between $s^{\mathcal{D}}$ and $\varphi(s^{\mathcal{D}})$ match. Further, $t^L \in \mathsf{leaves}(s^{\mathcal{D}})$ iff $\varphi(s^{\mathcal{D}})(v_{t^L}) = 1$. Thus, by construction, the $\mathcal{A}_{\mathcal{O}^L_{\not{C}}}$ axioms, which embody the leaf-only operators, will also derive $\mathcal{A}(\varphi(s^{\mathcal{D}}))(d_{s^L}) = 1$.

"⇐": $\mathcal{A}(\varphi(s^{\mathcal{D}}))(d_{s^L}) = 1$ iff: (a) $\mathcal{A}(\varphi(s^{\mathcal{D}}))(v_{s^L}) = 1$ by the frame axioms, or (b) $\mathcal{A}(\varphi(s^{\mathcal{D}}))(v_{s^L}) = 0$ but s^L can be derived with the $\mathcal{A}_{\mathcal{O}^L_{\not{C}}}$-axioms.

(a) By definition of φ, it holds that v_{s^L} iff $s^L \in \mathsf{leaves}(s^{\mathcal{D}})$, which implies $s^L \in \mathsf{leaves}^*(s^{\mathcal{D}})$.

(b) $\varphi(s^{\mathcal{D}})$ and $s^{\mathcal{D}}$ match in the center variables. Furthermore, it holds that if $\varphi(s^{\mathcal{D}})(v_{t^L}) = 1$ then $t^L \in \mathsf{leaves}(s^{\mathcal{D}})$. Thus, the leaf-only operators corresponding to the axioms in $\mathcal{A}_{\mathcal{O}^L_{\not{C}}}$ that make d_{s^L} true in $\mathcal{A}(\varphi(s^{\mathcal{D}}))$ must be applicable in $s^{\mathcal{D}}$, which implies that $s^L \in \mathsf{leaves}^*(s^{\mathcal{D}})$.

□

We next prove the following auxiliary Lemma 3 to prove Theorem 1.

Lemma 3. *Function $\varphi : S^{\mathcal{F}} \to S^{dec}$ is bijective.*

Proof. We need to show that φ is injective and surjective.

φ is injective, since no two different decoupled states can map to the same state. If two decoupled states $s^{\mathcal{D}}_1$ and $s^{\mathcal{D}}_2$ differ in their center state, $\mathsf{center}(s^{\mathcal{D}}_1) \neq \mathsf{center}(s^{\mathcal{D}}_2)$, then the resulting states will also differ in their center variables, $\varphi(s^{\mathcal{D}}_1)[C] \neq \varphi(s^{\mathcal{D}}_2)[C]$. Additionally, if these two states have a different set of leaf states, then there is at least one leaf state s^L that is reached in only one of the two decoupled states. Consequently, $\varphi(s^{\mathcal{D}}_1)(s^L) \neq \varphi(s^{\mathcal{D}}_2)(s^L)$.

φ is surjective, since for every state $s \in S^{dec}$, there exists a decoupled state $s^{\mathcal{D}} \in S^{\mathcal{F}}$ such that $\varphi(s^{\mathcal{D}}) = s$. Given a state $s \in S^{dec}$, we can construct a decoupled state $s^{\mathcal{D}}$ with $\mathsf{center}(s^{\mathcal{D}}) = s[C]$ and $\mathsf{leaves}(s^{\mathcal{D}}) = \{s^L \mid s(v_{s^L}) = 1\}$. Then $\varphi(s^{\mathcal{D}}) = s'$, where $s'[C] = \mathsf{center}(s^{\mathcal{D}}) = s[C]$ and $s'(v_{s^L}) = 1$ if $s^L \in \mathsf{leaves}(s^{\mathcal{D}})$ (so if $s(v_{s^L}) = 1$) and 0 otherwise. So $s'(v_{s^L}) = s(v_{s^L})$, which shows that $\varphi(s^{\mathcal{D}}) = s' = s$. □

Theorem 1. *Let $\Pi = \langle \mathcal{V}, \mathcal{I}, \mathcal{G}, \mathcal{O} \rangle$ be a SAS$^+$ planning task and $\mathcal{F}$ be a factoring for Π. Then the FDR state space of $\Pi^{dec}_{\mathcal{F}}$ and the decoupled state space of Π are isomorphic, i.e., $\Theta(\Pi^{dec}_{\mathcal{F}}) \sim \Theta^{\mathcal{D}}(\Pi, \mathcal{F})$.*

Proof. Let $\Theta^{\mathcal{D}}(\Pi, \mathcal{F}) = \langle S^{\mathcal{F}}, \mathcal{O}^G, T^{\mathcal{F}}, \mathcal{I}^{\mathcal{F}}, S^{\mathcal{F}}_{\mathcal{G}} \rangle$ and $\Theta(\Pi^{dec}_{\mathcal{F}}) = \langle S^{dec}, \mathcal{O}^{dec}, T^{dec}, \mathcal{I}^{dec}, S^{dec}_{\mathcal{G}} \rangle$.

We consider the function φ which we have shown to be bijective in Lemma 3. We need to show that 1. $\varphi(\mathcal{I}^{\mathcal{F}}) = \mathcal{I}^{dec}$, 2. $s^{\mathcal{D}} \in S^{\mathcal{F}}_{\mathcal{G}}$ iff $\varphi(s^{\mathcal{D}}) \in S^{dec}_{\mathcal{G}}$, and 3. $s^{\mathcal{D}} \xrightarrow{o} t^{\mathcal{D}} \in T^{\mathcal{F}}$ iff $\varphi(s^{\mathcal{D}}) \xrightarrow{o^{dec}} \varphi(t^{\mathcal{D}}) \in T^{dec}$.

1. Let $\varphi(\mathcal{I}^{\mathcal{F}}) = s$. It holds that $s[C] = \mathsf{center}(\mathcal{I}^{\mathcal{F}}) = \mathcal{I}[C] = \mathcal{I}^{dec}[C]$. Furthermore, $s(v_{s^L}) = 1$ iff $s^L \in$

$\mathsf{leaves}(\mathcal{I}^{\mathcal{F}})$. Since $s^L \in \mathsf{leaves}(\mathcal{I}^{\mathcal{F}})$ iff $s^L = \mathcal{I}[L]$ and $\mathcal{I}^{dec}(v_{s^L}) = 1$ iff $s^L = \mathcal{I}[L]$ it holds that $s(v_{s^L}) = \mathcal{I}^{dec}(v_{s^L})$ for all $s^L \in S^{\mathcal{L}}$. Hence, $\varphi(\mathcal{I}^{\mathcal{F}}) = \mathcal{I}^{dec}$.

2. "$\Rightarrow$": Let $s^{\mathcal{D}} \in S^{\mathcal{F}}_{\mathcal{G}}$. It follows that: (a) $\mathcal{G}[C] \subseteq \mathsf{center}(s^{\mathcal{D}})$, thus $\mathcal{G}[C] \subseteq s[C]$; (b) For every leaf $L \in \mathcal{L}$ there exists $s^L \in \mathsf{leaves}^*(s^{\mathcal{D}})$ such that $\mathcal{G}[L] \subseteq s^L$. By Lemma 2, this implies the truth of the corresponding d_{s^L} variable in $\mathcal{A}(\varphi(s^{\mathcal{D}}))$, which further implies the truth of the derived variable $d_{\mathcal{G}[L]}$ for all $L \in \mathcal{L}$ due to the $\mathcal{A}^L_{\mathcal{G}}$ axioms. Therefore, $\varphi(s^{\mathcal{D}}) \in S^{dec}_{\mathcal{G}}$.

   "$\Leftarrow$": Let $\varphi(s^{\mathcal{D}}) \in S^{dec}_{\mathcal{G}}$. It follows that: (a) $\mathcal{G}[C] \subseteq \varphi(s^{\mathcal{D}})[C]$, thus $\mathcal{G}[C] \subseteq \mathsf{center}(s^{\mathcal{D}})$; (b) all derived variables $d_{\mathcal{G}[L]}$ are true in $\mathcal{A}(\varphi(s^{\mathcal{D}}))$, implying that there is at least one d_{s^L} variable where $\mathcal{G}[L] \subseteq s^L$ that is true in $\mathcal{A}(\varphi(s^{\mathcal{D}}))$. Hence, by Lemma 2, such a leaf state s^L must be contained in $\mathsf{leaves}^*(s^{\mathcal{D}})$. Therefore, $\varphi(s^{\mathcal{D}}) \in S^{\mathcal{F}}_{\mathcal{G}}$.

3. "$\Rightarrow$": Let $s^{\mathcal{D}} \xrightarrow{o} t^{\mathcal{D}} \in T^{\mathcal{F}}$. Then $o \in \mathcal{O}^G$ and thus $o^{dec} \in \mathcal{O}^{dec}$. We need to show that $\varphi(s^{\mathcal{D}}) \xrightarrow{o^{dec}} \varphi(t^{\mathcal{D}}) \in T^{dec}$.

   Applicability: Since o is applicable in $s^{\mathcal{D}}$, it holds that $pre(o)[C] \subseteq \mathsf{center}(s^{\mathcal{D}})$, and for each leaf L there exists a reached leaf state s^L such that $pre(o)[L] \subseteq s^L$. Since o^{dec} has the same center preconditions as o, and $\mathsf{center}(s^{\mathcal{D}}) = \varphi(s^{\mathcal{D}})[C]$, we know that $pre(o^{dec})[C] \subseteq \varphi(s^{\mathcal{D}})$.

   By Lemma 2 it holds that $\mathcal{A}(\varphi(s^{\mathcal{D}}))(d_{s^L}) = 1$ for all $s^L \in \mathsf{leaves}^*(s^{\mathcal{D}})$. Together with the $\mathcal{A}^L_{pre}$-axioms, this implies that all $d_{pre(o)[L]}$ variables are true in $\mathcal{A}(\varphi(s^{\mathcal{D}}))$, and thus o^{dec} is applicable in $\varphi(s^{\mathcal{D}})$.

   Successor: We show that $\varphi(t^{\mathcal{D}}) = \varphi(s^{\mathcal{D}})[\![o^{dec}]\!]$.

   $\varphi(t^{\mathcal{D}})[C] = \varphi(s^{\mathcal{D}}[\![o]\!])[C] = \varphi(s^{\mathcal{D}})[\![o^{dec}]\!][C]$, since the preconditions and effects on the center variables are the same in o and o^{dec}, and φ is the identity function when projected onto the center variables.

   The construction of the conditional effects establishes that a variable v_{t^L} is true in $\varphi(s^{\mathcal{D}})[\![o^{dec}]\!]$ iff there exists a leaf state s^L such that $s^L \in preimg(t^L, o)$ and d_{s^L} is true in $\mathcal{A}(\varphi(s^{\mathcal{D}}))$. By Lemma 2 we know that $s^L \in \mathsf{leaves}^*(s^{\mathcal{D}})$ iff d_{s^L} is true in $\mathcal{A}(\varphi(s^{\mathcal{D}}))$, and $t^L \in \mathsf{leaves}(t^{\mathcal{D}})$ iff there exists $s^L \in preimg(t^L, o)$. Consequently, v_{t^L} is true in $\varphi(s^{\mathcal{D}})[\![o^{dec}]\!]$ iff $t^L \in \mathsf{leaves}(t^{\mathcal{D}})$. Thus, $\varphi(t^{\mathcal{D}})[L] = \varphi(s^{\mathcal{D}}[\![o^{dec}]\!])[L]$ for all $L \in \mathcal{L}$.

   Hence, $\varphi(s^{\mathcal{D}}) \xrightarrow{o^{dec}} \varphi(t^{\mathcal{D}}) \in T^{dec}$.

   "$\Leftarrow$": Let $\varphi(s^{\mathcal{D}}) \xrightarrow{o^{dec}} \varphi(t^{\mathcal{D}}) \in T^{dec}$. Then $o^{dec} \in \mathcal{O}^{dec}$ and thus $o \in \mathcal{O}^G$. We need to show that $s^{\mathcal{D}} \xrightarrow{o} t^{\mathcal{D}} \in T^{\mathcal{F}}$.

   Applicability: Given the applicability of o^{dec} in $\varphi(s^{\mathcal{D}})$, it follows that $pre(o)[C] \subseteq \varphi(s^{\mathcal{D}})$ and for each leaf L where $pre(o)[L] \neq \emptyset$, it holds that $\mathcal{A}(\varphi(s^{\mathcal{D}}))(d_{pre(o)[L]}) = 1$.

   Considering that o has the same center preconditions as o^{dec}, and that $\varphi(s^{\mathcal{D}})[C] = \mathsf{center}(s^{\mathcal{D}})$, we see that $pre(o)[C] \subseteq \mathsf{center}(s^{\mathcal{D}})$.

   Since all $d_{pre(o)[L]}$ variables are true, this implies the truth of at least one d_{s^L} variable for each such L in $\mathcal{A}(\varphi(s^{\mathcal{D}}))$. By construction, it holds that $pre(o)[L] \subseteq s^L$ for such s^L. By Lemma 2, if $\mathcal{A}(\varphi(s^{\mathcal{D}}))(d_{s^L})$, then $s^L \in \mathsf{leaves}^*(s^{\mathcal{D}})$, which consequently proves the applicability of o in $s^{\mathcal{D}}$.

   Successor: We show that $t^{\mathcal{D}} = s^{\mathcal{D}}[\![o]\!]$.

   $\mathsf{center}(t^{\mathcal{D}}) = \varphi(t^{\mathcal{D}})[C] = \varphi(s^{\mathcal{D}})[\![o^{dec}]\!][C] = \mathsf{center}(s^{\mathcal{D}}[\![o]\!])$, since the preconditions and effects on the center variables are the same in o^{dec} and o, and φ is the identity function when projected onto the center variables.

   By the definition of φ, it holds that $s^L \in \mathsf{leaves}(s^{\mathcal{D}})$ iff v_{s^L} is true in $\varphi(s^{\mathcal{D}})$. Furthermore, we know that a variable v_{t^L} is true in $\varphi(t^{\mathcal{D}})$ iff there exists a variable d_{s^L} which is true in $\mathcal{A}(\varphi(s^{\mathcal{D}}))$ such that $s^L \in preimg(t^L, o)$. Since $\mathsf{leaves}(t^{\mathcal{D}})$ includes exactly the states t^L where $s^L \in \mathsf{leaves}(s^{\mathcal{D}})$ and $s^L \in preimg(t^L, o)$, it follows that $\mathsf{leaves}(t^{\mathcal{D}}) = \mathsf{leaves}(s^{\mathcal{D}}[\![o]\!])$.

   Hence, $s^{\mathcal{D}} \xrightarrow{o} t^{\mathcal{D}} \in T^{\mathcal{F}}$.

□