

Sliding Mode Control based on Backstepping Approach for an UAV Type-Quadrotor

H. Bouadi, M. Bouchoucha, and M. Tadjine

Abstract—In this paper; we are interested principally in dynamic modelling of quadrotor while taking into account the high-order nonholonomic constraints in order to develop a new control scheme as well as the various physical phenomena, which can influence the dynamics of a flying structure. These permit us to introduce a new state-space representation. After, the use of Backstepping approach for the synthesis of tracking errors and Lyapunov functions, a sliding mode controller is developed in order to ensure Lyapunov stability, the handling of all system nonlinearities and desired tracking trajectories. Finally simulation results are also provided in order to illustrate the performances of the proposed controller.

Keywords—Dynamic modelling, nonholonomic constraints, Backstepping, Sliding mode.

I. INTRODUCTION

UNMANNED aerial vehicles (UAV) have shown a growing interest thanks to recent technological projections, especially those related to instrumentation. They made possible the design of powerful systems (mini drones) endowed with real capacities of autonomous navigation at reasonable cost.

Despite the real progress made, researchers must still deal with serious difficulties, related to the control of such systems, particularly, in the presence of atmospheric turbulences. In addition, the navigation problem is complex and requires the perception of an often constrained and evolutionary environment, especially in the case of low-altitude flights.

Nowadays, the mini-drones invade several application domains [4]: safety (monitoring of the airspace, urban and interurban traffic); natural risk management (monitoring of volcano activities); environmental protection (measurement of air pollution and forest monitoring); intervention in hostile sites (radioactive workspace and mine clearance), management of the large infrastructures (dams, high-tension lines and pipelines), agriculture and film production (aerial shooting).

In contrast to terrestrial mobile robots, for which it is often possible to limit the model to kinematics, the control of aerial robots (quadrotor) requires dynamics in order to account for gravity effects and aerodynamic forces [3].

H. Bouadi and M. Bouchoucha are with Control and Command Laboratory, EMP, BEB, 16111, Algiers, Algeria (e-mail: hakimusavj@yahoo.fr, mouloud_bouchoucha@yahoo.fr).

M. Tadjine is with Electrical Engineering Department, ENP, 10, Ave Hassen Badi, BP.182, EL-Harrah, Algiers, Algeria (e-mail: tadjine@yahoo.fr).

In [7], authors propose a control-law based on the choice of a stabilizing Lyapunov function ensuring the desired tracking trajectories along (X, Z) axis and roll angle. However, they do not take into account nonholonomic constraints. In [9], authors do not take into account frictions due to the aerodynamic torques nor drag forces or nonholonomic constraints. They propose a control-law based on backstepping in order to stabilize the complete system (i.e. translation and orientation). In [1], authors take into account the gyroscopic effects and show that the classical model-independent PD controller can stabilize asymptotically the attitude of the quadrotor aircraft. Moreover, they used a new Lyapunov function, which leads to an exponentially stabilizing controller based upon the PD^2 and the compensation of coriolis and gyroscopic torques. While in [2] the authors develop a PID controller in order to stabilize altitude.

Others papers; presented the sliding mode and high-order sliding mode respectively like an observer [14] and [15] in order to estimate the unmeasured states and the effects of the external disturbances such as wind and noise.

In this paper, based on the vectorial model form presented in [2] we are interested principally in the modelling of quadrotor to account for various parameters which affect the dynamics of a flying structure such as frictions due to the aerodynamic torques, drag forces along (X, Y, Z) axis and gyroscopic effects which are identified in [2] for an experimental quadrotor and for high-order nonholonomic constraints [11]. Consequently, all these parameters supported the setting of the system under more complete and more realistic new state-space representation, which cannot be found easily in the literature being interested in the control laws synthesis for such systems.

Then, we present a control technique based on the development and the synthesis of a control algorithm based upon sliding mode based on backstepping approach ensuring the locally asymptotic stability and desired tracking trajectories expressed in term of the center of mass coordinates along (X, Y, Z) axis and yaw angle, while the desired roll and pitch angles are deduced from nonholonomic constraints unlike to [9].

Finally all synthesized control laws are highlighted by simulations which gave results considered to be satisfactory.

II. MODELLING

A. Quadrotor Dynamic Modelling



Fig. 1 Typical example of a quadrotor

The quadrotor have four propellers in cross configuration. The two pairs of propellers (1,3) and (2,4) as described in Fig. 2, turn in opposite directions. By varying the rotor speed, one can change the lift force and create motion. Thus, increasing or decreasing the four propeller's speeds together generates vertical motion. Changing the 2 and 4 propeller's speed conversely produces roll rotation coupled with lateral motion. Pitch rotation and the corresponding lateral motion; result from 1 and 3 propeller's speed conversely modified. Yaw rotation is more subtle, as it results from the difference in the counter-torque between each pair of propellers.

Let $E(O, X, Y, Z)$ denote an inertial frame, and $B(o', x, y, z)$ denote a frame rigidly attached to the quadrotor as shown in Fig. 2.

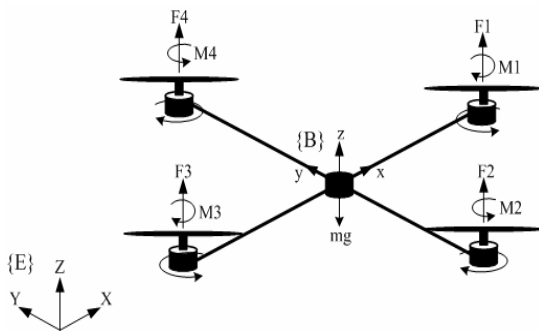


Fig. 2 Quadrotor configuration

We will make the following assumptions:

- The quadrotor structure is rigid and symmetrical.
- The center of mass and o' coincides.
- The propellers are rigid.
- Thrust and drag are proportional to the square of the propellers speed.

Under these assumptions, it is possible to describe the fuselage dynamics as that of a rigid body in space to which come to be added the aerodynamic forces caused by the rotation of the rotors.

Using the formalism of Newton-Euler, the dynamic equations are written in the following form:

$$\begin{cases} \dot{\xi} = v \\ m\ddot{\xi} = F_f + F_t + F_g \\ \dot{R} = RS(\Omega) \\ J\dot{\Omega} = -\Omega \wedge J\Omega + \Gamma_f - \Gamma_a - \Gamma_g \end{cases} \quad (1)$$

ξ is the position of the quadrotor center of mass with respect to the inertial frame. m is the total mass of the structure and $J \in R^{3 \times 3}$ is a symmetric positive definite constant inertia matrix of the quadrotor with respect to B .

$$J = \begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix} \quad (2)$$

Ω is the angular velocity of the airframe expressed in B :

$$\Omega = \begin{pmatrix} 1 & 0 & -\sin \theta \\ 0 & \cos \phi & \cos \theta \sin \phi \\ 0 & -\sin \phi & \cos \phi \cos \theta \end{pmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (3)$$

In the case when the quadrotor performs many angular motions of low amplitude Ω can be assimilated to $[\dot{\phi} \ \dot{\theta} \ \dot{\psi}]^T$.

R is the homogenous matrix transformation [12].

$$R = \begin{pmatrix} C\theta C\psi & C\psi S\theta S\phi - S\psi C\phi & C\psi S\theta C\phi + S\psi S\phi \\ C\theta S\psi & S\psi S\theta S\phi + C\psi C\phi & S\psi S\theta C\phi - C\psi S\phi \\ -S\theta & S\phi C\theta & C\phi C\theta \end{pmatrix} \quad (4)$$

Where C and S indicate the trigonometric functions **cos** and **sin** respectively. $S(\Omega)$ is a skew-symmetric matrix; for a given vector $\Omega = [\Omega_1 \ \Omega_2 \ \Omega_3]^T$ it is defined as follows:

$$S(\Omega) = \begin{pmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{pmatrix}$$

F_f is the resultant of the forces generated by the four rotors.

$$F_f = \begin{pmatrix} \cos \phi \cos \psi \sin \theta + \sin \phi \sin \psi \\ \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\ \cos \phi \cos \theta \end{pmatrix} \sum_{i=1}^4 F_i \quad (5)$$

$$F_i = K_p \omega_i^2 \quad (6)$$

Where K_p is the lift coefficient and ω_i is the angular rotor speed.

F_t is the resultant of the drag forces along (X, Y, Z) axis.

$$F_t = \begin{pmatrix} -K_{f_{tx}} & 0 & 0 \\ 0 & -K_{f_{ty}} & 0 \\ 0 & 0 & -K_{f_{tz}} \end{pmatrix} \dot{\xi} \quad (7)$$

Such as $K_{f_{tx}}, K_{f_{ty}}$ and $K_{f_{tz}}$ are the translation drag coefficients.

F_g is the gravity force.

$$F_g = [0 \ 0 \ -mg]^T \quad (8)$$

Γ_f is the moment developed by the quadrotor according to the body fixed frame. It is expressed as follows:

$$\Gamma_f = \begin{bmatrix} d(F_3 - F_1) \\ d(F_4 - F_2) \\ K_d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{bmatrix} \quad (9)$$

d is the distance between the quadrotor center of mass and the rotation axis of propeller and K_d is the drag coefficient.

Γ_a is the resultant of aerodynamics frictions torques.

$$\Gamma_a = \begin{bmatrix} K_{f_{ax}} & 0 & 0 \\ 0 & K_{f_{ay}} & 0 \\ 0 & 0 & K_{f_{az}} \end{bmatrix} \Omega^2 \quad (10)$$

$K_{f_{ax}}, K_{f_{ay}}$ and $K_{f_{az}}$ are the frictions aerodynamics coefficients.

Γ_g is the resultant of torques due to the gyroscopic effects.

$$\Gamma_g = \sum_{i=1}^4 \Omega \wedge J_r \begin{bmatrix} 0 \\ 0 \\ (-1)^{i+1} \omega_i \end{bmatrix} \quad (11)$$

Such as J_r is the rotor inertia.

Consequently the complete dynamic model which governs the quadrotor is as follows:

$$\begin{cases} \ddot{\phi} = \frac{1}{I_x} \{ \dot{\theta} \dot{\psi} (I_y - I_z) - K_{f_{ax}} \dot{\phi}^2 - J_r \bar{\Omega} \dot{\theta} + dU_2 \} \\ \ddot{\theta} = \frac{1}{I_y} \{ \dot{\phi} \dot{\psi} (I_z - I_x) - K_{f_{ay}} \dot{\theta}^2 + J_r \bar{\Omega} \dot{\phi} + dU_3 \} \\ \ddot{\psi} = \frac{1}{I_z} \{ \dot{\theta} \dot{\phi} (I_x - I_y) - K_{f_{az}} \dot{\psi}^2 + K_d U_4 \} \\ \ddot{x} = \frac{1}{m} \{ (C\phi S\theta C\psi + S\phi S\psi) U_1 - K_{f_{tx}} \dot{x} \} \\ \ddot{y} = \frac{1}{m} \{ (C\phi S\theta S\psi - S\phi C\psi) U_1 - K_{f_{ty}} \dot{y} \} \\ \ddot{z} = \frac{1}{m} \{ (C\phi C\theta) U_1 - K_{f_{tz}} \dot{z} \} - g \end{cases} \quad (12)$$

With U_1, U_2, U_3 and U_4 are the control inputs of the system which are written according to the angular velocities of the four rotors as follows:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{pmatrix} K_p & K_p & K_p & K_p \\ -K_p & 0 & K_p & 0 \\ 0 & -K_p & 0 & K_p \\ K_d & -K_d & K_d & -K_d \end{pmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (13)$$

and

$$\bar{\Omega} = (\omega_1 - \omega_2 + \omega_3 - \omega_4)$$

B. Nonholonomic Constraints

Taking into account nonholonomic constraints for our system is of major importance as are in compliance with physical laws and define the coupling between various states of the system.

From the equations of the translation dynamics (12) we can extract the expressions of the high-order nonholonomic constraints:

$$\begin{cases} \tan \theta = \frac{\left(\ddot{x} - \frac{K_{f_{tx}}}{m} \dot{x} \right) \cos \psi + \left(\ddot{y} - \frac{K_{f_{ty}}}{m} \dot{y} \right) \sin \psi}{\ddot{z} + g - \frac{K_{f_{tz}}}{m} \dot{z}} \\ \sin \phi = \frac{-\left(\ddot{x} - \frac{K_{f_{tx}}}{m} \dot{x} \right) \sin \psi + \left(\ddot{y} - \frac{K_{f_{ty}}}{m} \dot{y} \right) \cos \psi}{\sqrt{\left(\ddot{x} - \frac{K_{f_{tx}}}{m} \dot{x} \right)^2 + \left(\ddot{y} - \frac{K_{f_{ty}}}{m} \dot{y} \right)^2 + \left(\ddot{z} + g - \frac{K_{f_{tz}}}{m} \dot{z} \right)^2}} \end{cases} \quad (14)$$

C. Rotor Dynamic

The rotor is a unit constituted by D.C-motor actuating a propeller via a reducer. The D.C-motor is governed by the following dynamic equations:

$$\begin{cases} V = ri + L \frac{di}{dt} + k_e \omega \\ k_m i = J_r \frac{d\omega}{dt} + C_s + k_r \omega^2 \end{cases} \quad (15)$$

The different parameters of the motor are defined such:

V : motor input.

k_e, k_m : electrical and mechanical torque constant respectively.

k_r : load constant torque.

r : motor internal resistance.

J_r : rotor inertia.

C_s : solid friction.

Then, the model chosen for the rotor is as follows:

$$\dot{\omega}_i = bV_i - \beta_0 - \beta_1 \omega_i - \beta_2 \omega_i^2 \quad (16)$$

$$i \in [1, 4]$$

with:

$$\beta_0 = \frac{C_s}{J_r}, \beta_1 = \frac{k_e k_m}{r J_r}, \beta_2 = \frac{k_r}{J_r} \text{ and } b = \frac{k_m}{r J_r}$$

III. SLIDING MODE CONTROL OF THE QUADROTOR

The choice of this method is not fortuitous considering the major advantages it presents:

- It ensures Lyapunov stability.
- It ensures the robustness and all properties of the desired dynamics.
- It ensures the handling of all system nonlinearities.

The model (12) developed in the first part of this paper can be rewritten in the state-space form:

$\dot{X} = f(X) + g(X, U) + \delta$ and $X = [x_1 \dots x_{12}]^T$ is the state vector of the system such as:

$$X = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, x, \dot{x}, y, \dot{y}, z, \dot{z}]^T \quad (17)$$

From (12) and (17) we obtain the following state representation:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a_1 x_4 x_6 + a_2 x_2^2 + a_3 \bar{\Omega} x_4 + b_1 U_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = a_4 x_2 x_6 + a_5 x_4^2 + a_6 \bar{\Omega} x_2 + b_2 U_3 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = a_7 x_2 x_4 + a_8 x_6^2 + b_3 U_4 \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = a_9 x_8 + U_x \frac{U_1}{m} \\ \dot{x}_9 = x_{10} \\ \dot{x}_{10} = a_{10} x_{10} + U_y \frac{U_1}{m} \\ \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = a_{11} x_{12} + \frac{C x_1 C x_3 U_1}{m} - g \end{cases} \quad (18)$$

$$\begin{cases} a_1 = \left(\frac{I_y - I_z}{I_x} \right), a_2 = \frac{-K_{f_{ax}}}{I_x}, a_3 = \frac{-J_r}{I_x} \\ a_4 = \left(\frac{I_z - I_x}{I_y} \right), a_5 = \frac{-K_{f_{ay}}}{I_y}, a_6 = \frac{J_r}{I_y} \\ a_7 = \left(\frac{I_x - I_y}{I_z} \right), a_8 = \frac{-K_{f_{az}}}{I_z}, a_9 = \frac{-K_{f_{az}}}{m}, a_{10} = \frac{-K_{f_{vy}}}{m}, a_{11} = \frac{-K_{f_{vz}}}{m} \\ b_1 = \frac{d}{I_x}, b_2 = \frac{d}{I_y}, b_3 = \frac{1}{I_z} \end{cases} \quad (19)$$

$$\begin{cases} U_x = C x_1 S x_3 C x_5 + S x_1 S x_5 \\ U_y = C x_1 S x_3 S x_5 - S x_1 C x_5 \end{cases} \quad (20)$$

The state representation of the system under this form has never been developed before.

From high-order nonholonomic constraints developed in (14), roll (ϕ) and pitch (θ) angles depend not only on the yaw angle (ψ) but also on the movements along (X, Y, Z) axis and their dynamics. However the adopted control strategy is summarized in the control of two subsystems; the first relates to the position control while the second is that of the attitude control as shown it below the synoptic scheme:

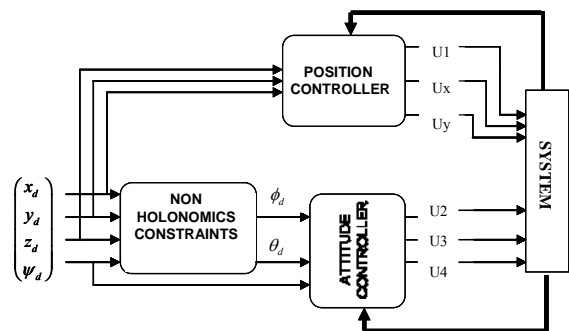


Fig. 3 Synoptic scheme of the proposed controller

In this section we develop a sliding mode controller for the quadrotor based on Backstepping approach using the technique presented in [13].

Using the backstepping approach as a recursive algorithm for the control-laws synthesis, we simplify all the stages of calculation concerning the tracking errors and Lyapunov functions in the following way:

$$z_i = \begin{cases} x_{id} - x_i & / i \in \{1, 3, 5, 7, 9, 11\} \\ x_i - \dot{x}_{(i-1)d} - \alpha_{(i-1)} z_{(i-1)} & / i \in \{2, 4, 6, 8, 10, 12\} \end{cases} \quad (21)$$

with $\alpha_i > 0 \quad \forall i \in [1, 12]$

$$\text{and: } V_i = \begin{cases} \frac{1}{2} z_i^2 & / i \in \{1, 3, 5, 7, 9, 11\} \\ \frac{1}{2} (V_{i-1} + z_i^2) & / i \in \{2, 4, 6, 8, 10, 12\} \end{cases} \quad (22)$$

The choice of the sliding surfaces is based upon the synthesized tracking errors which permitted us the synthesis of stabilizing control laws, so from (21) we define:

$$\begin{cases} S_\phi = z_2 = x_2 - \dot{x}_{1d} - \alpha_1 z_1 \\ S_\theta = z_4 = x_4 - \dot{x}_{3d} - \alpha_3 z_3 \\ S_\psi = z_6 = x_6 - \dot{x}_{5d} - \alpha_5 z_5 \\ S_x = z_8 = x_8 - \dot{x}_{7d} - \alpha_7 z_7 \\ S_y = z_{10} = x_{10} - \dot{x}_{9d} - \alpha_9 z_9 \\ S_z = z_{12} = x_{12} - \dot{x}_{11d} - \alpha_{11} z_{11} \end{cases} \quad (23)$$

Such as $S_\phi, S_\theta, S_\psi, S_x, S_y$ and S_z are the dynamic sliding surfaces.

To synthesize a stabilizing control law by sliding mode, the necessary sliding condition ($S\dot{S} < 0$) must be verified; so the synthesized stabilizing control laws are as follows:

$$\begin{cases} U_2 = \frac{1}{b_1} \{-q_1 \text{sign}(S_\phi) - k_1 S_\phi - a_1 x_4 x_6 - a_2 x_2^2 - a_3 \bar{\Omega} x_4 + \ddot{\phi}_d + \alpha_1 (\dot{\phi}_d - x_2)\} \\ U_3 = \frac{1}{b_2} \{-q_2 \text{sign}(S_\theta) - k_2 S_\theta - a_4 x_2 x_6 - a_5 x_4^2 - a_6 \bar{\Omega} x_5 + \ddot{\theta}_d + \alpha_3 (\dot{\theta}_d - x_4)\} \\ U_4 = \frac{1}{b_3} \{-q_3 \text{sign}(S_\psi) - k_3 S_\psi - a_7 x_2 x_4 - a_8 x_6^2 + \ddot{\psi}_d + \alpha_5 (\dot{\psi}_d - x_6)\} \\ U_x = \frac{m}{U_1} \{-q_4 \text{sign}(S_x) - k_4 S_x - a_9 x_8 + \ddot{x}_d + \alpha_7 (\dot{x}_d - x_8)\} / U_1 \neq 0 \\ U_y = \frac{m}{U_1} \{-q_5 \text{sign}(S_y) - k_5 S_y - a_{10} x_{10} + \ddot{y}_d + \alpha_9 (\dot{y}_d - x_{10})\} / U_1 \neq 0 \\ U_z = \frac{m}{C\phi C\theta} \{-q_6 \text{sign}(S_z) - k_6 S_z - a_{11} x_{12} + \ddot{z}_d + \alpha_{11} (\dot{z}_d - x_{12}) + g\} \end{cases} \quad (24)$$

Such as $(q_i, k_i) \in \mathbb{R}^{+2}$.

Proof

We know a priori from (21) and (22) that:

$$\begin{cases} V_2 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \\ z_2 = x_2 - \dot{x}_{1d} - \alpha_1 z_1 \end{cases} \quad (25)$$

And from (23):

$$S_\phi = z_2 = x_2 - \dot{x}_{1d} - \alpha_1 z_1 \quad (26)$$

So:

$$V_2 = \frac{1}{2} z_1^2 + \frac{1}{2} S_\phi^2 \quad (27)$$

With:

$$\begin{cases} \dot{V}_2 = z_1 \dot{z}_1 + S_\phi \dot{S}_\phi \\ \dot{V}_2 = z_1 \dot{z}_1 + S_\phi \{a_1 x_4 x_6 + a_2 x_2^2 + a_3 \bar{\Omega} x_4 + b_1 U_2 - \ddot{\phi}_d - \alpha_1 (\dot{\phi}_d - x_2)\} \end{cases} \quad (28)$$

The chosen law for the attractive surface is the time derivative of (26) satisfying ($S_\phi \dot{S}_\phi < 0$):

$$\begin{aligned} \dot{S}_\phi &= -q_1 \text{sign}(S_\phi) - k_1 S_\phi \\ &= \dot{x}_2 - \ddot{x}_{1d} - \alpha_1 \dot{z}_1 \\ &= a_1 x_4 x_6 + a_2 x_2^2 + a_3 \bar{\Omega} x_4 + b_1 U_2 - \ddot{\phi}_d - \alpha_1 (\dot{\phi}_d - x_2) \end{aligned} \quad (29)$$

As for the Backstepping approach, the control input U_2 is extracted:

$$U_2 = \frac{1}{b_1} \{-q_1 \text{sign}(S_\phi) - k_1 S_\phi - a_1 x_4 x_6 - a_2 x_2^2 - a_3 \bar{\Omega} x_4 + \ddot{\phi}_d + \alpha_1 (\dot{\phi}_d - x_2)\} \quad (30)$$

The same steps are followed to extract U_3, U_4, U_x, U_y and U_1 .

IV. SIMULATION RESULTS

The simulation results are obtained based on the following real parameters [2]:

$$\begin{aligned} K_p &= 2.9842 \times 10^{-5} \text{ N.m / rad / s} \\ K_d &= 3.2320 \times 10^{-7} \text{ N.m / rad / s} \\ m &= 486 \text{ g} \\ d &= 25 \text{ cm} \\ J &= \text{diag}(3.8278; 3.8288; 7.6566) \times 10^{-3} \text{ N.m / rad / s}^2 \\ K_{f\theta} &= \text{diag}(5.5670; 5.5670; 6.3540) \times 10^{-4} \text{ N / rad / s} \\ K_{f\psi} &= \text{diag}(5.5670; 5.5670; 6.3540) \times 10^{-4} \text{ N / m / s} \\ J_r &= 2.8385 \times 10^{-5} \text{ N.m / rad / s}^2 \\ \beta_0 &= 189.63 \\ \beta_1 &= 6.0612 \\ \beta_2 &= 0.0122 \\ b &= 280.19 \end{aligned}$$

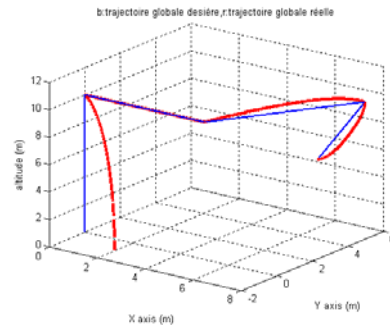


Fig. 4 Global tracking trajectory of the quadrotor by sliding mode

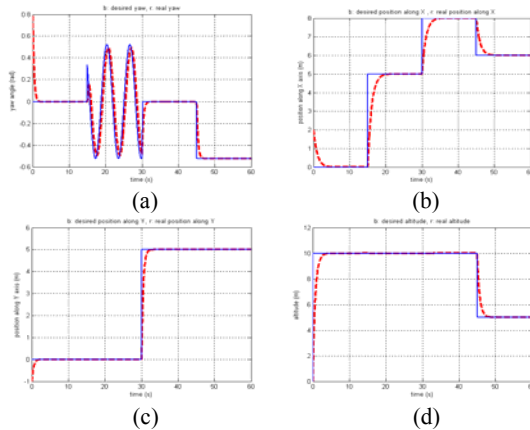


Fig. 5 Tracking simulation results of the desired trajectories along yaw angle (ψ) and (X, Y, Z) axis

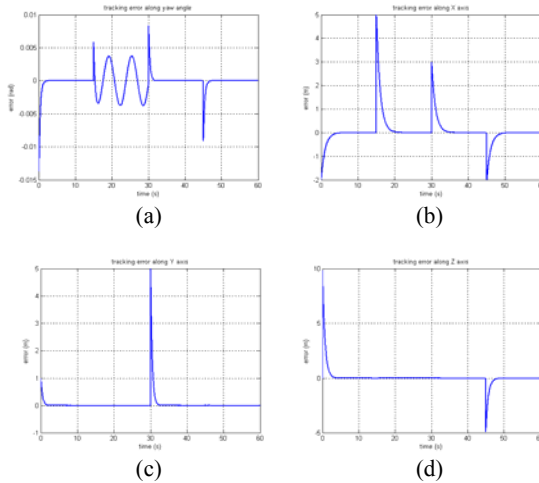


Fig. 6 Tracking errors according yaw (ψ) angle and (X, Y, Z) respectively

Fig. 4 shows the tracking of desired trajectory by the real one and the evolution of the quadrotor in space and its stabilization.

Fig. 5 highlights the tracking of the desired trajectories along yaw angle (ψ) and (X, Y, Z) axis respectively. The response time of the system is about 3s and the tracking in yaw presents a rather weak permanent error when the desired trajectory is dynamic.

Fig. 6 represents the errors made on the desired trajectory tracking.

V. CONCLUSION

In this paper, we presented stabilizing control laws synthesis by sliding mode based on backstepping approach. Firstly, we start by the development of the dynamic model of the quadrotor taking into account the different physics phenomena which can influence the evolution of our system in the space and secondly by the development of the high-order nonholonomic constraints imposed to the system motions; this says these control laws allowed the tracking of the various desired trajectories expressed in term of the center

of mass coordinates of the system in spite of the complexity of the proposed model. As prospects we hope to develop other control techniques in order to improve the performances and to implement them on a real system.

ACKNOWLEDGMENT

The authors would like to thank Dr. Taha Chettibi, PhD students Mohamed Guatni and Karim Souissi for their help and their fruitful discussions about the stability problem of under-actuated systems.

REFERENCES

- [1] A.Tayebi, S.Mcgilvray, 2004 "Attitude stabilisation of a four rotor aerial robot", IEEE conference on decision and control, December 14-17, Atlantis Paradise Island, Bahamas 1216-1217.
- [2] Derafa L. Madani t. and Benallegue A, 2006 « dynamic modelling and experimental identification of four rotor helicopter parameters » ICIT Mumbai, India.
- [3] Guenard N. Hamel t. Moreau V, 2004 « modélisation et élaboration de commande de stabilisation de vitesse et de correction d'assiette pour un drone »CIFA.
- [4] Hamel T. Mahoney r. Lozano r. Et Ostrowski j, 2002. "Dynamic modelling and configuration stabilization for an X4-flyer." In the 15ème IFAC world congress', Barcelona, Spain.
- [5] Olfati. S, 2001 "nonlinear control of under actuated mechanical systems with application to robotics and aerospace vehicles », PhD thesis, MIT.
- [6] P. Pounds, R.Mahony, 2002 "Design of a four rotor "ICRA, Auckland.
- [7] R.Lozano, P.Castillo, and A.Dzul, 2004 "global stabilization of the PVTOL: real time application to a mini aircraft" International Journal of Control, Vol 77, Number 8, pp 735-740, may.
- [8] S. Bouabdellah, P.Murrieri and R.Siegwart, 2004 "Design and control of an indoor micro quadrotor" ICRA 2004, New Orleans (USA), April.
- [9] S. Bouabdellah and R.Siegwart, 2005 "Backstepping and sliding mode techniques applied to an indoor micro quadrotor" Proceeding of the 2005 IEEE, ICRA, Barcelona, Spain, April.
- [10] S.Bouabdellah, A.Noah and R.Siegwart, 2004 "modelling of the (OS4) quadrotor" modelling course, EPFL, may.
- [11] S. Yazir, M. Mekki and T. Chettibi, 2007 "Modélisation dynamique directe et inverse d'un quadrotor» Accepted in CGE, Algiers, Algeria, April.
- [12] W.Khalil, Dombre, 2002 "modelling, identification and control of robots" HPS edition.
- [13] A.Benchaib, F.Boudjema and A.Rachid"sliding mode flux observer based on backstepping approach for induction motor" World Automation congress WAC'98, Alaska, USA, may 1998.
- [14] A.Mokhtari, A.Benallegue and A.Belaidi "polynomial linear quadratic Gaussian and sliding mode observer for a quadrotor unmanned aerial vehicle" Journal of Robotics and Mechatronics Vol.17 No.4, 2005.
- [15] A.Mokhtari, N.K.M'sirdi, K.Meghriche and A.Belaidi "feedback linearization and linear observer for a quadrotor unmanned aerial vehicle" Advanced Robotics, Vol.20, No.1, pp. 71-91, 2006.