# Optimal Synthesis of Multipass Heat Exchanger without Resorting to Correction Factor 

Bharat B. Gulyani, Anuj Jain, Shalendra Kumar


#### Abstract

Customarily, the LMTD correction factor, $\mathrm{F}_{\mathrm{T}}$, is used to screen alternative designs for a heat exchanger. Designs with unacceptably low $F_{T}$ values are discarded. In this paper, authors have proposed a more fundamental criterion, based on feasibility of a multipass exchanger as the only criteria, followed by economic optimization. This criterion, coupled with asymptotic energy targets, provide the complete optimization space in a heat exchanger network (HEN), where cost-optimization of HEN can be performed with only Heat Recovery Approach temperature (HRAT) and number-of-shells as variables.


Keywords-heat exchanger, heat exchanger networks, LMTD correction factor, shell targeting.

## I. INTRODUCTION

THE shell and tube heat exchangers are one of the most widely used equipment in the process industry. In order to achieve better heat transfer efficiency these heat exchangers are often made as multipass. Presently, the correction factor, $\mathrm{F}_{\mathrm{T}}$, is used to screen alternative designs for the exchangers before resorting to detailed design calculations. The value of $\mathrm{F}_{\mathrm{T}}$ adopted for the design is selected using ad hoc criteria, $\mathrm{F}_{\mathrm{T}}$ $>0.8$.

It has been more than seven decades since Bowman et al [1] proposed the use of LMTD correction factor in the design of multipass shell-and-tube heat exchangers. All the other criteria proposed later [2], [3], [4], [5], [6], [7] are modifications of $\mathrm{F}_{\mathrm{T}}$ criteria to make it computationally efficient. In all criteria proposed for calculating the number of shells in a multipass exchanger, the implicit constraint is that $F_{T}$ should not fall below a certain value ( 0.75 or 0.8 ).

## II. THE $\mathrm{F}_{\mathrm{T}}$ CORRECTION FACTORS

In case of the simplest shell and tube heat exchanger __ the 1-2 type, the liquid in one tube pass flows in counter flow while in the other pass flows in parallel relative to shell fluid. The method of calculation of log mean temperature difference, LMTD, for counter flow as well as parallel flow is well established [8]. For the design of multipass exchangers where both types of flow coexist, an analytical expression for

Bharat B. Gulyani is associate professor and head, chemical engineering department at BITS Pilani, Dubai Campus, Dubai International Academic City, Dubai (UAE) (Phone: 971553034760; Fax: 97144200844; e-mail: gulyanibb@yahoo.com).

Anuj Jain is professor department of applied mechanics, at MN National Institute of technology, Allahabad, India.

Shalendra Kumar is professor, mechanical engineering, at National Institute of Technology, Jamshedpur, India.
estimating actual mean temperature difference was developed by [9] and later modified by [1]. In this design practice, a
correction factor $\mathrm{F}_{\mathrm{T}}$ is introduced into the basic heat exchanger design equation, equation (1), to take into account the above phenomena,
$\mathrm{Q}=\mathrm{UA}$ (LMTD) $\mathrm{F}_{\mathrm{T}}$, where $0<\mathrm{F}_{\mathrm{T}}<1$
The $\mathrm{F}_{\mathrm{T}}$ factor can be represented as the ratio of actual mean temperature difference in a 1-2 exchanger to counter flow LMTD for the same terminal temperatures. The physical significance of $F_{T}$ is given by many authors [10], [11], [12].
$\mathrm{F}_{\mathrm{T}}$ is usually expressed as a function of two dimensionless parameters R and S defined below:
Heat Capacity ratio, $R=\frac{T_{1}-T_{2}}{t_{2}-t_{1}}$
Thermal Effectiveness, $\quad S=\frac{\mathrm{t}_{2}-\mathrm{t}_{1}}{\mathrm{~T}_{1}-\mathrm{t}_{1}}$
The analytical equations for estimating $F_{T}$ as a function of $R$ and $S$ for 1-2 and 2-4 exchangers are given by [8]. Design charts based on this method are available and are compiled by TEMA [13].
$\mathrm{F}_{\mathrm{T}}$ is used to screen alternative designs before resorting to detailed design calculations. Designs with unacceptably low $\mathrm{F}_{\mathrm{T}}$ values are discarded. A commonly used rule of thumb requires $\mathrm{F}_{\mathrm{T}} \geq 0.8$ for the design to be considered practical. However, the use of this ad-hoc criterion for 1-2 exchanger is arbitrary (and restrictive), and can lead to poor designs if not used with caution [2]. Frank [14] recommended that the 1-2 exchangers should not be designed where $F_{T}$ factors approach a vertical slope, as a small departure from the design point can result in precipitous decline of $\mathrm{F}_{\mathrm{T}}$. Thus, the advice to the designer to refrain from designing with $\mathrm{F}_{\mathrm{T}} \leq 0.8$ comes mainly because of steep slopes of the $\mathrm{F}_{\mathrm{T}}$ curves in that region, which prohibits the designer to estimate $\mathrm{F}_{\mathrm{T}}$ correctly.

## III. DESIGN FOR MULTIPASS EXCHANGERS

All the approaches till date to design multipass exchangers have been $F_{T}$-centric. Designers often encounter situations where either the $F_{T}$ is too low or the slope of $F_{T}$ versus $S$ curve is too large. If this happens, the designer is advised to consider multipass exchangers. A brief summary of these approaches is given below.

## A. The Traditional Approach (explicit $F_{T}$ approach)

Traditionally, the designer would approach a problem requiring multiple shells by trial and error. By assuming a
number of shells, usually one in the first instance, the $F_{T}$ is evaluated. If the $F_{T}$ is not acceptable then the number of shells in series is progressively increased until a satisfactory value of $\mathrm{F}_{\mathrm{T}}$ is obtained for each shell.

## B. Method of Ahmad et al.

Ahmad et al. [2] have given an analytical expression for calculating number of shells directly,
$N=\frac{\ln \frac{(1-R S)}{(1-S)}}{\ln W}$
where, N is real (non-integer) number of shells, and
$W=\frac{R+1+\sqrt{R^{2}+1}-2 R X_{P}}{R+1+\sqrt{R^{2}+1}-2 X_{P}}$
Equation (4) gives a value of N that satisfies the chosen value of $X_{P}$. The problem now is - what should be the design value of $\mathrm{X}_{\mathrm{P}}$ ? And how it will affect temperature cross and consequently, $\mathrm{F}_{\mathrm{T}}$. Though [2] emphasized the importance of temperature cross in exchanger design, they didn't explain how $X_{P}$ accounts for temperature cross. Their choice of value of $X_{P}=0.9$ is based on $F_{T}=0.75$ at $R=1$, which is again arbitrary. What if a designer wants to use a lower value of $X_{P}$ ?

## C. Method based on not allowing Temperature Cross

This method is based on a dimensionless parameter G which explicitly accounts for temperature cross in the exchanger [15], [3]. It is defined as,
$G=\frac{T_{2}-t_{2}}{T_{1}-t_{1}}$
$F_{T}$ decreases moderately with decreasing positive $G$ values, but falls sharply both where the temperature meet $(\mathrm{G}=0)$ and where the G values are negative (temperature cross).
The parameter G is related to parameter R and S , by the equation, $G=1-S(1+R)$
For any value of R there exists a minimum asymptotic value of $G$ (corresponding to $F T=-\infty$ ), say $G_{\text {min }}$, which represents the maximum temperature cross theoretically feasible in 1-2 exchanger.
The expression for $\mathrm{G}_{\text {min }}$ is,
$G_{\text {MIN }}=\frac{\sqrt{R^{2}+1}-(R+1)}{\sqrt{R^{2}+1}+(R+1)}$
It is shown by [4] that for $\mathrm{G}=0, \mathrm{~F}_{\mathrm{T}}$ is always above 0.8 ( $0.8<\mathrm{F}_{\mathrm{T}}<0.83$ ), an acceptable value. Thus, following the criteria that temperature cross is not allowed in each shell ensures that $\mathrm{F}_{\mathrm{T}}>0.8$. Using this fact, a simple equation is derived below to calculate the number of shells.
$\mathrm{N}=\frac{\ln \left(\frac{\mathrm{R}+\mathrm{G}}{1+\mathrm{RG}}\right)}{\ln (\mathrm{R})}$
Equation (9) can be used to estimate the number of shells for given R and G (calculated from the terminal temperatures).

## D. Method based on allowing Temperature Cross

Some authors recommend that in order to get the minimum number of shells it is necessary to allow some temperature cross [16]. For such cases a comprehensive criteria has been developed by [4]. A 1-2 exchanger designed for $\mathrm{G}<\mathrm{G}_{\min }$ will not be feasible. Any increment in G from $\mathrm{G}_{\text {min }}$ will make the exchanger feasible, and improve exchanger effectiveness and $\mathrm{F}_{\mathrm{T}}$. Let the desired increment be Y . Then
$G=G_{\min }+Y$
where Y is a constant set by the designer. Now, the expression for estimating the number of shells can be written as
$N=\frac{\ln \frac{\left(1-R G_{N}\right)}{\left(R+G_{N}\right)}}{\ln W}$
Where $\mathrm{G}_{\mathrm{N}}$ is G for multipass exchanger, and
$W=\frac{\sqrt{R^{2}+1}(R+1-R Y)-(R+1)(R-1-R Y)}{\sqrt{R^{2}+1}(R+1-Y)-(R+1)(R-1-Y)}$
$Y$ can be correlated with $X_{P}$ as,
$Y=\frac{2(R+1)\left(1-X_{P}\right)}{\sqrt{R^{2}+1}+(R+1)}$
Alternatively,
$X_{P}=\frac{(R+1)(2-Y) \sqrt{R^{2}+1}}{2(R+1)}$
Y is chosen by the designer's decision on how much temperature cross he is going to allow in the design. Author also recommends th(6) to be compatible with the existing design practices ( $\mathrm{F}_{\mathrm{T}}>.0 .75$; or $\mathrm{X}_{\mathrm{P}}=0.9$ ), a value of Y in the range 0.1 to 0.15 may be selected.

## E. Methods based on $F_{T}$ slopes

An additional method of avoiding areas of steep slopes in the $\mathrm{F}_{\mathrm{T}}$ chart is to consider a constant $\mathrm{F}_{\mathrm{T}}$ slope. Ahmad et al. [2] have presented a constant slope criterion in a graphical form. However, their criterion, which is good to guarantee to stay away from those regions, is very complex to use and evaluate, as the authors recognized in their paper.

## IV. FALLACY OF $\mathrm{F}_{\mathrm{T}}$ CRITERIA

Despite all the arguments put forth in favor of this criterion ( $\mathrm{F}_{\mathrm{T}}$ should be higher than a recommended minimum value), we show here that this cardinal rule of multipass exchanger design is fallacious, unnecessarily restrictive, and misleading. It is also shown by the authors that such restriction does not lead to economically optimum HEN. It is proposed that to obtain cost-optimal HEN consisting of multipass units, only the feasibility of a multipass exchanger should be a constraint.

In advancing the arguments in favor of a minimum $F_{T}$ value (in each shell) for shell and tube exchanger, it is implicitly assumed that the change in S (and/ or R) affects only $\mathrm{F}_{\mathrm{T}}$ independent of everything else. This is an incorrect notion. This fact is illustrated in the following paragraphs.

The Eqn. for $\mathrm{F}_{\mathrm{T}}$ for a 1-2 exchanger in terms of R and S is given as,

$$
\begin{equation*}
F_{T}=\frac{\sqrt{R^{2}+1}}{R-1} \frac{\ln \frac{(1-S)}{(1-R S)}}{\ln \frac{(1-S)+(1-R S)+S \sqrt{R^{2}+1}}{(1-S)+(1-R S)-S \sqrt{R^{2}+1}}} \tag{15}
\end{equation*}
$$

The design equation for the exchanger can be written as,
$Q=U A(L M T D) F_{T}$
The counter flow LMTD (used in above equation) can be written as,

$$
\begin{equation*}
\text { LMTD }=\left(\frac{\left(T_{2}-t_{2}\right)-\left(T_{1}-t_{1}\right)}{\ln \left(\frac{T_{2}-t_{2}}{T_{1}-t_{1}}\right)}\right)=\left(\frac{g S((R-1)}{\ln \left(\frac{1-S}{1-R S}\right)}\right) \tag{17}
\end{equation*}
$$

Where g is the greatest temperature difference in the exchanger,

$$
\begin{equation*}
g=T_{1}-t_{1} \tag{18}
\end{equation*}
$$

The effective temperature difference in a 1-2 exchanger is thus,

$$
\begin{align*}
& \Delta T_{\text {eff }}=(L M T D) F_{T}=\left(\frac{g S((R-1)}{\ln \left(\frac{1-S}{1-R S}\right)}\right) \times \\
& \frac{\sqrt{R^{2}+1}}{R-1} \frac{\ln \frac{(1-S)}{(1-R S)}}{\ln \frac{(1-S)+(1-R S)+S \sqrt{R^{2}+1}}{(1-S)+(1-R S)-S \sqrt{R^{2}+1}}} \\
& \text { Or } \Delta T_{e f f}=\frac{g S \sqrt{R^{2}+1}}{\ln \frac{(1-S)+(1-R S)+S \sqrt{R^{2}+1}}{(1-S)+(1-R S)-S \sqrt{R^{2}+1}}} \tag{19}
\end{align*}
$$

It is $\Delta T_{\text {eff }}$, not $\mathrm{F}_{\mathrm{T}}$ alone, which affects the area of the exchanger, and consequently, the cost.
To expose the fallacy of steep fall in driving force for $\mathrm{F}_{\mathrm{T}}<$ 0.8 , let us look at the behavior of $\mathrm{F}_{\mathrm{T}}$, LMTD, and $\Delta T_{\text {eff }}$ with S (for given R ). It is known that for a given value of R , there exists a $S_{\text {max }}$ beyond which the 1-2 exchanger becomes thermodynamically infeasible, and going for higher number of shells is the only option.
Following figures show the variation of LMTD, $\mathrm{F}_{\mathrm{T}}$, and $\Delta T_{\text {eff }}$ with S for different R values (Figure 1 for $\mathrm{R}=1$, figure 2 for $\mathrm{R}=2$, figure 3 for $\mathrm{R}=3$, and figure 4 for $\mathrm{R}=4$ ).


Fig. 1 Variation of correction factor FT, LMTD, and effective temperature difference with S for $\mathrm{R}=1.01$


Fig. 2 Variation of correction factor FT, LMTD, and effective temperature difference with S for $\mathrm{R}=2.0$

## $R=3.0$



Fig. 3 Variation of correction factor FT, LMTD, and effective temperature difference with S for $\mathrm{R}=3.0$


Fig. 4 Variation of correction factor FT, LMTD, and effective temperature difference with S for $\mathrm{R}=4.0$

It can be seen from these figures that for all R values, with increasing $\mathrm{S} ; \mathrm{F}_{\mathrm{T}}$, LMTD and $\Delta T_{\text {eff }}$ all decrease. It is also to be noted that the fall in value of LMTD and $\Delta T_{\text {eff }}$ is gradual and not steep. The fall in the value of LMTD with S is almost linear. The fall in the value of $\Delta T_{\text {eff }}$ with S is steep only in the region very near to infeasibility ( $S \cong S_{\max }$ ), where the $\mathrm{F}_{T}$ values are far below the recommended value.

## V. NUMBER OF SHELLS CALCULATION BASED ON FEASIBILITY CRITERION

Therefore, the value of $\mathrm{F}_{\mathrm{T}}$ need not be a limiting criterion in estimating number of shells. In fact, we need not calculate $F_{T}$ at all, when determining total number of shells in HEN. The feasibility of a multipass exchanger should be the only criteria, with overall cost as the final arbiter. The procedure is outlined below.
The equation for estimating number of shells is given as,
$\mathrm{N}=\frac{\ln \left(\frac{1-\mathrm{RS}}{1-\mathrm{S}}\right)}{\ln \left(\frac{1-\mathrm{RS}_{1}}{1-\mathrm{S}_{1}}\right)}$
Where $S_{1}$ is the temperature effectiveness in each shell of multipass exchanger having overall effectiveness S .

The maximum value of effectiveness for given $R$ is given as,
$S_{M A X}=\frac{2}{\sqrt{R^{2}+1}+(R+1)}$
$N=\frac{\ln \left(\frac{1-R S}{1-S}\right)}{\ln \left[\frac{\sqrt{R^{2}+1}-(R-1)}{\sqrt{R^{2}+1}+(R-1)}\right]}$
The above equation gives the absolute minimum number of shells needed in an exchanger. The above equation gives "real" number of shells, which has to be rounded-up, resulting in improved effectiveness.

When real number of shells N is rounded up to integer number of shells, M , an improvement in S and consequently $\mathrm{F}_{\mathrm{T}}$ happens. This improvement is calculated from the equation,
$S_{\text {improved }}=\frac{1-\left(\frac{1-R S}{1-S}\right)^{1 / M}}{R-\left(\frac{1-R S}{1-S}\right)^{1 / M}}$
To demonstrate the validity of above proposal, following case studies are taken up from [6] and [7].
Moita et al. [6] applied various approaches to shells estimation to a set of seven exchangers (E1 to E7), reproduced in table 1 below. Six case studies are taken up from [7], also reproduced in Table 1.
For multipass exchangers, the capital cost equation is given as,
$C=a+b N\left(\frac{A}{N}\right)^{c}=a+b N^{1-c} A^{c}$

Substituting $S_{M A X}$ for $S_{1}$ in Eqn (20) we get,
Table I
DATA FOR CASE STUDY EXCHANGERS

| Exchanger | $\mathrm{T}_{1}$ | $\mathrm{T}_{2}$ | $\mathbf{t}_{1}$ | $\mathrm{t}_{2}$ | Q (kW) | $\begin{gathered} \mathrm{U} \\ \left(\mathrm{~kW} / \mathrm{m}^{2} / \mathrm{K}\right. \\ ) \end{gathered}$ | Cost law coefficients |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | a | b | C |
| Moita et al. (2004) |  |  |  |  |  |  |  |  |  |
| E1 | 562 | 92 | 26 | 120 | 2000 | 0.1 | 0 | 7000 | 0.65 |
| E2 | 381.2 | 314 | 0 | 336 | 2000 | 0.1 | 0 | 7000 | 0.65 |
| E3 | 410 | 110 | 0 | 360 | 2000 | 0.1 | 0 | 7000 | 0.65 |
| E4 | 560 | 125 | 1 | 88 | 2000 | 0.1 | 0 | 7000 | 0.65 |
| E5 | 540 | 162 | 10 | 174 | 2000 | 0.1 | 0 | 7000 | 0.65 |
| E6 | 388 | 322 | 0 | 330 | 2000 | 0.1 | 0 | 7000 | 0.65 |
| E7 | 394 | 329 | 0 | 325 | 2000 | 0.1 | 0 | 7000 | 0.65 |
| Ponce-Ortega et al. (2008) |  |  |  |  |  |  |  |  |  |
| E8 | 410 | 110 | 0 | 360 | 2000 | 0.1 | 8600 | 670 | 0.83 |
| E9 | 500 | 270 | 40 | 195 | 2000 | 0.1 | 0 | 7000 | 0.65 |
| E10 | 500 | 130 | 40 | 180 | 2000 | 0.1 | 0 | 7000 | 0.65 |
| E11 | 570 | 150 | 50 | 150 | 2000 | 0.1 | 0 | 7000 | 0.65 |
| E12 | 570 | 150 | 0 | 200 | 2000 | 0.1 | 0 | 7000 | 0.65 |
| E13 | 400 | 320 | 120 | 330 | 2000 | 0.1 | 0 | 7000 | 0.65 |

The calculations are summarized in following tables.

Table II
CALCULATION RESULTS FOR CASE STUDY EXCHANGERS

| Ex | $\mathbf{R}$ | $\mathbf{S}$ | $\mathbf{L M T D}$ | $\mathbf{F}_{\mathbf{T}}$ | $\mathbf{S}_{\max }$ | $\mathbf{G}_{\boldsymbol{m i n}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E1 | 5.000 | 0.1754 | 197.72 | 0.6851 | 0.1802 | -0.0812 |
| E2 | 0.200 | 0.8814 | 138.68 | 0.6593 | 0.9010 | -0.0812 |
| E3 | 0.833 | 0.8780 | 76.10 | Infeasibl <br> e | 0.6380 | -0.1696 |
|  |  |  |  |  |  |  |
| E4 | 5.000 | 0.1556 | 260.34 | 0.8874 | 0.1802 | -0.0812 |
| E5 | 2.305 | 0.3094 | 243.53 | 0.7789 | 0.3438 | -0.1362 |
| E6 | 0.200 | 0.8505 | 154.02 | 0.7797 | 0.9010 | -0.0812 |
| E7 | 0.200 | 0.8249 | 166.46 | 0.8317 | 0.9010 | -0.0812 |
| E8 | 0.833 | 0.8780 | 76.10 | Infeasibl | 0.6380 | -0.1696 |
|  |  |  |  | e |  |  |
| E9 | 1.484 | 0.3370 | 265.74 | 0.9089 | 0.4680 | -0.1625 |
| E10 | 2.643 | 0.3043 | 181.31 | 0.5422 | 0.3092 | -0.1263 |
| E11 | 4.200 | 0.1923 | 222.98 | 0.8142 | 0.2101 | -0.0927 |
| E12 | 2.100 | 0.3509 | 243.67 | 0.6608 | 0.3686 | -0.1427 |
| E13 | 0.381 | 0.7500 | 123.83 | 0.7589 | 0.8160 | -0.1268 |

Table III

| Hx | Ft | $\mathbf{N}_{\text {shells }}$ by Eqn. (22) | Integer shells | S per shell | $\begin{aligned} & \mathbf{N}_{\text {shells }} \\ & \text { for } \\ & \mathbf{G}=0 \end{aligned}$ | $\mathrm{F}_{\mathrm{T}} \text { per }$ shell | $\mathbf{A c c}_{\text {cc }}$ | $\mathrm{A}_{12}$ | $\begin{gathered} \text { Cost }_{12} \text { (in } \\ \cdot 000) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E1 | 0.6851 | 0.90 | 1 | 0.1754 | 1.18 | 0.6851 | 101.15 | 147.64 | 179.924 |
|  | 0.6851 | 0.90 | 2 | 0.1330 | 1.18 | 0.9485 | 101.15 | 106.64 | 185.612 |
| E2 | 0.6593 | 0.92 | 1 | 0.8814 | 1.20 | 0.6593 | 144.22 | 218.74 | 232.300 |
|  | 0.6593 | 0.92 | 2 | 0.6716 | 1.20 | 0.9463 | 144.22 | 152.40 | 234.095 |
| E3 |  | 3.06 | 4 | 0.5666 | 4.32 | 0.7594 | 262.82 | 346.10 | 508.508 |
|  |  | 3.06 | 5 | 0.5061 | 4.32 | 0.8599 | 262.82 | 305.65 | 507.151 |
|  |  | 3.06 | 6 | 0.4573 | 4.32 | 0.9066 | 262.82 | 289.89 | 522.277 |
| E4 | 0.8874 | 0.63 | 1 | 0.1556 | 0.83 | 0.8874 | 76.82 | 86.57 | 127.171 |
|  | 0.8874 | 0.63 | 2 | 0.1086 | 0.83 | 0.9757 | 76.82 | 78.73 | 152.391 |
| E5 | 0.7789 | 0.76 | 1 | 0.3094 | 1.05 | 0.7789 | 82.13 | 105.44 | 144.557 |
|  | 0.7789 | 0.76 | 2 | 0.2141 | 1.05 | 0.9543 | 82.13 | 86.06 | 161.460 |
| E6 | 0.7797 | 0.81 | 1 | 0.8505 | 1.07 | 0.7797 | 129.86 | 166.55 | 194.581 |
|  | 0.7797 | 0.81 | 2 | 0.6290 | 1.07 | 0.9589 | 129.86 | 135.42 | 216.796 |
| E7 | 0.8317 | 0.74 | 1 | 0.8249 | 0.97 | 0.8317 | 120.15 | 144.47 | 177.399 |
| E8 |  | 3.06 | 4 | 0.5666 | 4.32 | 0.7594 | 262.82 | 346.10 | 117.232 |
|  |  | 3.06 | 5 | 0.5061 | 4.32 | 0.8599 | 262.82 | 305.65 | 110.375 |
|  |  | 3.06 | 6 | 0.4573 | 4.32 | 0.9066 | 262.82 | 289.89 | 109.064 |
| E9 | 0.9089 | 0.51 | 1 | 0.3370 | 0.72 | 0.9089 | 75.26 | 82.81 | 123.550 |
| E10 | 0.5422 | 0.95 | 1 | 0.3043 | 1.31 | 0.5422 | 110.31 | 203.46 | 221.619 |
|  | 0.5422 | 0.95 | 2 | 0.2223 | 1.31 | 0.9289 | 110.31 | 118.75 | 199.050 |
| E11 | 0.8142 | 0.75 | 1 | 0.1923 | 1.00 | 0.8142 | 89.69 | 110.16 | 148.735 |
| E12 | 0.6608 | 0.88 | 1 | 0.3509 | 1.22 | 0.6608 | 82.08 | 124.20 | 160.801 |
|  | 0.6608 | 0.88 | 2 | 0.2483 | 1.22 | 0.9373 | 82.08 | 87.57 | 163.303 |
| E13 | 0.7589 | 0.80 | 1 | 0.7500 | 1.09 | 0.7589 | 161.51 | 212.81 | 228.189 |
|  | 0.7589 | 0.80 | 2 | 0.5272 | 1.09 | 0.9518 | 161.51 | 169.68 | 251.028 |

Following observations can be made from above results:

1. Obeying the restriction on minimum $\mathrm{F}_{\mathrm{T}}$ value may result in expensive exchangers. In E 1 , E 2 , and E 12 minimum cost exchangers have $\mathrm{F}_{\mathrm{T}}<0.75$, while in E5, E6, and E13 the minimum cost exchangers have $\mathrm{F}_{\mathrm{T}}<0.8$.
2. Note that E3 and E8 are same except cost law coefficients. Ponce-Ortega et al. [7] report that the minimum cost exchanger will always have $\mathrm{F}_{\mathrm{T}}>0.8$ which is an incorrect conclusion. To illustrate this point, following table lists four exchangers, all with same terminal temperatures but different cost laws:

Table IV
DATA FOR ILLUSTRATING EFFECT OF COST LAW COEFFICIENTS

| Exchanger | $\mathbf{T}_{\mathbf{1}}$ | $\mathbf{T}_{\mathbf{2}}$ | $\mathbf{t}_{\mathbf{1}}$ | $\mathbf{t}_{\mathbf{2}}$ | $\mathbf{Q}(\mathbf{k W})$ | $\mathbf{U}\left(\mathbf{k W} / \mathbf{m}^{2} / \mathbf{K}\right)$ | Cost law coefficients |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | $\mathbf{c}$ |  |  |
| E3 | 410 | 110 | 0 | 360 | 2000 | 0.1 | 0 | 7000 | 0.65 |
| E8 | 410 | 110 | 0 | 360 | 2000 | 0.1 | 8600 | 670 | 0.83 |
| E14 | 410 | 110 | 0 | 360 | 2000 | 0.1 | 8600 | 670 | 0.70 |
| E15 | 410 | 110 | 0 | 360 | 2000 | 0.1 | 8600 | 670 | 0.60 |

Table V
Results of exchangers of Table 4.

| Hx | $\mathbf{N}_{\text {shells }} \mathbf{b y}$ <br> Eqn. (22) | Integer <br> shells | S per <br> shell | $\mathbf{N}_{\text {shells }}$ <br> for <br> $\mathbf{G}=\mathbf{0}$ | $\mathbf{F}_{\mathbf{T}}$ per <br> shell | Acc | $\mathbf{A}_{\mathbf{1 2}}$ | Cost $_{\mathbf{1 2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| E3 | 3.06 | 4 | 0.5666 | 4.32 | 0.7594 | 262.82 | 346.10 | 508.508 |
|  | 3.06 | 5 | 0.5061 | 4.32 | 0.8599 | 262.82 | 305.65 | 507.151 |
|  | 3.06 | 6 | 0.4573 | 4.32 | 0.9066 | 262.82 | 289.89 | 522.277 |
|  |  |  |  |  |  |  |  |  |
| E8 | 3.06 | 4 | 0.5666 | 4.32 | 0.7594 | 262.82 | 346.10 | 117.232 |
|  | 3.06 | 5 | 0.5061 | 4.32 | 0.8599 | 262.82 | 305.65 | 110.375 |
|  | 3.06 | 6 | 0.4573 | 4.32 | 0.9066 | 262.82 | 289.89 | 109.064 |
|  |  |  |  |  |  |  |  |  |
| E14 | 3.06 | 4 | 0.5666 | 4.32 | 0.7594 | 262.82 | 346.10 | 69.432 |
|  | 3.06 | 5 | 0.5061 | 4.32 | 0.8599 | 262.82 | 305.65 | 68.225 |
|  | 3.06 | 6 | 0.4573 | 4.32 | 0.9066 | 262.82 | 289.89 | 69.285 |
| E15 | 3.06 | 4 |  |  |  |  |  |  |
|  | 3.06 | 5 | 0.5666 | 4.32 | 0.7594 | 262.82 | 346.10 | 47.542 |
|  | 3.06 | 6 | 0.4573 | 4.32 | 0.9066 | 262.82 | 289.89 | 49.779 |

## VI. Applications To Hen Design

In HEN design practice, various targets are set prior to synthesis of HEN. These targets are:

1. Minimum energy (utility) targets
2. Minimum network area target
3. Minimum units target
4. Minimum shells target

The shells target and units target are not mutually exclusive, and in using exchanger capital cost equation for multipass exchangers (Eqn. 24) shells target replaces units target.

In the cost equation, both exchanger area and number-ofshells appear. It has been argued previously [17] that "total minimum network area" as an independent target is not advisable. It is also shown in the above work that one can replace area by number of shells in assessing practically feasible heat recovery.
The minimum energy targets are HRAT-dependent. The absolute limit on energy recovery is placed by asymptotic energy targets which correspond to HRAT $=0$.
Hence, based on above discussion, the problem of design optimization of HEN (consisting of multipass exchangers) can be put in most simplified manner possible. It requires only two bounds on optimization space - asymptotic energy targets, and shells target based on equation (22). It must be emphasized here that both the targets can only be approached, but never achieved. Thus, they provide the completely defined, widest thermodynamically feasible optimization space with just two
variables - HRAT and number-of-shells.

## VII. Conclusions

The following points can be made concerning the criteria and the approach advocated in this paper:

1. The correction factor $\mathrm{F}_{\mathrm{T}}$ is a misleading parameter when used to restrict design options available for optimization.
2. A new criterion for 1-2 exchanger feasibility has been proposed that does not relate to $\mathrm{F}_{\mathrm{T}}$. Instead, it is based on the premise that all feasible multipass exchangers must be considered for cost-based optimization.
3. The approach and the equations introduced in the paper are useful in simplifying the task of synthesis and optimization of heat exchanger networks consisting of multipass exchangers.

## NOTATION

A heat exchanger area, $\mathrm{m}^{2}$
F LMTD correction factor, dimensionless
G Dimensionless temperature approach, $\frac{\left(T_{2}-t_{2}\right)}{\left(T_{1}-t_{1}\right)}$
$\mathrm{G}_{1} \mathrm{G}$ for one shell
LMTD log mean temperature difference, $K$
M integer number of shells
N real (non-integer) number of shells
$R$ heat capacity ratio, $\frac{\left(T_{1}-T_{2}\right)}{\left(t_{2}-t_{1}\right)}$ dimensionless
$S$ temperature effectiveness, $\frac{\left(t_{2}-t_{1}\right)}{\left(T_{1}-t_{1}\right)}$ dimensionless
$\mathrm{S}_{1} \mathrm{~S}$ for one shell
$\mathrm{T}_{1}$ hot fluid inlet temperature, K
$\mathrm{T}_{2}$ hot fluid outlet temperature, K
$t_{1}$ cold fluid inlet temperature, K
$\mathrm{t}_{2}$ cold fluid outlet temperature
$\Delta \mathrm{T}_{\text {eff }}$ actual mean temperature difference, K
$\mathrm{X}_{\mathrm{P}} \mathrm{Ahmad}$ et al.'s parameter, dimensionless
U overall heat transfer coefficient

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