Network of Coupled Stochastic Oscillators and One-way Quantum Computations

Eugene Grichuk, Margarita Kuzmina, and Eduard Manykin

Abstract—A network of coupled stochastic oscillators is proposed for modeling of a cluster of entangled qubits that is exploited as a computation resource in one-way quantum computation schemes. A qubit model has been designed as a stochastic oscillator formed by a pair of coupled limit cycle oscillators with chaotically modulated limit cycle radii and frequencies. The qubit simulates the behavior of electric field of polarized light beam and adequately imitates the states of two-level quantum system. A cluster of entangled qubits can be associated with a beam of polarized light, light polarization degree being directly related to cluster entanglement degree. Oscillatory network, imitating qubit cluster, is designed, and system of equations for network dynamics has been written. The constructions of one-qubit gates are suggested. Changing of cluster entanglement degree caused by measurements can be exactly calculated.

Keywords—network of stochastic oscillators, one-way quantum computations, a beam of polarized light.

I. INTRODUCTION

QUANTUM computations is the interdisciplinary research field undergoing active development Currently both quantum physicians and information theory experts focused their attention on theoretical analysis and experimental realizations of quantum computation algorithms. After the discovery of Shor's quantum algorithm for large number factorization it become clear that quantum algorithms are capable to provide an effective solution to some mathematical problems for which exist no effective classical algorithms. The development theoretical foundation of quantum calculations stimulated the appearance of quantum informatics, a new research field arisen at the intersection of quantum physics and information theory [2-4].

Quantum computation algorithms are based on evolution of some quantum system and exploitation of quantum physics laws for computation performance. Sometimes it permits to realize a specific type of algorithm parallelization that is not inherent to traditional parallel algorithms, including neural network ones. The majority of quantum computation schemes is based on construction of a set one-qubit and two-qubit gates controlling evolution of quantum system. One-qubit gates provide modification of single qubit states whereas two-qubit gates specify qubit interactions. The information readout is implemented via a sequence of measurements over qubit states. The measurements inevitably destroy a coherent state of quantum system, and it is a matter of great difficulties.

In 2001 a significantly new type of quantum computation scheme has been proposed – so called one-way, or cluster quantum computations (CQC) [5 -7]. The feature of one-way QC is that the sequence of measurements, necessary to readout the information from qubit cluster, is explicitly included in CQC computation scheme. So, each qubit cluster, initially prepared in maximally entangled state, undergoes irreversible evolution via one-qubit measurements in the process of computations. As a result the cluster can be used for computations only once. The choice of measurement sequence just defines the quantum computation algorithm itself. As it turned out, the CQC scheme is ideally suitable for realization of Grover's algorithm [2]. The significant feature of CQC is that the information processing in the schemes is really performed at classical level, although quantum physics principles have been used in preparation of a cluster of entangled qubits. So, it seems natural to expect that CQC computation schemes could be formulated in terms of evolution of proper artificial neural network. In the paper we just try to develop a network approach to the problem. The preliminary design of proper oscillatory model of single qubit was of one of our goals. The qubit model should be capable to adequately imitate the features of two-level quantum system. Our oscillator qubit is constructed as a pair of coupled limit cycle oscillators with chaotically modulated limit cycle sized and frequencies. After that we designed a network of stochastic coupled oscillators as a model of cluster of entangled qubits. Further we associated the qubit model with a beam of polarized light, that is one of adequate physical realization of qubit, and related cluster entanglement to light polarization degree. Thus cluster entanglement destruction, caused by measurements, was related to polarization degree increasing due to external optical device actions on polarized light beam. The designed one–qubit gates just imitate actions of typical optical devices on polarized light beam.

Eugene Grichuk is a post graduate student of National Research Nuclear University "MEPI" (e-mail: es@t-25.ru);

Margarita Kuzmina is with Keldysh Institute of Applied Mathematics RAS, Miusskaya Sq.4, 125047 Moscow, Russia (phone: 7-499-972 3491; fax: 7-499-972 0737; e-mail: mg.kuzmina@gmail.com;

Eduard Manykin is with Russian Research Center "Kurchatov Institute", Kurchatov Sq. 1, 123182, 123182 Moscow, Russia (phohe: 7-495-196 9107; e-mail: emnany@isssph.kiae.ru.

World Academy of Science, Engineering and Technology
International Journal of Nuclear and Quantum Engineering
Vol:4, No:9, 2010

II. Quantum and Classical Levels of Description of Quasi-monochromatic Polarized Fight.

A qubit (quantum bit of information) can be described as a two-level quantum-mechanical system that can be either in a pure or in a mixed quantum state. Mixed state is understood as a state of statistical ensemble of identical quantum systems and is described by density operator (density matrix) $\hat{\rho}$ satisfying the conditions

$$\det \hat{\rho} \geq 0, \quad Tr\hat{\rho} = 1. \tag{1}$$

In the case of pure qubit state, defined by a column state function $\langle \psi |$, the density operator is reduced to one-dimensional projector onto the state $\langle \psi |$, $\hat{\rho}_\psi = |\psi\rangle\langle\psi|$. Here we use traditional notations: operator $A = |\psi\rangle\langle\varphi|$ of rank 1 acts on state $\langle \chi |$ by formula $A|\chi\rangle = |\psi\rangle\langle\varphi|\chi\rangle$, where $\langle\varphi|\psi\rangle$ is the inner product. If one uses the basis $\{\vec{e}_x, \vec{e}_y\} = \{(1\ 0)^T, (0\ 1)^T\}$, it is convenient to introduce the basis of Pauli matrices in real space of Hermitian matrices and present the density operator in the form

$$\hat{\rho} = \frac{1}{2}(\hat{I} + p_x\hat{\sigma}_x + p_y\hat{\sigma}_y + p_z\hat{\sigma}_z) = \frac{1}{2}\begin{pmatrix} 1+p_z & p_x - ip_y \\ p_x + ip_y & 1-p_z \end{pmatrix}, \tag{2}$$

where

$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$p_x = Tr(\hat{\rho}\hat{\sigma}_x), \quad p_y = Tr(\hat{\rho}\hat{\sigma}_y), \quad p_z = Tr(\hat{\rho}\hat{\sigma}_z), \tag{3}$$

and $P = (p_x, p_y, p_z)$ are Stokes parameters. From the condition $\det\hat{\rho} \geq 0$ it follows the restriction on P: $p_x^2 + p_y^2 + p_z^2 \leq 1$. Pure states are characterized by the condition $p_x^2 + p_y^2 + p_z^2 = 1$ and form the Bloch sphere (which is known as Poincare sphere in optics).

The Stoke parameters are also used for polarization description of classical electromagnetic radiation in terms of intensity, degree of polarization, shape and orientation of the polarization ellipse. Moreover, a beam of quasi-monochromatic light can be equivalently described both at quantum level (as an ensemble of photons) and at classical level, in frames of classical electromagnetic field theory. At quantum level of description a beam of quasi-monochromatic light is considered as photon beam propagating in a direction specified by vector $\vec{k}$. It can be described as statistical ensemble of photons with moment $\vec{p} = (ch\omega / c)\vec{k}$ and polarization state defined by two-dimensional unit vector $\vec{e}$,

located in the plane orthogonal to $\vec{k}$. Stokes parameters characterize the ensemble in a mixed state from the viewpoint of its representation by a superposition of two sub-ensembles in pure states with polarization vectors $\vec{e}_x$ and $\vec{e}_y$. In the case of coherent superposition of the sub-ensembles we have a beam of fully polarized photons, in the case of completely non-coherent superposition – a beam of unpolarized photons, and in an intermediate case of partially coherent superposition – a beam of partially polarized photons.

From the viewpoint of classical electrodynamics a beam of quasi-monochromatic light is a plane quasi-monochromatic electromagnetic wave, specified by propagation vector $\vec{k}$. Electrical field vector $\vec{E}(t)$ of the electromagnetic wave, located in the plane orthogonal to $\vec{k}$ (electromagnetic wave transversality), can be written as

$$\vec{E}(t) = E_x e^{i\alpha} \cdot \vec{e}_x + E_y e^{i(\alpha+\delta)} \cdot \vec{e}_y, \ (\vec{e}_x, \vec{k}) = (\vec{e}_y, \vec{k}) = 0.$$

For adequate description of light polarization in terms of wave electrical field one should consider $\vec{E}(t)$ is as a two-dimensional stationary random function of time. Let $\bar{\vec{E}}(t)$ be the mean of random function $\vec{E}(t)$ and so $\tilde{E}(t) = \vec{E}(t) - \bar{\vec{E}}(t)$ be the fluctuation of $\vec{E}(t)$. For stationary random functions the mean $\bar{\vec{E}}(t)$ coincides with the mean over time, $\langle \vec{E}(t) \rangle$, that is

$$\bar{\vec{E}}(t) = \langle \vec{E}(t) \rangle \equiv \lim_{T\to\infty} (2T)^{-1} \int_{-T}^{T} \vec{E}(t)dt.$$

There is the following relation between the coherence matrix $\hat{J}$ of quasi-monochromatic light beam in the basis $\{\vec{e}_x, \vec{e}_y\}$

$$\hat{J} = [J_{mn}] = \frac{1}{2}\begin{pmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{pmatrix}, \tag{4}$$

and correlation matrix $\hat{D}$ of random function $\vec{E}(t)$:

$$J_{mn} = D_{mn}(0) = \tilde{D}_{mn}(0) + \langle E_m \rangle\langle E_n \rangle,$$
$$\tilde{D}_{mn}(\tau) = \langle \tilde{E}_m(t+\tau)\tilde{E}_n^*(t+\tau) \rangle \tag{5}$$

In optics Stokes parameters, denoted as Q, U, V, characterize polarization state of light beam. In the basis $\{\vec{e}_x, \vec{e}_y\}$ there is the following relation between light intensity I, Stokes parameters and coherence matrix $\hat{J}$:

$$I = J_{11} + J_{22}, \quad Q = J_{11} - J_{22},$$
$$U = J_{12} - J_{21}, \quad V = -i(J_{12} - J_{21}), \tag{6}$$

World Academy of Science, Engineering and Technology
International Journal of Nuclear and Quantum Engineering
Vol:4, No:9, 2010

or

$$I = \langle E_x \rangle^2 + \langle E_y \rangle^2, \quad Q = \langle E_x \rangle^2 - \langle E_y \rangle^2,$$
$$U = 2\langle E_x E_y \cos(\delta) \rangle, \quad V = 2\langle E_x E_y \sin(\delta) \rangle. \tag{7}$$

The Stokes parameters describe form and orientation of polarization ellipse – the projection onto the plane, orthogonal to vector $\vec{k}$, of the curve, that the end of random vector $\vec{E}(t)$ traces out in the space. If we denote by β the angle between $\vec{e}_x$ and large ellipse semiaxis and by ε the angle, characterizing the ratio of ellipse semiaxes (so as $tg(\varepsilon) = |E_{min}| / |E_{max}|$), then the following relations are valid

$$tg(2\theta) = U/Q, \quad \sin(2\varepsilon) = \frac{V}{(Q^2 + U^2 + V^2)^{1/2}}, \tag{8}$$
$$I = E_{max}^2 + E_{min}^2 = (Q^2 + U^2 + V^2)^{1/2}.$$

The optical Stokes parameters Q, U, V, are related to Stokes parameters p_x, p_y, p_z, defined in eq. (3), as

$$p_x = U/I, \quad p_y = V/I, \quad p_z = Q/I. \tag{9}$$

The inequality $Q^2 + U^2 + V^2 \leq I^2$ is fulfilled for optical Stokes parameters (the equality takes place in the case of fully polarized light). The value $p = (Q^2 + U^2 + V^2)^{1/2} / I$ defines the degree of light polarization (the part of fully polarized component of light in total light beam).

So, as we can see, polarization state of classical quasi-monochromatic light preserves all the properties of quantum photon beam as two-level quantum system. Therefore, the attempt to design a qubit model capable to imitate electric field behavior of quasi-monochromatic light beam seems reasonable. It allows to formulate one-way quantum computations in terms of controllable dynamics of artificial neural network, qubit model being network processing unit.

III. QUBIT MODEL AS A PAIR OF COUPLED OSCILLATORS

We design qubit model as a pair of chaotically modulated coupled limit cycle oscillators. Let the initial dynamical equations for unperturbed pair of uncoupled limit cycle oscillators be written as

$$\dot{u}_{1,2} = [\rho_{1,2}^2 + i\omega_{1,2} - |u_{1,2}|^2] u_{1,2} \tag{10}$$

where $u_{1,2} = x_{1,2} + iy_{1,2}$ are complex-valued dynamical variables, ρ_1, ρ_2 are radii of circular limit cycles and ω_1, ω_2 are own oscillator frequencies of two identical oscillators [11-13]. Consider further chaotically modulated oscillators which limit cycle radii $\tilde{\rho}_1$, $\tilde{\rho}_2$ and own frequencies $\tilde{\omega}_1$, $\tilde{\omega}_2$ defined as

$$\tilde{\rho}_{1,2} = \rho_{1,2} + \xi_{1,2}(t), \quad \tilde{\omega}_{1,2} = \omega_{1,2} + \eta_{1,2}(t), \tag{11}$$

where $\xi_{1,2}(t)$ and $\eta_{1,2}(t)$ are stationary random functions with zero means. Then the system of ODE governing internal dynamics of two linearly coupled stochastic oscillators can be written as

$$\dot{u}_1 = [\tilde{\rho}_1^2 + i\tilde{\omega}_1 - |u_1|^2] u_1 + \kappa(u_2 - u_1),$$
$$\dot{u}_2 = [\tilde{\rho}_2^2 + i\tilde{\omega}_2 - |u_2|^2] u_2 - \kappa(u_2 - u_1), \tag{12}$$

where

$$u_1 = x_1 + iy_1, \quad u_2 = x_2 + iy_2, \tag{13}$$

and $\kappa = |\kappa| e^{i\delta}$ is the strength of oscillator coupling. The variable $U = u_1 + u_2$, defining oscillation superposition of two oscillators, will be of main interest for qubit behavior. So it is convenient to rewrite system (12) for variables $v_1 = 0.5(u_1 + u_2)$, $v_2 = 0.5(u_1 - u_2)$:

$$\dot{v}_1 = 0.5\{[\tilde{\rho}_1^2 + i\tilde{\omega}_1 - |v_1 + v_2|^2](v_1 + v_2) + [\tilde{\rho}_1^2 + i\tilde{\omega}_1 - |v_1 - v_2|^2](v_1 - v_2)\}$$
$$\dot{v}_2 = 0.5\{[\tilde{\rho}_2^2 + i\tilde{\omega}_2 - |v_1 + v_2|^2](v_1 + v_2) - [\tilde{\rho}_1^2 + i\tilde{\omega}_1 - |v_1 - v_2|^2](v_1 - v_2)\} - 4\kappa v_2, \tag{14}$$

where

$$v_1 = x + iy, \quad v_2 = z + iu. \tag{15}$$

Four-dimensional dynamical system (14) has been constructed in such a manner, that the projection of its trajectory onto (x, y)-plane imitates the behavior of electrical field $\vec{E}(t)$ of superposition of two light beams in the states of right and left circular polarization. In the case of coherent superposition of two oppositely circularly polarized beams the summary beam will be in some state of full polarization, whereas in the case of completely non-coherent superposition it will be in the unpolarized state. Three typical examples of $\vec{E}(t)$ behavior of summary beam in the case of coherent superposition of two inner beam components are shown in fig. 1 – 3. The electric field of circularly polarized light depicted in fig. 1, is obtained at zero intensity of the second sub-beam. It corresponds to pure qubit state $|1\rangle$ (photon ensemble of fully circular polarized photons). The electric field of linearly polarized light, depicted in fig. 2, is obtained as a result of coherent superposition of two oppositely circularly polarized sub-beams at phase difference $\delta = 0$ between the electric field components $\vec{E}^{1,2}(t)$. The case of full elliptic polarization, shown in fig. 3, is obtained in the case of coherent beam superposition at $\delta \neq 0$. At last, the electrical field of unpolarized light, obtained in the case of non-coherent superposition of two identical oppositely circularly polarized beams, is presented in fig.4.

World Academy of Science, Engineering and Technology
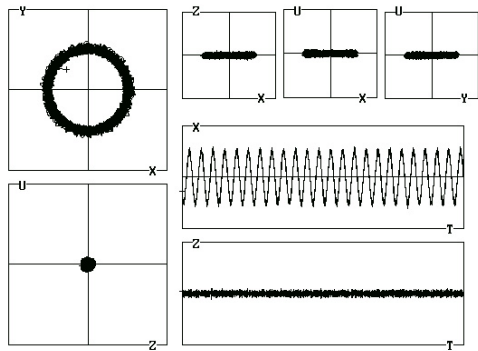International Journal of Nuclear and Quantum Engineering
Vol:4, No:9, 2010

Fig. 1. Pure qubit state $|1\rangle$
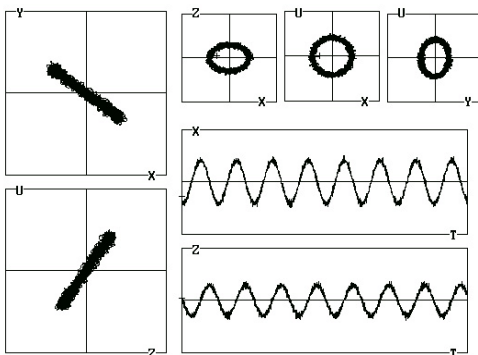(Ensemble of circular polarized photons)

Fig. 2. Pure qubit state $|1\rangle + |0\rangle$
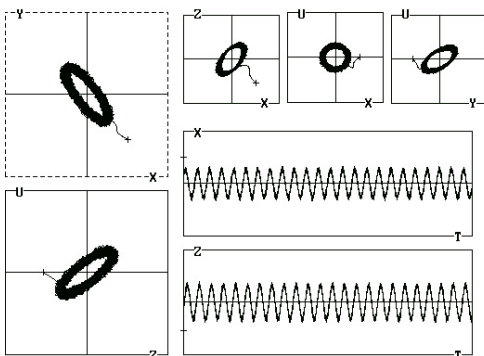(Ensemble of linearly polarized photons)

Fig. 3. Pure qubit state $\alpha\,|1\rangle + \beta\,|0\rangle$
(Ensemble of elliptically polarized photons)

It is important, that in the case incoherent superposition of strongly monochromatic oppositely circular polarized beams the mixed state (unpolarized light beam) just corresponds to entangled qubit state (polarization entangled state of two oppositely circularly polarized photons).

So, the designed oscillatory model of qubit correctly simulates both pure quantum mechanical state of photon (electric field behavior of light beam in different states of full polarization) and mixed (polarization entangled) quantum mechanical state of photon (electric field behavior of unpolrized photon beam).
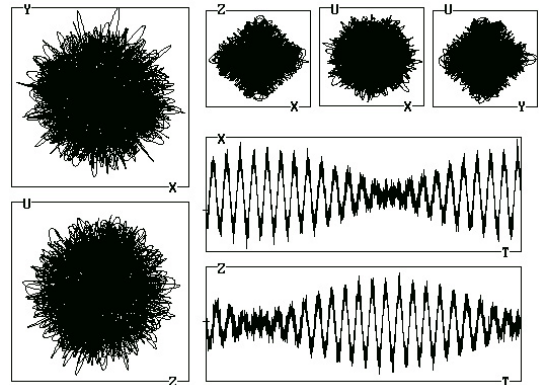

Fig. 4. Mixed qubit state , corresponding to $\hat{\rho} = 0.5 \cdot diag[1,1]$
(Ensemble of unpolarized photons in polarization entangled state)

IV. DYNAMICAL EQUATIONS FOR OSCILLATORY NETWORK, SIMULATING A QUBIT CLUSTER

Remind that one-way computation schemes are based on gradual qubit cluster entanglement destruction during a sequence of one-qubit measurements, realized via a set of one-qubit gates. We designed single qubit as stochastic oscillator, imitating the behavior of electric field of classical electromagnetic wave. We further model a cluster of entangled qubits as a network of coupled stochastic oscillators. A sequence of one-qubit measurements can be just realized as a sequence of optical device actions, transforming single oscillator states (modifying polarization of light beam, corresponding the oscillator).

In the frames of our optical interpretation of CQC computation process we further associate a cubit cluster in the state maximal initial entanglement with a beam of quasi-monochromatic unpolarized light, consisting of N independent unpolarized sub-beams. And we relate an oscillatory network of N stochastic oscillators to the total light beam, each network oscillator being a model of sub-beam in a mixed (polarization entangled) state. Single oscillator state can be changed in response to action of external optical device. It is convenient to interpret the device action as measurement. Obviously, as a result of a sequence of measurements the oscillatory network state will be changed in a discrete manner.

Let $\vec{V}^{\,j} = (x^j, y^j, z^j, u^j)^T$ is four–component variable, specifying oscillator state, and $\alpha^j = \{\tilde{\rho}_1^{\,j}, \tilde{\rho}_2^{\,j}, \tilde{\omega}_1^{\,j}, \tilde{\omega}_2^{\,j}, \kappa^j\}$ is the collection of internal oscillator parameters (see eq. (14)). Then system of equations governing network dynamics can be written as [11-13]

World Academy of Science, Engineering and Technology
International Journal of Nuclear and Quantum Engineering
Vol:4, No:9, 2010

$$\dot{\vec{V}}^j = \vec{f}(\vec{V}^j;\,\alpha^j) + \sum_{k=1}^{N} \hat{W}^{jk}\cdot(\vec{V}^k - \vec{V}^j) + \vec{F}(\vec{V}^j;\,\beta^j), \qquad (15)$$

$$j = 1, \dots N,$$

where $[\hat{W}^{jk}]$ is the matrix characterizing oscillatory network coupling and $\vec{F}(\vec{V};\,\beta^j)$ is a four-component function, specifying external action on j-th network oscillator. In general problems of quantum computations $[\hat{W}^{jk}]$ can be defined via system of two-qubit gates. However, in the case of one-way quantum computation schemes, which are of main interest for us at the moment, $\hat{W}^{jk} \equiv 0$. Now we shortly describe typical optical device actions and give the example of function $\vec{F}(\vec{V};\,\beta^j)$ construction.

V. EXAMPLES OF ONE-QUBIT GATES

One-qubit gates should imitate the actions of typical optical devices that modify polarization of light. A polarizer is just one of widely used optical devices that transforms light polarization. It converts a beam of arbitrarily polarized light into the beam with well-defined light polarization, for instance, linear polarization. Let β be the angle between the direction of polarizer plane of polarization and direction of $\vec{e}_x$-vector, and $\vec{E}$ be electric field vector of incident light beam. Then electric field vector $\vec{E}'$ of transmitted light can be written as $\vec{E}' = \hat{A}\vec{E}$ (where $\hat{A}$ is so called Jones matrix of optical device). For instance, in the basis $\{\vec{e}_x, \vec{e}_y\}$ the Jones matrix of ideal absorptive linear polarizer can be written as

$$\hat{A}_{LP} = e^{-2\pi i n d/\lambda}\begin{pmatrix} \cos^2(\beta) & \cos(\beta)\sin(\beta) \\ \cos(\beta)\sin(\beta) & \cos^2(\beta) \end{pmatrix}, \qquad (16)$$

where β is the angle between $\vec{e}_x$ and the plane of transmitted polarization, n and d are refractive index of polarizer matter and polarizer thickness, correspondingly.

In the frames of our model the one-qubit gate, imitating qubit transmission through linear polarizer of some finite thickness d, can be defined by a function $\vec{F}(\vec{V};\,\theta,\Delta t)$ that is nonzero only during finite time interval $\Delta t = t_2 - t_1$, $\Delta t \square\, d$. The analytical expression for such $\vec{F}(\vec{V};\,\theta,\Delta t)$ can be written as

$$\vec{F}(\vec{V};\,\theta,\Delta t) = \begin{bmatrix} \vec{F}_1 \\ \vec{F}_2 \end{bmatrix} = \frac{d}{dt}\left\{ H(t)\cdot\begin{bmatrix} \hat{A}_{LP}\vec{v}_1 \\ \hat{A}_{LP}\vec{v}_2 \end{bmatrix}\right\}, \qquad (17)$$

$$H(t) = 0.5\{th(\gamma(t - t_1)) - th(\gamma(t - t_2))\},\ \gamma\,\square\,1, \qquad (18)$$

$$\vec{V} = \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \end{bmatrix},\ \vec{v}_1 = \begin{bmatrix} x \\ y \end{bmatrix},\ \vec{v}_2 = \begin{bmatrix} z \\ u \end{bmatrix}. \qquad (19)$$

As one can see from fig. 5, after qubit transmission through linear polarizer, its initial linear polarization has been transformed into another type of linear polarization. Besides absorptive polarizers, there exist also so called beam-splitting polarizers that split the unpolarized light beam into two light beams of opposite polarization states.

Similarly functions $\vec{F}(\vec{V};\,\beta^j)$, corresponding to actions of phase-shifters (polarization rotators) and optical compensators can also be designed. Phase-shifters transform a linearly polarized light beam into a beam of circularly polarized light via creating of additional phase difference between two components of electric field $\vec{E}$. Matrix $\hat{A}$ of optical
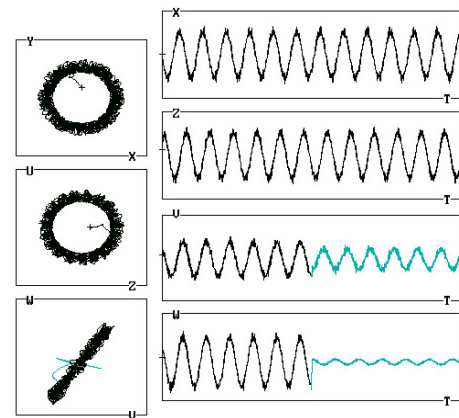

Fig. 5 Single qubit under action of one-qubit gate, imitating passing through linear polarizer (oscillations after transmission are shown by curves of green color)

compensator in the complex-valued basis $\{\vec{e}^+, \vec{e}^-\}$, $\vec{e}^\pm = (1/\sqrt{2})(\vec{e}_x \pm i\vec{e}_y)$ can be written as

$$\hat{A}_c = \begin{pmatrix} \cos(\delta/2) & \pm\sin(\delta/2) \\ \mp\sin(\delta/2) & \cos(\delta/2) \end{pmatrix}. \qquad (20)$$

At last, let network oscillator be subject to a measurement via transmission through circular polarizer. In the basis $\{\vec{e}^+, \vec{e}^-\}$, $\vec{e}^\pm = (1/\sqrt{2})(\vec{e}_x \pm i\vec{e}_y)$, the matrix $\hat{A}_{CP}$ of partial circular polarizer is diagonal one and can be written as

$$\hat{A}_{CP} = diag(e^{\alpha/2}, e^{-\alpha/2}) \qquad (21)$$

where α is the ratio of damping factor of right circular polarized light to that one of left circular polarized light in the polarizer medium. In the case of ideal polarizer, transmitting only the light with right circular polarization, we obviously obtain $\hat{A}_{CP} = diag[1,\,0]$.

World Academy of Science, Engineering and Technology
International Journal of Nuclear and Quantum Engineering
Vol:4, No:9, 2010

VI. CALCULATION OF CLUSTER ENTANGLEMENT DEGREE CHANGING UNDER A SEQUENCE OF MEASUREMENTS

One-way quantum computation algorithm is defined through a sequence of measurements for entangled qubit cluster. In our model an entangled qubit cluster is implied as a beam of quasi-monochromatic unpolarized light composed of N independent sub-beams (beam components) of quasi-monochromatic unpolarized light which frequencies belong to different non-intersecting narrow frequency intervals. To characterize statistical properties of unpolarized or partially polarized light beam (mixed state of quantum system) it is necessary to use Stokes parameters (I, Q, U, V), defined in eq. (6) - (7). In particular, there is the obvious simple relation between light polarization degree $p = (Q^2 + U^2 + V^2)^{1/2}/I$ of total beam and entanglement degree e of corresponding qubit cluster: $e = 1 - p$. The optical interpretation of qubit cluster permits to consider a sequence of measurements over sigle qubits as a sequence of actions of optical devices on unpolarized light sub-beams. The results of such actions in terms of Stokes parameters can be accurately calculated with the help of known methods of classical ellipsometry [14]. The general result is that light polarization degree is gradually increased when unpolarized light is transmitted through a sequence of non-depolarizing optical devices. Correspondingly, entanglement degree of initially maximally entangled cluster is decreased. Moreover, in view of optical interpretation of entangled qubit cluster the results of entanglement degree changing under a sequence of measurements can be accurately calculated in terms of Stokes parameters.

Another advantage of the relation between cubit cluster and a beam of classical polarized light is that one can associate one-way quantum computation algorithm with state evolution of a feed-forward neural-like network – network of stochastic oscillators. The network oscillators are not mutually connected, but the network itself undergoes discrete sequence of state changing as a result of external actions on single oscillators. Artificial neural networks of the type might be regarded as quantum-inspired networks.

At last it should be noted, that sometimes the study of quantum evolution of entangled qubit cluster can be reduced to simple calculation of Stokes parameter changing in the problem of polarized light beam transmission through a series of optical devices, transforming beam light polarization state.

VII. CONCLUSIONS

The following results are obtained in the paper:

● oscillatory model of qubit is designed in the form of pair of coupled stochastic oscillators (limit cycle oscillators with chaotically modulated oscillation amplitudes and frequencies); the model simulates the behavior of electric field of polarized light beam; it adequately imitates both pure and mixed states of two-level quantum system;

● network of N stochastic oscillators is designed as a model of entangled qubit cluster – a computation resource in one-way quantum computation schemes; a beam of polarized light can serve as an example of adequate physical realization of the entangled qubit cluster; the oscillatory network can be used for detailed study of electric field behavior of polarized light beam, composed of N independent sub-beams;

● one-qubit gates in one-way quantum computation scheme can be modeled as actions of typical optical devices, modifying polarization of light;

● the optical interpretation one-qubit gates provides tools for exact calculation of gradually decreased cluster entanglement degree;

● system of equations, governing dynamics of oscillatory network, subject to external actions on single network oscillators, is written;

● as it follows from the approach, developed in the paper, quantum-mechanical qubit cluster evolution is equivalent to state evolution of feed-forward neural-like network of stochastic oscillators; in the case of one-way quantum computation schemes oscillatory network state is changed in a discrete manner in response to external actions on network oscillators.

REFERENCES

[1] P.W.Shor, Young, "Polynomial time algorithms for prime factorization and discrete algorithms on a quantum computer, *SIAM J. Sci. Statist. Comput.*, 1997, v.26, p. 1484.

[2] A.S.Holevo, "Probabilistic and statistical aspects of quantum theory". Nauka, Moscow, 1980; North Holland Translation – 1982.

[3] A.S.Holevo, " Quantum probability and quantum statistics", VINITI, 1991, v.83, Moscow (in Rusian).

[4] M.A.Nielsen and I.L.Chuang, *Quantum computation and quantum information*, Cambridge Univ. Press, 2000.

[5] R.Raussendorf and H.J.Breigel, "A one way quantum computer", P*hys. Rev. Lett.* , 2001, V.86, p.5188.

[6] R.Raussendorf and H.J.Breigel, "Computational model underlying the one-way quantum computer", *Quant. Inf. Comp.*, 2002, v.6, p. 443; *quant-ph/* 0108063 (2001).

[7] R.Raussendorf and H.J.Breigel, Computational model underlying the one-way quantum computer: concepts and summary", in: Th.Beth ans G. Leuch, *Quantum Information processing*, Wiley-VCH, 2003.

[8] L.K.Grover, "A fast quantum mechanical algorithm for database search", *Proc. 28 Annual ACM Symp. on Theory of Computing*, 1996, p.22.

[9] I.L.Chuang, L.M.K.Vandersypen, X.Zhou, D.W.Leung, S.Lloyd, "Experimental realization of a quantum algorithm", *Nature*, 1998, v. 393, p.143.

[10] L.M.K. Vandersypen, I.L.Chuang, "NMR technique for quantum control and computation", *arXiv: quant-ph/*0404064 v.2, 2004.

[11] M.G. Kuzmina, E.A. Manykin and I.I. Surina, "Oscillatory network with self-organized dynamical connections for synchronization-based visual image segmentation", *BioSystems*, 2004, vol. 76, pp. 43-53.

[12] E.S Grichuk, M.G. Kuzmina, E.A. Manykin, "Oscillatory Network for Synchronization-based Adaptive Image Segmentation"*, Proc. of WCCI 2006*, Vancouver, BC, Canada; *Proc. of Int. Joint Conf on Neural Networks 2006,* p.4529.

[13] E.S.Grichuk, M.G.Kuzmina, E.A.Manykin, "Adaptive image processing via synchronization in self-organizing oscillatory network", *Proceedings of European Computing Conference (ECC 2007)*, Athens, Greece; *Lecture Notes in Electrical Engineering*, (Eds. N.Mastorakis, V.Mladenov, V.T.Kontargyri), Berlin: Springer, 2009, v.1, p.103.

[14] R.M.A.Azzam, N.M.Bashara, "Ellipsometry and polarized light", North-Holland Publishing Company, Amsterdam-New-York, Oxford, 1977.