



On Neutrosophic $\alpha_{(\gamma,\beta)}$ -Continuous Functions, Neutrosophic $\alpha_{(\gamma,\beta)}$ -Open (Closed) Functions in Neutrosophic Topological Spaces

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Abstract: In the present script, we explain the neutrosophic $\alpha_{(\gamma,\beta)}$ -continuous function in the neutrosophic topological spaces. We analyze their behaviour; study the various relations and properties existing among them.

Further we like to extend the study to neutrosophic $\alpha_{(\gamma,\beta)}$ -open function, $\alpha_{(\gamma,\beta)}$ -closed function, neutrosophic $(\gamma_{ne}, \beta_{ne})$ - irresolute functions and neutrosophic $\alpha_{(\gamma,\beta)}$ - homeomorphism in the neutrosophic topological spaces. The relationship among them can be studied in detail. The neutrosophic $(\alpha_\gamma, \beta_{ne})$ -continuous function, neutrosophic $(\gamma_{ne}, \alpha_\beta)$ -open(closed) function, neutrosophic α_γ -limit point, neutrosophic α_γ -derived set and neutrosophic α_γ -neighbourhood point are explained and utilized to obtain various remarkable properties. They are explored through the specified examples.

Key Words: neutrosophic γ -open set (closed set), neutrosophic α_γ -open set (closed set), neutrosophic $\alpha_{(\gamma,\beta)}$ -continuous function, neutrosophic $\alpha_{(\gamma,\beta)}$ -open(closed) function, neutrosophic $\alpha_{(\gamma,\beta)}$ -homeomorphism.

1.Introduction and motivation

The following notations have been used through this paper: neut- neutrosophic, neut topo spa- neutrosophic topological space, neutro- neutrosophy, topo spa- topological space, se-set, topol-topology, ses-sets, spa-space, spas-spaces, fuz-fuzzy

Many philosophies have been raised for vagueness counting the argument of probability, the approach of fuz ses, the ideology of intuitionistic fuz ses, the idea of rough ses, and so on. Even though numerous novel approaches have been extended as a development of these concepts there are still various problems, main complications arise due to the insufficiency of parameters.

Lugojan [11] studied the generalized topology during the year 1982. The concept of fuz topo spas was dealt by Chang [5]. Neut se is a generalization of a classic se, a fuz se and a Intuitionistic fuzzy se. The word Neutro means skill on neutrals. Neut method is derived from Fuzzy logic or Intuitionistic fuz logic. In 1965, Zadeh [21] familiarized the fuz se and in 1983 Atanassov [2-4] presented the Intuitionistic fuz se. A novel division of the idea named Neutro was introduced by Smarandache [15-18] in 1999 by totaling a self-governing indeterminacy-affiliation function.

In a neut set, the indeterminacy is calibrated obviously. The certainty-affiliation function, indeterminacy- affiliation function, and erroneous- affiliation function is ultimately self-regulating. Wang [19] described the lone valued neut set and then offered the se-formularized process and a variety of resources of lone valued neu ses.

Salama [12-14], initiated neut topo spas which was a generality of Intuitionistic fuz topo spa exposed by Coker [6] also a neut se as well the degree of membership, the degree of indeterminacy then the degree of non- membership of respective element.

N. Kalaivani and G. Sai Sundara Krishnan [9] introduced the α_γ -open sets, α_γ - Ti spaces with the help of α -open sets and γ -open sets. Their properties were studied. They also introduced $\alpha_{(\gamma,\beta)}$ -continuous functions, $\alpha_{(\gamma,\beta)}$ -irresolute functions, [8] $\alpha_{(\gamma,\beta)}$ -open(closed) functions [7] and studied them in detail.

To fill up the gap existing in the neu theory, now we want to introduce γ -open ses, neu α_γ -open ses, γ Ti spas, α_γ - Ti spas, $\alpha_{(\gamma,\beta)}$ -continuous functions and $\alpha_{(\gamma,\beta)}$ -open (closed) functions in neutrosophic Topological Spaces.

In our earlier article [10] the insight of neu γ -open ses, neu α_γ -open ses, that are created through neu α_γ -open ses are deliberated besides few of their fundamental properties were discovered. The

connection among these neut γ Ti spas in addition neut α_γ -Ti spas were incarnated over drawings besides evaluated their behaviour.

In third chapter we investigate neut $\alpha_{(\gamma,\beta)}$ -continuous functions and analyze their properties. The neut $\alpha_{(\gamma,\beta)}$ -open functions, neut $\alpha_{(\gamma,\beta)}$ -closed functions and neut $\alpha_{(\gamma,\beta)}$ -homeomorphism are introduced and analyzed in the fourth, fifth and sixth chapters respectively. The rapport amongst them is examined in the sixth chapter.

All over this study, consider that \mathcal{Z}_{ne} designate the neut topo spa $(\mathcal{Z}_{ne}, \tau_{ne})$ and $\gamma: \tau_{ne} \rightarrow P(\mathcal{Z}_{ne})$ be an operation on τ_{ne} .

2.Preliminaries

The theory of neut ses which is a tool for dealing with uncertainties was exposed by Smarandache [15-18]. Salama [12-14], Alblowi [1] familiarized the thought of neut topo spas.

The neut se, its complement, inclusion relation, union, intersection, neut topology, neut open set, neut closed se were introduced by Salama et al [12-14]. The neut functions was revealed by Turnali and Coker [20].

Definition 2.1.[10] Let $(\mathcal{Z}_{ne}, \tau_{ne})$ be a neut topo spa. A maneuver γ on the topo τ_{ne} is a charting from τ_{ne} into the power set $P(\mathcal{Z}_{ne})$ of \mathcal{Z}_{ne} such that $M \subseteq M^\gamma$ for each $M \in \tau_{ne}$. M^γ designates the charge of γ at M . It is symbolized by $\gamma: \tau_{ne} \rightarrow P(\mathcal{Z}_{ne})$.

Definition 2.2. [10] A neut subse B of a neut topo spac is supposed to remain a neut γ -open se contingent upon for individual $z \in B$, there prevails a neut open se V , aforesaid that $z \in V$ along with $V^\gamma \subseteq B$. $\tau_{(ne)\gamma}$ designates the set of all neut γ -open ses.

Definition 2.3. [10] A neut subse B of a neut topo spa is thought to remain a neut γ -closed se in $(\mathcal{Z}_{ne}, \tau_{ne})$ in case that $\mathcal{Z}_{ne} - B$ is a neut γ -open se in $(\mathcal{Z}_{ne}, \tau_{ne})$.

Definition 2.4. [10] An operation γ is thought to be neut open if, for every single neut open neighbourhood V of \mathcal{Z}_{ne} , there occurs a neut γ -open se P akin that $z \in P$ besides $P \subseteq V^\gamma$.

Definition 2.5.[10] Agree $(\mathcal{Z}_{ne}, \tau_{ne})$ be a neut topo spa in addition B be a neut subse of $(\mathcal{Z}_{ne}, \tau_{ne})$. Formerly neut γ -interior of B is the congregation of entire neut γ -open ses encompassed in B in addition it is indicated through $\tau_{(ne)\gamma}\text{-int}(B)$.

$$\tau_{(ne)\gamma}\text{-int}(B) = \{V: V \text{ is a neut } \gamma\text{-open se then } V \subseteq B\}.$$

Definition 2.6. [10] Agree $(\mathcal{Z}_{ne}, \tau_{ne})$ be a neut topo spa also B be a neut subse of $(\mathcal{Z}_{ne}, \tau_{ne})$. Formerly neut γ -closure of B is the intersection of entire neut γ -closed ses contained in B in addition it is symbolized by means of $\tau_{(ne)\gamma}\text{-cl}(B)$.

$$\tau_{(ne)\gamma}\text{-cl}(B) = \{M: M \text{ is a neut } \gamma\text{-closed se besides } B \subseteq M\}.$$

Definition 2.7. [10] A spa $(\mathcal{Z}_{ne}, \tau_{ne})$ is termed a neut γT_0 spa if for each distinct point $s, t \in \mathcal{Z}_{ne}$ there lies a neut γ -open se M alike that $s \in M$ and $t \notin M^\gamma$ or $t \in M$ besides $s \notin M^\gamma$.

Definition 2.8. [10] A spa $(\mathcal{Z}_{ne}, \tau_{ne})$ is labeled a neut γT_1 spa if for each distinct point $s, t \in \mathcal{Z}_{ne}$ there endures neut γ -open ses M, N containing s, t respectively aforesaid that $t \notin M^\gamma$ and $s \notin N^\gamma$.

Definition 2.9. [10] A spa $(\mathcal{Z}_{ne}, \tau_{ne})$ is termed a neut γT_2 spa if for each distinct point $s, t \in \mathcal{Z}_{ne}$ there occurs a neut γ -open ses M, N like that $s \in M, t \in N$ and $M^\gamma \cap N^\gamma = \emptyset$.

Definition 2.10. [10] Let $(\mathcal{Z}_{ne}, \tau_{ne})$ be a neut top spa. Formerly a neut subse member H of \mathcal{Z}_{ne} is aforesaid to be a neut γ generalized closed se (ne γg -closed set) if $\tau_{(ne)\gamma}\text{-cl}(H) \subseteq M$ whensoever $H \subseteq M$ besides M is a neut γ -open se in $(\mathcal{Z}_{ne}, \tau_{ne})$.

Remark 2.1. [10] After the definition 4.4, Each one neut γ -closed se in $(\mathcal{Z}_{ne}, \tau_{ne})$ is a neut γ generalized closed se. Yet, the conflicting statement need not be exact.

Definition 2.11. [10] A neut top spa $(\mathcal{Z}_{ne}, \tau_{ne})$ is labeled as a neut $\gamma T_{\frac{1}{2}}$ spa in case that each single neut γ generalized -closed se belonging to the $(\mathcal{Z}_{ne}, \tau_{ne})$ is a neut γ -closed se.

Definition 2.12. [10] A neut conventional se H in a neut top spa $(\mathcal{Z}_{ne}, \tau_{ne})$ is concluded as a neut α_γ -open se contingent upon $H \subseteq \tau_{(ne)\gamma}\text{-int}(\tau_{(ne)\gamma}\text{-cl}(\tau_{(ne)\gamma}\text{-int}(H)))$.

Theorem 2.1. [10] Let $(\mathcal{Z}_{ne}, \tau_{ne})$ be a neut top spa and $\{A_\kappa: \kappa \in J\}$ be the group of neut α_γ - open ses in $(\mathcal{Z}_{ne}, \tau_{ne})$. Formerly $\cup_{\kappa \in J} A_\kappa$ is also a neut α_γ - open se.

Definition 2.13. [10] Agree $(\mathcal{Z}_{ne}, \tau_{ne})$ be a neut top spa in addition P be a subse of \mathcal{Z}_{ne} . At that time P is supposed to be neut α_γ - closed se on condition that $\mathcal{Z}_{ne}-P$ is a neut α_γ - open se.

Definition 2.14. [10] A subse member M of \mathcal{Z}_{ne} is noted to be a neut α_γ - closed se in the event that $\mathcal{Z}_{ne} - M$ is a neut α_γ -open se, which is unvaryingly, Agree $(\mathcal{Z}_{ne}, \tau_{ne})$ be a neut top spa then γ - an activity on τ_{ne} in addition M be a subse member of \mathcal{Z}_{ne} . Formerly M is a neut α_γ - closed se subject to $M \supseteq \tau_{(ne)\gamma}\text{-cl}(\tau_{(ne)\gamma}\text{-int}(\tau_{(ne)\gamma}\text{-cl}(M)))$.

Definition 2.15. [10] Endorse $(\mathcal{Z}_{ne}, \tau_{ne})$ as a top spa along with Q as a neut subse of $(\mathcal{Z}_{ne}, \tau_{ne})$. Then neut $\tau_{(ne)\alpha_\gamma}$ -interior of Q is the unification of all neut α_γ -open ses accommodated within Q and it is symbolized by $\tau_{(ne)\alpha_\gamma}\text{-int}(Q)$.

$$\tau_{(ne)\alpha_\gamma}\text{-int}(Q) = \cup \{U : U \text{ is a neut } \alpha_\gamma \text{ - open se and } U \subseteq Q \}.$$

Definition 2.16. [10] Confer $(\mathcal{Z}_{ne}, \tau_{ne})$ to be a top spa along with C be a neut subse of $(\mathcal{Z}_{ne}, \tau_{ne})$. At that time, $\tau_{(ne)\alpha_\gamma}$ -closure of C is the intersection of all neut α_γ -closed ses consisting of C and it is indicated by $\tau_{(ne)\alpha_\gamma}\text{-cl}(C)$.

$$\tau_{(ne)\alpha_\gamma}\text{-cl}(C) = \cap \{F: F \text{ is a neut } \alpha_\gamma \text{ -closed se and } C \subseteq F \}.$$

Remark 2.2. [10] (i) If M is a neut subse of $(\mathcal{Z}_{ne}, \tau_{ne})$. Then $\tau_{(ne)\alpha_\gamma}\text{-cl}(M)$ is a neutrosophic α_γ - closed se containing M .

(ii) M is a neut α_γ -closed se in the event $\tau_{(ne)\alpha_\gamma}\text{-cl}(M) = M$.

Definition 2.17. [10] A top spa $(\mathcal{Z}_{ne}, \tau_{ne})$ is entitled a neut $\alpha_\gamma T_0$ spa if for each different points $p, q \in \mathcal{Z}_{ne}$ nearby exists a α_γ - open set, P like that $p \in P$ and $q \notin P$ or $q \in P$ besides $p \notin P$.

Definition 2.18. [10] A top spa $(\mathcal{Z}_{ne}, \tau_{ne})$ is termed a neut $\alpha_\gamma T_1$ spa if for each dissimilar points $p, q \in \mathcal{Z}_{ne}$ nearby exists neut α_γ -open ses, P, Q enclosing p and q commonly alike that $q \notin P$

and $p \notin Q$.

Definition 2.19. [10] A top $\text{spa}(\mathcal{Z}_{ne}, \tau_{ne})$ is described a neut α_γ T_2 spa if for each distinctive points $p, q \in \mathcal{Z}_{ne}$ nearby exists neut α_γ -open ses, P, Q akin that $p \in P, q \in Q$ and $P \cap Q = \emptyset$.

Definition 2.20. [10] Agree $(\mathcal{Z}_{ne}, \tau_{ne})$ be a top spa. Then a neut subse member M of \mathcal{Z}_{ne} is forenamed to be neut α_γ -g- closed se if $\tau_{(ne)\alpha_\gamma}\text{-cl}(M) \subseteq P$ whenever $M \subseteq P$ and P is a neut α_γ -closed se in $(\mathcal{Z}_{ne}, \tau_{ne})$.

Remark 2.3. [10] After the definition 4.4 explanation, Individual neut α_γ -closed conventional se of $(\mathcal{Z}_{ne}, \tau_{ne})$ is a neut α_γ -g-closed se. Nevertheless, the antipode need not be appropriate.

Definition 2.21. [10] A top $\text{spa}(\mathcal{Z}_{ne}, \tau_{ne})$ is termed as a neut α_γ $T_{1/2}$ spa supposing that individual neut α_γ g- closed se of $(\mathcal{Z}_{ne}, \tau_{ne})$ is a neut α_γ - closed se.

Theorem 2.2. [10] The top $\text{spa}(\mathcal{Z}_{ne}, \tau_{ne})$ is a neut α_γ $T_{1/2}$ spa on condition that for every single $m \in \mathcal{Z}_{ne}$, $\{m\}$ is a neut α_γ - closed se or a neut α_γ - open se in $(\mathcal{Z}_{ne}, \tau_{ne})$.

Theorem 2.3. [10] Agree $(\mathcal{Z}_{ne}, \tau_{ne})$ be a top spa also $M \subseteq \mathcal{Z}_{ne}$. At that time the succeeding statements hold:

- (i) $\tau_{(ne)\alpha_\gamma}\text{-int}(\mathcal{Z}_{ne} - M) = \mathcal{Z}_{ne} - \tau_{(ne)\alpha_\gamma}\text{-cl}(M)$
- (ii) $\tau_{(ne)\alpha_\gamma}\text{-cl}(\mathcal{Z}_{ne} - M) = \mathcal{Z}_{ne} - \tau_{(ne)\alpha_\gamma}\text{-int}(M)$

Theorem 2.4. [10] Accredited $(\mathcal{Z}_{ne}, \tau_{ne})$ be a top spa. Supposing that a neut subse member M of \mathcal{Z}_{ne} is assumed to be a neut α_γ g- closed se, thereupon $\tau_{(ne)\alpha_\gamma}\text{-cl}(M) - M$ does not enclose a non-void neut α_γ - closed se.

Theorem 2.5. [10] The top $\text{spa}(\mathcal{Z}_{ne}, \tau_{ne})$ is a neut α_γ $T_{1/2}$ spa on condition that for every single $m \in \mathcal{Z}_{ne}$, $\{m\}$ is a neut α_γ - closed se or a neut α_γ - open se in $(\mathcal{Z}_{ne}, \tau_{ne})$.

3. Neutrosophic $\alpha_{(\gamma,\beta)}$ -continuous functions

In this chapter we investigate neutrosophic $\alpha_{(\gamma,\beta)}$ -continuous functions as well analyze their properties.

3.1.(i) Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is aforesaid to be a neut $\alpha_{(\gamma,\beta)}$ -continuous function given for respective neut α_β - open set U of \mathcal{Y}_{ne} , the contrary image $f_{ne}^{-1}(U)$ is a neut α_γ -open se in \mathcal{Z}_{ne} .

(ii) Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is supposed to be a neut $(\alpha_\gamma, \beta_{ne})$ -continuous function with the condition that the converse appearance of apiece neut β_{ne} -open se in $(\mathcal{Y}_{ne}, \sigma_{ne})$

abides to be a neut α_γ -open se of $(\mathcal{Z}_{ne}, \tau_{ne})$.

(iii) **Definition** A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is aforesaid to be a neut $(\gamma_{ne}, \beta_{ne})$ -continuous function in case that the reverse icon of every single neut β_{ne} -open se of $(\mathcal{Y}_{ne}, \sigma_{ne})$ continues to be a neut γ_{ne} -open se of $(\mathcal{Z}_{ne}, \tau_{ne})$.

3.2. Example Given $\mathcal{Z}_{ne} = \{h_1, h_2, h_3\}, \tau_{ne} = \{1_{ne}, \mathcal{Z}_{ne}, F_1, F_3, F_4, F_5\}$

, $\mathcal{Y}_{ne} = \{g_1, g_2, g_3\}$ and $\sigma_{ne} = \{1_{ne}, \mathcal{Z}_{ne}, H_1, H_3, H_4, H_5\}$ were

$$F_1 = \{z, (0.2, 0.6, 0.1), (0.1, 0.1, 0.1), (0.3, 0.1, 0.1)\},$$

$$F_3 = \{z, (0.8, 0.6, 0.3), (0.4, 0.5, 0.1), (0.1, 0.1, 0.3)\},$$

$$F_4 = \{z, (0.8, 0.6, 0.3), (0.4, 0.5, 0.1), (0.1, 0.1, 0.1)\},$$

$$F_5 = \{z, (0.7, 0.6, 0.3), (0.1, 0.1, 0.1), (0.1, 0.1, 0.1)\}$$

$$H_1 = \{y, (0.2, 0.6, 0.1), (0.1, 0.1, 0.1), (0.3, 0.1, 0.1)\},$$

$$H_3 = \{y, (0.7, 0.6, 0.3), (0.4, 0.5, 0.1), (0.1, 0.1, 0.3)\},$$

$$H_4 = \{y, (0.7, 0.6, 0.3), (0.4, 0.5, 0.1), (0.1, 0.1, 0.1)\},$$

$$H_5 = \{y, (0.8, 0.6, 0.3), (0.1, 0.1, 0.1), (0.1, 0.1, 0.1)\}$$

Define an operation γ on τ_{ne} such that $(U)^\gamma = \begin{cases} U \cup \{h_3\} & \text{if } U \neq \{h_1\} \\ U & \text{if } U = \{h_1\} \end{cases}$

Then $\tau_{(ne)\alpha_\gamma} = \{1_{ne}, \mathcal{Z}_{ne}, F_1, F_3, F_5\}$.

Define an operation β on σ_{ne} such that $(U)^\beta = \begin{cases} U \cup \{g_3\} & \text{if } U \neq \{g_1\} \\ U & \text{if } U = \{g_1\} \end{cases}$

Then $\tau_{(ne)\alpha_\beta} = \{1_{ne}, \mathcal{Z}_{ne}, H_1, H_3, H_5\}$

Define $f_{ne}: \mathcal{Z}_{ne} \rightarrow \mathcal{Y}_{ne}$ as $f_{ne}(h_1) = g_1, f_{ne}(h_2) = g_2$ and $f_{ne}(h_3) = g_3$ Formerly the overturned copy of restricted neut α_β -open se acts as a neut α_γ -open se beneath f_{ne} . Thence f_{ne} endures to be a neut $\alpha_{(\gamma, \beta)}$ -continuous function.

The subsequent 3.3. Remark and 3.4. Remark display that the thought of neut $\alpha_{(\gamma, \beta)}$ -continuous functions and neut $(\gamma_{ne}, \beta_{ne})$ -irresolute functions are self-governing nevertheless after \mathcal{Z}_{ne} is a neutrosophic γ -regular spa and \mathcal{Y}_{ne} is a neut β -regular spa together the thoughts concur.

3.3. Remark The insights of neut $\alpha_{(\gamma, \beta)}$ -continuous functions besides neut $(\gamma_{ne}, \beta_{ne})$ -irresolute functions are self-regulating.

Given $\mathcal{Z}_{ne} = \{h_1, h_2, h_3\}, \tau_{ne} = \{1_{ne}, \mathcal{Z}_{ne}, F_1, F_2, F_3, F_4, F_5\}$, $\mathcal{Y}_{ne} = \{g_1, g_2, g_3\}$ and σ_{ne}

$$\begin{aligned}
 &= \{1_{ne}, \mathcal{Z}_{ne}, H_1, H_3, H_4, H_5\} \text{ were} \\
 F_1 &= \{z, (0.2, 0.6, 0.3), (0.1, 0.1, 0.1), (0.3, 0.1, 0.1)\}, \\
 F_2 &= \{z, (0.8, 0.5, 0.1), (0.1, 0.1, 0.1), (0.1, 0.1, 0.1)\}, \\
 F_3 &= \{z, (0.8, 0.6, 0.3), (0.1, 0.1, 0.1), (0.1, 0.1, 0.1)\}, \\
 F_4 &= \{z, (0.8, 0.6, 0.3), (0.4, 0.5, 0.1), (0.1, 0.1, 0.1)\}, \\
 F_5 &= \{z, (0.7, 0.6, 0.3), (0.1, 0.1, 0.1), (0.1, 0.1, 0.1)\} \\
 H_1 &= \{y, (0.2, 0.6, 0.1), (0.1, 0.1, 0.1), (0.3, 0.1, 0.1)\}, \\
 H_3 &= \{y, (0.7, 0.6, 0.3), (0.4, 0.5, 0.1), (0.1, 0.1, 0.3)\}, \\
 H_4 &= \{y, (0.7, 0.6, 0.3), (0.4, 0.5, 0.1), (0.1, 0.1, 0.1)\}, \\
 H_5 &= \{y, (0.8, 0.6, 0.3), (0.1, 0.1, 0.1), (0.1, 0.1, 0.1)\}
 \end{aligned}$$

Define an operation γ on τ_{ne} such that $(U)^\gamma = \begin{cases} U \cup \{h_3\} \text{ if } U \neq \{h_1\} \\ U \text{ if } U = \{h_1\} \end{cases}$

Then $\tau_{(ne)\alpha_\gamma} = \{1_{ne}, \mathcal{Z}_{ne}, F_1, F_3, F_5\}$.

Define an operation β on σ_{ne} such that $(V)^\beta = \begin{cases} V \text{ if } g_2 \notin V \\ cl(V) \text{ if } g_2 \in V \end{cases}$

Define $f_{ne}: \mathcal{Z}_{ne} \rightarrow \mathcal{Y}_{ne}$ as $f_{ne}(h_1) = g_1, f_{ne}(h_2) = g_2$ and $f_{ne}(h_3) = g_3$

Formerly f_{ne} is a neut $(\gamma_{ne}, \beta_{ne})$ irresolute function. But $f_{ne}^{-1}(\{g_1, g_2\}) = \{h_1, h_2\}$, is not a neut α_γ -open se under f_{ne} . Henceforth f_{ne} is not a neut $\alpha_{(\gamma, \beta)}$ -continuous function.

3.4. Remark If \mathcal{Z}_{ne} is a neut γ -regular spa and \mathcal{Y}_{ne} is a neut β - regular spa, at that moment the conception of neut (γ, β) - irresoluteness in addition neut $\alpha_{(\gamma, \beta)}$ -continuity concur.

3.5. Definition A neut member H of \mathcal{Z}_{ne} is supposed to be a neut α_γ -neighbourhood of a point $t \in \mathcal{Z}_{ne}$ if there befalls a neut α_γ - open se G like that $t \in G \subseteq H$.

3.6. Theorem A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma, \beta)}$ -continuous function designed for each r of \mathcal{Z}_{ne} , the opposing statement of each neut α_β -neighbourhood of $f_{ne}(r)$ is a neut α_γ -neighbourhood of r .

Proof. Assume $r \in \mathcal{Z}_{ne}$ in addition B be a neut α_γ -neighbourhood of $f_{ne}(r)$. By the assertion of the 3.5. Definition there ensues a $V \in \sigma_{(ne)\alpha_\beta}$ such that $f_{ne}(r) \in V \subseteq B$. This deduces that $r \in f_{ne}^{-1}(V) \subseteq f_{ne}^{-1}(B)$. Since f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -continuous function, $f_{ne}^{-1}(V) \in \tau_{(ne)\alpha_\gamma}$. Henceforward $f_{ne}^{-1}(B)$ is a neut α_γ -neighbourhood of r .

Conversely, let $B \in \sigma_{(ne)\alpha_\beta}$, $A = f_{ne}^{-1}(B)$ and $r \in A$. Later by the announcement of the 3.5. Definition, there arises a set $A_r \in \tau_{(ne)\alpha_\gamma}$ similar that $r \in A_r \subseteq A$. This supposes that $A = \cup_{r \in A} A_r$. Through the assertion of the 2.1. Theorem, A is a neut α_γ -open se of \mathcal{Z}_{ne} . Accordingly, f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -continuous function.

3.7. Theorem A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma,\beta)}$ -continuous function provided that for separate point m of \mathcal{Z}_{ne} besides respective neutrosophic α_β -neighbourhood B of $f_{ne}(m)$, there is a neutrosophic α_γ -neighbourhood A of m alike that $f_{ne}(A) \subseteq B$.

Proof. Let $n \in \mathcal{Z}_{ne}$ also B acts as a neut α_β -neighbourhood of $f_{ne}(n)$. Later in there lies a set $O_{f(n)} \in \sigma_{(ne)\alpha_\beta}$ such that $f_{ne}(n) \in O_{f(n)} \subseteq B$. It follows that $n \in f_{ne}^{-1}(O_{f(n)}) \subseteq f_{ne}^{-1}(B)$. By hypothesis, $f_{ne}^{-1}(O_{f(n)}) \in \tau_{(ne)\alpha_\gamma}$. Let $A = f_{ne}^{-1}(B)$. Then it trails that A is a neut α_γ -neighbourhood of n and $f_{ne}(A) = f_{ne}(f_{ne}^{-1}(B)) \subseteq B$.

Conversely, let $U \in \sigma_{(ne)\alpha_\beta}$. Let $W = f_{ne}^{-1}(U)$. Let $n \in W$. Formerly $f_{ne}(n) \in U$. Thus U is a neut α_β -neighbourhood of $f_{ne}(n)$ and hence there exists a neut α_γ -neighbourhood V_n of n akin that $f_{ne}(V_n) \subseteq U$. Accordingly it trails that $n \in V_n \subseteq f_{ne}^{-1}(f_{ne}(V_n)) \subseteq f_{ne}^{-1}(U) = W$. Since V_n is a neut α_γ -neighbourhood of n , which implies that there exists a $W_n \in \tau_{(ne)\alpha_\gamma}$ like that $n \in W_n \subseteq W$.

This implies that $W = \cup_{n \in W} W_n$. Through 2.1. Theorem Statement, W is a neut α_γ -open se of Z_{ne} .

Consequently f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -continuous function.

3.8. Theorem Accredit $f_{ne}: (Z_{ne}, \tau_{ne}) \rightarrow (Y_{ne}, \sigma_{ne})$ remain a function. Formerly the ensuing assertions are comparable:

- (i) $f_{ne}: (Z_{ne}, \tau_{ne}) \rightarrow (Y_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma,\beta)}$ -continuous function;
- (ii) $f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-cl}(D)) \subseteq \sigma_{(ne)\alpha_\beta}\text{-cl}(f_{ne}(D))$ holds for every member D belonging to Z_{ne} ;
- (iii) For respective single neut α_β -closed se V of Y_{ne} , $f_{ne}^{-1}(V)$ is a neut α_γ -closed se belonging to Z_{ne} .

Validation. (i) \Rightarrow (ii) Given $s \in f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-cl}(D))$ besides V be a neut α_β -open se comprising s . By dint of the 3.7. Theorem, at this juncture ensues a point $y \in Z_{ne}$ along with a neut α_γ -open se U comparable that $y \in U$ with $f_{ne}(y) = s$ and $f_{ne}(U) \subseteq V$. Subsequently $y \in \tau_{s\alpha_\gamma}\text{-cl}(D)$, $U \cap D \neq \emptyset$ besides henceforth $\emptyset \neq f_{ne}(U \cap D) \subseteq f_{ne}(U) \cap f_{ne}(D) \subseteq V \cap f_{ne}(D)$. This implies that $s \in \sigma_{(ne)\alpha_\beta}\text{-cl}(f_{ne}(D))$. Therefore $f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-cl}(D)) \subseteq \sigma_{(ne)\alpha_\beta}\text{-cl}(f_{ne}(D))$.

(ii) \Rightarrow (iii) Given V be a neut α_β -closed se in Y_{ne} . Then $\sigma_{(ne)\alpha_\beta}\text{-cl}(V) = V$. Through (ii), $f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-cl}(f_{ne}^{-1}(V))) \subseteq \sigma_{(ne)\alpha_\beta}\text{-cl}(f_{ne}(f_{ne}^{-1}(V))) \subseteq \sigma_{(ne)\alpha_\beta}\text{-cl}(V) = V$ holds. Consequently $\tau_{(ne)\text{-cl}}(f_{ne}^{-1}(A)) \subseteq f_{ne}^{-1}(V)$ and $f_{ne}^{-1}(V) = \tau_{(ne)\alpha_\gamma}\text{-cl}(f_{ne}^{-1}(V))$. Henceforth $f_{ne}^{-1}(V)$ is a neut α_γ -closed se in Z_{ne} .

(iii) \Rightarrow (i) Contemplate B as a neut α_β -open se in Y_{ne} . Deliberate $V = Y_{ne} - B$. Thereupon V is a neut α_β -closed se in Y_{ne} . Through (iii) $f_{ne}^{-1}(V)$ is a neut α_γ -closed se in Z_{ne} . Later $f_{ne}^{-1}(B) = Z_{ne} - f_{ne}^{-1}(Y_{ne} - B) = Z_{ne} - f_{ne}^{-1}(V)$ is a neut α_γ -open se of Z_{ne} . Hereafter f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -continuous function.

3.9. Theorem Agree $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ be a neut $\alpha_{(\gamma, \beta)}$ - continuous also injective function.

Assuming that \mathcal{Y}_{ne} is a neut $\alpha_{\beta} T_2$ spa (respectively neut $\alpha_{\beta} T_1$ spa), formerly \mathcal{Z}_{ne} is a neut $\alpha_{\gamma} T_2$ spa (respectively neut $\alpha_{\gamma} T_1$ spa).

Proof. Suppose \mathcal{Y}_{ne} a neut $\alpha_{\beta} T_2$ spa. Given i and j be two distinct points of \mathcal{Z}_{ne} . Formerly, there presents dual neut α_{β} -open ses U, V akin that $f_{ne}(i) \in U, f_{ne}(j) \in V$ in addition $U \cap V = \emptyset$. Meanwhile f_{ne} is a neut $\alpha_{(\gamma, \beta)}$ - continuous function, for U along with V , in view there occurs two neut α_{γ} - open ses I and J such that $i \in I$ and $j \in J, f_{ne}(I) \subseteq U$ and $f_{ne}(J) \subseteq V$, infers that $I \cap J = \emptyset$. Henceforth \mathcal{Z}_{ne} is a neutrosophic $\alpha_{\gamma} T_2$ spa. In the similar technique it can be evinced that \mathcal{Z}_{ne} is a neut $\alpha_{\gamma} T_1$ spa whenever \mathcal{Y}_{ne} is a neut $\alpha_{\beta} T_1$ spa.

3.10. Theorem Accredited $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ and $g_{Ma}: (\mathcal{Y}_{ne}, \sigma_{ne}) \rightarrow (\mathcal{X}_{ne}, \delta_{ne})$ be two functions. Supposing that f_{ne} is a neut $\alpha_{(\gamma, \beta)}$ -continuous function further g_{ne} is a neut $\alpha_{(\beta, \delta)}$ -continuous function, previously $g_{ne} \circ f_{ne}: (\mathcal{X}_{ne}, \delta_{ne}) \rightarrow (\mathcal{Z}_{ne}, \tau_{ne})$ is a neut $\alpha_{(\gamma, \delta)}$ -continuous function.

Proof. Manifestation trails from the 3.1. Definition.

3.11. Definition Approve D be a neut subse of \mathcal{Z}_{ne} then z be any point in \mathcal{Z}_{ne} . At that time z is called a neut α_{γ} -limit point of D suppose that $U \cap (D - \{z\}) \neq \emptyset$, for any neut α_{γ} -open set U encompassing z . The collection of all neut α_{γ} -limit points of D is commanded as a neut α_{γ} -derived set of A as well it is indicated as $d_{(ne)\alpha_{\gamma}}(D)$.

3.12. Remark Agree L, M be some subsets of \mathcal{Z}_{ne} . At that time,

- (i) if $L \subseteq M$, then $d_{(ne)\alpha_\gamma}(L) \subseteq d_{(ne)\alpha_\gamma}(M)$.
- (ii) $r \in d_{(ne)\alpha_\gamma}(L)$ if and only if $r \in \tau_{(ne)\alpha_\gamma}(L)\text{-cl}(L - \{r\})$.

Proof. Confirmation tracks after the declaration of 3.11. Definition.

3.13. Theorem Accept L and M are any two neutrosophic subsees of \mathcal{Z}_{ne} . At that moment the ensuing information hold good.

- (i) $L \cup d_{(ne)\alpha_\gamma}(L) \subseteq \tau_{(ne)\alpha_\gamma}\text{-cl}(L)$;
- (ii) $d_{(ne)\alpha_\gamma}(L \cup M) = d_{(ne)\alpha_\gamma}(L) \cup d_{(ne)\alpha_\gamma}(M)$;
- (iii) $\bigcup_i d_{(ne)\alpha_\gamma}(L_i) = d_{(ne)\alpha_\gamma}(\bigcup_i L_i)$;
- (iv) $d_{(ne)\alpha_\gamma}(d_{(ne)\alpha_\gamma}(L)) \subseteq d_{(ne)\alpha_\gamma}(L)$;
- (v) $\tau_{(ne)\alpha_\gamma}\text{-cl}(d_{(ne)\alpha_\gamma}(L)) = d_{(ne)\alpha_\gamma}(L)$

Proof. (i) In case $l \in L \cup d_{(ne)\alpha_\gamma}(L)$, then to establish that $l \in \tau_{(ne)\alpha_\gamma}\text{-cl}(L)$. If $l \in L$ formerly $l \in \tau_{(ne)\alpha_\gamma}\text{-cl}(L)$. If $l \notin L$, at that time to indicate that $l \in \tau_{(ne)\alpha_\gamma}\text{-cl}(L)$. Or else, at that time there is a neut α_γ - closed se C comprising L but not encompassing l . Then $l \in \mathcal{Z}_{ne} - C$, which is a neut α_γ -open se furthermore $U \cap L = \emptyset$. This suggests that $l \notin d_{(ne)\alpha_\gamma}(L)$. This strangeness displays that $x \in \tau_{(ne)\alpha_\gamma}\text{-cl}(L)$. Henceforth $L \cup d_{(ne)\alpha_\gamma}(L) \subseteq \tau_{(ne)\alpha_\gamma}\text{-cl}(L)$.

(ii) Let $l \in d_{(ne)\alpha_\gamma}(L \cup M)$. Through the 3.11.Definition, $\emptyset \neq U \cap ((L \cup M) - \{l\}) = U \cap ((L - \{l\}) \cup (M - \{l\})) = [U \cap (L - \{l\})] \cup [U \cap (M - \{l\})]$ and hence either $l \in d_{(ne)\alpha_\gamma}(L)$ or $d_{(ne)\alpha_\gamma}(M)$.

(M).Therefore $d_{(ne)\alpha_\gamma} (L \cup M) \subseteq d_{(ne)\alpha_\gamma} (L) \cup d_{(ne)\alpha_\gamma} (M)$. The outcome $d_{(ne)\alpha_\gamma} (L) \cup d_{(ne)\alpha_\gamma} (M) \subseteq d_{(ne)\alpha_\gamma} (L \cup M)$, tracks by the 3.12. (i) Remark outcome.

(iii) Validation trails on or after the 3.12. Remark and (ii) outcome.

(iv) Assuming that $l \notin d_{(ne)\alpha_\gamma} (L)$. Then $l \notin \tau_{(ne)\alpha_\gamma}\text{-cl} (L - \{l\})$. At this moment, there ensues a neut α_γ - open set U like that $l \in U$ and $U \cap (L - \{l\}) = \emptyset$. To attest that $l \notin d_{(ne)\alpha_\gamma} (d_{(ne)\alpha_\gamma} (L))$. Supposing on the conflict that $l \in d_{(ne)\alpha_\gamma} (d_{(ne)\alpha_\gamma} (L))$. Then $l \in \tau_{(ne)\alpha_\gamma}\text{-cl} (d_{(ne)\alpha_\gamma} (L) - \{l\})$. Since $l \in U$, $U \cap (d_{(ne)\alpha_\gamma} (L) - \{l\}) \neq \emptyset$. Subsequently there is a $q \neq l$ so that $q \in U \cap (d_{(ne)\alpha_\gamma} (L))$. This suggests that $q \in (U - \{l\}) \cap (d_{(ne)\alpha_\gamma} (L) - \{l\})$. Later $((U - \{l\}) \cap (d_{(ne)\alpha_\gamma} (L) - \{l\})) \neq \emptyset$, an illogicality to the datum that $U \cap (d_{(ne)\alpha_\gamma} (L) - \{l\}) = \emptyset$. This hints that $l \notin d_{(ne)\alpha_\gamma} (d_{(ne)\alpha_\gamma} (L))$ and after this $d_{(ne)\alpha_\gamma} (d_{(ne)\alpha_\gamma} (L)) \subseteq d_{(ne)\alpha_\gamma} (L)$.

(v) This trails after the 3.11. Definition.

3.14. Theorem A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma,\beta)}$ -continuous function contingent upon $f_{ne}(d_{(ne)\alpha_\gamma}(A)) \subseteq \sigma_{(ne)\alpha_\beta}\text{-cl} (f_{ne}(A))$, for entire $A \subseteq \mathcal{Z}_{ne}$.

Proof. Given $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma,\beta)}$ - continuous function. Agree that $A \subseteq \mathcal{Z}_{ne}$, and $z \in d_{(ne)\alpha_\gamma}(A)$. Assume that $f_{ne}(z) \notin f_{ne}(A)$ and let V denote a neut α_β - neighbourhood of $f_{ne}(z)$. Meanwhile f_{ne} is a neutrosophic $\alpha_{(\gamma,\beta)}$ -continuous function, by means of 3.6. Theorem, there arises a neut α_γ - neighbourhood U of z like that $f_{ne}(U) \subseteq V$. From $z \in d_{(ne)\alpha_\gamma}(A)$, it trails that $U \cap A \neq \emptyset$. There arises at least a factor $c \in U \cap A$, suggests that $f_{ne}(c) \in f_{ne}(A)$ and $f_{ne}(c) \in V$. Meanwhile $f_{ne}z \notin f_{ne}(A)$ and $f_{ne}(c) \neq f_{ne}(z)$. Therefore, each neut α_β - neighbourhood of $f_{ne}(z)$ encompasses a

component $f_{ne}(c) \in f_{ne}(A)$ unlike from $f_{ne}(z)$. Henceforward, $f_{ne}(z) \in d_{(ne)\alpha_\beta}(f_{ne}(A))$. By dint of the assertion of 3.13. (i) Theorem $f_{ne}(d_{(ne)\alpha_\gamma}(A)) \subseteq \sigma_{(ne)\alpha_\beta}^{-1} \text{cl}(f_{ne}(A))$.

On the contrary, assume that f_{ne} is not a neut $\alpha_{(\gamma,\beta)}$ -continuous function. Then via 3.7. Theorem, there occurs $z \notin Z_{ne}$ and a neut α_β - neighbourhood V of $f_{ne}(z)$ so that every single neut α_γ -neighbourhood U of z covers at least one member $c \in U$, for which $f_{ne}(c) \notin V$. Let $A = \{c \in Z_{ne} : f_{ne}(c) \notin V\}$. Since $f_{ne}(z) \in V$, therefore $z \notin A$ and hence $f_{ne}(z) \notin f_{ne}(A)$. Since $f_{ne}(A) \cap (V - f_{ne}(z)) = \emptyset$, therefore $f_{ne}(z) \notin d_{(ne)\alpha_\beta}(f_{ne}(A))$. It surveys that $f_{ne}(z) \in f_{ne}(d_{(ne)\alpha_\beta}(A)) - (f_{ne}(A) \cup d_{s\alpha_\beta}(f_{ne}(A))) \neq \emptyset$, which is a flaw to the given condition. Henceforth f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -continuous function.

3.15. Theorem Let $f_{ne}: (Z_{ne}, \tau_{ne}) \rightarrow (Y_{ne}, \sigma_{ne})$ be a neut one-to-one function. Then f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ - continuous function on condition that $f_{ne}(d_{(ne)\alpha_\gamma}(A)) \subseteq d_{(ne)\alpha_\beta}(f_{ne}(A))$, for all that $A \subseteq Z_{ne}$.

Proof. Given $A \subseteq Z_{ne}$, $z \in d_{(ne)\alpha_\gamma}(A)$ and V is a neut α_β -neighbourhood of $f_{ne}(z)$. By the reason of f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -continuous function by the announcement of 3.7. Theorem, at this juncture befalls a neut α_γ - neighbourhood U of z similar that $f_{ne}(U) \subseteq V$. Nevertheless $z \in d_{(ne)\alpha_\gamma}(A)$ stretches there happens a component $c \in U \cap A$ corresponding that $c \neq z$, $f_{ne}(c) \in f_{ne}(A)$ then by the cause of f_{ne} is one-to-one, $f_{ne}(c) \neq f_{ne}(z)$. Consequently, every neut α_β -neighbourhood V of $f_{ne}(z)$ comprises a component $f_{ne}(c)$ of $f_{ne}(A)$ dissimilar from $f_{ne}(z)$. Accordingly, $f_{ne}(z) \in d_{(ne)\alpha_\beta}(f_{ne}(A))$.

Thus $f_{ne} (d_{(ne)\alpha_\gamma}(A)) \subseteq d_{(ne)\alpha_\beta} (f_{ne}(A))$, for altogether $A \subseteq \mathcal{Z}_{ne}$. Contrary portion trails from the 3.14. Theorem.

4. Neutrosophic $\alpha_{(\gamma,\beta)}$ -open functions

4.1. Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is said to be a neut $\alpha_{(\gamma,\beta)}$ -open function akin that for entire neut α_γ -open se $M \in \tau_{(ne)\alpha_\gamma}$, the image $f_{ne} (M) \in \sigma_{(ne)\alpha_\beta}$.

4.2. Example Given

$\mathcal{Z}_{ne} = \{h_1, h_2, h_3\}, \tau_{ne} = \{1_{ne}, \mathcal{Z}_{ne}, F_1, F_3, F_4, F_5\}, \mathcal{Y}_{ne} = \{g_1, g_2, g_3\}$ and $\sigma_{ne} = \{1_{ne}, \mathcal{Z}_{ne}, H_2, H_5\}$ were

$$F_1 = \{z, (0.2,0.6,0.1), (0.1,0.1,0.1), (0.3,0.1,0.1)\},$$

$$F_3 = \{z, (0.8,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.3)\},$$

$$F_4 = \{z, (0.8,0.6,0.3), (0.4,0.5,0.1), (0.1,0.1,0.1)\},$$

$$F_5 = \{z, (0.7,0.6,0.3), (0.1,0.1,0.1), (0.1,0.1,0.1)\}$$

$$H_2 = \{y, (0.2,0.6,0.1), (0.1,0.1,0.1), (0.3,0.1,0.1)\},$$

$$H_5 = \{y, (0.8,0.6,0.3), (0.1,0.1,0.1), (0.1,0.1,0.1)\}$$

Construe an operation γ on τ_{ne} analogous that $(U)^\gamma = cl(U)$

Delineate an operation β on σ_{ne} alike that $(V)^\beta = cl(B)$

Define $f_{ne}: \mathcal{Z}_{ne} \rightarrow \mathcal{Y}_{ne}$ as $f_{ne}(h_1) = g_1, f_{ne}(h_2) = g_3$ and $f_{ne}(h_3) = g_2$. Then the image of each one neut α_γ -open se is a neut α_β -open se under f_{ne} .

Later f_{Ma} is a neut $\alpha_{(\gamma,\beta)}$ -open function.

4.3. Theorem Suppose that $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma,\beta)}$ -open function in addition supposes that $g_{ne}: (\mathcal{Y}_{ne}, \sigma_{ne}) \rightarrow (\mathcal{X}_{ne}, \delta_{ne})$ is a neut $\alpha_{(\beta,\delta)}$ -open function, at that juncture the composition $g_{ne} \circ f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{X}_{ne}, \delta_{ne})$ is a neut $\alpha_{(\gamma,\delta)}$ -open function.

Proof. Validation trails after the 4.1. Definition statement.

4.4. Theorem A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma, \beta)}$ -open function in situation that for entire $z \in \mathcal{Z}_{ne}$, and for every $A \in \tau_{(ne)\alpha_\gamma}$ akin that $z \in A$, there ensues a $B \in \sigma_{(ne)\alpha_\beta}$ such that $f_{ne}(z) \in B$ and $B \subseteq f_{ne}(A)$.

Proof. Consider A as a neut α_γ -open set and $z \in \mathcal{Z}_{ne}$. At that juncture $f_{ne}(z) \in f_{ne}(A)$. Then $f_{ne}(A)$ is a neut α_β -neighbourhood of $f_{ne}(z)$ in \mathcal{Y}_{ne} . Formerly by the declaration of 3.7. Theorem there present a neut α_γ -open neighbourhood $B \in \sigma_{(ne)\alpha_\beta}$ akin that $f_{ne}(z) \in B \subseteq f_{ne}(A)$.

In reverse let $A \in \tau_{(ne)\alpha_\gamma}$ alike that $z \in A$. At that moment by belief, there is a $B \in \sigma_{(ne)\alpha_\beta}$ alike that $f_{ne}(z) \in B \subseteq f_{ne}(A)$. So $f_{ne}(A)$ is a neut α_β -neighbourhood of $f_{ne}(z)$ in \mathcal{Y}_{ne} and this infers that $f_{ne}(A) = \bigcup_{f_{ne}(z) \in f_{ne}(A)} B$. Formerly through 2.1. Theorem $f_{ne}(A)$ is a neut α_β -open set in \mathcal{Y}_{ne} .

Henceforward f_{ne} is a neut $\alpha_{(\gamma, \beta)}$ -open function.

4.5. Theorem A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma, \beta)}$ -open function in the event that for entire $z \in \mathcal{Z}_{ne}$, in addition for every single neut α_γ -neighbourhood U of $z \in \mathcal{Z}_{ne}$, there present a neut α_β -neighbourhood V of $f_{ne}(z)$ alike that $V \subseteq f_{ne}(U)$.

Proof. Given U be a neut α_γ -neighbourhood of $z \in \mathcal{Z}_{ne}$. At that time via the statement of 3.1. Definition there occurs a neut α_γ -open set W such that $z \in W \subseteq U$. This mentions that $f_{ne}(z) \in f_{ne}(W) \subseteq f_{ne}(U)$. Then f_{ne} is a neut $\alpha_{(\gamma, \beta)}$ -open function, $f_{ne}(W)$ is a neut α_β -open set. Henceforward $V = f_{ne}(W)$ is a neut α_β -neighbourhood of $f_{ne}(z)$ and $V \subseteq f_{ne}(U)$.

Conversely, let $U \in \tau_{(ne)\alpha_\gamma}$ and $x \in U$. Then U is a neut α_γ -neighbourhood of x also thence, there prevails a neut α_β -neighbourhood V of $f_{ne}(x)$ akin that $f_{ne}(x) \in V \subseteq f_{ne}(U)$. That is, $f_{ne}(U)$ is a neut α_β -neighbourhood of $f_{ne}(x)$. Thus $f_{ne}(U)$ is a neut α_β -neighbourhood to each of its points. Accordingly, $f_{ne}(U)$ is a neut α_β -open se. Thence f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -open function.

4.6. Theorem A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma,\beta)}$ -open function in case that if $f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-int}(A)) \subseteq \sigma_{(ne)\alpha_\beta}\text{-int}(f_{ne}(A))$, for each $A \subseteq \mathcal{Z}_{ne}$.

Proof. Let $x \in \tau_{(ne)\alpha_\gamma}\text{-int}(A)$. Then there occurs a $U \in \tau_{(ne)\alpha_\gamma}$ alike that $x \in U \subseteq A$. So $f_{ne}(x) \in f_{ne}(U) \subseteq f_{ne}(A)$. Meanwhile f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -open function, $f_{ne}(U)$ is a neut α_β -open set in \mathcal{Y}_{ne} . Later $f_{ne}(x) \in \sigma_{(ne)\alpha_\beta}\text{-int}(f_{ne}(A))$. Thus $f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-int}(A)) \subseteq \sigma_{(ne)\alpha_\beta}\text{-int}(f_{ne}(A))$.

Contrariwise, let $U \in \tau_{(ne)\alpha_\gamma}$ and hereafter $f_{ne}(U) = f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-int}(U)) \subseteq \sigma_{(ne)\alpha_\beta}\text{-int}(f_{ne}(U)) \subseteq f_{ne}(U) \subseteq \sigma_{(ne)\alpha_\beta}\text{-int}(f_{ne}(U)) \subseteq f_{ne}(U)$. This implies that $f_{ne}(U)$ is a neut α_β -open se. Thusly f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -open function.

4.7. Theorem A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma,\beta)}$ -open function on the supposition that $\tau_{(ne)\alpha_\gamma}\text{-int}(f_{ne}^{-1}(B)) \subseteq f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-int}(B))$, for each $B \subseteq \mathcal{Y}_{ne}$.

Proof. Given B be a neut subse of \mathcal{Y}_{ne} . Apparently $\tau_{(ne)\alpha_\gamma}\text{-int}(f_{ne}^{-1}(B))$ is a neut α_γ -open se belonging to \mathcal{Z}_{ne} . Also $f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-int}(f_{ne}^{-1}(B))) \subseteq f_{ne}(f_{ne}^{-1}(B)) \subseteq B$. Subsequently f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -open function besides by 4.6. Theorem $f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-int}(f_{ne}^{-1}(B))) \subseteq \sigma_{(ne)\alpha_\beta}\text{-int}(B)$.

Hence $\tau_{(ne)\alpha_\gamma}\text{-int}(f_{ne}^{-1}(B)) \subseteq f_{ne}^{-1}(f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-int}(f_{ne}^{-1}(B))))$. This implies that $\tau_{(ne)\alpha_\gamma}\text{-int}(f_{ne}^{-1}(B)) \subseteq f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-int}(B))$ for all $B \subseteq \mathcal{Y}_{ne}$.

Contrarywise, accredit that $A \subseteq \mathcal{Z}_{ne}$, at that time $\tau_{(ne)\alpha_\gamma}\text{-int}(A) \subseteq \tau_{(ne)\alpha_\gamma}\text{-int}(f_{ne}^{-1}(f_{ne}(A))) \subseteq f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-int}(f_{ne}(A)))$. This implies that $f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-int}(A)) \subseteq f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-int}(f_{ne}^{-1}(f_{ne}(A)))) \subseteq f_{ne}(f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-int}(f_{ne}(A)))) \subseteq \sigma_{(ne)\alpha_\beta}\text{-int}(f_{ne}(A))$. Consequently $f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-int}(A)) \subseteq \sigma_{(ne)\alpha_\beta}\text{-int}(f_{ne}(A))$, for all $A \subseteq \mathcal{Z}_{ne}$. By 4.6. Theorem, f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -open function.

4.8. Theorem A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma,\beta)}$ -open function on the supposition that $f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-cl}(D)) \subseteq \tau_{(ne)\alpha_\gamma}\text{-cl}(f_{ne}^{-1}(D))$, for all $D \subseteq \mathcal{Y}_{ne}$.

Proof. Agree D be a neut subse of \mathcal{Y}_{ne} . Through 4.7. Theorem, $\tau_{(ne)\alpha_\gamma}\text{-int}(f_{ne}^{-1}(\mathcal{Y}_{ne} - D)) \subseteq f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-int}(\mathcal{Y}_{ne} - D))$. Then $\tau_{(ne)\alpha_\gamma}\text{-int}(\mathcal{Z}_{ne} - f_{ne}^{-1}(D)) \subseteq f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-int}(\mathcal{Y}_{ne} - D))$. As $\sigma_{(ne)\alpha_\beta}\text{-int}(D) = \mathcal{Y}_{ne} - \sigma_{(ne)\alpha_\beta}\text{-cl}(\mathcal{Y}_{ne} - D)$, therefore $\mathcal{Z}_{ne} - \tau_{(ne)\alpha_\gamma}\text{-cl}(f_{ne}^{-1}(D)) \subseteq f_{ne}^{-1}(\mathcal{Y}_{ne} - \sigma_{(ne)\alpha_\beta}\text{-cl}(D))$ or $\mathcal{Z}_{ne} - \tau_{(ne)\alpha_\gamma}\text{-cl}(f_{ne}^{-1}(D)) \subseteq \mathcal{Z}_{ne} - f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-cl}(D))$. Hence $f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-cl}(D)) \subseteq \tau_{(ne)\alpha_\gamma}\text{-cl}(f_{ne}^{-1}(D))$.

Conversely, let $D \subseteq \mathcal{Y}_{ne}$ and hence, $f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-cl}(\mathcal{Y}_{ne} - D)) \subseteq \tau_{(ne)\alpha_\gamma}\text{-cl}(f_{ne}^{-1}(\mathcal{Y}_{ne} - D))$.

Then $\mathcal{Z}_{ne} - \tau_{(ne)\alpha_\gamma}\text{-cl}(f_{ne}^{-1}(\mathcal{Y}_{ne} - D)) \subseteq \mathcal{Z}_{ne} - f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-cl}(\mathcal{Y}_{ne} - D))$. Hence $\mathcal{Z}_{ne} - \tau_{(ne)\alpha_\gamma}\text{-cl}(\mathcal{Z}_{ne} - f_{ne}^{-1}(D)) \subseteq f_{ne}^{-1}(\mathcal{Y}_{ne} - \sigma_{(ne)\alpha_\beta}\text{-cl}(\mathcal{Y}_{ne} - D))$. This gives that $\tau_{(ne)\alpha_\gamma}\text{-int}(f_{ne}^{-1}(D)) \subseteq f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-int}(D))$.

Using 4.5. Theorem, it follows that f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -open function.

4.9. Theorem Let $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ and $g_{ne}: (\mathcal{Y}_{ne}, \sigma_{ne}) \rightarrow (\mathcal{X}_{ne}, \delta_{ne})$ be two functions such that $g_{ne} \circ f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{X}_{ne}, \delta_{ne})$ is a neut $\alpha_{(\gamma, \delta)}$ -continuous function. Formerly

(i) Assuming that g_{ne} is a neut $\alpha_{(\beta, \delta)}$ -open injection at that time f_{ne} is a neut $\alpha_{(\gamma, \beta)}$ -continuous function.

(ii) Supposing that f_{ne} is a neut $\alpha_{(\gamma, \beta)}$ -open surjection at that time g_{ne} is a neut $\alpha_{(\beta, \delta)}$ -continuous function.

Proof. (i) Approve $U \in \sigma_{(ne)\alpha_\beta}$. Meanwhile g_{ne} is a neut $\alpha_{(\beta, \delta)}$ -open function, at that time $g_{ne}(U) \in \zeta_{(ne)\alpha_\delta}$. Subsequently g_{ne} is injective besides $g_{ne} \circ f_{ne}$ is a neut $\alpha_{(\gamma, \delta)}$ -continuous function, $(g_{ne} \circ f_{ne})^{-1}(g_{ne}(U)) = (f_{ne}^{-1} \circ g_{ne}^{-1})(g_{ne}(U)) = f_{ne}^{-1}(g_{ne}^{-1}(g_{ne}(U))) = f_{ne}^{-1}(U)$ is a neut α_γ -open function of \mathcal{Z}_{ne} . This demonstrates that f_{ne} is a neut $\alpha_{(\gamma, \beta)}$ -continuous function.

(ii) Accredited $V \in \zeta_{(ne)\alpha_\delta}$. Meanwhile $g_{ne} \circ f_{ne}$ is a neut $\alpha_{(\gamma, \delta)}$ -continuous function, at that time $(g_{ne} \circ f_{ne})^{-1}(V) \in \tau_{(ne)\alpha_\gamma}$. As well f_{ne} is a neut $\alpha_{(\gamma, \beta)}$ -open function, accordingly $f_{ne}((g_{ne} \circ f_{ne})^{-1}(V))$ is a neut α_β -open set prevailing in \mathcal{Y}_{ne} . By reason of f_{ne} is surjective, we obtain $(f_{ne} \circ (g_{ne} \circ f_{ne})^{-1})(V) = (f_{ne} \circ (f_{ne}^{-1} \circ g_{ne}^{-1}))(V) = ((f_{ne} \circ f_{ne}^{-1}) \circ g_{ne}^{-1})(V) = g_{ne}^{-1}(V)$. It trails that $g_{ne}^{-1}(V) \in \sigma_{(ne)\alpha_\beta}$. This evidences that g_{ne} is a neut $\alpha_{(\beta, \delta)}$ -continuous function.

5. Neutrosophic $\alpha_{(\gamma, \beta)}$ -Closed Functions

5.1.(i) Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is supposed to be a neut $\alpha_{(\gamma, \beta)}$ -closed function provided that the image set $f_{ne}(A)$ is a neut α_β -closed set for entire neutrosophic α_γ -closed subset A of \mathcal{Z}_{ne} .

(ii) Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is supposed to be a neut $(\gamma_{ne}, \alpha_{\beta})$ -open(closed) function supposing that the icon of each neut β_{ne} -open(closed) se prevailing in \mathcal{Z}_{ne} is a neut α_{β} -open(closed) se prevailing in \mathcal{Y}_{ne} .

(iii) Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is thought to be a neut $(\gamma_{ne}, \beta_{ne})$ -irresolute function in case that $f_M^{-1}(P)$ is a neut γ_{ne} -open se be present in \mathcal{Z}_{ne} for each β_{ne} -open set P survives in \mathcal{Y}_{ne} .

5.2. Example Given $\mathcal{Z}_{ne} = \{h_1, h_2, h_3\}, \tau_{ne} = \{1_{ne}, \mathcal{Z}_{ne}, F_1, F_3, F_4, F_5\}$

, $\mathcal{Y}_{ne} = \{g_1, g_2, g_4\}$ and $\sigma_{ne} = \{1_{ne}, \mathcal{Z}_{ne}, H_1, H_4, H_5, H_6\}$ were

$$F_1 = \{z, (0.2, 0.6, 0.3), (0.1, 0.1, 0.1), (0.3, 0.1, 0.1)\},$$

$$F_3 = \{z, (0.8, 0.6, 0.3), (0.1, 0.1, 0.1), (0.1, 0.1, 0.1)\},$$

$$F_4 = \{z, (0.8, 0.6, 0.3), (0.4, 0.5, 0.1), (0.1, 0.1, 0.1)\},$$

$$F_5 = \{z, (0.7, 0.6, 0.3), (0.1, 0.1, 0.1), (0.1, 0.1, 0.1)\}$$

$$H_1 = \{y, (0.2, 0.6, 0.1), (0.1, 0.1, 0.1), (0.3, 0.1, 0.1)\},$$

$$H_4 = \{y, (0.7, 0.6, 0.3), (0.4, 0.5, 0.1), (0.1, 0.1, 0.3)\},$$

$$H_5 = \{y, (0.8, 0.6, 0.3), (0.4, 0.5, 0.1), (0.1, 0.1, 0.1)\},$$

$$H_6 = \{y, (0.7, 0.6, 0.3), (0.4, 0.5, 0.1), (0.1, 0.1, 0.1)\}$$

$$\text{Characterize an operation } \gamma \text{ on } \tau_{ne} \text{ alike that } (U)^\gamma = \begin{cases} U \cup \{h_3\} & \text{if } U \neq \{h_1\} \\ U & \text{if } U = \{h_1\} \end{cases}$$

$$\text{Specify an operation } \beta \text{ on } \sigma_{ne} \text{ aforesaid that } (V)^\beta = \begin{cases} V & \text{if } g_2 \notin V \\ cl(V) & \text{if } g_2 \in V \end{cases}$$

Define $f_{ne}: \mathcal{Z}_{ne} \rightarrow \mathcal{Y}_{ne}$ as $f_{ne}(h_1) = g_1$, $f_{ne}(h_2) = g_2$ and $f_{ne}(h_3) = g_3$.

Then the appearance of each one neut α_γ -closed se is a neut α_β -closed se under f_{ne} .

Henceforth f_{ne} is a neut $\alpha_{(\gamma, \beta)}$ -closed function.

5.3. Theorem Accredited $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ be a neut $\alpha_{(\gamma, \beta)}$ -closed function, previously the succeeding declarations hold good.

- (i) Assume $g_{ne}: (\mathcal{Y}_{ne}, \sigma_{ne}) \rightarrow (\mathcal{X}_{ne}, \delta_{ne})$ is a neut $\alpha_{(\beta, \delta)}$ -closed function, then $g_{ne} \circ f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{X}_{ne}, \delta_{ne})$ is a neut $\alpha_{(\gamma, \delta)}$ -closed function;
- (ii) $\sigma_{(ne)\alpha_\beta}\text{-cl}(f_{ne}(A)) \subseteq f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-cl}(A))$, for each subset A of \mathcal{Z}_{ne} ;
- (iii) $\sigma_{(ne)\alpha_\beta}\text{-cl}(\sigma_{(ne)\alpha_\beta}\text{-int}(\sigma_{(ne)\alpha_\beta}\text{-cl}(f_{ne}(A)))) \subseteq f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-cl}(A))$, for all neut subset A of \mathcal{Z}_{ne} ;
- (iv) for all neut subset B of \mathcal{Y}_{ne} and to each neut α_γ -open set A of \mathcal{Z}_{ne} encompassing $f_{ne}^{-1}(B)$, there occurs a neut α_β -open set C in \mathcal{Y}_{ne} comprising B alike that $f_{ne}^{-1}(C) \subseteq A$.

Proof. Confirmations are alike to the proofs of the 4.3,4.4,4.5 and 4.6. Theorems.

5.4. Theorem Let $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ be a neut bijective function. Previously the ensuing assertions are analogous:

- (i) f_{ne} is a neut $\alpha_{(\gamma, \beta)}$ -closed function;
- (ii) f_{ne} is a neut $\alpha_{(\gamma, \beta)}$ -open function;
- (iii) f_{ne}^{-1} is a neut $\alpha_{(\beta, \gamma)}$ -continuous function.

Proof. (i) \Rightarrow (ii) Substantiation trails after the declarations of 4.1. Definition and 5.1. Definition.

(ii) \Rightarrow (iii) Specify that A is a neut α_γ -closed se in \mathcal{Z}_{ne} . Formerly $\tau_{(ne)\alpha_\gamma}\text{-cl}(A) = A$. By the effect of

(ii) and 4.8.Theorem, $f_{ne}^{-1}(\sigma_{(ne)\alpha_\beta}\text{-cl}(f_{ne}(A))) \subseteq \tau_{(ne)\alpha_\gamma}\text{-cl}(f_{ne}^{-1}(f_{ne}(A)))$ infers that $\sigma_{(ne)\alpha_\beta}\text{-cl}(f_{ne}$

$(A)) \subseteq f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-cl}(A))$. Consequently $\sigma_{(ne)\alpha_\beta}\text{-cl}(f_{ne}^{-1})^{-1}(A) \subseteq (f_{ne}^{-1})^{-1}(A)$, aimed at each single

subse A of \mathcal{Z}_{ne} , it trails that f_{ne}^{-1} is a neut $\alpha_{(\gamma, \beta)}$ -continuous function.

(iii) \Rightarrow (i) Specify that A is a neut α_γ -closed se of \mathcal{Z}_{ne} . Formerly $\mathcal{Z}_{ne} - A$ is a neut α_γ -open se lying

in \mathcal{Z}_{ne} . By reason of f_{ne}^{-1} is an $\alpha_{(\gamma, \beta)}$ -continuous function, $(f_{ne}^{-1})^{-1}(\mathcal{Z}_{ne} - A)$ is a neut α_β -open se in

\mathcal{Y}_{ne} . Nevertheless $(f_{ne}^{-1})^{-1}(\mathcal{Z}_{ne} - A) = f_{ne}(\mathcal{Z}_{ne} - A) = \mathcal{Y}_{ne} - f_{ne}(A)$. Consequently $f_{ne}(A)$ is a neut α_β -closed set lying in \mathcal{Y}_{ne} . This one demonstrates that the function f_{Ma} is a neut $\alpha_{(\gamma,\beta)}$ -closed function.

5.5. Definition Let $id: \tau \rightarrow P(X)$ remain as the identity maneuver. A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is held to be a neut $\alpha_{(id,\beta)}$ -closed func if designed at any neut α -closed set F of \mathcal{Z}_{ne} , $f_{ne}(F)$ is a neut α_β -closed set in \mathcal{Y}_{ne} .

5.6. Theorem Supposing that $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a bijective function also $f_{ne}^{-1}: (\mathcal{Y}_{ne}, \sigma_{ne}) \rightarrow (\mathcal{Z}_{ne}, \tau_{ne})$ is a neut $\alpha_{(id,\beta)}$ -continuous function, formerly f_{ne} is a neut $\alpha_{(id,\beta)}$ -closed function.

Proof. Authentication tracks next to the descriptions of 5.1. Definition besides 5.5. Definition.

5.7. Theorem Supposing that f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -continuous function. Formerly

(i) Suppose that A is a neut α_γ g -closed set in $(\mathcal{Z}_{ne}, \tau_{ne})$, later the image $f_{ne}(A)$ is a neut α_β g -closed set.

(ii) Given B be a neut α_β g -closed set of $(\mathcal{Y}_{ne}, \sigma_{ne})$, later the set $f_{ne}^{-1}(B)$ is a neut α_γ g -closed set.

Proof. (i) Contemplate V as a neut α_β -open set prevailing in \mathcal{Y}_{ne} alike that $f_{ne}(A) \subseteq V$. By means of 3.8. Theorem statement, $f_{ne}^{-1}(V)$ is a neut α_γ -open set encompassing A . By postulation $\tau_{(ne)\alpha_\gamma}$ -cl $(A) \subseteq f_{ne}^{-1}(V)$, so $f_{ne}(\tau_{(ne)\alpha_\gamma}$ -cl $(A)) \subseteq V$. Since f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ -closed function, $f_{ne}(\tau_{(ne)\alpha_\gamma}$ -cl

(A) is a neut α_β -closed se comprising $f_{ne}(A)$ entails that $\sigma_{(ne)\alpha_\beta}\text{-cl}(f_{ne}(A)) \subseteq \sigma_{(ne)\alpha_\beta}\text{-cl}(f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-cl}(A))) = f_{ne}(\tau_{(ne)\alpha_\gamma}\text{-cl}(A)) \subseteq V$. Hence $f_{ne}(A)$ is a neut $\alpha_\beta g$ -closed se.

(ii) Assume U be a neut α_γ - open se of \mathcal{Z}_{ne} akin that $f_{ne}^{-1}(B) \subseteq U$ for any subse B in \mathcal{Y}_{ne} . Put $F = \tau_{(ne)\alpha_\gamma}\text{-cl}(f_{ne}^{-1}(B)) \cap (\mathcal{Z}_{ne} - U)$. It trails from the 2.2.(ii) Remark and 2.3. Theorem, that F is a neut α_γ -closed se in \mathcal{Z}_{ne} . Meanwhile f_{ne} is a neut $\alpha_{(\gamma,\beta)}$ - closed function, $f_{ne}(F)$ is a neut $\alpha_{(\gamma,\beta)}$ - closed se in \mathcal{Y}_{ne} . By the 2.4 Theorem declaration then by the 3.8.(ii) Theorem declaration in addition the subsequent insertion $f_{ne}(F) \subseteq \sigma_{(ne)\alpha_\beta}\text{-cl}(B) - B$, it is gained that $f_{ne}(F) = \emptyset$, and henceforth $F = \emptyset$. This infers that $\tau_{(ne)\alpha_\gamma}\text{-cl}(f_{ne}^{-1}(B)) \subseteq U$. Therefore $f_{ne}^{-1}(B)$ is a neut $\alpha_\gamma g$ -closed se.

5.8. Theorem Given $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma,\beta)}$ -continuous and neut $\alpha_{(\gamma,\beta)}$ - closed function. Then

(i) With the condition that f_{ne} is a neut injective function moreover \mathcal{Y}_{ne} is a neut $\alpha_\beta T_{\frac{1}{2}}$ at that juncture \mathcal{Z}_{ne} is a neut $\alpha_\gamma T_{\frac{1}{2}}$ space.

(ii) Conceding that f_{ne} is a surjective function besides \mathcal{Z}_{ne} is a neut $\alpha_\gamma T_{\frac{1}{2}}$ formerly \mathcal{Y}_{ne} is a neut $\alpha_\beta T_{\frac{1}{2}}$ space.

Proof. (i) Accept A be a neut $\alpha_\gamma g$ -closed se of \mathcal{Z}_{ne} . Formerly via 5.7. Theorem statement (i), $f_{ne}(A)$ is a neutrosophic $\alpha_\beta g$ - closed se. Accordingly, by postulation A is a neut α_γ -closed se in \mathcal{Z}_{ne} . So \mathcal{Z}_{ne} is a neut $\alpha_\gamma T_{\frac{1}{2}}$ space.

(ii) Contemplate B as a neut $\alpha_\beta g$ -closed se lying in \mathcal{Y}_{ne} . At that moment it surveys after the 5.7.(ii) Proposition and the supposition that $f_{ne}^{-1}(B)$ is a neut α_γ -closed se. Hence f_{ne} is a neut

$\alpha_{(\gamma,\beta)}$ -closed function, implies that $f_{ne}(f_{ne}^{-1}(B)) = B$ is a neut α_β -closed se in \mathcal{Y}_{ne} . Therefore \mathcal{Y}_{ne} is a neut $\alpha_\beta T_{\frac{1}{2}}$ space.

5.9. Remark Each neut $(\gamma_{ne}, \beta_{ne})$ -irresolute function is a neut $(\alpha_\gamma, \beta_{ne})$ -continuous function. Nonetheless, the conflicting statement need not be factual.

6. Neutrosophic $\alpha_{(\gamma,\beta)}$ - Homeomorphism

6.1. Definition A function $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ is a neut $\alpha_{(\gamma,\beta)}$ -homeomorphism, if f_{ne} is a bijective, neut $\alpha_{(\gamma,\beta)}$ -continuous function and f_{ne}^{-1} is a neut $\alpha_{(\beta,\gamma)}$ -continuous function.

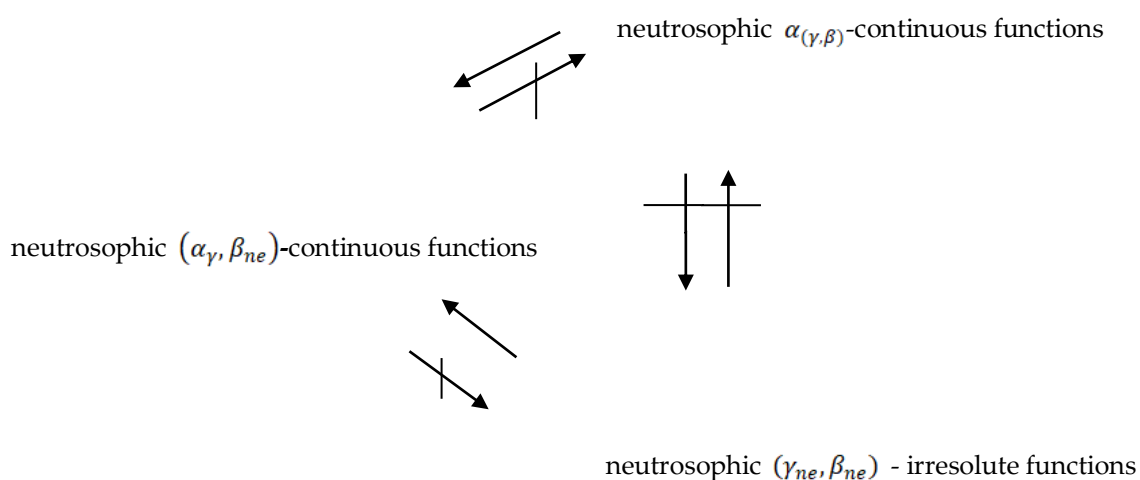
6.2. Remark Each bijective, neut $\alpha_{(\gamma,\beta)}$ -continuous and neut $\alpha_{(\gamma,\beta)}$ - closed function is a neut $\alpha_{(\gamma,\beta)}$ -homeomorphism.

6.3. Theorem Let $f_{ne}: (\mathcal{Z}_{ne}, \tau_{ne}) \rightarrow (\mathcal{Y}_{ne}, \sigma_{ne})$ be a neut $\alpha_{(\gamma,\beta)}$ -homeomorphism. If \mathcal{Z}_{ne} is a neut $\alpha_\gamma T_{\frac{1}{2}}$ space then \mathcal{Y}_{ne} is a neut $\alpha_\beta T_{\frac{1}{2}}$ space.

Proof. Assume $\{y\}$ as a singleton se of \mathcal{Y}_{ne} . Then there befalls a point z of \mathcal{Z}_{ne} alike that $y = f_{ne}(z)$. Via 2.5. Theorem Announcement, it trails that the singleton se $\{y\}$ is furthermore a neut α_β -open se or else a neut α_β -closed se. Accordingly \mathcal{Y}_{ne} is a neut $\alpha_\beta T_{\frac{1}{2}}$ spa.

6.4. Remark Each neut $\alpha_{(\gamma,\beta)}$ -open (closed) function is a neut $(\gamma_{ne}, \alpha_\beta)$ -open (closed) function. Nonetheless, the opposing statement must not be exact. Then the succeeding comment shows the connotation amongst the neut $\alpha_{(\gamma,\beta)}$ -open (closed) functions, neut $(\gamma_{ne}, \alpha_\beta)$ -open (closed) functions and neut (γ_{ne}, β_s) - open (closed) functions.

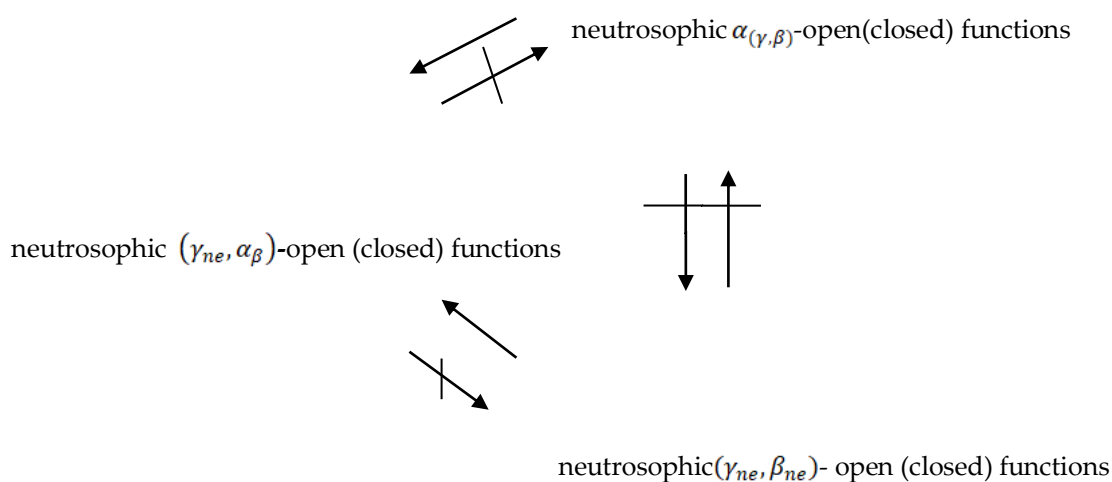
6.5. Remark From the 6.1.(i), (ii), (iii) Definitions 6.3. and 6.4. Remarks the subsequent illustrative inferences 2. Figure is attained:



$A \rightarrow B$ represents $A \text{ infer } B$, $A \nrightarrow B$ represents A does not infer B .

2. Figure: Relationship between neutrosophic continuous functions

6.6. Remark As of the 4.1.,5.1. (i), (ii), (iii) Definitions and 5.9. Remark the ensuing pictorial inferences 3. Figure is gained:



$A \rightarrow B$ symbolizes A indicates at B , $A \nrightarrow B$ signifies A does not indicate at B .

3. Figure: Association between neutrosophic open (closed) functions

Conclusion and Future study

In this article the observation of neut $\alpha_{(\gamma,\beta)}$ -continuous functions that are created over neut α_γ -open sets are considered and many of their basic properties are detailed. Also, the neut $\alpha_{(\gamma,\beta)}$ -open (closed) functions are declared besides inspected their rudimentary properties. Neut α_γ -derived sets, neut α_γ -frontier besides neut α_γ -kernel are described also experienced to create the ideas of several neut continuous functions and neut open(closed) functions. The connection amongst these neut $\alpha_{(\gamma,\beta)}$ -continuous functions, neut $\alpha_{(\gamma,\beta)}$ -open functions, neut $\alpha_{(\gamma,\beta)}$ -closed functions are illustrated. Further the concept of the neut $(\gamma_{ne}, \alpha_\beta)$ -open(closed) function, neut $(\gamma_{ne}, \beta_{ne})$ -irresolute function, neut $(\alpha_\gamma, \beta_{ne})$ -continuous function and neut $(\gamma_{ne}, \beta_{ne})$ -continuous function, neut α_γ -neighbourhood of a point, neut α_γ -limit point, Composition of functions, neut α_γ -derived set and neut $\alpha_{(\gamma,\beta)}$ -continuous, injective function are detailed and utilized for deriving numerous highly significant results. Contra neut $\alpha_{(\gamma,\beta)}$ -continuous functions, contra neut $(\alpha_\gamma, \beta_{ne})$ -continuous function and contra neut $(\gamma_{ne}, \beta_{ne})$ -continuous function can be studied as a future work.

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