

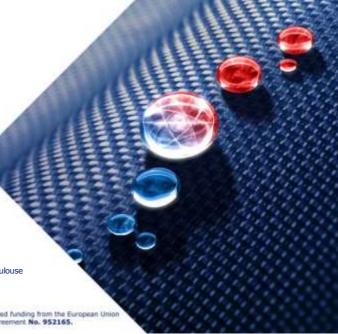
QMCkl: A Unified
Approach to Accelerating
Quantum Monte Carlo
Codes

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The TREX European Center of Excellence







Partners

























Codes

- CHAMP
- QMC=Chem
- TurboRVB
- NECI
- QuantumPackage
- GammCor



CHAMP (Claudia Filippi)

- Wavefunction optimization: Jastrow, Cl, MOs
- Ground/Excited states
- Geometry optimization



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- Molecular and Periodic systems
- JAGP, Pfaffian, ...
- LRDMC



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QMC=Chem (Michel Caffarel + Me!)

- DMC as "Post-Full-CI" energy calculations (CIPSI)
- Very large CI expansions (millions of determinants)
- Designed with HPC in mind
- Highly optimized with W. Jalby's group (UVSQ) in 2011-2013



- TREX CoE: Targeting REal chemical accuracy at the eXascale
- Started in Oct. 2020
- Objective: Make codes ready for exascale systems



- TREX CoE: Targeting REal chemical accuracy at the eXascale
- Started in Oct. 2020
- Objective: Make codes ready for exascale systems
- How: Instead of re-writing codes, provide libraries
 - One library for high-performance (QMCkl)
 - One library for exchanging information between codes (TREXIO)



The QMC kernel library (QMCkI)





- Progress in quantum chemistry requires codes with new ideas/algorithms
- New ideas/algorithms are implemented by physicists/chemists
- Different scientists have different programming language knowledge/preference
- Exascale machines are horribly complex to program





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Question

Is it reasonable to ask physicists/chemists to write codes for exascale machines?



3

 $(from \ \underline{https://github.com/jeffhammond/dpcpp-tutorial})$

$$Z_{n+1} = Z_n + aX_n + Y_n$$

```
do i=1, n

Z(i) = Z(i) + A * X(i) + Y(i)

end do
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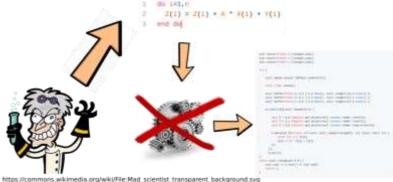


A compiler¹ that can read an average researcher's code and transform it into highly efficient code on an exascale machine.

¹Wikipedia: A compiler is a computer program that translates computer code written in one programming language (the source language) into another language (the target language)

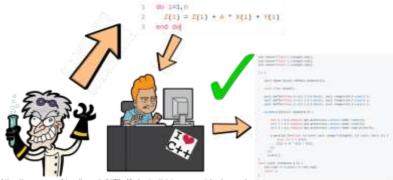


Artificial Intelligence was not ready in 2021 when we started the project ...





... so we decided to use *Natural Intelligence*, and add a human layer between the machine and the researchers: a biological compiler



https://commons.wikimedia.org/wiki/File:Mad_scientist_transparent_background.svg



- Identify the common computational kernels of QMC
- Implement these kernels in a human-readable library (QMC experts) Bio-
- compile the human-readable library in a HPC-library (HPC experts)
- Scientists can link either library with their codes



For scientists

- The choice of the programming language is not imposed to the scientist
- The code can stay easy to understand by the physicists/chemists
 Performance-related aspects are delegated to the library
- Codes will not die with a change in hardware
- Scientific code development does not break the performance
- Scientists don't lose control on their codes



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Separation of concerns

- Scientists will never have to manipulate low-level HPC code
- HPC experts will not be required to be experts in theoretical physics
- Better re-use of the optimization effort among the community



The QMCkl Documentation library



- The API is C-compatible: QMCkl appears to scientists like a C library =⇒ can be used in all other languages
- System functions in programmed C (memory allocation, thread safety, etc)
- Computational kernels programmed in simple Fortran for readability
- A lot of documentation (remember: the HPC compiler is a human!)



Literate programming is a programming paradigm introduced by Donald Knuth in which a computer program is given an explanation of its logic in a natural language, such as English, interspersed with snippets of macros and traditional source code, from which compilable source code can be generated. (Wikipedia)



Literate programming with *org-mode*:

- Here, comments are more important than code
- Can add graphics, LATEXformulas, tables, etc
- Documentation always synchronized with the code
- Some functions can be generated by embedded scripts
- Web site auto-generated when code is pushed

Instead of writing comments documenting code, we write code illustrating documentation.



Literate programming with org-mode

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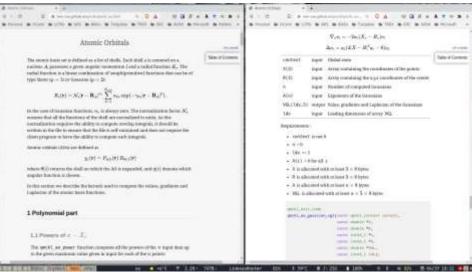
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At each QMC step, we need to evaluate $E_{loc}(\mathbf{r}_1,\ldots,\mathbf{r}_N)=\frac{\hat{H}\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_N)}{\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_N)}$:

- $\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_N)$
- Δ_i Ψ($r_1, ..., r_i, ..., r_N$): kinetic energy
- $\nabla \mathcal{W}(r_1, \ldots, r_i, \ldots, r_N)$: drift in the stochastic process



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Kernels implemented and well tested today

- AOs: $\chi(r)$, $\nabla \chi(r)$, $\Delta \chi(r)$
- MOs: $\varphi(r)$, $\nabla \varphi(r)$, $\Delta \varphi(r)$
- Jastrow correlation factor (eN, ee, eeN)
- Inverses of small matrices



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- $\blacksquare \Psi(\mathbf{r}_1,\ldots,\mathbf{r}_N)$
- Δ_i Ψ($r_1,...,r_i,...,r_N$): kinetic energy
- $\nabla t \Psi(r_1, ..., r_i, ..., r_N)$: drift in the stochastic process

Kernels implemented and well tested today

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- Jastrow correlation factor (eN, ee, eeN)
- Inverses of small matrices

Work in progress

Everything else required to compute \Psi, \nabla\Psi and \Delta\Psi.

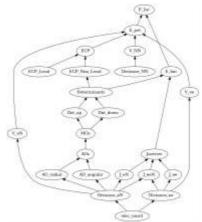


$$E_{\text{loc}}(R) = E_{\text{pot}}(R) + E_{\text{kin}}(R)$$

$$E_{\text{pot}}(R) = V_{\text{ee}}(R) + V_{\text{eN}}(R) + V_{\text{NN}}(R) + V_{\text{ECP}}(R)$$

$$E_{\text{kin}}(R) = -\frac{1}{2} \frac{\Delta \Psi(R)}{\Psi(R)}$$

$$\Psi(R) = \Phi(R) J(R)$$
...



All the graph is invalidated updated when the electron coordinates are changed.



Algorithms



Before computing anything, QMCkl needs to be given a trial wave function.

Setting wave function parameters

- Wave function exchange between codes is a major difficulty
- Our solution:
 - Define a standard format for wavefunction parameters
 - TREXIO: TREX Input/Output library (see Evgeny Posenitskiy's presentation)

Initialization of QMCkl

Two ways:

1 Control: Each array can be set by hand

2 Simplicity: Read all the wave function parameters from a TREXIO file



Atomic Orbitals

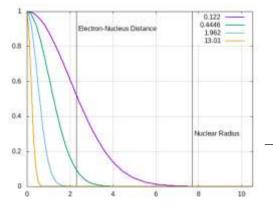
$$R_{s}(\mathbf{r}) = N_{s}|\mathbf{r} - R_{A}|^{n_{s}} \sum_{k=1}^{N_{b}rim} a_{ks} f_{ks} \exp(-\gamma_{ks}|\mathbf{r} - R_{A}|^{p}).$$

Flexible

- Software like GAMESS use different normalization factors for d orbitals
- Implementing Slater-type orbitals is a minor modification (in the very long to-do list)
- Contribution from the FHI-AIMS group for the evaluation of numerical AOs
- Separation of the radial and angular components packed in shells
- Efficient computation of powers of x, y, z to maximize data re-use







- Definition of an atomic radius for each nucleus beyond which all AOs are zero (VGL^a).
- Primitives are sorted in ascending order of the exponents.
- Only non-zero elements are computed

^aVGL: value, gradients, Laplacian



Molecular Orbitals

$$\varphi_{i}(\mathbf{r}_{j}) = \sum_{k=1}^{\infty} A_{ik} \chi_{k}(\mathbf{r}_{j}) \qquad B_{1} = A \cdot C_{1}$$

$$\nabla_{\chi} \varphi_{i}(\mathbf{r}_{j}) = \sum_{k=1}^{\infty} A_{ik} \nabla_{\chi} \chi_{k}(\mathbf{r}_{j}) \qquad B_{2} = A \cdot C_{2}$$

$$\nabla_{\chi} \varphi_{i}(\mathbf{r}_{j}) = \sum_{k=1}^{\infty} A_{ik} \nabla_{\chi} \chi_{k}(\mathbf{r}_{j}) \qquad B_{3} = A \cdot C_{3}$$

$$\nabla_{\chi} \varphi_{i}(\mathbf{r}_{j}) = \sum_{k=1}^{\infty} A_{ik} \nabla_{\chi} \chi_{k}(\mathbf{r}_{j}) \qquad B_{4} = A \cdot C_{4}$$

$$\Delta \varphi_{i}(\mathbf{r}_{j}) = \sum_{k=1}^{\infty} A_{ik} \Delta \chi_{k}(\mathbf{r}_{j}) \qquad B_{5} = A \cdot C_{5}$$



Sparse / dense matrix multiplication

- QMC=Chem (2013): https://doi.org/10.1002/jcc.23216
- Exploits the common sparse character of the AO matrices:
 - When $\chi(r) = 0$ because r is too far, all the derivatives are also zero
 - Quadratic scaling
- Can be fully vectorized
 - >60% of peak performance on Sandy-Bridge CPUs

	Smallest system	B-Strand	β-Strand TZ	1ZE7	1AMB
N N _{basis}	158 404	434 963	434 2934	1056 2370	1731 3892
% of non-zero ⁿ MO coefficients n_{ij} $(A_{ij} \neq 0)$	81.3% (99.4%)	48,4% (76.0%)	73,4% (81,9%)	49.4% (72.0%)	37,1% (66.1%)
Average % of non-zero basis functions $\chi_i(\mathbf{r}_j)$ $(B_{1ij} \neq 0)$	36.2%	14.8%	8.2%	5.7%	3.9%
Average number of non-zero elements per column of B _{1,j}	146	142	241	135	152



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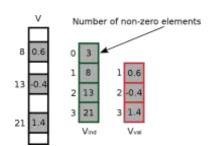
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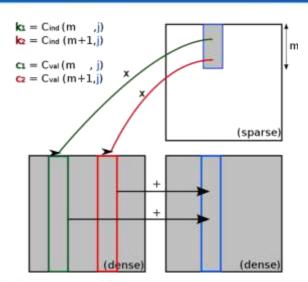
Sparse / dense matrix multiplication

```
do j=1, point num
   mo vgl(:, :, j) = 0. d0
   do k=1, ao num
      if (ao vgl(k, 1, j) /= 0, d0) then
         c1 = ao vgl(k, 1, j)
         c2 = ao vgl(k, 2, i)
         c3 = ao vgl(k, 3, j)
         c4 = ao vgl(k, 4, i)
         c5 = ao vgl(k, 5, j)
         do i=1, mo num
             mo vgl(i, 1, j) = mo vgl(i, 1, j) + coefficient t(i, k) * c1
             mo vgl(i, 2, j) = mo vgl(i, 2, j) + coefficient t(i, k) * c2
             mo vgl(i, 3, j) = mo vgl(i, 3, j) + coefficient t(i, k) * c3
             mo vgl(i, 4, j) = mo vgl(i, 4, j) + coefficient t(i, k) * c4
             mo vgl(i, 5, j) = mo vgl(i, 5, j) + coefficient t(i, k) * c5
         end do
      end if
   end do
```



Sparse / dense matrix multiplication

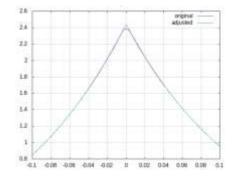






$$\varphi_{\text{cusp }i}(\mathbf{r}) = \varphi_i(\mathbf{r}) - \varphi_{\text{SA}i}(\mathbf{r}) + \sum_{k=0}^{23} f_k |\mathbf{r} - \mathbf{R}_A|^k$$
, where $|\mathbf{r} - \mathbf{R}_A| < r_{\text{cusp},A}$

- φ_{SA}i: contributions of the s AOs centered at A to MO φ_i.
- 3 conditions:
 - Electron-nucleus cusp at $|\mathbf{r} \mathbf{R}_{\mathcal{A}}| = 0$
 - Continuity of the MO: $\varphi_{\text{cusp }i} = \varphi_i$ when $|\mathbf{r} \mathbf{R}_A| = r_{\text{cusp},A}$
 - Continuity of the gradient: $\nabla \varphi_{\text{cusp }i}(\mathbf{r}) = \nabla \varphi_i(\mathbf{r})$ when $|\mathbf{r} \mathsf{R}_A| = r_{\text{cusp }A}$



3-body component of the Jastrow factor

$$J_{\text{een}}(\mathbf{r}, \mathbf{R}) = \sum_{a=1}^{N} \sum_{j=1}^{N} \sum_{p=2}^{N} \sum_{k=0}^{N} \sum_{j=1}^{N} \sum_{p=2}^{N} \sum_{k=0}^{N} \sum_{j=1}^{N} \sum_{k=0}^{N} \sum_{j=1}^{N} \sum_{k=0}^{N} \sum_{j=1}^{N} \sum_{k=0}^{N} \sum_{j=1}^{N} \sum_{k=0}^{N} \sum_{j=1}^{N} \sum_{k=0}^{N} \sum_{j=1}^{N} \sum_{k=0}^{N} \sum_{k=0}^{N} \sum_{j=1}^{N} \sum_{k=0}^{N} \sum_{$$

can be rewritten as

$$J_{\text{een}}(\mathbf{r}, \mathbf{R}) = \begin{pmatrix} \mathbf{v}_{\text{bord}} & \mathbf{v}_{\text{bord}} & \mathbf{v}_{\text{bord}} & \mathbf{v}_{\text{bord}} \\ \mathbf{v}_{\text{een}}(\mathbf{r}, \mathbf{R}) & \mathbf{v}_{\text{bord}} & \mathbf{v}_{\text{bord}} & \mathbf{v}_{\text{elec}} \\ \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} \\ \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} \\ \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} \\ \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} \\ \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} \\ \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} \\ \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} & \mathbf{v}_{\text{lkpa}} &$$

with

$$\overline{P}_{i,k,a,l} = \sum_{j=1}^{N_{elec}} \overline{r}_{i,k,j} \overline{R}_{j,a,l}. (GEMM)$$



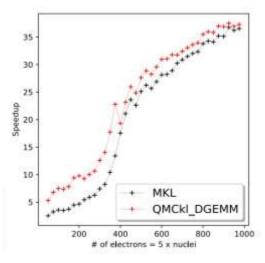
3-body component of the Jastrow factor

$$\nabla_{im} J_{\text{een}}(\mathbf{r}, \mathsf{R}) = \frac{\sum_{k=0}^{\infty} \sum_{k=0}^{\infty} \sum_{k=0$$

with

$$\overline{G_{i,m,a,l}} = \frac{\partial (R_{ia})}{\partial r_i}, \qquad \overline{g_{i,m,j,k}} = \frac{\partial (r_{ij}h)}{\partial r_i}, \qquad \text{and } \overline{Q_{i,m,a,k,l}} = \frac{\sum_{j=1}^{m} \overline{g_{i,m,j,k}}}{\overline{g_{i,m,j,k}}} \overline{R_{j,a,l}}$$







HPC implementations



- MAQAO, developed by the UVSQ team, is used to help us optimize the CPU code
 - Loop-level diagnostics
 - Vectorization ratio
 - Hints to improve efficiency
- Algorithms rewritten in C:
 - C compilers are usually more mature than Fortran on new hardware
 - Access to more low-level features than Fortran (pinned memory, alignment, inline assembly, etc)
 - Precision can be changed on-the-fly: switch to single-precision if possible
- Specialization:
 - Specialization for s, p and d AOs
 - Inverse of small matrices hard-coded for 2×2 to 5×5
 - Small matrix multiplication
 -





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                    and make out the partners have
```



- GPU library has the same functions, suffixed with device
- Two different flavours: OpenMP or OpenACC
- Possibility to use CPU and GPU library together in the same code
- In early development, not fully integrated to our codes yet (work in progress)
- Although the kernels are fast on Nvidia GPUs, GPU acceleration is not clear because of data transfer
 - Maybe efficient on next generation of hardware
- On GPU, brute-force CuBLAS DGEMM is faster than sparse AO-MO transformation. Energy efficiency?



GPU: major difficulties with OpenMP

- Tensor core instructions are not generated in OpenMP kernels =⇒≤ 50% peak DP
- Conflict between OpenMP runtime of the code and of QMCkl-GPU =⇒
 - Need to compile the code with GPU compiler (Nvfortran)
 - May not compile, or with low CPU efficiency
 - Our solution: decouple QMCkl-CPU and QMCkl-GPU and recover CPU performance with QMCkl-CPU
- RocBLAS ~ CuBLAS, but some OpenMP kernels have 10× lower performance on AMD GPUs than Nvidia (under investigation...)
- Unreliable software stack: =⇒ Compared to CPU, very inefficient in human resources
- Open Question:
 - Should we have opted instead for vendor-specific implementations? (Cuda, HIP)



```
1  $ tar -zxvf qmckl.tar.gz
2  $ cd qmckl
3  $ ./configure --enable-hpc
4  $ make -j 32
5  $ make check
6  $ make install
```

- QMCkl has been used in
 - C/C++
 - Fortran
 - Python
 - Julia
 - Rust

- Very few dependencies:
 - BLAS/Lapack (CPU)
 - TREXIO (optional) with HDF5 (optional)
- BSD license: very permissive. You can distribute the tar. gz with your code
- Hosted on GitHub:

https://github.com/trex-coe/qmckl



Integration into TREX codes



- Single-core benchmark: C₆₀, Hartree-Fock/cc-pVQZ/ECP(BFD)
 - Time for a single MC step (all-electrons)
 - 4140 AOs, 120 MOs, 240 electrons

CPU	Compiler	QMCkl	milliseconds	Speedup
Intel(R) Core(TM) i7	ifort/mkl	-	24.58	
(8-core Laptop, 2.8GHz)	ifort/mkl	gcc12	24.06	1.02x
	ifort/mkl	icx	23.85	1.03x
	gfortran/openblas	-	30.58	
	gfortran/openblas	gcc12	26.04	1.17x
ARM Neoverse V1	gfortran/armpl	-	41.24	
(80 cores, 3GHz)	gfortran/armpl	gcc12	31.91	1.29x



- Single-core benchmark: C₆₀, Hartree-Fock/cc-pVXZ/ECP(BFD)
 - Short VMC run
 - 4140 AOs, 120 MOs, 240 electrons

Basis	# AOs	Compiler	QMCkl	seconds	Speedup
cc-pVDZ	840	ifort/mkl	-	315.45	
			gcc12	218.29	1.45x
			icx	212.35	1.49x
cc-pVTZ	2040	ifort/mkl	-	565.67	
			gcc12	287.32	1.97x
			icx	271.68	2.08x
cc-pVQZ	4140	ifort/mkl	-	993.42	
			gcc12	462.74	2.15x
			icx	441.32	2.25x



Other possible applications beyond accelerating QMC

- Reproducibility of QMC calculations (Jastrow factors)
- 3D visualization software:
 - AO or MO visualization
 - Interpretative methods like AIM or ELF
- Numerical integration
 - Computation of density grids for DFT with gradients
 - Jastrow factor in transcorrelated methods (Quantum Package)
- Teaching QMC algorithms in Jupyter notebooks
- Implementation of QMC methods in traditional quantum chemistry software



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Example: Evaluate MOs on a grid

```
import qmckl
import numpy as np
def main (trexio filename):
  context = qmckl.context create()
                                                     # Create a OMCkl context
  qmckl. trexio read(context, trexio filename)
                                                      # Read the TREXIO file into the context
  nucl num = qmckl.get nucleus num(context)
                                                                # Get the number of nuclei
  nucl coord = qmckl.get nucleus coord(context, 'N', nucl num*3) # Get the nuclear coordinates
  nucl_coord = np.reshape(nucl_coord, (3, nucl num))
  mo num
         = qmckl.get mo basis mo num(context)
                                                                # Get the number of MOs
  point = setup grid points (nucl coord)
  point num = len(point)
  qmckl.set point(context, 'N', point num, np.reshape(point, (point num*3))) # Give points to QMCkl
  mo value = qmckl.get mo basis mo value(context, point num*mo num) # Get the values of the MOs
  qmckl.context destroy(context)
                                                   # Free QMCkl resources
```





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