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PEDAGOGICAL CONTENT KNOWLEDGE OF TUTORS: A PREDICTOR OF PRE-SERVICE TEACHERS' MATHEMATICS LEARNING ACHIEVEMENT

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Abstract

The study investigates if tutors' pedagogical content knowledge (PCK) is a predictor of pre-service teachers' mathematics learning performance in some selected colleges of education in Northern Ghana. The design employs a cross-sectional survey design. A questionnaire was used to gather data for both pre-service-teachers and tutors for the study including the determination of their validity and reliability. This is followed by the use of descriptive statistics for the analysis. Outcome of the findings revealed that tutors' PCK has a strong influence on pre-service teachers' mathematics performance. The findings also showed that knowledge of learners and learning is a PCK component of tutors. Similarly, it emerged that student teachers have challenges in following instructions when learning mathematics. It further showed majority of the tutors did not adopt appropriate measures to address the difficulties pre-service teachers encounter while learning mathematics. It is therefore recommended that tutors in the colleges of education update their knowledge of the various PCK components so as to improve their classroom and professional practices. Management of the CoE should ensure the institutional enforcement of policies that improve learning performance in mathematics.

Keywords: *PCK, pre-service teachers, Learning, performance, Non-Science COEs,*

1. Introduction

The impact of the teacher is vital if any instructional programme is to be successful and stand the test of time. Teachers are the agents responsible for ensuring the success of such programmes. However, the behaviour of the teacher in the classroom is an important determiner of their professionalism, which is classified into teaching skills and subject-specific knowledge. The teacher's performance in these two broad areas of their professional traits is the driving force that aids their appraisal.

Alexander (1992) posits that numerous studies have revealed that a teacher's knowledge is a portent force in teaching and learning, but It remains pervasive and individualistic, which is dynamic in nature. At the moment, lots of debates rage on over the kind of knowledge teachers should possess to be successful in the classroom. This is because many believe that successful teachers' specialised knowledge for teaching is drawn from the learners, but there has been a controversy on how this knowledge is specified and measured, thereby making it more elusive.

Mewborn (2003) takes the view that the numerous studies that have been undertaken are an attempt to identify the relationships among teachers' mathematics knowledge, instructional practices, and student learning. According to Tchoshanov, Quinones and

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Shakirovo (2017), research on teacher knowledge was initiated by the work of Shulman (1986) and it focused on teacher knowledge as a major predictor of students' learning performance.

Danisman and Tanisli (2017) outline the kinds of knowledge teachers must have to demonstrate effective teaching and ease pre-service teachers' difficulties. According to Dasnisman and Tanisli, Shulman described this knowledge as pedagogic content knowledge (PCK). They added that Shulman explains PCK as a combination of subject matter knowledge and pedagogy that embraces everything required to handle a subject or concept in ways that improve pre-service teachers' learning achievement.

To Maher, Muir, and Chick (2006), "effective mathematics teaching requires knowledge of mathematical content, knowledge of pre-service teachers' thinking, and knowledge of how to represent content so that it can make sense to others" (p.1). Maher, Muir, and Chick explained that PCK is how content is translated from the teachers' knowledge into instructional content. Shulman describes this pedagogical content knowledge as a combination of subject matter knowledge and pedagogy that embraces everything required to handle a subject or concept in ways that improve learners' achievements.

Many other scholars have equally argued about the components of teaching knowledge possessed by teachers and have made many suggestions regarding this (Danisman & Tanisli, 2017). Danisman and Tanisli (2017) outlined a six-category framework for effective teachers involving general liberal education, personal performance, subject matter, general pedagogic knowledge, subject-matter-specific pedagogical knowledge, and the foundation of the teaching profession. According to Grossman, PCK consists of subject matter knowledge, knowledge of pre-service teachers, curriculum knowledge, and knowledge of instructional strategies. Smith and Neale (1989) classify pedagogical content knowledge into four components: knowledge of teaching strategy, knowledge of pre-service teachers' knowledge of curriculum materials, and knowledge of shaping and content elaboration.

An, Kulm, and Wu (2004) and Hill, Ball, and Chilling (2008) had similar components: student knowledge, curriculum knowledge, subject matter knowledge, and knowledge of pedagogy. Finally, Hashweh (2005) proposed a very detailed model: learners' knowledge, knowledge of instructional strategies, content knowledge, knowledge of purpose, curriculum knowledge, and resources, knowledge of measurement and assessment, and knowledge of context and pedagogy.

From the various models proposed by the authors, knowledge of pre-service teachers occurred in almost all of the models, since methods of teaching and teaching strategies are important for pre-service teachers' academic achievement. Hill and Chin (2018) pointed out that for knowledge of pre-service teachers to be considered an important characteristic of teachers, three types of teacher knowledge should be considered: the knowledge that is a representation of a stable characteristic that has a meaningful difference; also, that knowledge should be related to a similar kind of knowledge which shows instructional quality in expected ways; and finally, the knowledge gives a forecast of learning achievement.

In the absence of such rigorous and indisputable evidence, there is little assurance that teacher knowledge of pre-service teachers, as theorised by Shulman and others, will assist teachers to easily execute both present and long-term instructional decision-making, as hypothesised and that knowledge of pre-service teachers is worth developing in teacher preparation and assessment at licensure (Hill & Chin, 2018).

It is well known that the majority of Ghanaian pre-service teachers achieve extremely poor results in mathematics, including pre-service teachers in teacher training institutions. Many believe that the issue of pre-service teachers' learning achievement and difficulties are influenced by their teachers' PCK in mathematics, and have proposed that improving teachers' knowledge would improve pre-service teachers' achievement in mathematics (Panourana, Hodgen, & Pillay, 2015).

A critical study of the TIMSS 2007 report in which six African countries participated indicates that Ghana's overall performance of Junior High School two (JHS 2) students' mathematics test results was poor and was significantly below the international benchmark (Mereku, Ghartey, & Anamoah-Mensah, 2008). According to Mereku et al. (2008), Ghana occupied the 44th position out of the forty-nine countries that participated. They further asserted that on the international benchmark for mathematics, geometry was one of the subject areas tested in mathematics. The TIMSS report indicates that Ghanaian pre-service teachers are very weak in using geometric facts and theorems in mathematics. Even though the teaching and learning of mathematics are of considerable international interest, there is little research in the domain to improve pre-service teachers' learning (Mammana & Villani, 1998).

Learning achievement in mathematics as assessed by TIMSS 2011 has been below average (below the international benchmark of 400) among African students, including Ghana (Foy, Arora, & Stanco, 2013). Mullis, Martin, Foy, and Arora (2012) also reported that for the TIMSS 2011 study on the mathematics achievement of 4th and 8th graders, it examined whether learners, teachers, or the school factor was responsible for the poor achievement of students in mathematics. The study confirmed that the education system in Ghana is not different from other education systems where Ghanaian pre-service teachers were compared.

However, like in the TIMSS 2007 report, the result indicates that pre-service teachers' poor achievement of Ghanaian pre-service teachers in mathematics was partly attributed to inadequate teacher preparation. This poor achievement was blamed on the school factor. Thus, teachers emphasise lower-order thinking skills rather than higher-order skills of thinking. This means that improved mathematics learning performance contributes to achieving the SDG 4.3.

Since teachers of the basic schools are trained by the colleges of education where this study was conducted, it is believed that the study's findings will bring out desirable results that can be replicated. The study therefore sought to examine if tutors' PCK is a predictor of the pre-service teachers' mathematics learning achievement in some selected colleges of education in Northern Ghana.

1.1 Statement of the Problem

Several research studies have examined issues related to teacher knowledge and pre-service teachers' learning achievement from early grade level to higher levels of education, but little research focuses on teachers and their classroom practices (Bosu, 2010; Danisman & Tanisli, 2017). There are still numerous problems in the classroom, yet there are limited studies in this regard.

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Considering the fact that teachers need to be "competent and effective in their subject matter," many teachers do not have the adequate pedagogical knowledge required to teach for pre-service teachers' long-term achievement (Gonzalez & Maxwell, 2018). According to Tchoshanov (2011), teachers lack essential knowledge for teaching mathematics based on studies conducted throughout America. This problem has a negative impact on learners' achievement because teacher ineffectiveness in their content areas hinders pre-service teachers' learning (Gonzalez & Maxwell, 2018). The major cause of this problem is teachers' inadequate pedagogical content knowledge.

Bosu (2010), acknowledging the work of Shulman (1987), opines that modern standards of instruction for teaching all learners require deep reflection by teachers, knowledge of the subject matter, and pedagogy, which enables teachers to develop concept mapping or mind mapping of concepts for learners to interconnect ideas with each other to overcome misconceptions bordering them. Golemark (1994) reports that the ability of the teachers to deliver instructions in this manner means the teacher must understand pre-service teachers' learning styles and adopt appropriate pedagogic strategies to support learning achievement.

A study by Kandjinga (2018) in Namibia has shown that very little information is available to support mathematics teachers' subject matter content knowledge and pedagogical content knowledge of mathematics. According to Kandjinga (2018), up till now, the only projects carried out are Teacher Education Development Studies in Mathematics (TEDS-M) and Cognitively Activating (COACTIV) conducted by Tatto et al. (2012) and Kraus et al. (2008b) respectively. This situation is not different from what pertains in Ghana as there are few studies available on the pedagogical content knowledge of teachers as a predictor of pre-service teachers' mathematics learning achievement. It is the result of this low pedagogical content knowledge of teachers in Ghana that Transforming Teacher Education and Learning (T-TEL) through the Ministry of Education (2015) organised a five-year professional development programme for tutors in colleges of education. The aim of the programme was to improve pre-service teachers' learning outcomes and update tutors' PCK in colleges. Despite this professional programme, tutors' PCK as a predictor of pre-service teachers' learning performance in the non-science colleges of education still needs further research.

Similarly, literature has shown that few studies that have been conducted only investigated secondary school mathematics teachers' subject matter knowledge and pedagogical content knowledge, as many of the studies targeted primary school teachers (Kandjinga, 2018). Contrarily, these studies focused either on a specific topic or were conducted in a particular school. These particular studies have indicated that most mathematics teachers have demonstrated insufficient pedagogical content knowledge in their teaching. Hurell (2013) waded into this debate, adding that most teachers have inadequate subject matter knowledge, which makes them less confident during lesson delivery in the mathematics classroom.

In Ghana, few studies have been conducted regarding the pedagogical content knowledge of teachers as a predictor of pre-service teachers' mathematics learning achievement. However, the few studies that have been conducted investigated the pedagogical content knowledge of teachers in other disciplines. These studies include: Amuah's (2021) pedagogical content knowledge of Religious and Moral Education Teachers in Komenda-Edina-Equafo-Abirem. Bosu's (2010) study on pedagogical content knowledge of Accounting Teachers in Senior High Schools in the Central Region of Ghana. These studies confirmed the fact that some mathematics and other subject teachers have very limited knowledge of pedagogical content knowledge, which is reflected in pre-service teachers' performance.

In Northern Ghana, no study has been conducted on the PCK of tutors, the influence of PCK on learning performance, and measures adopted to address mathematics learning challenges. This study, therefore, examined tutors' PCK, its influence on mathematics learning performance, challenges and measures adopted to address challenges of learning mathematics among tutors and pre-service teachers of selected non-science Colleges of Education.

The purpose of the study was to investigate the pedagogical content knowledge of tutors on pre-service teachers' mathematics learning performance among non-science colleges of education in Northern Ghana.
The objectives of this paper are to determine the pedagogical content knowledge of mathematics tutors in teaching as well as to examine the influence of tutors' PCK on pre-service teachers' mathematics learning.

The results of this study will be useful because it will add little knowledge to the literature on PCK in mathematics education. The study also intends to benefit pre-service teachers with modern pedagogical approaches to teaching, which will in the long run be replicated in their future instruction. It will be of great help to tutors on how to adopt a blended PCK strategy in teaching to achieve learning outcomes among student teachers.

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1.2 Research Questions

1. What pedagogical content knowledge do tutors have in teaching?
2. What influence does tutors' pedagogical content knowledge have on pre-service teachers' performance?

Hypotheses

The research hypothesis formulated was tested at a significance level of 0.05.

H_O1: Pedagogical content knowledge of tutors does not significantly predict pre-service teachers' mathematics learning performance.

H_A1: Pedagogical content knowledge of tutors significantly predicts pre-service teachers' mathematics learning performance.

2. REVIEW OF RELATED LITERATURE

2.1. Components of Teachers' Pedagogical Content Knowledge

Teacher education in mathematics is noted to be embedded with two main tensions that negatively affect pre-service teachers' attempts to learn to teach mathematics for understanding (Wilmot, 2008). The first of these tensions is attributed to pre-service

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teachers' weak understanding of the subject matter, and the second is the duration of the pedagogical courses in teacher education progrommes (Wilmot, 2008).

According to Darling-Hammond (2000), the key factor to pre-service teachers' academic success in today's classrooms solely relies on the pedagogical content knowledge. Because of this, the kind of knowledge desirable for mathematics success has interested researchers, and many studies are being carried out to ascertain the effectiveness of teachers' pedagogical content knowledge. The different kinds of the pedagogical content knowledge possess by the teachers are considered as follow:

2.2. Subject Matter/Content Knowledge

According to Danisman and Tanisli (2017), their view of subject matter/content knowledge is the basic teaching knowledge and the essential knowledge base for pre-service teachers' attainments in mathematics. Sidhu et al. (2011) succinctly asserts that the content knowledge of a teacher is what they know about what they teach, while pedagogical knowledge is the teacher's knowledge about the methodology of teaching. Teachers' deep content knowledge of mathematics leads to their effectiveness in teaching. A teacher who is knowledgeable about the content enables them to have the requisite knowledge base for teaching mathematics.

2.3. Curricular Knowledge

Curricular knowledge provides the teacher with knowledge about how subjects are developed, the concepts and the relationships between the topics. With curriculum knowledge, teachers need to have knowledge of the correlation that exists between content knowledge and curricular knowledge and how they relate to each other. According to Shulman's model, having curricular knowledge provides teachers with the means and strategies to carry out the work expected of them. It is not enough for teachers to know how subjects and concepts are developed and how the topics relate to each other and to other subjects; there is a need to know how the scope of the topics in the curriculum in the current and subsequent years (Danisman & Tanisli, 2017). The authors added that curriculum knowledge is divided into two categories, such as goals and objectives of the curriculum and special programme knowledge.

Danisman and Tanisli (2017) further illustrated that knowledge of the curriculum is put into two types: horizontal curriculum knowledge and vertical curriculum knowledge of a particular subject. Horizontal curriculum knowledge is having knowledge about skills and concepts concerning a particular subject at the same level of learning, and vertical curriculum knowledge is the knowledge relating to skills and concepts in a particular subject across all levels of learning. It is not only content knowledge that teachers are required to possess, but they are equally required to have curriculum knowledge and its related aims and goals (Danisman & Tanisli, 2017).

2.4. Knowledge of Learners

According to Gonzalez and Maxwell (2018), knowledge of the learners is about "prior knowledge of specific topics," misconceptions about learners, and their learning difficulties. This means a teacher needs to know learners' prior knowledge, learners' learning difficulties in specific concepts, and manage the learning process effectively. Shulman (1986) reported that teachers need to ask questions about how difficult or easy it is to learn some concepts based on the knowledge of learners of varied backgrounds who come into the learning environment.

Knowledge of learners explains the information that teachers know about learners' learning procedures and their knowledge of concepts, processes, styles, difficulties, and misunderstandings about learning (Danisman & Tanisli, 2017). This means that knowledge of learners is seen as the potential difficulties that learners may encounter in the classroom while learning a specific subject.

2.5. Knowledge of Teaching Methods and Strategies

This is a way of transferring content knowledge in mathematics in a manner that learners may easily grasp. This means teachers require the appropriate and best strategies and knowledge representation to foster learning.

According to Shulman (1987), having ideas about methods and strategies of teaching can transform learning in the sense that it is the presentation of subject matter in a way that can be understood. Shulman further broke down the knowledge of teaching methods and strategies into sub-teams and concepts, stating that knowing what enhances learning in a discipline or difficulties in it, representation, illustrations, and verbal expositions will bring meaningful learning of a concept while doing away with misconceptions. The appropriateness of activities and effective use of resources, including the use of verbal expositions, citing instances with illustrations and the use of analogies that pace up learners' comprehension of concepts, are all part of instructional strategies (Dasnisman & Tanisli, 2017).

2.6. Knowledge of Educational Goals and Philosophical Basis

The fifth component of Shulman's model is knowledge of educational goals and philosophical basis. This component means that teachers need to be familiar with educational principles and social expectations that they need to sort out as educators. Philosophy is a vital tool for the development of all kinds of curriculum. Shah (2019) posits that philosophical bases determine the objectives, content, classroom delivery, and evaluation process of curriculum.

The result is that without philosophy, educators will have no direction as to what and how to organize and implement whatever is to be achieved within the school system. It is true that philosophy influences and to some extent outlines our educational decisions, choices and alternatives but that cannot be achieved without educational goals and objectives of the curriculum. Philosophy provides the starting point in the development of curriculum as it reflects on the total needs of the children, environment, schools and the society (Shah, 2019). One can infer from this that a source of development of curriculum is philosophy.

For a curriculum to developed, it involves different aspects. Some of these aspects include the objectives of learning, sources of the contents/subject matter, nature of pedagogical practices, characteristics of the leaner, assessment procedures among others (Shah, 2019).

2.7. Knowledge of Educational Context

The sixth category, knowledge of the educational context, comprises the schools, classrooms, communities, and culture. Knowledge of context being a component of PCK recognises a situation where personal beliefs and educational programmes of teachers appear to be based mainly on their previous experience as learners (Bosu, 2010). According to Bosu (2010), there are very good relationships between teachers' images of teaching and their classroom practices. This means a change in image of the teacher

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can often be linked to changes in behaviour of both teacher and pupils. Teachers need to be innovative in using a range of specific management techniques, and be knowledgeable about the circumstances in which a given technique should best be used to promote student learning. The context for practice influences the knowledge about classroom organization and management that is most relevant for teachers to use. A major issue in the education of teachers is problem of how to fuse theoretical knowledge with practical knowledge for the development of functioning professionals.

Jones and Moreland (2015) reported that teachers must understand the appropriate subject-related contexts for their learners. Jones and Moreland citing an instance stated that "in our classroom research, we have found that if problems are too openly defined, or there is limited teacher and student understanding of the context, learners may lose their way" (p. 8). A serious problem area is where teachers are overly constrained with tasks, as teacher over-specification may lead to lack of student ownership and control of classroom activities.

It is appropriate that learners should be involved in setting mathematics classroom learning agenda since this makes class tasks more meaningful and buy-in from the learners. This does not mean that anything goes. Teachers need to be cautious in their decision taking. As result teachers must therefore involve children in problem-solving tasks where issues are made clear to them and presented to them in a way in which they can relate. Thus, to bring relevance to an activity means to teach in context (Jones & Moreland, 2003).

2.8. The Pedagogical Content Knowledge Model

Pedagogical content knowledge is a combination of subject content knowledge and pedagogy as a teaching strategy for classroom management and organization. The concept of PCK allows teachers to be aware of pre-service teachers' different learning needs so as to address them appropriately. Teachers who have experience seem to have formed a conceptual framework in which knowledge and beliefs about mathematics, the subject matter, teaching and learning, and learners are related in a coherent manner, while their teaching behaviour seems consistent with this framework (Brickhouse, 1990). Most teachers are not able to forecast what the learners already know, the kinds of questions pre-service teacher find difficult, how to go about instructions and what questions they might ask.

Pre-service teachers, therefore, encounter difficulties tailoring representations to meet the needs of learners (Zembal, Starr & Krajcik, 1999). Most pre-service teachers encounter misconceptions in concepts and principles, which can be difficult to learn because misconceptions form a strong foundation over subject knowledge. The misconceptions are dealt with so that learning becomes sensible and coherent and has utility for pre-service teachers in everyday life. Although teachers may be aware of pre-service teachers' challenges, they frequently overlook their misconceptions or struggle to find ways to respond to them due to a lack of knowledge required to assist the pre-service teachers in overcoming those challenges.

Apart from PCK of materials and activities that interact with knowledge of pre-service teachers' "naive ideas," teachers need strategies that they will teach that will promote alternatives to those ideas (Cochran-Smith, 1999). Teachers' knowledge about students' ideas in real situations and how to support them construct a complex understanding of the nature of the subject that they are learning is an additional kind of PCK for teaching the subject. An explicit pictorial representation of PCK which is the intersection between pedagogy and content is as delineated by Tsafe [2013] in Figure 1 illustrating PCK and some of its different components.

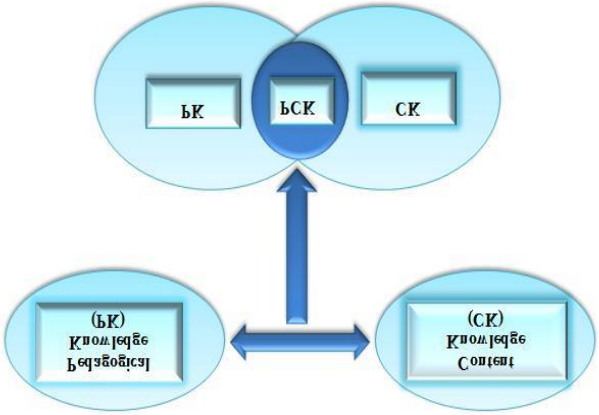

Figure 1. A Pictorial Representation of Pedagogical Content Knowledge

2.8.1. Influence of Pedagogical Content Knowledge of Tutors on the Performance of Pre-service Teachers in Mathematics

Hill, Rowan, and Ball (2005) have examined the relationship between teacher characteristics, behaviours, and knowledge and student achievement. According to Hill et al. (2005) studies in classroom level educational research, attempts to predict student achievement based on teacher characteristics had their roots in what was referred to as process-product literature in teaching a large body of research studies describing relationships between teacher behaviours and student achievement. Affective factors such as teacher appearance and enthusiasm predict student achievement, but going beyond these, researchers in this tradition are of the view that what teachers do in their classroom may impact student performance.

Hill, et al. (2005) observed that by the late 70s, scholars did gather evidence that teacher behaviours did impact on pre-service teachers' achievement gains. They added that giving emphasis in class time to active academic instruction rather than classroom management, student choice/game time, personal adjustment, or non-academic subjects was found to be one consistent correlate of student achievement gains; so was presenting materials in a structured format via advance organizers, making salient linkages explicit, and calling attention to main ideas.

The impact of teachers' pedagogic content knowledge on their students' gains also depends on their efficacy. Teacher efficacy is their ability to promote pre-service teachers' learning performance (Gonzalez & Maxwell, 2018). It is the extent to which teachers believe that they can promote pre-service teachers' learning, even though conditions may be difficult (Gonzalez & Maxwell, 2018). They added that if a teacher has high confidence in their ability, pre-service teachers' learning and performance can be successful. According to Gonzalez and Maxwell (2018), experience and knowledge of teachers are vital to engaging pre-service teachers in meaningful and effective mathematical practises in the classroom in order that a deep understanding of mathematics is achieved.

This paper attempts to find out if a mathematics teacher's effectiveness lies in their education and professional development

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practises in the classroom, for which this paper seeks to find out teachers' PCK as a factor of pre-service teachers' achievements. Many studies have been conducted to find out if teachers' knowledge is a predictor of pre-service teachers' learning gain. While some studies used proxy measures such as state certification, number of mathematics education courses taken, and years of teaching mathematics, they found that such proxy measures are neither good measures of teachers' knowledge nor good predictors of learners' attainment (Pournara, Hodgen, & Pillay, 2015). They, however, agreed that there is some evidence of their predictive power in secondary school mathematics. For instance, Darling-Hammond (2000) and Brewer (2000), cited by Pournara, Hodgen & Pillay (2015), found that there is a positive relationship between state certification and learning gains in the United States (US). Another study found that there was a positive relationship between the number of mathematics courses taken and pre-service teachers' achievement, even though the impacts were very small (Pournara, Hodgen & Pillay, 2015).

Several attempts have been made to use a more direct measure of teacher knowledge, which has taken on different dimensions. Some studies have tested teachers' knowledge of the same or similar level of content area as their learners. For instance, Pournara, Hodgen and Pillay (2015) cite in Harbison and Hanushek (1992), who administered the same test to grade four learners and their teachers in rural Brazil. According to Belize, Mullen, et al. (1996), teachers' scores on the national primary school-leaving examination for mathematics were good predictors of their Grade three learners' mathematics scores. In the United States and Germany, large research studies have developed measures that attempt to separate different components of teacher knowledge (Hill et al., 2008; Krauss et al., 2008).

In the Study of Instructional Improvement, Hill et al. (2005) found that the mathematical knowledge-for-teaching of Grade One and Grade Three teachers was a stronger predictor of learner attainment than proxy measures, such as the number of courses taken in mathematics or mathematics methodology, or years of teaching experience, or average daily length of mathematics lessons. Charalambous (2011) wrote: the argument that teachers' profound knowledge of mathematics—manifested in their breadth and depth of knowledge—matters for teaching mathematics well is not new.
In fact, centuries ago, Aristotle argued that "teaching is the highest form of understanding" to emphasise the depth of knowledge necessary for teaching. Similarly, a Chinese proverb holds that if teachers want to give student a cup of water, they need to have a bucket of water of their own (Kulm & Wu, 2004), an assertion that points to the breadth of knowledge teachers must possess. Although few would refute that teachers' profound understanding of mathematics constitutes a critical resource for teaching mathematics effectively, the association between tutors' knowledge and the quality of their instruction, and especially the association between tutors' knowledge and the establishment of mathematically rich and challenging environments, largely remains undisputed (Charalambous, 2011).

In fact, many studies have explicitly or more tacitly attributed certain deficiencies in teachers' instruction to the teachers' impoverished knowledge. A longitudinal study conducted at Michigan State University (Schram, Wilcox, Lanier, & Lappan, 2008) investigated the nature and extent of the changes in pre-service elementary teachers' beliefs and knowledge about mathematics and teaching and learning mathematics as a result of a series of innovative mathematics content courses, a mathematics methods course, and a curriculum seminar in the USA. The data was collected from 24 pre-service teachers during their two-year teacher preparation programme and their first year of teaching. The pre-service teachers took three content courses that were specifically oriented to exploring ideas about numbers, geometry, probability, and statistics as well as the relationships between them. Schram et al. noted that at the end of the courses, the pre-service teachers' views about mathematics had changed; initially, they thought that mathematics was a meaningless series of symbols and rules, but by the end of the courses, they appreciated the value of conceptual understanding of mathematics.

Furthermore, they liked the way the instructor set up the learning environment. However, they were unable to transfer what they experienced in the courses to their instruction. Some of them still held onto their traditional view of mathematics and emphasised procedural knowledge rather than conceptual understanding when teaching mathematical facts and procedures. It was recommended that mathematics should be taught with current innovation. Ball, Thames, and Phelps (2008) investigated content knowledge for teaching at Michigan University, USA. The study indicated that teacher educators should provide opportunities for pre-service teachers to evaluate their understanding and knowledge of teaching and learning maths during their teacher preparation programs.

Ozden (2008) investigated the effect of the amount and quality of pedagogical content knowledge on the quality of understanding of pre-service teachers and also came to the same conclusion that they are inseparable. Penso (2010) examines the pedagogical content knowledge of student teachers of biology during their teaching practise in a school in Haifa, Israel. Penso's study gives credence to research-based findings that PCK knowledge of tutors is key to effective knowledge acquisition by pre-service teachers, but goes further to recommend the need to increase the teacher educators' awareness of the important role of didactic processes aimed at exposing the student teachers to their pupils' learning difficulties and helping them deal with them effectively. Maryani and Martanigsih (2015) examined the correlation between a teacher's pedagogical content knowledge and a student's motivation in a primary school in Yogyakarta, Indonesia. The results showed there is a relationship between the teacher's pedagogical content knowledge and pre-service teachers' motivation for learning.

Lucenario, Yangco, Punzalan, and Espinosa (2016) investigated the effectiveness of Pedagogical Content Knowledge-Guided Lesson Study (PCKLS) as an intervention to develop PCK competencies among teachers and consequently enhance pre-service teachers' performance in terms of conceptual understanding and problem-solving skills in Chemistry in the Philippines. The study recommends that this intervention be used across.

Chemistry topics are covered in other science classes as well, including Mathematics, Biology, Earth and Environmental Science, and Physics. All this and much other research only highlights the correlation between the content knowledge of the teacher and the final product (the knowledge passed on to the student: Hashim, Saili and Noh (2015)), hence the need to ensure tutors in our various teacher training institutions have a full grasp of their pedagogical content knowledge to ensure that pre-service teachers after their training can go back and in turn impart the right knowledge to their pre-service teachers.

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3. Methods and the Study Area

The study adopts positivists research paradigm. Positivism holds the assumption of the absolutism of an objective reality, which can be measured using set variables that can be used to establish cause-and-effect relationships (Creswell & Creswell, 2018). Therefore, the positivist philosophical paradigm inspires the application of a purely quantitative approach to the study. The quantitative approach is the most appropriate as it offers the opportunity to use numerical data to perform both descriptive and inferential statistical analysis, thus assessing the pedagogical content knowledge of tutors as a predictor of pre-service teachers' mathematics learning performance.

This study employed a cross-sectional design. The selected non-science Colleges of Education was primarily obtained and described phenomena such as pedagogical content knowledge and pre-service mathematics learning achievement among tutors and students of the selected non-science Colleges of Education. A cross-sectional design prospectively allows the researcher to sample a cross-section of tutors of non-science CoE while assessing a cross-section of variables such as pedagogical content knowledge and mathematics learning achievement simultaneously. Cross-sectional designs are typically economical, easy, and rapid to conduct, and can potentially allow measurement of exposures to many risk factors (Sedgwick, 2014). Furthermore, cross-sectional designs are not liable to measurement bias arising from loss to follow-up, as data collection is a one-time exercise (Sedgwick, 2014).

However, cross-sectional designs, like other research designs, are not without limitations. First of all, cross-sectional designs, although they can be used to test associations, are not suitable for establishing causal relationships because of a lack of temporal dimension (Thelle & Laake, 2015). However, this temporal limitation of cross-sectional designs can be addressed by retrospective assessments, though subject to recall bias (Thelle & Laake, 2015).

The study was conducted in the Northern sector of Ghana, specifically among non-science CoE. There are five non-science CoE in Northern Ghana, namely, McCoy, Tumu, Gbewaa, Gambaga and Evangelical Presbyterian Colleges of Education. The study population consists of all tutors and pre-service teachers of CoE in Northern Ghana. However, tutors and level 300 pre-service teachers of the five non-science CoE in Northern Ghana formed the target population. The five non-science CoE were purposively selected because they all offered general programmes and hence were considered non-science CoE, and were not mathematics biased or oriented.

3.1. Sample and sampling Procedure

Sample size determination in quantitative research remains crucial, particularly for the purposes of achieving adequate power and hence representativeness of the study. The sample size for this study was determined using Gay and Diehl's (1992) sample size determination approach. Gay and Diehl (1992) advanced that the sample size of a heterogeneous population can be determined by sampling 20% of the sub-populations as small as 500 and below and 10% of sub-populations as large as 1000.

Therefore, for the tutors; $n = 30 \times 0.20 = 6$
For the pre-service teachers, $n = 1241 \times 0.1 \approx 124$

Assuming 20% non-response, $n = 6 \times 1.2 \approx 10$ for tutors (to the nearest tenths) and $n = 124 \times 1.2 \approx 150$ for the pre-service teachers. Therefore, 10 mathematics tutors and 150 level 300 pre-service teachers were selected to participate in the study.

The sampling technique is the specific sampling method deployed in the sampling process. Using a convenience sampling technique, three of the five non-science CoE (Gbewaa, Tumu, and Gambaga) were initially selected. Convenient sampling of the three CoE was necessary because of proximity and temporal human and financial resource constraints. Guided by the sample size calculated for each of the tutors and pre-service mathematics teachers, a purposive sampling technique was used to select the study participants who were either mathematics tutors or level 300 student teachers. The purposive sampling technique was appropriate because the researcher had an interest in specific characteristics of the target population. The target population was from non-science CoEs that offered mathematics only as a core subject, and therefore were suitable to offer responses consistent with the scope and objectives of the study.

3.2. Ensuring Reliability and Validity

The validity and reliability of the research questionnaire were tested. The validity measures the degree to which an instrument is able to achieve the purpose for which it was designed (Creswell & Creswell, 2018). Reliability of a research instrument, however, measures the consistency or repeatability of the responses to the scales of the questionnaire (Creswell & Creswell, 2018).

For validity and reliability of the questionnaire, it was pre-tested on 2 mathematics tutors and 20 pre-service teachers in different CoE, who had similar socio-demographic characteristics as the sampled population. The selected CoEs for the pilot study included St. John Boscos and Bagabaga. The justification for the pilot test was to unpack ambiguity, poorly worded items, unclear statements and removing double worded questions. Face and content validity of the questionnaire were assessed and improved using responses and suggestions of the pre-test group and the researcher's academic supervisor.

The questionnaire was, therefore, revised to suit the purposes of the study. Data from the pre-test was used to determine the reliability (internal consistency) of the research instrument using Cronbach's alpha (α) coefficient test. The overall of the research instrument for the tutors was 0.615 and that of the pre-service teachers was 0.874. The α for the scales of PCK of tutors, and influence of PCK on learning performance were 0.702 and 0.784 respectively. These values were considered acceptable (De-Vellis, 1991).

3.3. Ethical Consideration and Data collection Procedure

A letter of introduction was taken to the principals of the colleges in the study. This afforded the researcher the opportunity to gain access, assistance, and collaboration from the respondents for the administration of the questionnaire. Written informed consent was obtained from the study participants before the questionnaire was administered. The survey was conducted in line with the declarations of Helsinki.

A total of 10 questionnaires for tutors and 150 questionnaires for pre-service teachers were sent to the field. Informed written consent was obtained before the questionnaire was distributed. The questionnaire was administered by the researcher to the respondents in collaboration with the heads of the mathematics departments of the respective schools. Collaboration with the heads

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of departments was done to ensure the high accessibility of the study respondents and response rate.

Administration of the questionnaire was done at different times in each of the colleges selected for the study. The questionnaire was distributed to the eligible tutors and pre-service teachers in the selected colleges to be completed and returned the following week. In one of the colleges visited, the researcher had to wait to collect the completed questionnaires personally from respondents who had finished with their responses. However, respondents who were unable to complete the questionnaire on the same day of data collection were directed to submit them to the heads of department for onward delivery to the researcher. The administration and collection of the returned questionnaire lasted eight weeks.

3.4. Data Analysis

The data was cleaned and coded for analysis. The coding of the questionnaire was done based on the scoring keys: strongly disagree (SD = 1), disagree (D = 2), neutral (N = 3), agree (A = 4) and strongly agree (SA = 5). The Statistical Package for Social Sciences (SPSS) software package version 25 was used to analyse the data. Descriptive statistics tools such as frequencies, percentages, minimum, maximum, arithmetic mean, and standard deviation were used to analyse the socio-demographic characteristics, pedagogical content knowledge, and its influence on pre-service teachers' mathematics learning. The descriptive statistics presented a baseline description of the data (Bosu, 2010).
The Summary statistics were computed for each of the scales or variables (PCK of tutors, influence of PCK on mathematics learning performance measured in the study. In interpreting the analysis of 5-point Likert scales for each of the scales of the research instrument, scores ranging from 1.00–1.80 indicated that the respondents strongly disagree (SD); 1.81-2.60 indicated that the respondents disagree (D); 2.61-3.40 indicated that the respondents were neutral (N); 3.41-4.20 indicated that the respondents agree (A) and 4.21–5.00 indicated that the respondents strongly agree (SA).

4. Results

4.1. Socio-Demographic Characteristics of the Respondents

The sociodemographic data of respondents include gender, age, academic qualification and duration of teaching. In the distribution of teacher-respondents by gender in the study, a total of ten (10) respondents were present to answer questions on the instrument. Out of this, 9 (90%) were males while 1(10%) was a female. The result indicated that males form the majority in the study. For the age grouping of respondents, the majority of the respondents five (5) constituting 50% represents 40-49 years while age range of 50-59 years represents 4(40%). However, the age group of 30-39 years had 1(10%). The next section describes academic qualification.

On academic qualification of tutor respondents, majority of the respondents 10(100%) were masters holders. None of the respondent holds first degree or PhD. The duration of teaching mathematics, indicated majority (40%) of the tutors have been teaching mathematics for 6–10 years, while the duration of teaching mathematics ranged from 1–5 years, 11–15 years, and 16–20 years, all represent 20%.

For the distribution of pre-service teacher-respondents by gender in the study, a total of 148 respondents were present to answer questions on the questionnaire. Out of this, 87 (58.8%) were males while 61(41.2%) were females. This indicates that males form the majority in the study. On the distribution by age, majority of the respondents, 141 constituting 95.3% represents 20-29 years age range while age range 30-39 years represents 7(4.7%). The next section presents pedagogical content knowledge of tutors.

4.2. Pedagogical Content Knowledge of Tutors

Table 4.7 displays the pedagogical content knowledge of the tutors. As shown in Table 4.7, the majority of the tutors agreed (3.41 ± 1.316) to add an effective combination of content and pedagogy for teaching and learning. Also, in terms of using suitable subject matter styles of presentation apt for the abilities and interests of the pre-service teachers, the majority of the tutors strongly disagreed (1.50 ± 0.527). For instructional strategies/techniques meeting the learning needs of pre-service teachers, the majority of the tutors were in strong disagreement (1.80 ± 0.632).

However, the majority of them were in agreement (3.43 ± 1.252) in terms of using different assessment strategies for assessing understanding and diagnosing learning performance problems. In terms of possessing characteristics for teaching and addressing complex learning problems, the most of the tutors were in strong disagreement (1.40 ± 0.516). The a number of the tutors agreed (3.60 ± 1.516) to being able to choose appropriate teaching methods for specific concepts. For the ability to create a purposeful learning environment during teaching, the majority of the tutors indicated strong disagreement (1.70 ± 0.483). Also, in terms of facilitating critical thinking and problem-solving strategies, the majority of the tutors strongly agreed (3.60 ± 1.521). Table 4.1, summarizes tutors' responses as shown

Table 4.1: Pedagogical Content Knowledge of the Mathematics Tutors

Variable	N	M	SD
I can choose appropriate teaching methods to teach a specific concept.	10	3.60	1.51
I can effectively combine content and pedagogy in teaching and learning process.	10	3.41	1.31
I can use different assessment strategies to assess pre-service teachers understanding and diagnose their learning performance.	10	3.43	1.25
My instructional strategies and techniques meet the learning needs of pre-service teachers.	10	1.80	0.63
I am able to establish a purposeful learning environment during teaching.	10	1.70	0.48
I am able to facilitate critical thinking and problem-solving strategies in student teachers by establishing connection with their life experience.	10	1.70	0.48
My subject matter presentation style suits the different abilities and interests of student teachers.	10	1.50	0.52
I have the needed teaching characteristics for teaching and addressing complex issues of pre-service teachers learning.	10	1.40	0.51

Source: Field Data, (2021)

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As shown in Table 4.7, the majority of the tutors strongly disagreed with adding an effective combination of content and pedagogy for teaching and learning. This finding contradicts the conclusions of researchers like Darling-Hammond (2000), who posits that the key to pre-service teachers' performance in mathematics is solely dependent on the pedagogical content knowledge of the tutor. Also, in terms of using suitable subject matter styles of presentation apt for the abilities and interests of the pre-service teachers, the majority of the tutors strongly disagreed. This research finding does not support the assertion by Danisman and Tanisli (2017), who opined that subject matter/content knowledge is the basic teaching knowledge and the essential knowledge base for pre-service teachers' attainments in mathematics.

Thus, it does not support Sidhu's et al. (2011) claim that teachers' deep content knowledge in mathematics leads to their effectiveness in teaching. However, the majority of them (respondents) showed neutrality in terms of using different assessment strategies for assessing understanding and diagnosing learning performance problems. According to Shulman (1987), having curricular knowledge provides teachers with the means and strategies to carry out the work expected of them, but a multi-faceted assessment process is the panacea to pre-service teachers' difficulty with mathematics. It is not enough for teachers to know how subjects and concepts are developed and how the topics relate to each other and to other subjects; there is a need to know the scope of the topics in the curriculum, but it is important to develop an effective methodology and ensure a holistic application of content and pedagogical content knowledge (Danisman & Tanisli, 2017).

4.3. Influence of Tutors' Pedagogical Content Knowledge on Pre-Service Teachers' Mathematics Learning

Table 4.8 shows the influence of tutors' pedagogical content knowledge on pre-service teachers' mathematics learning. As shown in Table 4.8, the majority of the pre-service teachers disagreed (2.08 ± 0.993) that their tutor's style of teaching impacts positively on learning outcomes. Also, majority of the pre-service teachers were neutral (2.76 ± 1.238) on whether their tutors' varied pedagogies enhance understanding of mathematics. In terms of tutors combining content and pedagogy, the majority of the pre-service teachers showed neutrality (2.97 ± 1.284).

The majority of them were neutral (3.08 ± 1.327) as to whether the use of methods by their tutor influenced quality learning. However, the majority of the pre-service teachers agreed (3.48 ± 1.322) that using the tutor's think pair-share strategies enhances learning of mathematics concepts. Regarding participation in group activities to enhance learning, the majority of them were neutral (3.14 ± 1.271). The majority indicated disagreement (2.58 ± 1.251) on whether participation in group presentations impacts their mathematics learning performance positively.

Table 4.8: Influence of Tutors' Pedagogical Content Knowledge on Pre-Service Teachers' Mathematics Learning

Variable	N	M	SD
My colleague and I do engage in think pair-share strategies used by the tutor in class to learn mathematics concepts.	148	3.48	1.32
When I participate in group activities during lessons I understand better.	148	3.14	1.27
My tutor's use of only methods influence quality learning in me.	148	3.08	1.32
My tutor's combination of content and pedagogical practices in his/her lessons make me achieve high in mathematics.	148	2.97	1.28
My tutor's varied pedagogies employ in lessons enhance my mathematics understanding.	148	2.76	1.23
When I participate in group presentation activities in class I learn better.	148	2.58	1.25
My tutor's style of teaching positively influences my learning outcome.	148	2.08	0.99

Source: Field Data, (2021)

Table 4.8 shows the influence of tutors' pedagogical content knowledge on pre-service teachers' mathematics learning. Hill et al. (2005) concluded from their research that a teacher's PCK did impact on pre-service teachers' achievement gains. A majority of the pre-service teachers disagree that their tutor's style of teaching impacts positively on the learning outcome. Also, the majority of the pre-service teachers were neutral on whether their tutors' varied pedagogies enhanced their understanding of mathematics. Many of the students polled strongly believed that using the tutor's think pair-share strategies to enhance learning of mathematics concepts is a far more effective strategy for learning mathematics.

Many students took a neutral stance when it came to encountering difficulties while studying independently, and they also downplayed the role of concentration loss in their inability to understand mathematics. However, the majority of pre-service teachers admit to having difficulty following directions. In terms of sequencing and completion of tasks, the majority of the pre-service teachers also agreed to having challenges. Furthermore, the majority of pre-service teachers agreed that they had difficulty understanding certain mathematical terms and concepts.

4.4. Discussion of Results

This study was designed to answer two thematic research questions on the study topic: pedagogical content knowledge of tutors: a predictor of pre-service teachers' mathematics learning performance. Data was therefore analyzed on: the socio-demographic characteristics of the respondents, pedagogical content knowledge of tutors, and influence of tutors' pedagogical content knowledge on students' mathematics learning.

Majority of the tutors strongly disagreed to adding effective combination of content and pedagogy for teaching and learning. This contradicts the conclusions of researchers like Darling-Hammond (2000) who posits that the key to students performance in mathematics is solely relies on the pedagogical content knowledge of the tutor. Also, in terms of using suitable subject matter style presentation apt for the abilities and interests of the students, majority of the tutors strongly agreed. This research finding does not support the assertion by Danisman and Tanisli (2017) who opined that subject matter/content knowledge is the basic teaching knowledge and the essential knowledge base for students' attainments in mathematics.

Thus, it does not also support Sidhu et al (2011) claim that teachers' deep content knowledge in mathematics leads to their effectiveness in teaching. However, majority of them (respondents)

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showed neutrality in terms of using different assessment strategies for assessing understanding and diagnosing learning performance problems. According to Shulman (1987), having curricular knowledge provides teacher with the means and strategies to carry out the work expected of them, but a multi-faceted assessment process is the panacea to students' difficulty with mathematics. It is not enough for teachers to know how subjects and concepts are developed and how the topics relate to each other and to other subjects but there is the need to know how the scope of the topics in the curriculum and how important to develop effective methodology and ensure a holistic application of content and pedagogical content knowledge (Danisman & Tanisli, 2017).

The influence of tutors' pedagogical content knowledge on students' mathematics learning, Hill, et al (2005) concluded from their research that teacher's PCK impacts on students' achievement gains. Majority of the students disagreed that their tutor's style of teaching impacts positively on learning outcome. Also, majority of the students were neutral on whether their tutors' varied pedagogies enhance understanding of mathematics. Many a student surveyed strongly believed that using the tutor's think pair-share strategies enhance learning of mathematics concepts is a much more effective strategy in learning mathematics.

5. Conclusion and Recommendation

5.1. Conclusion

This study revealed that when teachers are committed to demonstrating various components of PCK, it will go a long way to deepen their professional competence. The study has indicated that tutors' PCK has no significant influence on the pre-service teachers' mathematics learning performance. Perhaps, the pre-service teachers' inability to juxtapose or properly link their understanding through teachers' exposition of various PCK components might be the reason. Though not significant, the implication is that tutors must continue to demonstrate various components of PCK in their classroom practices. The use of think-pair-share as a pedagogical strategy in teaching has been influential to pre-service teachers' mathematics learning performance.

Challenges with following instructions, sequencing and completing tasks in class and understanding certain concepts or topics in class are some of the difficulties pre-service teachers face in mathematics learning. This means that pre-service teachers will continue to face this problem as long as it remains unresolved.

The tutors did not acknowledge the use of appropriate mitigating measures in respect of resolving the major challenges pre-service teachers face when learning mathematics. Until pragmatic measures are put in place to eliminate most of the challenges associated with learning mathematics, student tutors will continue to perform poorly. If conscious efforts are not made to solve these problems, the authorities who are in charge of this situation will continuous to face problems.

5.2. Recommendations

The following recommendations were made regarding the outcome of the this study: Ghana Tertiary Education Commission (GTEC), when revising the curricular of CoE, should place emphasis on PCK of tutors, major challenges of learning mathematics, and strategies that can be used to mitigate such challenges. GTEC should supervise the acquisition of knowledge and skills necessary for addressing the challenges of mathematics learning among tutors and pre-service teachers of CoE.

Management of the CoE should ensure the institutional enforcement of policies that improve learning performance in mathematics. CoE's management should implement a reward system for tutors and pre-service teachers who excel in mathematics learning performance. Tutors in the CoE should be familiar with the various PCK components so as to improve their classroom and professional practices.Tutors should endeavour to understand and practise the instructional strategies required to ensure improved mathematics learning performance practices.

REFERENCES

1. Aaron, P. G. (1995). Differential diagnoses of reading disabilities. *Journal of School Psychology Review*, *24* (3), 345-360.
2. Akpo, S. E. (2012). *The impact of teacher-related variables on pre-service teachers' junior secondary certificate (JSC) mathematics results in Namibia*. Unpublished Ph. D Thesis, University of South Africa, South Africa.
3. Alexander, R. (1992). *Policy and practice in primary education*. London: Routledge.
4. Amuah, U. (2021). A*ssessing pedagogical content knowledge of religious and moral education teachers at basic schools in Komenda-Edina-Equafo-Abrem Municipality*. A Thesis submitted to the Faculty of Educational Studies, University of Cape Coast, Cape Coast.
5. An, S. Kulm, G. & Wu, Z. (2004). The pedagogical content knowledge of middle school teachers in China and the U.S.A. *Journal of Mathematics Teacher Education*, *7*, 145–172.
6. Angiulli, A. D., & Siegel, L. S. (2003). Cognitive functioning as measured by the WISC-R: Do children with learning disabilities have distinctive patterns of performance? *Journal of Learning Disabilities*, *36*, 5-28.
7. Arslan, S. & Baki, M. (2017). Effects of mathematics content knowledge on pedagogical content knowledge. *Journal of Teacher Education and Educators*, *6*(1), 53-68.
8. Ary, D., Jacobs, L. C., & Razavieh, A. (1972). *Introduction to research in education* (4th ed.). New York: Holt, Rinehart and Winston Inc.
9. Australian National University (ANU) Pre-service teachers' Services (1994). *Guidelines for working effectively with pre-service teachers with learning disabilities*. Canberra: Australian National University.
10. Ball, D. (2000). Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education*, *51*(3):241-247.
11. Ball, D. L. (1990). The mathematical understanding that preservice teachers bring to teacher education. *Elementary School Journal*, *90*, 449–466.
12. Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator 29*(1), 14-17.
13. Baxter, J.A. & Lederman, N. G. (1999). Assessment and measurement of pedagogical content knowledge. In J. Gess-Newsome & N. Lederman (Eds.). *PCK and Science Education* (147-161). Kluwer: Academic Publishers.
14. Belize, G., Mullens, J. E., Murnane, R. J. & Willett, J. B. (1996). The contribution of training and subject matter

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DOI: 10.5281/zenodo.10726130

knowledge to teaching effectiveness: A multilevel analysis of longitudinal evidence from Belize. *Comparative Education Review, 40*, 139-57.
15. Bosu, L. (2010). *Assessing pedagogical content knowledge of accounting teachers in senior high schools in Central Region, Ghana*. Thesis submitted to Institute of Education, University of Cape Coast, Ghana.
16. Brickhouse, N. W. (1990). Teachers' beliefs about the nature of science and their relationship to classroom practice. *Journal of Teacher Education, 41*, 53-62.
17. Bryans, M. B. (2004). Teachers' orientations towards mathematics curriculum materials: Implications for teacher learning. *Journal for Research in Mathematics Education*, *35*(5), 352-388.
18. Buschang, R. (1999). *Validating measures of math teacher knowledge. California Educational (Eds.)*. S*cience Education*, 49, 147-161.
19. Charalambos, Y. (2011). Preservice teachers' mathematics knowledge for teaching basic schools. *African Journal Online, 10*, 8-32.
20. Chinn, S. (2002). *Math and dyslexia: A view from the United Kingdom*. Principal of Mark College, Somerest, UK. Retrieved November 8, 2020, from http://www.Sprakaloss.se/Chinn-mathsanddyslevia.html.
21. Cochran-Smith, M. (1999). Relationships of Knowledge and Practice: Teacher Learning in Communities. *Review of Research in Education*, *24*, 249–305.
22. Cohen L., Manion, L., & Morrison, K. (2007). *Research Methods in Education.* Taylor & Francis e-Library, New York.
23. Creswell, J. W. & Creswell, J. D. (2018). *Research Design: Qualitative, Quantitative, and Mixed Methods Approach* (5th ed.). Sage Publications, Inc.
24. Danisman, S. & Tanisli, D. (2017). Examination of mathematics teachers' pedagogical content Knowledge of probability. *Malaysian Online Journal of Educational Sciences*, *5*(2), 3-11.
25. Darling-Hammond, L. (2000). *Teacher quality and student achievement: A review of state Policy Evidence*. Retrieve from Education Policy and Archives Website: http://eppaa.asu.edu/epaa/v8nl.
26. Davis, B. & Simmt, E. (2006). Mathematics-for-Teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Journal of Educational Studies in Mathematics*, *61*(3), 293−319.
27. De Vellis, R. F. (1991). *Scale development: Theory and applications*. Newbury Park: Corwin Press, Inc.
28. Dunn, R, & Dunn, K. (1978). *Teaching pre-service teachers through their individual learning styles: A practical approach*. Reston, VA: Reston Publishing Company.
29. Fennema, E. & Frankel, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York, Macmillan.
30. Focus on the family. (2006). *Types of learning difficulties*. Retrieved December 1, 2008, from http://focusonthefamily.com/parenting/learningdifficulites.
31. Fraenkel, J. R., & Wallen, N. E. (2000). *How to design and evaluate research in Education* (4th ed.). New York: McGraw Hill, Higher Education.
32. Fullan, M. & Hargreaves, A. (1992). *What is worth fighting for in your school?* Toronto: Ontario Public School Teacher Federation.
33. Gay, L. R., & Diehl, R. (1992). Research Methods for Business and Management. In Hill, R. (1998) (Ed.), *What Sample Size is 'Enough' in Internet Survey? Interpersonal Computing and Technology*. An Electronic Journal for the 21st Century. Retrieved 18 April, 2021 from http://www.emoderators.com/ipct-j/1998/n3-3hill.hmt/.
34. Golemark, P. (1994). Putting teachers back into teachers' knowledge. TESOL *Quarterly*, *28*(2), 401-407.
35. Gonzalez, K. & Maxwell, G. M. (2018). Mathematics teachers efficacy, experience, certification and their impact on student achievement. *Journal of Instructional Pedagogies*, *2*, 2-8.
36. Halim, L. & Meerah, S. M. (2002). Science trainee teachers' pedagogical content knowledge and its influence on physics teaching. *Research in Science and Technological Education, 20*(2), 215-225.
37. Harris, J. B. & Philips, M. (2018). PCK and TPCK/TPACK*:* More than etiology. In E. Langran & J. Borup (Eds.), *Proceedings of Society for Information Technology and Teacher Education International Conference* (pp. 2109-2116). Washington, D.C., United States: Association for the Advancement of Computing in Education (AACE). Retrieved May 23, 2021 from https://www.learntechlib.org/primary/pp182817/.
38. Hashim, A, Saili, J. Noh, M. A. C. (2015). The relationship between pedagogical content knowledge and al-Quran Tajweed performance among students KKQ in Malaysia. *Procedia-Social and Behavioral Sciences, 197*, 1530 – 1537.
39. Hashweh, M. (2005). Teacher pedagogical constructions: A reconfiguration of pedagogical content knowledge. *Teachers and Teaching: Theory and Practice, 11*(3), 273-292.
40. Hill, H. C. & Chin, M. (2018). Connection between teachers' knowledge of pre-service teachers, instruction and achievement outcomes. *American Educational Research Journal*, *55*(5), 1076-1112.
41. Hill, H. C., Rowan, B. D. & Ball, L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371–406.
42. Hill, H.C, Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of pre-service teachers. *Journal for Research in Mathematics Education*, *39*(4), 372–400.
43. Hurrell, D. P. (2013). What teachers need to teach mathematics: An argument for a reconceptualised model. *Australian Journal of Teacher Education, 38*(11), 54-64.
44. Jacob, F., John, S. & Gwany, D. M. (2020). Teachers' pedagogical content knowledge and students' academic achievement: A theoretical overview. *Journal of Global Research in Education and Social Sciences, 14*(2), 14-44.
45. Jones, A. & Moreland, J. (2003). Considering pedagogical content knowledge in the context of research on teaching: An example from technology. *Waikato Journal of Education, 9*, 65 - 76

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46. Juttner, M. & Neuhaus, B. J. (2012). Development of items for a pedagogical content knowledge test based on empirical analysis of pupils' errors. *International Journal of Science Education, 34*(7), 1125–1143.
47. Juttner, M. Boone, W., Park, S. & Neuhaus, B. J. (2013). Development and use of a test instrument to measure biology teachers' content knowledge (CK) and pedagogical content knowledge (PCK). *Education, Assessment, Evaluation and Accountability*, *25*(1):45–67.
48. Kandjinga, J. K. (2018). *Investigating the grade 12 teachers' subject and pedagogical content knowledge of mathematics in some selected public schools in the Khomas education region, Namimbia.* Master of Education Thesis Submitted to University of Namimbia.
49. Kilic, H. (2009). *Pedagogical content knowledge of preservice secondary mathematics teachers*. Unpublished Ph.D Thesis, University of Georgia, Athens, Georgia.
50. Kleickmann, T., Richter, D., Kunter, M., Elsner, J., Besser, M., Krauss, S. & Baumert, J. (2013). Teachers´ content knowledge and pedagogical content knowledge: The role of structural differences in teacher education. *Journal of Teacher Education,64*(1), 90-106.
51. Kovarik, K. (2008). *Mathematics educators and teachers' perception of pedagogical content knowledge*. Dissertation at the Columbia University.
52. Krauss, S., Baumert, J., & Blum, W. (2008). Secondary mathematics teachers' pedagogical content knowledge and content knowledge: Validation of the COACTIV constructs. *The International Journal on Mathematics Education, 40*(5), 873–892.
53. Lee, Y., Capraro, R. M. & Capraro, M. M. (2018). Mathematics teachers subject matter knowledge and pedagogical subject matter knowledge in problem posing. *International Electric Journal of Mathematics Education, 13*(2), 75-90.
54. Lenhart, S.T. (2010). *The effect of teacher pedagogical content knowledge and the instruction of middle school geometry*. Unpublished Ph.D. Thesis, Liberty University, Virginia, USA.
55. Loughran, J., Berry, A. & Mulhall, P. (2007). Pedagogical content knowledge: What does it mean to science teachers? In R. Pinto & D. Couso (Eds.). *Contributions from science education research* (pp. 93-105). Springer.
56. Lucenario, J. L. S., Yangco, R. T., Punzalan, A. E. & Espinosa A. E. (2016). Pedagogical content knowledge guided lesson study: Effects on teacher competence and students' achievement in chemistry. *Education Research International, 1*, 1-9.
57. Madsen, J. A., & Oslon, K. J. (2005). Student teachers' use of learning theories to diagnose children learning difficulties. *Journal Article of Elementary Science Education, 17*(2), 35-55.
58. Magnusson, S., Krajcik, H. & Borko, H. (2001). Nature, sources and development of PCK. In J. Gess Newsome & N. G. Lederman (Eds.). *Examining PCK* (pp. 95–132). Dordrecht: Kluwer Academic Publishers.
59. Maher, N., Muir, T. & Chick, H. (2015). Examining PCK in secondary school mathematics. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia*), pp. 389–396. Sunshine Coast: MERGA.
60. Mammana, C. & Villani, V. (Eds.) (1998). *Perspective on the teaching of Geometry for the 21st Century*. Dordrecht: Kluwer.
61. Marks, R. (1990). Pedagogical content knowledge. From a mathematical case to a modified conception. *Journal of Teacher Education, 41*(3), 3-11.
62. Marshall, J. H. & Sorto, M. A. (2012). The effects of teachers' mathematics knowledge and pedagogy on student achievement. *International Review Education.* DOI 101007/s11159-012-927-6.
63. Maryani, I. & Martanigsih, S. (2015). Correlation between teacher's pedagogical content knowledge and student's motivation in primary school in Indonesia. *International Journal of Evaluation and Research in Education, 4*(1), 38-58.
64. Mereku, D. K., Ghartey, J. & Anamoah-Mensah, J. (2008). *TIMSS 2007 Ghana report: Findings from IEA's Trends in International mathematics and Science Study at eighth grade*. Accra, Ghana: Ministry of Education Science and Sports.
65. Mewborn, D.S. (2003). Teaching, teachers' knowledge and their professional development. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.). *A research companion to principles and standards for school mathematics* (pp. 45-52). Reston, VA: The National Council of Teachers of mathematics.
66. Ministry of Education. (2015). *Transforming teacher education and learning*. Accra, Ghana: Ministry of Education.
67. Mitchelmore, M. C., & White, P. (2000). Development of angle concepts by progressive abstraction and generalisation. *Educational Studies in Mathematics, 41*, 209 - 238.
68. Morine-Dershimer, G., & Kent, T. (1999). The complex nature and sources of teachers‟ pedagogical knowledge. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge* (pp.95-132). Dordrecht: Kluwer Academic Publishers.
69. Mullis, I. V. S., Martin, M. O., Foy, P. & Arora, A. (2012). *TIMSS 2011 international result in mathematics*. Boston College, TIMSS & PIRLS international Study Centre, International Association of Evaluation of Educational Achievement.
70. National Dissemination Centre for children with Disabilities. (2004). *Types of learning disabilities.* Retrieved March 22nd, 2021, fromhttp://en.wikipedia.org/wiki/learningdifficulties.
71. National Joint Committee on Learning Disabilities. (NJCLD, 2005). *Responsiveness to intervention and learning disabilities*. Retrieved February 14, 2021, from http://www.ldonline.org/njcld.
72. Nunnaly, J. C. (1978). *Psychometric theory* (2nd ed.). New York: McGraw-Hill.
73. Ornstein, A. & Hunkins, F. (1998). *Curriculum: Foundations, principle and issues*. Boston, MA: Allyn & Bacon.
74. Ozden, M. (2008). The effect of content knowledge on pedagogical content knowledge: The case of teaching

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DOI: 10.5281/zenodo.10726130

phases of matters. *Educational Sciences: Theory and Practice, 8*(2), 633-645.
75. Park, S. & Oliver, S. (2008). Revisiting the conceptualization of pedagogical content knowledge (PCK): PCK as a conceptual tool to understand teachers as professionals. *Research in Science Education, 38*, 261-284.
76. Penso, S. (2002). Pedagogical content knowledge: How do student teachers identify and describe the causes of their pupils' learning difficulties? *Asia Pacific of Teacher Education, 30*(1), 25-37.
77. Penso, S. (2010). Pedagogical content knowledge: How do student teachers identify and describe the causes of their pupils' learning difficulties. *Asian-Pacific Journal of Teacher Education. 30*(1), 25-37.
78. Pinamang, I. & Penrose, O.C. (2017). Pre-service teachers' content knowledge and pedagogical content knowledge in teaching geometric transformation. *African Journal of Educational Studies in Mathematics and Sciences, 13*, 63-70.
79. Pineda, C. (2002). Knowledge base for EFL/ESL educators: What does it mean? *Profile, 3*(1), 9-15.
80. Pourana, C., Hodgen, J., Adler, J. & Pillay, V. (2015). Can improving teachers' knowledge of mathematics lead to gain in learners' attainment in mathematics? *South African Journal of Education, 35*(2), 2-11.
81. Riccio, C. A. (2002). A comparison of multiple methods for the identification of children with reading disabilities. *Journal of Learning Disabilities, 12*(2), 23-47.
82. Rowan, B. Correnti, R. & Miller, R. J. (2002). Measuring teachers' pedagogical content knowledge in surveys: An exploratory study. *Sociology of Education, 8*, 235-256.
83. Rowan, B., Schilling, S. G., Ball, D. L., Miller, R., Atkins-Burnett, S., Camburn, E., H. (2001). Measuring teachers' pedagogical content knowledge in surveys: An exploratory study. *Sociology of Education, 8*, 235- 256.
84. Schram, P., Wilcox, S, Lanier, P. & Lappan, G. (2008*). Changing mathematical conceptions of preservice teachers: A content and pedagogical intervention*. Paper presented at the annual Meeting of the American Educational Research Association, New Orleans, LA.
85. Sedgwick, P. (2014). Cross sectional studies: Advantages and disadvantages. *BMJ* [Online], *348*, g2276. Available from: http://doi.org/10.1136/bmj.g2276 [Accessed 10 April 2021]
86. Shah, R. K. (2019). Philosophical foundation for curriculum development in Nepal. *International Educational Scientific Research Journal, 5*(3), 2 – 15.
87. Sherin, M. G. (2002). When teaching becomes learning. *Cognition and Instruction, 20*(2), 119-150.
88. Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.
89. Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform, *Harvard Educational Review, 57*(1), 1–22.
90. Shuttleworth, M. (2009). *Matched subjects design*. Explorable.com. Retrieved 15th March 2021 from http:/explorable.com/matched-subjects-design
91. Sidhu, G. K., Fook, C. Y. & Kaur, S. (2011). Assessing ESL teacher trainees' content, pedagogical, and professional knowledge base. *The Assessment Books, 4*(2), 3-18.
92. Smith, D. C. & Neale, D. C. (1989). The construction of subject matter knowledge in primary science teaching. *Teaching and Teacher Education, 5*(1), 1 – 20.
93. Strawhecker, J. (2005). Preparing elementary teachers to teach mathematics: How field experiences impact pedagogical content knowledge. *IUMPST Journal, 4*, 5-30.
94. Tasdan, B. T. & Celik, A. (2016). A conceptual framework for examining mathematics teachers' pedagogical content knowledge in the context of supporting mathematical thinking. *European Journal of Education Studies, 2*(5), 90-120.
95. Tchoshanov, M. (2011). Relationship between teacher knowledge of concepts and conncetions, teaching practice and student achievement in middle grades mathematics. *Education Studies in Mathematics, 76*(2), 141-164.
96. Tchoshanov, M., Quinones, M. C. Shakirova, K. B., Ibragimova, E. N. & Shakirova, L. R. (2017). Connection between topic specific teacher knowledge and student performance in lower secondary school mathematics. *International Journal of Science and Mathematics, 16*(5), 560 – 561.
97. Thelle, D. S. & Laake, P. (2015). *Research methodology in medical and biological sciences.* United Kingdom: Academic Publishers, 84 Theobald's Road, London WC1X 8RR, UK.
98. Tsafe A. K. (2013). Teacher pedagogical knowledge in mathematics: A tool for addressing learning problems. *Scientific Journal of Pure and Applied Science, 2*(1), 35-41.
99. Turnuklu ,E. & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary math teachers' perspectives. *IUMPS: The Journal, 1, 1*-13.
100. U.S. Department of Education. (2002). *The condition of education 2002.* Washington, DC: National Centre for Education Statistics, U.S. Government Printing Office.
101. Van Driel, J. H., Verloop, N. & de Vos, W. (1998). Developing science teachers' pedagogical content knowledge. *Journal of Research in Science Teaching, 35*(6), 673-695.
102. Wilmot, E. M, (2008). Learning to teach basic school mathematics: Lesson from a pedagogical course. *African Journal of Educational Studies in Mathematics and Science, 6*, 1-13.
103. Wilson, N., & McLean, S. (1994). *Questionnaire design: A practical introduction.* Newton Abbey: University Ulster Press.
104. Wilson, S., Floden, R. & Ferrini-Mundy J. (2002). Teacher preparation research: An insider's view from the outside. *Journal of Teacher Education, 53*(3), 190-204.
105. Zambal-Saul, C., Starr, M. & Krajcik, J. (1999). Constructing a framework for elementary science teaching using pedagogical content knowledge. In J. Gess-Newsome & N. Lederman (Eds.), *Examining pedagogical content knowledge* (337-357). Dordrecht: Kluwer Academic Publishers.
106. Zhang, Y. (2015). Pedagogical content knowledge in early mathematics: what teachers know and how it

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