

# Digital Redesign of Interval Systems via Particle Swarm Optimization

Chen-Chien Hsu, and Chun-Hui Gao

**Abstract**—In this paper, a PSO-based approach is proposed to derive a digital controller for redesigned digital systems having an interval plant based on resemblance of the extremal gain/phase margins. By combining the interval plant and a controller as an interval system, extremal GM/PM associated with the loop transfer function can be obtained. The design problem is then formulated as an optimization problem of an aggregated error function revealing the deviation on the extremal GM/PM between the redesigned digital system and its continuous counterpart, and subsequently optimized by a proposed PSO to obtain an optimal set of parameters for the digital controller. Computer simulations have shown that frequency responses of the redesigned digital system having an interval plant bare a better resemblance to its continuous-time counter part by the incorporation of a PSO-derived digital controller in comparison to those obtained using existing open-loop discretization methods.

**Keywords**—Digital redesign, Extremal systems, Particle swarm optimization, Uncertain interval systems.

## I. INTRODUCTION

MOST practical systems, such as flight vehicles, electric motors, and robots, are formulated in continuous-time uncertain settings. The uncertainties in these systems arise from unmodelled dynamics, parameter variation, sensor noises, actuator constraints, etc. These variations of the uncertain parameters generally do not follow any of the known probability distributions and are most often quantified in terms of bounds [27]-[28]. Hence, practical systems or plants are most suitably represented by continuous-time parametric interval models [9]-[10], rather than deterministic mathematical models.

With the fast advances in digital technologies and computers, digital control is getting more and more popular in recent years. As a result, digital controllers are increasingly being used to implement control systems, because they have many advantages over the analogue control in terms of better reliability, flexibility, low cost, performance, lower disturbance, etc [11]-[12].

On the other hand, digital redesign [6],[7],[8] provides an indirect way to design digital controller. The process of digital redesign begins with a design of an analogue controller by using any existing well-developed continuous-time robust controller design approaches such that the closed-loop system meets the performance specifications. The analog controller is then converted into a digital one by using any of the conventional discretization techniques [30], for example, Tustin transform, zero-order-hold transform, backward integral, Boxer-Thaler, etc. This kind of ‘open-loop’ design philosophy [16]-[18], where a sufficiently small sampling time is assumed, however, ignores the fact that the analog system is actually operating in a feedback configuration whose output is determined by the closed-loop system. As a result, poor suitability from the redesigned digital system is inevitable if an open-loop system only is considered. If a better system performance is required, closed-loop discretization methods [19], [20], [21], [22], [29] are required to derive the digital controllers. Unfortunately, the above-mentioned approaches dealt with deterministic mathematical models only. To the knowledge of the authors, there is no systematic approach available to solve the problems of digital redesign of uncertain continuous-time interval systems. As an attempt to improve the performance of the redesigned digital systems having an interval plant, this paper proposes a PSO-based approach to derive a digital controller based on frequency-response resemblance for the redesigned digital system and its continuous counterpart in terms of extremal gain/phase margins (GM/PM).

The paper is organized as follows. Section II introduces the uncertain interval systems. Derivations of extremal GM/PM for interval systems are introduced in Section III. Section IV proposes a PSO-based method to derive digital controllers for uncertain interval systems based on resemblance of extremal gain/phase margins (GM/PM). An illustrated example is demonstrated in Section V. Conclusions are drawn in Section VI.

## II. PROBLEM DESCRIPTION

Consider an uncertain interval plant in Fig. 1 given by:

$$G_p(s, \mathbf{a}, \mathbf{b}) = \frac{b_0 + b_1s + b_2s^2 + \dots + b_{n-1}s^{n-1}}{a_0 + a_1s + a_2s^2 + \dots + a_{n-1}s^{n-1} + s^n} = \frac{\widehat{N}(s)}{\widehat{D}(s)} \quad (1)$$

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where coefficient vectors  $\mathbf{a} = (a_0, a_1, a_2, \dots, a_{n-1})$  and  $\mathbf{b} = (b_0, b_1, b_2, \dots, b_{n-1})$  lie in the  $n$ -dimensional boxes

$$A = \{a : a_i \in [a_i^-, a_i^+], \forall i = 0, 1, 2, \dots, n-1\} \quad (2)$$

and

$$B = \{b : b_i \in [b_i^-, b_i^+], \forall i = 0, 1, 2, \dots, n-1\}, \quad (3)$$

respectively.

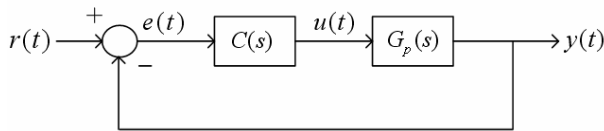


Fig. 1 A robust control system with an interval plant

Assume a fixed-structure controller described by a rational transfer function as:

$$C(s, \mathbf{p}, \mathbf{q}) = \frac{q_0 + q_1s + q_2s^2 + \dots + q_ms^m}{p_0 + p_1s + p_2s^2 + \dots + p_ms^m} \quad (4)$$

is designed so that the closed-loop system in Fig. 1 satisfies the performance specifications, where  $\mathbf{p} = [p_0, p_1, \dots, p_m]$  and  $\mathbf{q} = [q_0, q_1, \dots, q_m]$  designate the vectors of the controller parameters, respectively. When the controller  $C(s, \mathbf{p}, \mathbf{q})$  is placed in series with plant  $G_p(s, \mathbf{a}, \mathbf{b})$  and closed under unity feedback as shown in Fig. 1, the transfer function of the closed-loop system becomes:

$$\begin{aligned} G_{cl}(s, \mathbf{p}, \mathbf{q}, \mathbf{a}, \mathbf{b}) &= \frac{C(s, \mathbf{p}, \mathbf{q})G_p(s, \mathbf{a}, \mathbf{b})}{1 + C(s, \mathbf{p}, \mathbf{q})G_p(s, \mathbf{a}, \mathbf{b})} \\ &\equiv \frac{N(s, \mathbf{p}, \mathbf{q}, \mathbf{a}, \mathbf{b})}{D(s, \mathbf{p}, \mathbf{q}, \mathbf{a}, \mathbf{b})} = \frac{n_0 + n_1s + n_2s^2 + n_3s^3 + n_4s^4 + \dots}{d_0 + d_1s + d_2s^2 + d_3s^3 + d_4s^4 + \dots} \end{aligned} \quad (5)$$

When the continuous plant is subject to digital control, the sampled-data system of the interval plant preceded by a zero-order hold (ZOH) is illustrated in Fig. 2. The exact discrete equivalent plant of  $G_p(s)$  in Fig. 2 can be further represented in the purely discrete-time domain as:

$$G_h G_p(z) = \mathbf{Z} \left[ \frac{1 - e^{-Ts}}{s} G_p(s) \right] = (1 - z^{-1}) \mathbf{Z} \left[ \frac{G_p(s)}{s} \right] \quad (6)$$

Fig. 3 illustrates the redesigned digital control system for the continuous system having an interval plant  $G_p(s, \mathbf{a}, \mathbf{b})$  in Fig. 1, where  $C_d(z, \mathbf{p}, \mathbf{q})$  is the digital controller to be designed. Now the problem to derive an optimal digital controller for the redesigned digital system having an interval plant can be formulated as: *Given interval plant  $G_p(s, \mathbf{a}, \mathbf{b})$  and an analog controller  $C(s)$ , determine an optimal digital controller*

$C_d(z)$ , so that the performance of the redesigned digital system in Fig. 3 closely match that of its continuous counterpart in Fig. 1 from the view point of gain/phase margins.

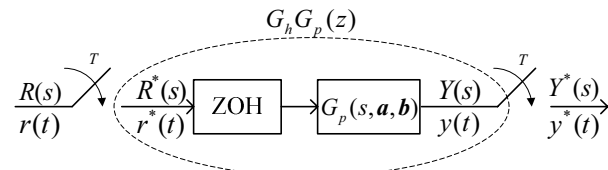


Fig. 2 A sampled plant with uncertain interval parameters

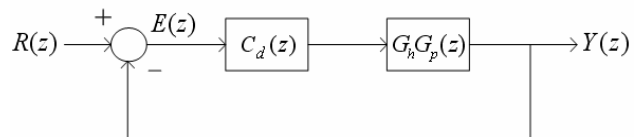


Fig. 3 Representation of a redesigned digital control system for Fig. 1

### III. EXTREMAL GAIN MARGIN AND PHASE MARGIN

Thanks to the perturbation of uncertain plant parameters, there are infinite set of frequency responses for the interval systems in Fig. 1. As a result, there are an infinite set of gain and phase margins (GM/PM). By investigating the extremal GM/PM, i.e. largest and smallest GM/PM, for the redesigned digital system and the original continuous system, we can formulate the design problem as an optimization problem to derive a set of optimal parameters for the digital controller.

#### A. Extremal Systems

Consider the interval plant  $G_p(s) = \frac{\overline{N}(s)}{\overline{D}(s)}$ . The boundary of the complex plane set for interval plant  $G_p(s)$  can be described as follows:

$$\begin{aligned} \partial G_p(s) &\subset \left( \frac{K_N(s)}{S_D(s)} \cup \frac{S_N(s)}{K_D(s)} \right) \\ &= \frac{\overline{N}_i(s)}{(1-\lambda)\overline{D}_j(s) + \lambda\overline{D}_k(s)} \cup \frac{(1-\lambda)\overline{N}_j(s) + \lambda\overline{N}_k(s)}{\overline{D}_i(s)} \\ &\equiv G_{PE}(s) \end{aligned} \quad (7)$$

where

$$\lambda \in [0, 1], (j, k) \in \{(1, 2), (1, 3), (2, 4), (3, 4)\}, i \in \{1, 2, 3, 4\}.$$

The set of points  $K_N$  and  $K_D$  are the vertices of  $\overline{N}(s)$  and  $\overline{D}(s)$ , respectively. The set of points  $S_N$  and  $S_D$  are the edges of  $\overline{N}(s)$  and  $\overline{D}(s)$ , respectively.  $G_{PE}(s)$  is referred to as the so-called extremal systems [14] of  $G_p(s)$ .

With the frequency domain properties [14], it is understood that the boundary of the complex plane set associated with the loop transfer function  $C(s)G_p(s)$  can be derived as follows:

$$\begin{aligned} \sigma(C(s)G_p(s)) &\subset G_c(s) \left( \frac{K_N(s)}{S_D(s)} \cup \frac{S_N(s)}{K_D(s)} \right) \\ &= C(s) \left( \frac{\hat{N}_i(s)}{(1-\lambda)\hat{D}_j(s) + \lambda\hat{D}_k(s)} \cup \frac{(1-\lambda)\hat{N}_j(s) + \lambda\hat{N}_k(s)}{\hat{D}_i(s)} \right) \\ &= C(s)G_{PE}(s) \\ &\equiv G_E(s) \end{aligned} \quad (8)$$

Alternatively, a set of 32 extremal systems for  $G_E(s)$  can be represented as:

$$G_{Ei}(s), \quad i=1,2,3,\dots,32, \quad \lambda \in [0,1]. \quad (9)$$

Substituting  $s = j\omega$  into the extremal system  $G_{Ei}(s)$ , we have

$$G_{Ei}(s, \lambda) \Big|_{s=j\omega} = G_{Ei}(j\omega, \lambda) = |G_{Ei}(j\omega, \lambda)| \angle G_{Ei}(j\omega, \lambda) \quad (10)$$

where  $i=1,2,3,\dots,32, \lambda \in [0,1]$ .

Based on the 32 extremal systems above, frequency responses envelopes of an interval system can be constructed where boundary of the magnitude envelope is calculated from the Kharitonov polynomials and Kharitonov segments, and boundary of the phase envelope is calculated from the Kharitonov polynomials.

#### B. Steps to Obtain Extremal GM/PM

Before the design can be performed, we need to calculate the extremal GM/PM of the interval system associated with the loop transfer function via the extremal systems. Steps to obtain the extremal GM/PM are detailed as follows:

##### Step 1 (Extremal systems)

Obtain the loop transfer function  $G(s) = G_p(s)C(s)$  as an interval system. 32 extremal systems associated with the interval system  $G(s)$  can be obtained via Eq. (10), denoted as  $G_{Ei}(s), i=1,2,3,\dots,32$ . Substituting  $s = j\omega$  into the extremal system  $G_{Ei}(s)$ , we have

$$G_{Ei}(s, \lambda) \Big|_{s=j\omega} = G_{Ei}(j\omega, \lambda) = |G_{Ei}(j\omega, \lambda)| \angle G_{Ei}(j\omega, \lambda), \quad (11)$$

where  $i=1,2,3,\dots,32, \lambda \in [0,1]$ .

##### • Step 2 (Extremal GM)

To obtain the smallest and largest gain margin of the loop transfer function  $G(s)$ , we calculate the frequency response of the extremal systems  $G_{Ei}(s)$  and locate the intersections on the real axis when phase is  $-180^\circ$  for each extremal system. Let

$$f_i(\omega, \lambda) = \angle G_{Ei}(\omega, \lambda) + 180^\circ = 0, \quad i=1,2,3,\dots,32. \quad (12)$$

Solving Eq.(12) and substituting the solution obtained into  $|G_{Ei}(\omega, \lambda)|$ , we have the intersections  $g_j, j=1,2,3,\dots$ , on the real axis. Thus, the intersection closest to point  $(-1,0)$  on the real axis for every extremal system  $G_{Ei}(s)$  can be obtained as:

$$\rho_i = \max \left\{ g_j \mid g_j = |G_{Ei}(\omega, \lambda)|_{f_i(\omega, \lambda)=0} \right\}, \quad j=1,2,3,\dots, \quad (13)$$

and the intersection farthest from point  $(-1,0)$

$$\sigma_i = \min \left\{ g_j \mid g_j = |G_{Ei}(\omega, \lambda)|_{f_i(\omega, \lambda)=0} \right\}, \quad j=1,2,3,\dots \quad (14)$$

on the real axis for every extremal system  $G_{Ei}(s)$  can be identified, respectively. Take

$$\alpha = \max \{ \rho_i \} \quad \text{and} \quad \beta = \min \{ \sigma_i \} \quad (15)$$

for all extremal system  $G_{Ei}(s), i=1,2,3,\dots,32$ . Therefore, the smallest gain margin  $GM_{c\_lower}$  and largest gain margin  $GM_{c\_upper}$  associated with the controller  $C(s)$  can be obtained as:

$$GM_{c\_lower} = 20 \log \left| \frac{1}{\alpha} \right| \quad \text{and} \quad GM_{c\_upper} = 20 \log \left| \frac{1}{\beta} \right| \quad (16)$$

, respectively.

##### • Step 3 (Extremal PM)

To obtain the smallest and largest phase margin of the loop transfer function  $G(s)$ , we need to locate the intersections on the unit circle of the frequency response for each extremal system  $G_{Ei}(s)$ . Let

$$h_i(\omega, \lambda) = |G_{Ei}(\omega, \lambda)| - 1 = 0, \quad i=1,2,3,\dots,32. \quad (17)$$

Solving Eq.(17) and substituting the solution obtained into  $\angle G_{Ei}(\omega, \lambda)$ , we have the intersections  $p_j, j=1,2,3,\dots$ , on the unit circle for each extremal system  $G_{Ei}(s)$ . Thus, the smallest angle between  $-180^\circ$  and intersections  $p_j$

$$q_i = \min \left\{ p_j \mid p_j = |G_{Ei}(w, \lambda)|_{h_i(w, \lambda)=0} \right\} \quad j = 1, 2, 3 \dots \quad (18)$$

and the largest angle between  $-180^\circ$  and intersections  $p_j$

$$\psi_i = \max \left\{ p_j \mid p_j = |G_{Ei}(w, \lambda)|_{h_i(w, \lambda)=0} \right\} \quad j = 1, 2, 3 \dots \quad (19)$$

for each extremal system  $G_{Ei}(s)$  can be identified, respectively. Take

$$k = \min \{q_i\} \text{ and } \chi = \max \{\psi_i\} \quad (20)$$

for all extremal system  $G_{Ei}(s)$ ,  $i = 1, 2, 3, \dots, 32$ . Therefore, the smallest phase margin  $PM_{c\_lower}$  and largest phase margin  $PM_{c\_upper}$  associated with the controller  $C(s)$  can be obtained as:

$$PM_{c\_lower} = \kappa + 180^\circ \text{ and } PM_{c\_upper} = \chi + 180^\circ \quad (21)$$

, respectively.

Based on the above steps, extremal GM/PM associated with the continuous loop transfer function containing the interval plant can be obtained. On the other hand, the derivation of the extremal GM/PM associated with the discrete-time loop transfer function is similar. By discretizing the 32 extremal systems  $G_{PE}(s)$  associated with the interval system, a set of 32 extremal systems in the discrete-time domain can be obtained in combination with a digital controller, based on which frequency responses can be calculated to derive the extremal GM/PM of the redesigned digital systems via steps 2 and 3.

#### IV. DIGITAL REDESIGN OF INTERVAL SYSTEMS VIA PARTICLE SWARM OPTIMIZATION

Thanks to its advantages of simplicity for implementation and high optimization efficiency for global optimization, a PSO-based approach [5] will be proposed in this section to derive an optimal digital controller  $C(z)$  based on resemblance of the extremal GM/PM for the redesigned digital system and its continuous counterpart.

##### A. Particle Swarm Optimization Algorithm

In a PSO [5] algorithm, a population of particles is first initialized with random positions  $x_i$  and velocities  $v_i$ . Fitness of the particles is evaluated by calculating the fitness value  $f(x_i)$ . The best position of each particle is set as  $Pbest$ . The  $Pbest$  with best value in the swarm is set as  $Gbest$ . As evolution continues, position for each particle is updated by using the following formula:

$$v_i(t+1) = w \times v_i(t) + c_1 \times rand \times (pbest_i - x_i(t))$$

$$+ c_2 \times rand \times (Gbest - x_i(t)) \quad (22)$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (23)$$

where

- $c_1, c_2$ : acceleration constants;
- $rand$ : random number between 0 and 1;
- $x_i(t)$ : the position of particle  $i$  at iteration  $t$ ;
- $v_i(t)$ : the velocity of particle  $i$  at iteration  $t$ ;
- $w$ : inertia weight factor;
- $Gbest$ : the best previous position among all the particles;
- $Pbest_i$ : the best previous position of particle  $i$ ;

##### B. Derivation of Digital Controller based on Extremal Case GM/PM

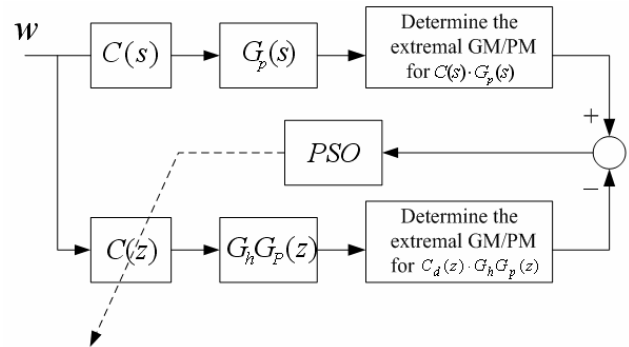


Fig. 4 Design of optimal digital controller for redesigned digital systems based on extremal GM/PM via a PSO-based approach

Fig. 4 shows a PSO-based approach to derive an optimal digital controller for the redesigned digital system based on deviation of extremal GM/PM for the redesigned digital system and its continuous counterpart. First of all, 32 extremal systems  $G_{Ei}(s, \lambda)$  shown in Eq. (9) associated with the 32 extremal controlled systems  $G_{PE}(s)$  and the analog controller  $C(s)$  need to be established for calculating the extremal GM/PM as a comparison basis. Subsequently, discretize the 32 extremal controlled systems  $G_{PE}(s)$  as discrete equivalent plants  $G_h G_{PE}(z)$ , based on which a digital controller  $C_d(z)$  is designed via the proposed PSO-based approach. It is hoped that extremal GM/PMs for the redesigned digital system and its continuous counterpart are closely matched with the use of the PSO-derived digital controller.

Assume that the digital controller has the form:

$$C(z, \mathbf{p}, \mathbf{q}) = \frac{q_0 + q_1 z + q_2 z^2 + \dots + q_m z^m}{p_0 + p_1 z + p_2 z^2 + \dots + p_m z^m} \quad (24)$$

where  $\mathbf{p} = [p_0, p_1, \dots, p_m]$  and  $\mathbf{q} = [q_0, q_1, \dots, q_m]$  represent the coefficients of the numerator and denominators of the controller. Let  $X$  be a particle representing a set of parameters for the digital controller:

TABLE I  
 EXTREMAL GM/PM OF THE REDESIGNED DIGITAL SYSTEM ASSOCIATED WITH VARIOUS DIGITAL CONTROLLERS

Methods	ZOH	Tustin	Proposed PSO-derived digital controller
Controller	$C_d(z) = \frac{9.5557z - 8.9707}{z - 0.1212}$	$C_d(z) = \frac{4.9921z - 4.3086}{z + 0.0267}$	$C_d(z) = \frac{9.94318z - 8.167}{1.3936z + 1.8158}$
Extremal GM Boundary	[1.7712dB, 8.3693dB]	[8.5753dB, 15.4088dB]	[17.8608dB, 22.7236dB]
Extremal PM Boundary	[9.2923°, 48.7159°]	[34.6461°, 60.4545°]	[45.6482°, 66.3518°]

$$X = [p_0 \ p_1 \ \dots \ p_m \ q_0 \ q_1 \ \dots \ q_m]$$

The initial particles are randomly generated from within the pre-defined range:

$$p_j \in [p_j^-, p_j^+], \quad q_j \in [q_j^-, q_j^+], \quad j = 1, 2, \dots, m$$

To accelerate the derivation process, one of the initial particles is replaced by a seeded particle obtained by converting the analog controller  $C(s)$  into  $C_d(z)$  via an existing discretization method.

#### V. ILLUSTRATED EXAMPLE

Consider the feedback control system shown in Fig. 1, where the plant is described by the interval transfer function [26]:

$$G_p(s) = \frac{[5,7]}{[0.09,0.11]s^3 + [0.9,1.2]s^2 + [0.75,1.2]s + [0.05,0.25]}$$

An analog controller  $C(s)$

$$C(s) = 0.6657 \frac{0.6804s + 1}{0.0474s + 1}$$

was designed for the interval plant, such that the resulting system has extremal GM/PM of:

$$[G_{c\_lower}, G_{c\_upper}] = [16.104dB, 23.751dB]$$

and

$$[P_{c\_lower}, P_{c\_upper}] = [45.2753^\circ, 66.9909^\circ],$$

respectively. For a sampling time of  $T = 0.1$ , determine a digital controller  $C(z)$ , such that the extremal GM/PM associated with the loop transfer function of the redesigned digital system in Fig. 3 closely match those of their continuous counterpart.

[Solution]:

By using the proposed PSO-based approach adopting control parameters of population size=30, max generation=500, inertia weight ( $w$ )=0.4,  $C_1 = C_2 = 2$ , and search space of the controller parameters as  $-10 < \alpha_i < 10$ ,  $i = \{1,2,3,4\}$ , an optimal controller

$$C_d(z) = \frac{9.94318z - 8.167}{1.3936z + 1.8158}$$

is evolutionarily obtained for  $T = 0.1$  via the proposed PSO-based approach. For comparison purpose, extremal GM/PMs of the redesigned digital system associated with various digital controllers are listed in Table I. As clearly indicated in Table I, there is a significant deviation on extremal GM/PMs between the redesigned digital system and its continuous counterpart by using conventional open-loop discretization methods, for example, ZOH and Tustin methods. The proposed PSO-derived digital controller, on the other hand, results in extremal GM/PM of [17.8608dB, 22.7236dB] and [45.6482°, 66.3518°], respectively, for the redesign digital system, which significantly resemble [16.104dB, 23.751dB] and [45.2753°, 66.9909°] of the continuous system. As demonstrated in this example, the proposed PSO-derived digital controller out-performs the other discretization methods with a better resemblance in terms of extremal GM/PM between the redesigned digital system and its continuous counterpart.

#### VI. CONCLUSION

The problem to digitally redesign a continuous-time system having an interval plant is generally difficult by conventional design methods. By formulating the design problem as an optimization problem subject to robust stability constraint, the proposed PSO-based approach can effectively derive an optimal digital controller for the redesigned digital system, preventing the problems encountered by conventional discretization methods. There is no restrictive condition under which the proposed approach is developed. Conventional design constraints on the higher-order interval plants and controller order are therefore removed. In general, the optimal controller can be obtained within a moderate number of iterations by using the proposed PSO-based approach without suffering from the inherent shortcomings. As demonstrated in this paper, the proposed approach provides a simple yet practical way in the design of an optimal digital controller for redesigned digital systems having an interval plant.

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