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Beyond the Kelly Criterion

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Abstract. This research conducts an in-depth examination of various risk management methodologies in financial trading, evolving from the high-risk, reward-driven YOLO Criterion to the advanced and nuanced Justus Criterion. It delves into how each methodology navigates the delicate equilibrium between growth opportunities and the inherent risks of significant losses, factoring in the dynamics of win probabilities and risk-reward scenarios. Introducing a layered analytical framework, this paper delineates the progression from the probabilistic foundations laid by the Kelly Criterion through to the adaptable constructs of the Tax and Just Criteria, culminating in the development of the bespoke Ad'Just Criterion. The pinnacle of this evolutionary journey is the Justus Criterion, which adeptly integrates these methodologies with insights into human behavioral patterns, particularly in terms of dealing with uncertainties in win probabilities, the aversion to losses, and the valuation of growth. Our findings indicate that the Justus Criterion presents a pragmatic, user-friendly strategy for trading that resonates with empirical data and the intrinsic risk preferences of traders, offering a comprehensive model for enhanced decision-making in financial markets.

1. Introduction

In the ever-shifting sands of financial trading, the pursuit of strategies that deftly balance profit maximization with risk mitigation remains a central endeavor, captivating the intellects of both practitioners and scholars. This pursuit has propelled an evolutionary journey from the reliance on gut-driven, high-stake gambles to the adoption of sophisticated, mathematically underpinned methodologies that navigate the twin complexities of market volatility and human psychology. This paper delineates this transformative trajectory, commencing with the audacious, yet simplistic, YOLO ("You Only Live Once") philosophy, epitomized by its unabashed embrace of all-in bets, and evolving to the nuanced Justus Criterion, which stands as a testament to the zenith of strategic ingenuity in trading, marrying advanced risk management tenets with deep behavioral finance insights.

At the heart of this evolution lies the Kelly Criterion, an influential paradigm born from the realms of information theory, which introduced a probabilistic foundation for the optimization of long-term capital growth. The elegance and analytical rigor of the Kelly Criterion paved the way for further innovations, notably the Tax Criterion, which heralded a systematic method for profit extraction, thus addressing the imperative of capital preservation alongside the quest for growth.

Emerging from this intellectual groundwork, the Just Criterion presented a novel dynamic bank/balance ratio, imbuing trading strategies with a newfound agility to adapt to the trader's evolving financial landscape and the capricious nature of market forces. This was further refined by the Ad'Just Criterion, which augmented this flexibility by permitting adjustments based on personal risk appetites, thereby ensuring alignment with individual trader profiles and market perspectives.

The zenith of this evolutionary arc is encapsulated in the Justus Criterion, a harmonious synthesis of quantitative rigor and psychological insight. By tackling the intricacies of win probability uncertainties, accommodating the psychological phenomenon of loss aversion, and refining the calculus of growth utility, the Justus Criterion unfurls a comprehensive framework. This framework transcends the mere mathematical intricacies of risk management to echo the nuanced interplay of human decision-making dynamics within the unpredictable theatre of trading.

In charting the course of these strategic evolutions, from their elemental beginnings to their current sophisticated incarnations, this paper seeks to shed light on the progressive refinement of risk management paradigms in trading. It is our objective to furnish traders with the acumen to traverse the multifaceted landscape of financial markets, armed with strategies that are not only analytically rigorous but also resonant with the subtle undercurrents of human behavior and market flux.

2. Background and Literature Review

The landscape of financial trading strategies, particularly in risk management, has been significantly shaped by pioneering works that blend mathematical rigor with practical applicability. Central to this discourse is the seminal paper by Kelly [1], which introduced a criterion for optimal bet sizing to maximize long-term wealth growth, fundamentally changing the approach to money management in gambling and investment scenarios alike. This criterion, known for its compelling mathematical foundation, has been widely adopted and adapted in the financial trading sphere.

Building on Kelly's foundational work, Thorp [2] extended the Kelly criterion's applications to the stock market, providing a more accessible and practical framework for traders and investors. Thorp's contributions not only validated the Kelly criterion's relevance in financial markets but also highlighted its potential to inform broader investment strategies.

Despite the widespread acclaim and adoption of the Kelly criterion, its application has been met with critical challenges, particularly concerning the inherent volatility and potential for significant drawdowns. This has spurred discussions and explorations around the concept of "fractional Kelly" betting, a strategy aimed at mitigating risk by leveraging only a fraction of the bet size recommended by the Kelly criterion. However, beyond the fractional Kelly approach, there has been a noticeable paucity of research directly addressing these challenges, leaving a gap in the literature concerning comprehensive solutions to the volatility and drawdown dilemmas posed by the Kelly framework.

A notable exception is the insightful analysis by Elliot Noma [3], which delves into the intricacies of how the Kelly bet size and the number of concurrent bets impact the maximum drawdown experienced by traders. Noma's work provides a valuable perspective on the trade-offs between aggressive bet sizing, as prescribed by the Kelly criterion, and the resultant risk of significant portfolio drawdowns.

Despite these contributions, the quest for a more nuanced and adaptive risk management strategy that transcends the limitations of the Kelly criterion and its fractional adaptations remains. This paper aims to bridge this gap by introducing novel criteria that build upon the Kelly framework while addressing its shortcomings in volatility and drawdown management.

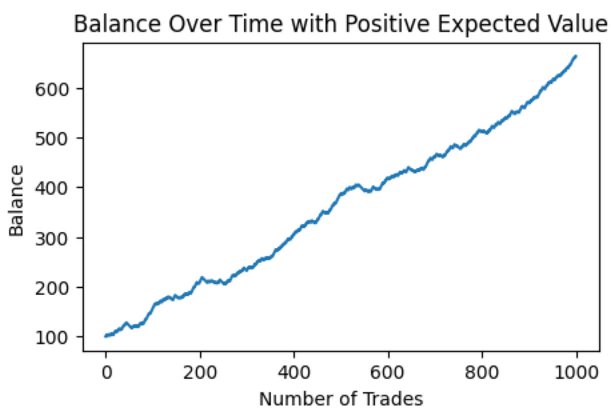
3. Mathematical Model Overview

The cornerstone of devising effective money management strategies in trading lies in a robust mathematical model, fundamentally anchored by two critical variables: the probability of a winning trade p and the risk-reward ratio b. These variables collectively determine the Expected Value EV of a trading strategy, encapsulating the average return per trade by accounting for both the frequency of wins and the proportion of gains to losses.

To elucidate this concept, consider a simplified model akin to a coin toss, where each trade has an equal probability of winning $p = 0.5$ and the risk-reward ratio is 2:1 ($b = 2$). In this scenario, the Expected Value EV is derived as follows:

$$EV = p \times b - (1 - p) = 0.5 \times 2 - (1 - 0.5) = 1 - 0.5 = 0.5$$

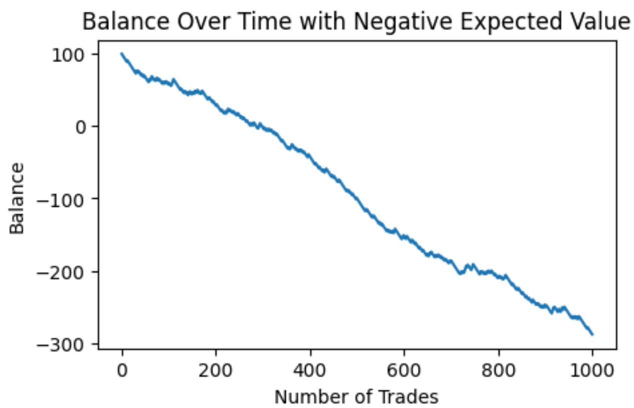
With an EV of 0.5, this model suggests an inherently profitable strategy, as indicated by the positive expected value.



An illustrative graph here depicts the trajectory of an account balance over time, assuming an initial capital of \$100 and engaging in 1000 trades, each with a \$1 risk. This graph demonstrates a consistent upward trend in capital, affirming the long-term profitability potential of strategies with a positive EV.

In contrast, consider a trading scenario with a lower win rate ($p = 0.2$), leading to a negative Expected Value ($EV = -0.4$). In this case, the model predicts a downward trajectory for the account balance, potentially leading to financial ruin. This starkly illustrates the critical importance of maintaining a positive EV for sustainable trading success.

This graph contrasts the previous scenario by showing the account balance over time for a strategy with a negative EV. Despite the minimized risk per trade, the balance trends downward, often dipping into negative territory, underscoring the detrimental impact of a negative EV on capital preservation.



Through these mathematical formulations and graphical simulations, we underscore the paramount significance of a positive Expected Value in the architecture of trading strategies. As we progress from the simplicity of fixed bets to the complexity of dynamic, nuanced strategies, the emphasis remains on harnessing positive EV strategies to refine money management tactics and bolster profitability.

4. Risk Management Strategies: A Hierarchical Approach

At the heart of this paper lies a comprehensive examination of risk management strategies, articulated through a hierarchical lens that progressively builds from foundational concepts to sophisticated models. This section endeavors to dissect and analyze a spectrum of criteria, each emblematic of a particular evolutionary stage in the quest for optimal trading strategy formulation. Starting from the high-risk, high-reward gambles characteristic of the YOLO Criterion, we trace the trajectory of thought and practice to the nuanced and calculated Justus Criterion, which synthesizes mathematical rigor with insights into human behavior and market dynamics.

This hierarchical exploration is not merely a chronological recounting of risk management strategies but a deliberate dissection of their underlying principles, practical applications, and the specific challenges they address within the trading domain. By delving into each criterion in turn, we aim to illuminate the continuous thread of innovation and adaptation that defines the field, highlighting how each model contributes to a more refined understanding of risk and reward.

- **The YOLO Criterion:** We begin with an examination of the allure and pitfalls of high-stakes betting strategies, setting the stage for the evolution that follows.
- **The Kelly Criterion:** Next, we explore the Kelly Criterion's probabilistic approach to bet sizing, a pivotal shift towards systematic risk management.
- **The Tax Criterion:** The narrative progresses to the Tax Criterion, which introduces considerations of systematic profit withdrawals and capital preservation.
- **The Just Criterion:** We then transition to the Just Criterion, which adapts the strategy to the trader's financial state, enhancing responsiveness to market conditions.
- **The Ad'Just Criterion:** The Ad'Just Criterion's focus on personalization and individual risk thresholds represents a further refinement, tailoring strategies to align with the trader's unique risk profile.
- **The Justus Criterion:** Finally, we arrive at the Justus Criterion, a comprehensive model that integrates quantitative analysis with behavioral finance, offering a holistic framework for decision-making in trading.

Navigating through these hierarchical levels will furnish readers with a thorough grasp of the risk management spectrum in trading, empowering them to customize strategies that align seamlessly with their trading ethos and risk preferences.

A. The YOLO Criterion: High Risk, High Reward

At the outset of exploring risk management strategies in trading, the YOLO (You Only Live Once) Criterion stands as a stark representation of high-risk, high-reward trading philosophy. This approach, historically likened to gambling, embodies the thrill of betting big on single, speculative investments, driven by the allure of potentially massive gains. The YOLO mentality, particularly among retail investors, gained mainstream attention in the mid-2010s as trading platforms democratized access to financial markets, enticing a younger demographic with the prospect of quick wealth through speculative trades in assets like cryptocurrencies and meme stocks.

Mathematically, the YOLO Criterion is characterized by a simplified model where the expected return E of a trade is calculated as:

$$E = m(bp - (1 - p))$$

With m representing the fixed amount risked per trade, b the risk-reward ratio, and p the probability of a win. The aggressive nature of this strategy suggests maximizing E by potentially risking the entire trading balance on each trade, a tactic that, while offering the promise of exponential gains, inherently carries the significant risk of total capital depletion.

The allure of YOLO trading, despite its evident risks, lies in its simplicity and the human inclination towards risk-taking in pursuit of substantial rewards. However, this approach is fundamentally flawed for sustainable trading due to the high probability of ruin, especially as the number of trades increases. The mathematical representation of risk, given by $1 - p^n$, where n is the trade count, illustrates how the likelihood of a total loss escalates with each successive trade.

As we transition from the YOLO Criterion, the question arises: how can traders optimize capital growth while mitigating the existential threat of ruin? This inquiry paves the way to more sophisticated risk management strategies, leading us to the Kelly Criterion. The Kelly Criterion offers a more balanced approach, providing a methodological framework to optimize bet sizes in alignment with winning probabilities, thereby ensuring a more sustainable path to capital growth.

B. The Kelly Criterion: Balancing Growth and Risk

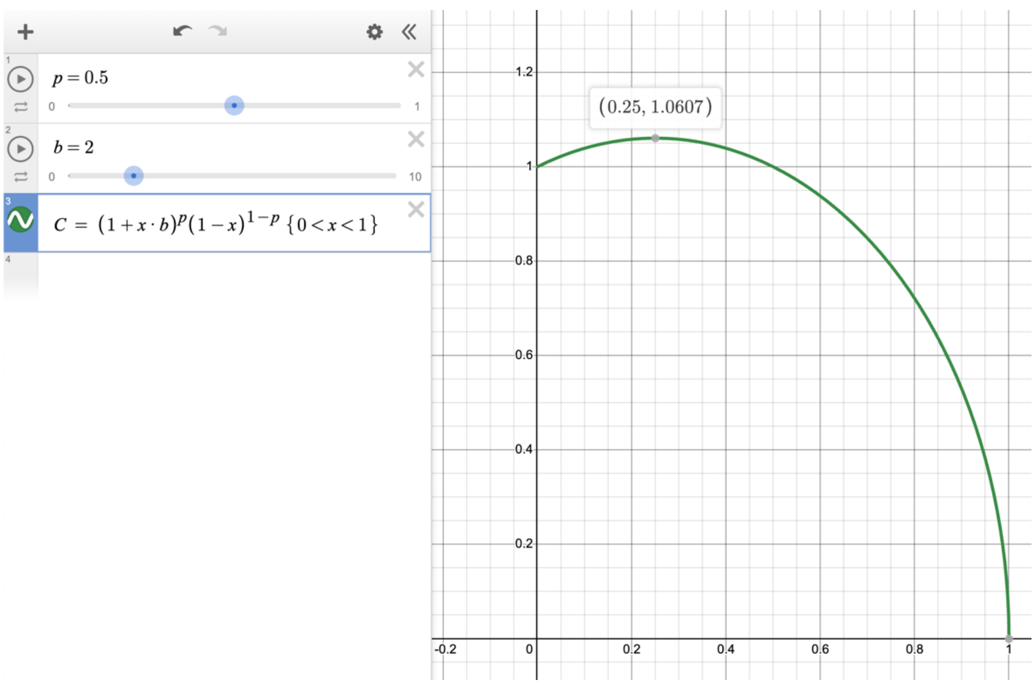
The Kelly Criterion, conceived by John L. Kelly in 1956, has evolved from its origins in telecommunications to a fundamental principle in trading risk management. It distinguishes itself by focusing on maximizing the exponential growth rate of capital rather than just the expected value, promoting a more sustainable path to wealth accumulation.

Mathematical Articulation

At the heart of the Kelly Criterion is the formula:

$$E = (1 + fb)^p \times (1 - f)^{1-p}$$

Where E represents the expected capital growth, f the fraction of capital to be risked on each trade, p the probability of a win, and b the risk-reward ratio. This equation emphasizes strategic capital growth over mere profit chasing.



Plotting $E = (1 + fb)^p (1 - f)^{1-p}$ in tools like Desmos.com for parameters such as $p = 0.5$, and $b = 2$, highlights the Kelly Criterion's optimal point at $f = 0.25$, suggesting a 25% capital risk per trade under these conditions.

Optimization Process

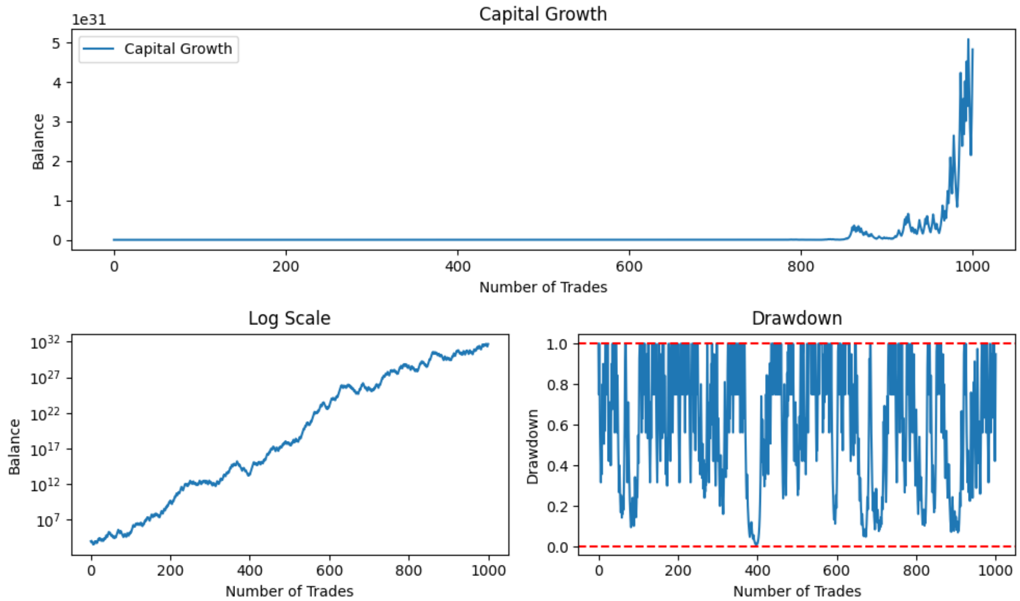
The quest for the optimal f involves differentiating the Kelly formula and solving for zero:

$$\frac{dE}{df} = 0 \Rightarrow \frac{d}{dx}(E) \ / E = 0$$
$$\Rightarrow \frac{d}{df}(\log(E)) = 0$$
$$\Rightarrow \frac{d}{df}(\mathrm{p}\log(1 + fb) + (1 - p)\log(1 - f))' = 0$$
$$\Rightarrow \frac{pb}{1 + fb} - \frac{1 - p}{1 - f} = 0$$
$$\Rightarrow f = p - \frac{(1 - p)}{b}$$

This result reaffirms the Kelly formula for the optimal fraction f of capital to risk.

Simulation Insights

Simulating 1000 trades with the Kelly-determined f showcases notable capital exponential growth potential.



However, an analysis of potential drawdowns under the Kelly Criterion unveils a critical caveat; maximum drawdowns can near 100%, particularly in long sequences of trades. This stark reality underscores a significant risk of capital depletion, challenging the criterion's viability for traders wary of substantial losses.

In response to these risks, the concept of Fractional Kelly emerges, advocating for risking only a portion of the stake suggested by the full Kelly Criterion. Despite this precaution, the threat of severe drawdowns persists, as they can still trend towards 100% over an extensive series of trades [3], underscoring the inherent volatility and risk of ruin associated with aggressive betting strategies.

Transitioning to the Tax Criterion

Acknowledging the Kelly Criterion's limitations, particularly its vulnerability to volatility and the absence of a profit withdrawal mechanism, paves the way for the Tax Criterion. This evolved strategy incorporates systematic profit withdrawals, addressing the Kelly Criterion's drawback of potentially exacerbating drawdown risks through a more balanced and pragmatic approach to risk management and capital growth.

C. The Tax Criterion: Systematic Profit Withdrawals

Building upon the foundations laid by the Kelly Criterion, the Tax Criterion emerges as a novel advance in trading risk management, introducing a systematic mechanism for profit withdrawal. This strategy is designed to mitigate against market volatility and safeguard accumulated gains from the capricious nature of trading markets.

Mathematical Articulation

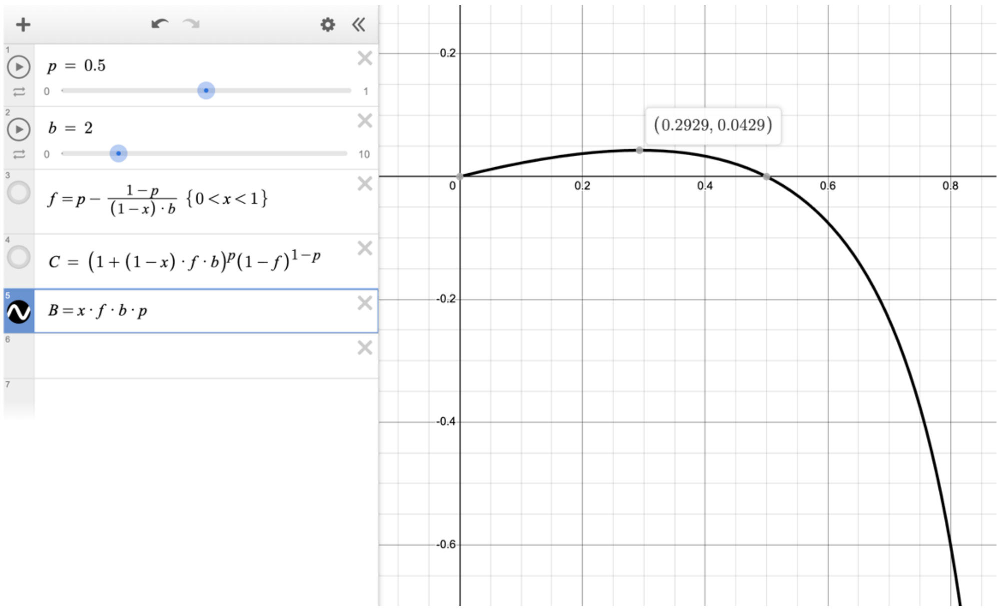
The Tax Criterion is mathematically formulated as follows, where C represents the expected capital growth after profit withdrawals and B signifies the bank's expected value after a win:

$$C = (1 + (1 - w)bf)^p \times (1 - f)^{1-p}$$

$$B = wbfp$$

Here, w denotes the fixed percentage of profit to be withdrawn from each winning trade, f the fraction of capital risked as determined by the Kelly Criterion, p the probability of a win, and b the risk-reward ratio. By optimizing f using the Kelly approach, the formula refines to:

$$f = p - \frac{1 - p}{(1 - w)b}$$



Graphical analysis with parameters such as $p = 50\%$ and $b = 2$ reveals an optimal strategy of withdrawing $w = 29.29\%$ of profits per winning trade, while risking $f = 14.65\%$ of capital per trade. This method allows traders to determine the ideal withdrawal rate ('tax rate') and risk fraction to balance returns and growth sustainability.

Optimization Process

To discover the optimal w, one must differentiate the Tax Criterion's formula with respect to w and solve for a maximum:

$$\Rightarrow \frac{dB}{dx}(B) \quad = 0 \Rightarrow \frac{d}{dx}(wbpf) \quad = 0$$

$$\Rightarrow \frac{d}{dx}\left(w - \frac{1-p}{pb} \times \frac{w}{1-w}\right) \quad = 0$$

$$\Rightarrow 1 - \frac{1-p}{pb} \times \frac{1}{(1-w)^2} = 0$$

$$\Rightarrow (1-w)^2 = \frac{1-p}{pb}$$

$$\Rightarrow w = 1 - \sqrt{\frac{1-p}{pb}}$$

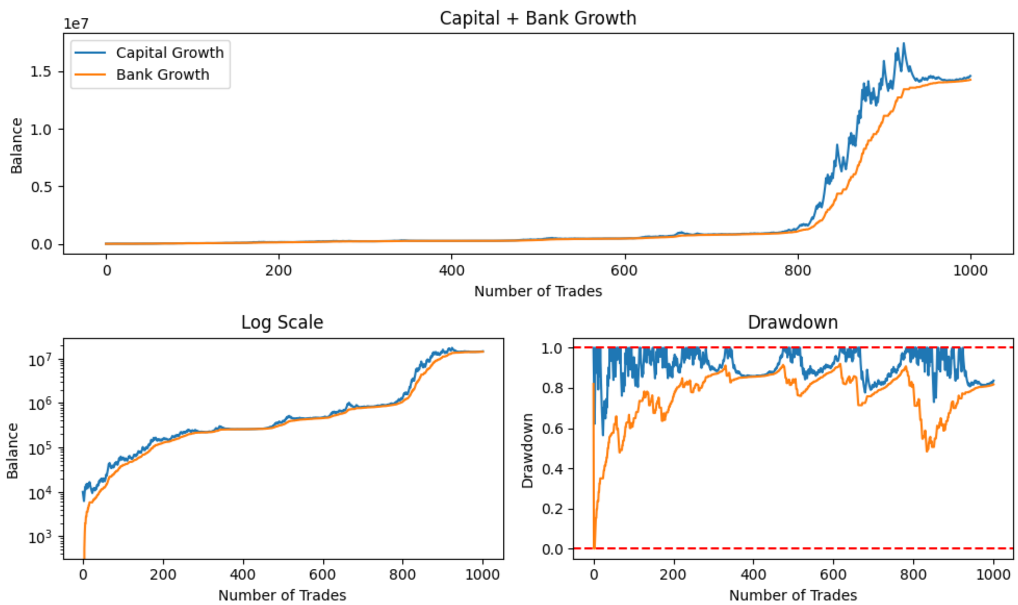
Linking the optimal w to f yields:

$$f = p - \frac{1-p}{(1-w)b} \Rightarrow f = p \times (1 - \sqrt{\frac{1-p}{pb}})$$

The Tax Criterion's essence is encapsulated in:

$$\begin{cases} w = 1 - \sqrt{\frac{1-p}{pb}} \\ f = pw \end{cases}$$

Simulation Insights



Simulations under the Tax Criterion highlight its effectiveness in curbing maximum drawdowns, contributing to a more resilient strategy. However, during extended periods of drawdowns, the strategy may show signs of stagnation, pointing to potential areas for further refinement.

Towards the Just Criterion

The Tax Criterion offers valuable insights into the challenge of managing growth during protracted drawdowns. Recognizing the need for an even more adaptable approach, the transition to the Just Criterion is made. The Just Criterion proposes dynamic tax adjustments based on the trader's current financial situation and market conditions, aiming to further refine the balance between capital growth and risk management.

D. The Just Criterion: Dynamic Taxation for Optimized Drawdown Recovery

The Just Criterion represents an evolutionary step in optimizing trading strategies, aiming to address the challenge of growth stagnation due to continuous taxation, particularly during downturns. It introduces a dynamic component, the bank/balance ratio r, which allows for the adjustment of the taxation rate in response to the changing relationship between capital and bank balance.

Mathematical Articulation

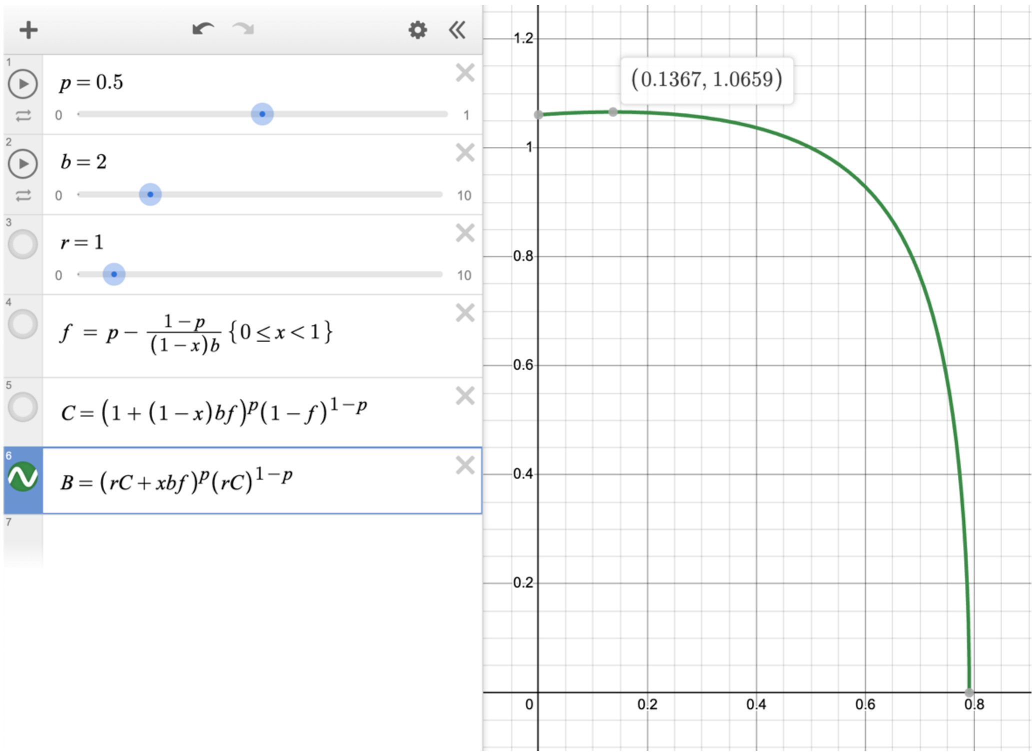
In a scenario where a trading account holds \$100 and a separate bank contains \$10,000, the Tax Criterion may dictate an \$8 tax withdrawal from a \$28 profit—a negligible amount in the context of the larger bank balance. The Just Criterion refines this approach by recalibrating the tax rate based on the dynamic bank/balance ratio r, defined as:

$$C = (1 + (1 - w)bf)^p \times (1 - f)^{1-p}$$

$$B = (rC + wbf)^p (rC)^{1-p}$$

Applying the Kelly Criterion for optimal capital growth, we adjust for the tax rate w to determine the optimal fraction f to be risked:

$$f = p - \frac{1 - p}{(1 - w)b}$$



Graphical analysis via Desmos reveals dynamic fluctuations in the optimal tax rate w as a function of the capital-to-bank ratio r. When r is at 0, the optimal tax rate aligns with the original Tax Criterion. As r gradually increases, a discernible transition in the optimal tax rate occurs, reflecting the Just Criterion's responsive nature to changes in the capital-to-bank balance. This transition reaches a critical point when r is equal to 1—where capital and bank balances are equivalent—at which the optimal tax rate stabilizes at 13.67%. Beyond this equilibrium, as r extends to 1.414 and above, the optimal tax rate progressively decreases to 0, signifying a full transition to the Full Kelly Criterion, where no taxation is applied, and the entire focus shifts to capital growth.

Optimization of the dynamic Just Tax Rate

To determine the optimal tax rate w in the Just Criterion, we start by setting the derivative of the expected bank return B, with respect to w to zero $\frac{dB}{dw} = 0$. This condition implies finding the stationary points of B which we approach by analyzing the derivative of the natural logarithm of B,

$$\frac{dB}{dw} = 0 \Rightarrow \frac{\frac{dB}{dw}}{B} = 0$$

$$\Rightarrow \frac{d}{dw}(\ln(B)) = 0$$

$$\Rightarrow \frac{d}{dw}\left(p \ln\left(C + \frac{wbf}{r}\right) + (1-p)\ln(C)\right) = 0$$

To facilitate the simplification process, we introduce the term $A = \frac{wbf}{r}$, which signifies the adjusted profit post-taxation. This allows us to express the equation in terms of A:

$$\Rightarrow \frac{d}{dw}(p \ln(C + A) + (1-p)\ln(C)) = 0$$

$$\Rightarrow \mathrm{p}\frac{\frac{dC}{dw} + \frac{dA}{dw}}{C + A} + (1-p)\frac{\frac{dC}{dw}}{C} = 0$$

$$\Rightarrow \mathrm{pC}\left(\frac{dC}{dw} + \frac{dA}{dw}\right) + (1-\mathrm{p})\frac{dC}{dw}(\mathrm{C} + \mathrm{A}) = 0$$

$$\Rightarrow \mathrm{pC}\frac{dC}{dw} + \mathrm{pC}\frac{dA}{dw} + (1-\mathrm{p})\mathrm{C}\frac{dC}{dw} + (1-\mathrm{p})\mathrm{A}\frac{dC}{dw} = 0$$

$$\Rightarrow \mathrm{C}\frac{dC}{dw} + \mathrm{pC}\frac{dA}{dw} + (1-\mathrm{p})\mathrm{A}\frac{dC}{dw} = 0$$

$$\Rightarrow \frac{dC}{dw} + \mathrm{p}\frac{dA}{dw} + (1-\mathrm{p})\mathrm{A}\frac{d}{dw}(\ln(\mathrm{C})) = 0$$

With further derivation and simplification, the differential equation can be simplified into this algebraic equation:

$$x^2 - kx^{p+1} + kjx^p - j - (1-p)b\frac{(1-x)(x-j)^2}{1+xb} = 0$$

Where $r_0 = \left(\frac{1-p}{pb}\right)^{p-1}$, $k = \frac{r}{r_0}$, $j = \frac{(1-p)}{bp}$ and $x = 1 - w$. For a detailed account of the derivation process, please refer to Appendix A.

Unfortunately, the complex equation is not readily solvable algebraically due to the presence of the variable exponent p. However we can approximate the solution by setting $p \approx 1$, yielding:

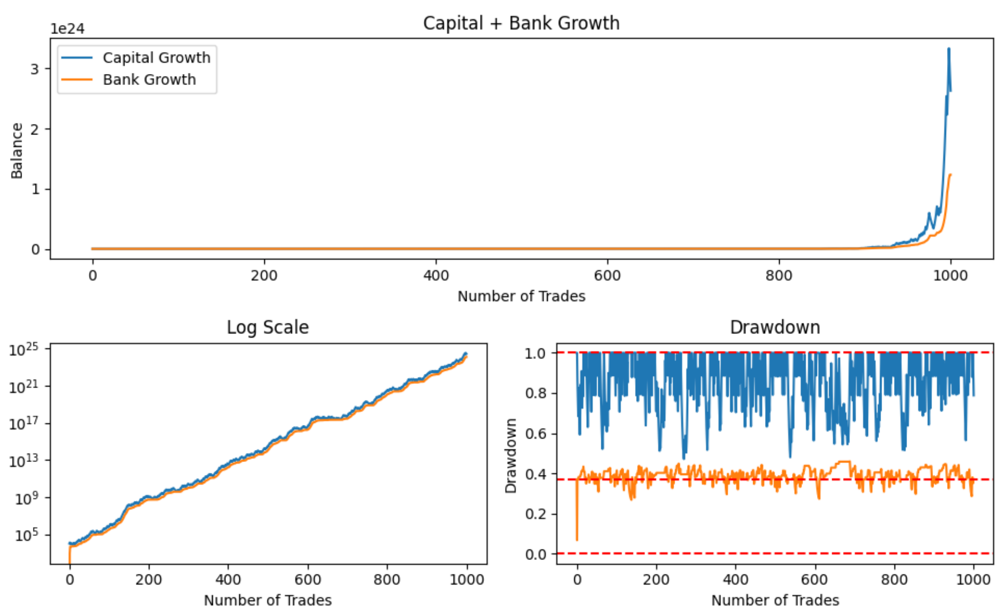
$$(1-k)x^2 + kjx - j = 0$$

The approximate solution become:

$$w \approx 1 + \Delta + \sqrt{\Delta^2 + \frac{2\Delta}{k}} \;\; with \;\; \Delta = \frac{j}{2k(1-k)}$$

Simulation Insights

On the simulations we applied the Bisection method to find solutions for specific values of p, b and r on each trade.



Simulation outcomes robustly demonstrate the Just Criterion's enhanced performance when juxtaposed with the Tax Criterion. In instances of pronounced trade downturns, the Just Criterion strategically pauses tax withdrawals, defaulting to the Full Kelly strategy to expedite capital recovery. This tactic underscores the Just Criterion's agility in adapting to market conditions. However, it is noteworthy that the Just Criterion, while effective, consistently manifests a higher average maximum drawdown—about 63%—over a wide array of simulated trade scenarios. Despite this, it remarkably upholds a growth trajectory on par with the Kelly Criterion, showcasing its potential for sustaining long-term capital increase even amidst substantial market fluctuations.

Understanding the Maximum Drawdown Threshold

The observation that the average maximum drawdown converges to 63% prompts a deeper analytical inquiry: what underlying dynamics within the Just Criterion dictate this specific threshold? To unravel this, we engage with mathematical sequences that accurately model the Just Criterion's trading behavior, placing particular emphasis on the fluctuations in the bank-to-capital ratio during periods of drawdown. This meticulous approach allows us to dissect the trade patterns and rigorously explore the mathematical underpinnings that give rise to this consistent drawdown figure. It is through this lens of systematic analysis that we seek to decode and articulate the factors contributing to the 63% average maximum drawdown, enhancing our comprehension of the Just Criterion's risk parameters.

In the Just Criterion's framework, let's define the sequence (C_n, B_n) that represents the expected Capital and Bank after n trades:

$$\begin{cases} C_{n+1} = C_n + bp(1-w_n)f_nC_n - (1-p)f_nC_n \\ B_{n+1} = B_n + bpw_nf_nC_n \end{cases}$$

Here $r_n = \frac{B_n}{C_n}$, signifies the ratio of the bank to the capital after n trades, $f_n = p - \frac{1-p}{(1-w_n)b}$ is the optimal fraction of capital to bet as per the Kelly Criterion, and $w_n = W(p, b, r_n)$ is the calculated tax rate for the nth trade. where $W(p, b, r_n)$ computes the tax rate according to the Just Criterion's directives.

To ensure the sequence's convergence and avoid divergence as n approaches infinity, we analyze the ratio of capital to bank by normalizing the total balance such that $C_n + B_n = 1$ for all n:

$$\begin{cases} C_{n+1} = \dfrac{C_n + bp(1-w_n)f_nC_n - (1-p)f_nC_n}{C_n + bp(1-w_n)f_nC_n - (1-p)f_nC_n + B_n + bpw_nf_nC_n} \\ \\ B_{n+1} = \dfrac{B_n + bpw_nf_nC_n}{C_n + bp(1-w_n)f_nC_n - (1-p)f_nC_n + B_n + bpw_nf_nC_n} \end{cases}$$

Starting with initial values $C_0 = 1$ and $B_0 = 0$, we can extrapolate the average Capital and Bank ratio at the limit:

$$C_\infty = \lim_{n\to\infty} C_n$$

$$B_\infty = \lim_{n\to\infty} B_n = 1 - C_\infty$$

Over the long term, the most severe scenario for a trading account is the complete erosion of capital. At any point n, the drawdown is bound by the capital C_n, leading to the maximum drawdown at that point being $DD_n = C_n$. Through simulation, we observe the average maximum drawdown converges to a value just a single successful trade above C_∞:

$$DD_{Max} = \frac{C_\infty + C_\infty f_\infty b(1 - w_\infty)}{1 + C_\infty f_\infty b}$$

This formula offers a mathematical understanding of the average maximum drawdown observed across simulated trade sequences. When we analyze the drawdown relative to various probabilities of winning p and risk-reward ratios b, we find that the drawdown gravitates toward 66% at the threshold of non-profitability. This figure represents a theoretical ceiling for drawdown in any trading scenario—indicating a loss of two-thirds of the total value.

Risk Reward Ratio

Wining Probability	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6	6,5	7	7,5	8	8,5	9	9,5	10
5%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
10%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	66%	67%
15%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	66%	66%	66%	66%	66%	66%	65%	65%	65%
20%	100%	100%	100%	100%	100%	100%	100%	100%	66%	66%	66%	65%	65%	65%	64%	64%	64%	63%	63%	62%
25%	100%	100%	100%	100%	100%	100%	66%	66%	65%	65%	64%	64%	63%	63%	62%	62%	61%	61%	61%	60%
30%	100%	100%	100%	100%	66%	66%	65%	65%	64%	63%	63%	62%	61%	61%	60%	60%	59%	59%	58%	58%
35%	100%	100%	100%	66%	66%	65%	64%	63%	62%	61%	61%	60%	59%	59%	58%	58%	57%	57%	56%	56%
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45%	100%	100%	66%	64%	63%	62%	61%	60%	59%	58%	57%	56%	56%	55%	55%	54%	54%	53%	53%	52%
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60%	100%	65%	62%	60%	58%	57%	56%	55%	54%	53%	52%	52%	51%	50%	50%	49%	49%	49%	48%	48%
65%	100%	63%	60%	58%	57%	55%	54%	53%	52%	51%	51%	50%	50%	49%	49%	48%	48%	47%	47%	47%
70%	66%	61%	59%	56%	55%	54%	52%	51%	51%	50%	49%	49%	48%	48%	47%	47%	47%	46%	46%	46%
75%	64%	60%	57%	55%	53%	52%	51%	50%	49%	49%	48%	48%								
80%	62%	58%	55%	53%	52%	50%	50%	49%												
85%	60%	55%	53%	51%	50%															
90%	57%	53%	51%																	
95%	53%																			

Expected Growth Formulation

In calculating the expected growth under the Just Criterion, we begin by considering the scenario where the account balance reaches its maximum drawdown, DD_{Max}, which in turn implies the bank's balance is $1 - DD_{Max}$. From this, we can derive the bank ratio $r_{max} = \frac{1-DD_{Max}}{DD_{Max}}$. By applying the function $w_{max} = W(p, b, r_{max})$, which calculates the optimal tax rate according to the Just Criterion, we can then ascertain the optimal risk fraction $f_{Max} = p - \frac{1-p}{(1-w_{max})b}$. This forms the foundation for determining the expected growth rate EG_∞:

$$EG_\infty = (C_{Max} + b(1 - w_{Max})f_{Max}C_{Max})^p(C_{Max} - f_{Max}C_{Max})^{1-p} + (B_{Max} + bw_{Max}f_{Max}C_{Max})^p(B_{Max})^{1-p}$$

	Risk Reward Ratio																			
Wining Probability	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6	6,5	7	7,5	8	8,5	9	9,5	10
5%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
10%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
15%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,2%	0,3%	0,5%	0,6%	0,8%	1,0%	1,3%
20%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,2%	0,5%	0,8%	1,2%	1,6%	2,0%	2,4%	2,9%	3,3%	3,8%	4,2%
25%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,5%	1,0%	1,5%	2,2%	2,8%	3,6%	4,3%	5,0%	5,8%	6,5%	7,3%	8,0%	8,8%
30%	0,0%	0,0%	0,0%	0,0%	0,0%	0,4%	1,1%	1,9%	2,9%	3,9%	5,0%	6,1%	7,2%	8,3%	9,4%	10,5%	11,6%	12,7%	13,7%	14,7%
35%	0,0%	0,0%	0,0%	0,0%	0,6%	1,6%	2,9%	4,4%	5,9%	7,4%	9,0%	10,5%	12,1%	13,6%	15,1%	16,6%	18,0%	19,4%	20,8%	22,2%
40%	0,0%	0,0%	0,0%	0,6%	2,0%	3,8%	5,7%	7,8%	9,9%	12,0%	14,0%	16,1%	18,1%	20,1%	22,1%	24,0%	25,8%	27,7%	29,5%	31,2%
45%	0,0%	0,0%	0,3%	1,9%	4,2%	6,7%	9,4%	12,2%	14,9%	17,6%	20,3%	22,9%	25,5%	28,0%	30,4%	32,8%	35,1%	37,4%	39,7%	41,9%
50%	0,0%	0,0%	1,3%	4,0%	7,2%	10,6%	14,1%	17,6%	21,1%	24,5%	27,8%	31,0%	34,2%	37,2%	40,3%	43,2%	46,1%	48,9%	51,6%	54,3%
55%	0,0%	0,3%	3,0%	6,8%	11,1%	15,5%	19,9%	24,2%	28,4%	32,5%	36,6%	40,5%	44,3%	48,1%	51,7%	55,3%	58,8%	62,2%	65,6%	68,9%
60%	0,0%	1,3%	5,4%	10,6%	15,9%	21,4%	26,7%	31,9%	37,0%	42,0%	46,8%	51,6%	56,2%	60,7%	65,1%	69,4%	73,6%	77,8%	81,8%	85,8%
65%	0,0%	2,9%	8,7%	15,2%	21,8%	28,3%	34,7%	41,0%	47,0%	53,0%	58,8%	64,4%	69,9%	75,3%	80,6%	85,8%	90,9%	95,8%	#####	#####
70%	0,2%	5,4%	12,9%	20,8%	28,7%	36,5%	44,1%	51,5%	58,7%	65,8%	72,6%	79,4%	85,9%	92,4%	98,7%	#####	#####	#####	#####	#####
75%	1,0%	8,7%	18,0%	27,5%	36,9%	46,1%	55,1%	63,8%	72,4%	80,7%	88,9%	96,9%								
80%	2,7%	12,9%	24,2%	35,5%	46,6%	57,4%	68,0%	78,4%												
85%	5,4%	18,3%	31,8%	45,2%	58,2%															
90%	9,3%	25,2%	41,2%																	
95%	14,9%																			

In a comparative analysis with the Kelly Criterion's optimal growth, the Just Criterion demonstrates superior expected growth exceeding that of the Half Kelly. It manages this while maintaining a long-term maximum drawdown that remains below the total ruin of 100%, a significant improvement in risk management efficiency.

	Risk Reward Ratio																			
Wining Probability	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6	6,5	7	7,5	8	8,5	9	9,5	10
5%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
10%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,1%	60,6%
15%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	59,3%	61,4%	63,2%	64,5%	65,8%	67,0%	68,1%	69,1%	70,1%
20%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	60,9%	63,1%	64,9%	66,4%	67,9%	69,2%	70,4%	71,5%	72,4%	73,3%	74,1%	74,8%
25%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	61,5%	63,9%	65,9%	67,7%	69,3%	70,6%	71,8%	72,9%	73,8%	74,7%	75,4%	76,1%	76,7%	77,3%
30%	0,0%	0,0%	0,0%	0,0%	59,6%	63,0%	65,4%	67,5%	69,2%	70,8%	72,1%	73,2%	74,2%	75,0%	75,8%	76,4%	77,0%	77,6%	78,1%	78,5%
35%	0,0%	0,0%	0,0%	59,7%	63,3%	65,9%	68,1%	69,9%	71,4%	72,7%	73,8%	74,7%	75,5%	76,2%	76,8%	77,3%	77,8%	78,3%	78,7%	79,0%
40%	0,0%	0,0%	0,0%	62,7%	65,7%	68,1%	70,0%	71,5%	72,8%	73,8%	74,7%	75,5%	76,1%	76,7%	77,2%	77,7%	78,1%	78,5%	78,8%	79,1%
45%	0,0%	0,0%	61,3%	64,8%	67,4%	69,5%	71,1%	72,4%	73,5%	74,4%	75,1%	75,8%	76,3%	76,8%	77,3%	77,6%	78,0%	78,3%	78,6%	78,9%
50%	0,0%	0,0%	63,2%	66,3%	68,6%	70,3%	71,7%	72,8%	73,7%	74,5%	75,1%	75,7%	76,2%	76,6%	77,0%	77,3%	77,6%	77,9%	78,2%	78,4%
55%	0,0%	60,8%	64,5%	67,2%	69,2%	70,7%	71,9%	72,8%	73,6%	74,3%	74,8%	75,3%	75,7%	76,1%	76,5%	76,8%	77,1%	77,3%	77,6%	77,8%
60%	0,0%	62,1%	65,4%	67,7%	69,4%	70,7%	71,7%	72,5%	73,2%	73,8%	74,3%	74,8%	75,2%	75,5%	75,8%	76,1%	76,4%	76,6%	76,9%	77,1%
65%	0,0%	63,0%	65,8%	67,8%	69,2%	70,4%	71,3%	72,0%	72,6%	73,2%	73,6%	74,1%	74,4%	74,8%	75,1%	75,4%	75,6%	75,8%	76,1%	76,3%
70%	59,8%	63,5%	65,8%	67,5%	68,8%	69,8%	70,6%	71,3%	71,9%	72,4%	72,8%	73,2%	73,6%	73,9%	74,2%	74,5%	74,8%	75,0%	75,3%	75,5%
75%	60,5%	63,5%	65,5%	66,9%	68,1%	69,0%	69,8%	70,5%	71,0%	71,5%	72,0%	72,4%								
80%	60,7%	63,1%	64,8%	66,1%	67,2%	68,1%	68,9%	69,6%												
85%	60,3%	62,2%	63,8%	65,2%	66,3%															
90%	59,3%	61,0%	62,7%																	
95%	57,4%																			

This refined approach enhances the understanding of the Just Criterion's growth potential relative to the Kelly Criterion, showcasing its efficacy in maximizing long-term growth without succumbing to the risk of total capital depletion.

Towards the Ad’Just Criterion

The Just Criterion marks a significant milestone, offering enhanced growth prospects while mitigating the risk of catastrophic drawdowns, thus outperforming the Half Kelly strategy. However, this advancement prompts a critical discussion about the applicability of universal risk management strategies. Notably, even though the Just Criterion prevents total capital erosion, the level of maximum drawdown it presents calls for a personalized consideration of risk tolerance unique to each investor.

At this crossroad, we shift focus to a more individualized strategy. The Ad'Just Criterion emerges from the flexibility inherent in the Just framework, evolving to meet the diverse needs of traders. It champions the customization of risk parameters, tailoring strategies to align with the specific financial goals and risk appetites of individual market participants. The Ad'Just Criterion is not just a new chapter in the development of trading strategies but a paradigm shift towards a personalized trading experience, where growth potential is harmonized with personal risk thresholds.

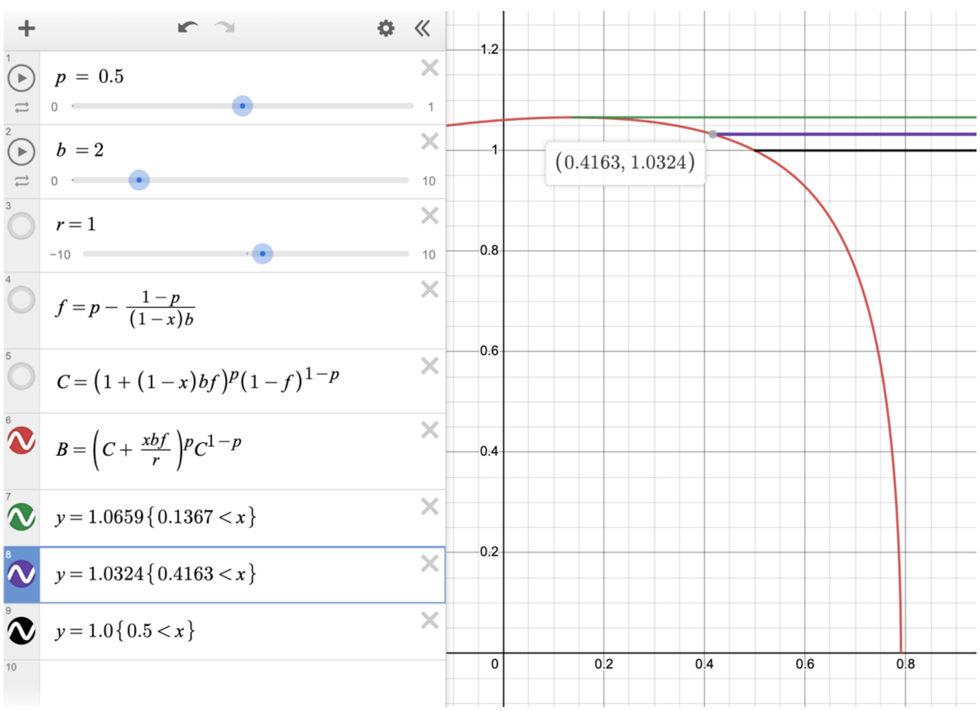
Recognizing the solid foundation laid by the Just Criterion, we now embark on a journey with the Ad'Just Criterion, poised to dissect its nuances and the way it enhances the interplay between strategic performance and individual risk management. The ensuing discussion will navigate through the sophisticated mechanisms of the Ad'Just Criterion, aiming to cultivate a trading strategy as unique and nuanced as the traders who adopt it.

E. The Ad'Just Criterion: Personalized Risk Management

The Ad'Just Criterion signifies a nuanced leap in risk management, integrating personalization into the existing Just Criterion framework. This advancement tailors strategies to the specific preferences of individual traders and institutions, with a particular focus on accommodating maximum drawdown limits—an essential component in risk evaluation.

a. Maximum Drawdown Constraint

For both individual traders and financial institutions, managing the maximum drawdown is a fundamental aspect of risk control, dictating the thresholds within which they are willing to operate. The Ad'Just Criterion caters to this need by providing a mechanism that allows for the establishment of custom drawdown limits. This customization ensures that trading strategies are tailored to align with the specific risk tolerances and regulatory requirements of the investor or institution, thereby upholding a disciplined approach to risk management.



Embracing more conservative strategies, such as adopting an Ad'Just Fraction of the Just Criterion's suggestions—say, at 50% of the calculated maximum from the breakeven tax rate w_0—substantially curtails risk. This adjusted approach, particularly when the account balance matches the bank's, modifies the tax rate to 41.63% from an original 13.67%. Simulation results reveal that this conservative strategy lowers the average maximum drawdown to 37% from the previous 63%.

Mathematical Articulation

By employing the Bisection method, we can first ascertain the peak expected bank return B and then determine a suboptimal yet risk-conscious point that resonates with a trader's risk-return preferences:

$$B = B_0 \left(\frac{B_{max}}{B_0}\right)^{\delta}$$

For a break-even withdrawal rate, the fractions f_0 and w_0 adjust to 0 and, respectively. Subsequently, B_0 is equated to r, transforming the equation to:

$$B = r^{1-\delta} {B_{max}}^{\delta}$$

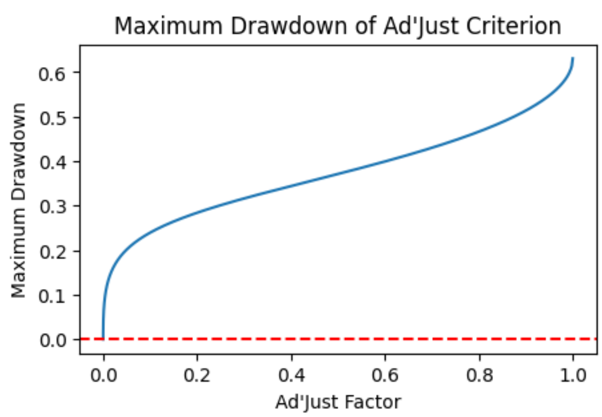
This strategic adjustment allows traders to align their risk management with their comfort levels while still aiming for optimal growth, providing a pragmatic balance between aggressive trading and cautious investment.

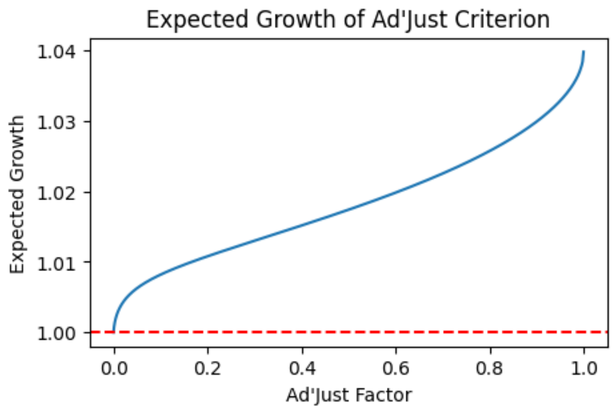
Simulation Insights

Simulating different Ad'Just Fractions of the maximum gives a lower potential average maximum drawdown:

Ad'Just Fraction δ	Maximum Drawdown DD_{max}	Expected Growth EG_{∞}
100%	-63%	+3.98%
50%	-37%	+1.74%
25%	-30%	+1.19%
10%	-24%	+0.82%
5%	-20%	+0.62%
2.5%	-17%	+0.47%
1%	-13%	+0.30%
0.5%	-10%	+0.21%
0.25%	-8%	+0.14%
0.1%	-6%	+0.07%

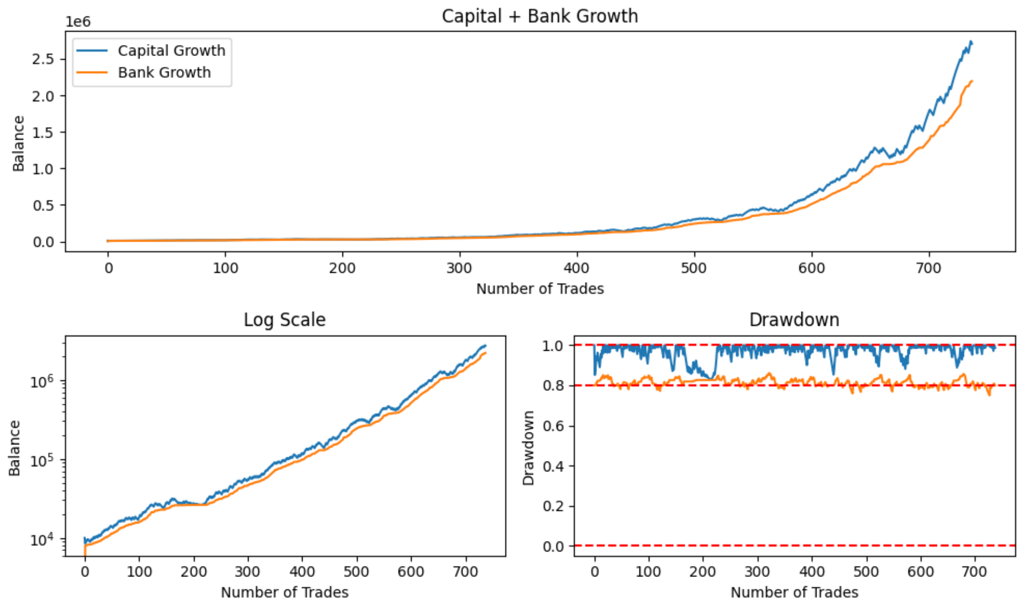
Plotting the values give us this curve for the Maximum Drawdown and Expected Growth:



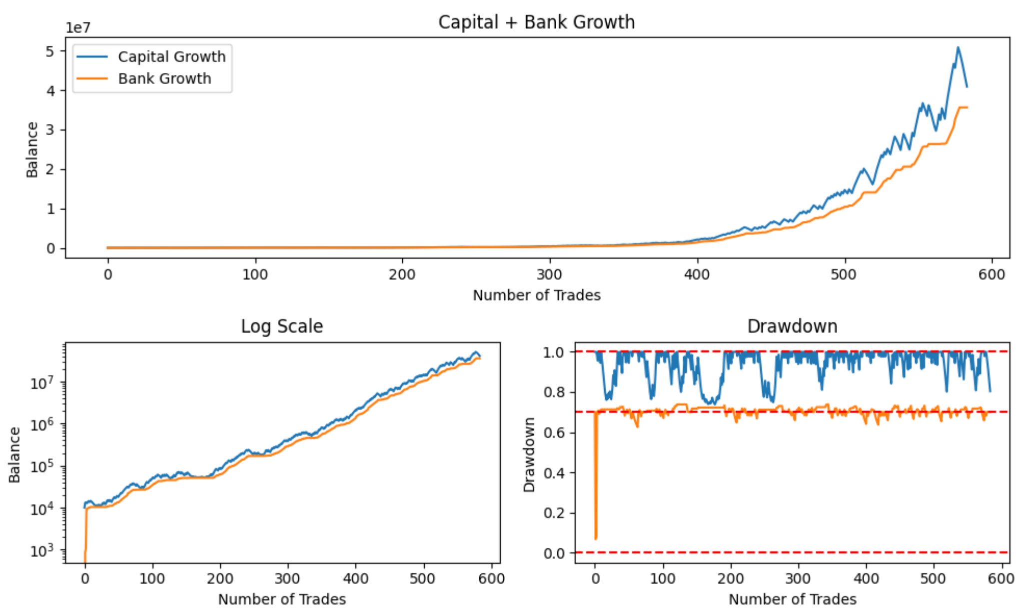


The generated curves serve as a strategic tool, enabling the calibration of the Ad'Just Criterion to meet specific maximum drawdown parameters while concurrently providing projections for expected growth per trade. This analytical capability is instrumental in projecting potential returns within designated risk boundaries. For example, with a strategy that limits maximum drawdown to -10% and encompasses 100 trades annually, one could reasonably forecast an approximate annual return of +23%.

Imagine embarking on an investment journey with an initial capital of $10,000 and an aspiration to grow it to $1 million. By adhering to a -20% maximum drawdown strategy, an estimated 737 trades would be required to achieve this goal. Conversely, initiating this venture with a smaller capital base of $1,000 and allowing for a -30% maximum drawdown would likely span approximately 583 trades to reach the million-dollar milestone. These scenarios provide tangible illustrations of how strategic risk management can significantly influence the trajectory and duration of capital growth within set drawdown constraints.



Additionally, here's the scenario of escalating $1,000 to $1 million under a -30% maximum drawdown constraint:



Crucially, it must be acknowledged that drawdowns have the potential to breach the 30% threshold prescribed by the formula, particularly when a series of losses outstrips the gains withdrawn after successful trades. To mitigate the risk of drawdowns surpassing their designated limits, a proactive strategy must be in place. This strategy could involve the judicious reduction of capital following a loss, effectively curtailing any surplus beyond the established drawdown threshold. Such a measure is designed to enforce the drawdown constraint actively, ensuring that the investment strategy remains within the bounds of the trader's risk parameters.

Transitioning from the discussion of drawdown constraints, it becomes apparent that while theoretical models may predict substantial growth, practical market conditions impose additional considerations. For instance, a model projecting a $10,000 balance with a -20% maximum drawdown potentially escalating to nonillion-dollar figures after 10,000 trades confronts the reality of market liquidity. In actual trading environments, such extreme growth is constrained by the market's capacity to absorb large orders, a factor often overlooked in theoretical simulations.

Setting Realistic Market Limits

While the Ad'Just Criterion adeptly customizes risk levels to match an investor's personal risk tolerance, it also invites a realistic perspective on market limitations. True risk management must account for the fact that the market may not be able to accommodate exceedingly large orders due to liquidity constraints. This realization compels the adoption of a maximum dollar risk strategy, not solely based on personal risk preferences but on the practicalities of market mechanics.

As we venture into strategies that prepare for liquidity-limited scenarios, it becomes essential to establish a ceiling on the dollar amount risked in each trade. This ceiling isn't dictated by personal risk aversion alone but by the liquidity that the market can realistically provide. Preparing for such scenarios ensures that trading strategies remain viable and sustainable, even when faced with the upper bounds of market capacities.

In the subsequent section, we'll explore these market-imposed limits further, delving into how the Ad'Just Criterion can adapt to not only personal risk profiles but also to the overarching realities of market dynamics. This balanced approach aims to align theoretical growth potential with the practical aspects of trading in real-world financial markets.

b. Maximum Dollar Risk Constraint

Transitioning to the discussion of maximum dollar risk, it is pivotal to recognize that trading strategies are not only influenced by personal risk preferences but also by the operational capabilities of the broader market. This becomes particularly evident when theoretical growth projections, such as escalating a $10,000 balance to nonillion-dollar figures, confront the reality of market liquidity constraints. Such exponential growth is not feasible due to the market's inability to provide liquidity for excessively large orders.

In real-world trading, it is imperative to set a cap on the maximum dollar amount risked per trade. This cap is determined not by the trader's risk tolerance alone but by the liquidity available in the market. As traders, we must anticipate scenarios where market conditions cannot support the execution of our orders due to a lack of liquidity. Preparing for these situations is crucial in ensuring that our trading strategies are both realistic and executable.

The Ad'Just Criterion recognizes the necessity of incorporating market-imposed limits into its risk management framework. By setting a maximum dollar risk threshold for each trade, traders can ensure that their strategies remain within the realm of what the market can accommodate. This limit ensures that trades are executed within strategic risk parameters and that the strategy remains adaptable to varying market conditions.

In the scenario of setting a maximum dollar risk per trade, denoted as m_{max} with respect to the current account balance B, the fraction f_{max} of capital at risk is expressed as $f_{max} = \frac{m_{max}}{B}$. This fraction signifies the optimal level of risk per trade. When a trade results in profit, denoted as m_{profit}, the Ad'Just Criterion facilitates the calculation of an appropriate tax rate w_{max} utilizing the Kelly Criterion's framework:

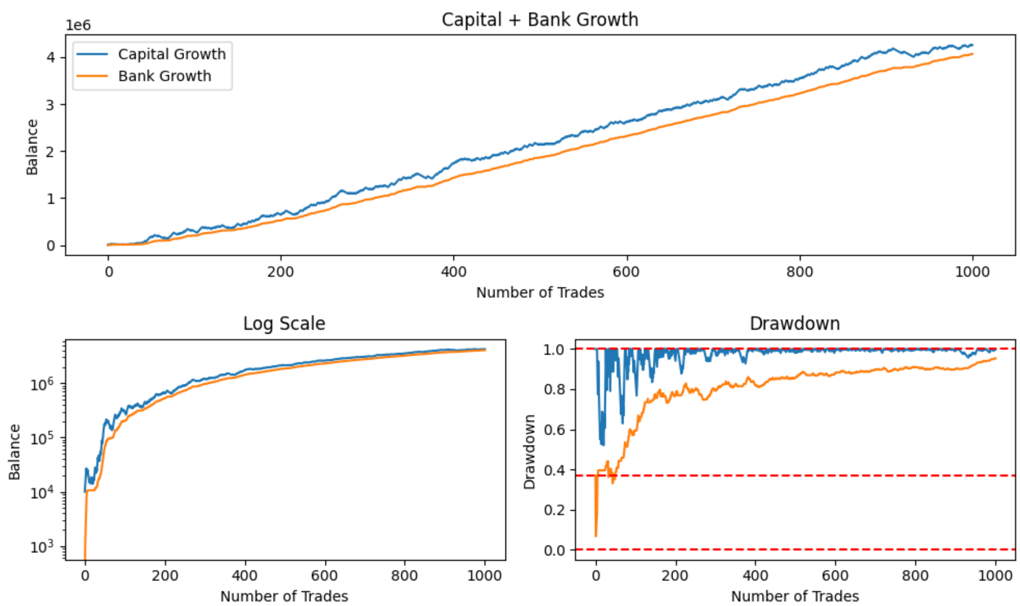
$$f_{max} = p - \frac{1 - p}{(1 - w_{max})\frac{m_{profit}}{m_{max}}}$$

This equation paves the way for an optimized withdrawal strategy, ensuring the trade's risk remains aligned with the trader's defined risk tolerance:

$$w_{max} = 1 - \frac{1-p}{(p - f_{max})\frac{m_{profit}}{m_{max}}}$$

By adhering to a pre-established maximum dollar risk per trade, traders can maintain their account balances within their strategic boundaries. This disciplined approach guarantees that each trade is executed within the trader's risk comfort zone, thus protecting the account from excessive exposure while striving for optimal capital growth.

Simulation Insights



In our simulation where we implement a maximum dollar risk of $10,000, a notable pattern emerges: once this threshold is reached, the account's growth trajectory becomes linear. This occurs as trades are executed at the upper limit of risk tolerance, and subsequent withdrawals are meticulously calculated to maintain account equilibrium. Such an approach guarantees the consistent realization of profits while respecting market-imposed ceilings, thereby ensuring the portfolio's durability and long-term growth potential.

Towards the Justus Criterion

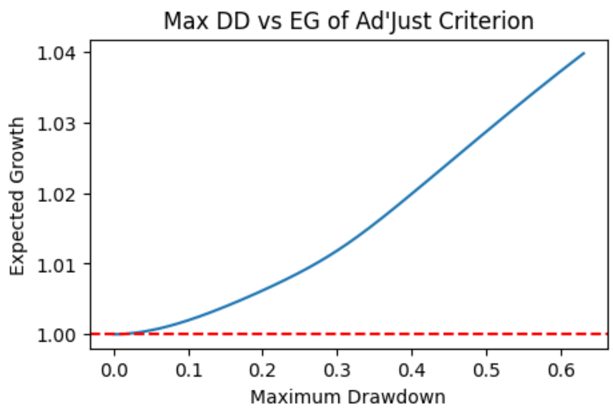
Through this practical application of the Ad'Just Criterion, we navigate the fine line between aggressive capital accumulation and the realities of market functionality. The imposition of a dollar risk ceiling is proven to be effective, ensuring that the trading strategy remains viable within the constraints of market liquidity and regulatory frameworks.

However, the journey of refining risk management strategies extends beyond numerical thresholds; it encompasses the more profound elements of trading psychology and decision-making under uncertainty. Herein, we introduce the Justus Criterion, an innovative strategy meticulously crafted to navigate the uncertainties of winning probabilities, to mitigate the psychological impact of loss aversion, and to enhance growth utility. This approach aims to align the mechanics of trading with the cognitive processes of traders, thereby optimizing the decision-making framework to account for both the unpredictable nature of markets and the subtleties of human behavior in the face of risk.

With the Justus Criterion, we aim to amalgamate the analytical precision of risk management with the nuanced realm of trader psychology. It represents not merely an evolution of strategic thinking but a groundbreaking approach that empathetically accounts for the trader's psychological landscape. Our forthcoming discussion will delve into the Justus Criterion, dissecting its role as the pinnacle of risk management strategies that holistically blend empirical science with the art of trading psychology, serving as a lighthouse amidst the stormy volatility of the financial markets.

F. The Justus Criterion: Harmonizing Risk and Human Behavior

The exploration of the Justus Criterion brings us to the intersection of quantitative risk management and the qualitative aspects of human psychology. A pivotal element in formulating a robust strategy is understanding the behavioral curve that emerges from varying maximum drawdown limits, which provides strategic insights into potential growth.

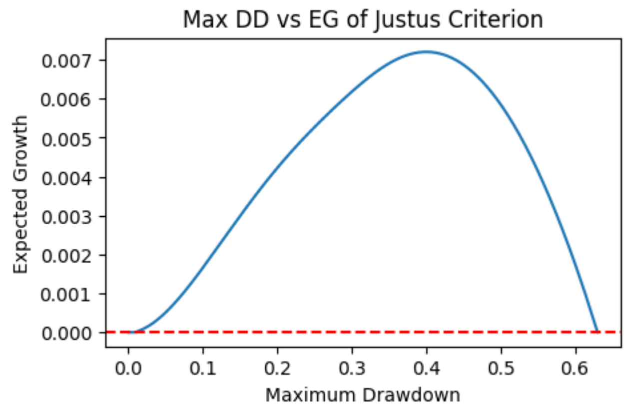


While an ideal mathematical model might advocate for a 63% maximum drawdown to foster sustained long-term growth, we must consider three essential factors that influence the practicality of such a strategy: the uncertainties inherent in win probability p and risk-reward ratio b, the psychological weight of loss aversion, and the relative utility derived from growth.

Reducing Uncertainty

Mitigating the effects of uncertainty necessitates a strategic approach that circumvents the theoretical maximum drawdown threshold. This involves exploring the intricate dynamics between Expected Growth EG and Drawdown DD, highlighting the critical point where a 63% drawdown equates to a null Expected Growth per unit of risk, as delineated by the pertinent equation:

$$\log(EG)\,(1 - \frac{DD}{DD_{Max}})$$

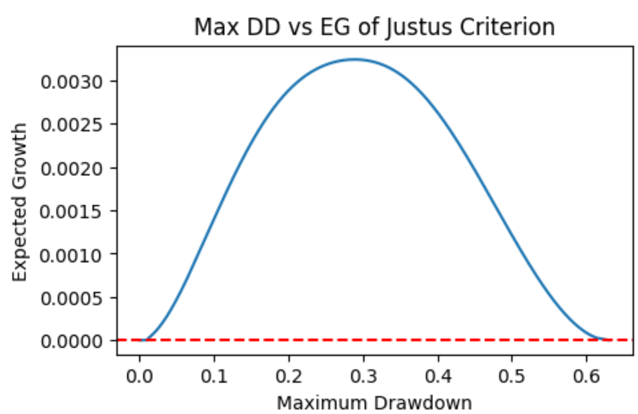


Our analysis identifies an optimal point at a 40% maximum drawdown, accompanied by a +2% expected growth rate. This represents an assertive yet statistically conscious approach that allows for uncertainties in winning probabilities.

Considering Loss Aversion

Addressing the cognitive bias of loss aversion requires modifying the loss term to reflect the psychological reality that losses typically feel more impactful than gains, often by a factor of two [4]:

$$\log(EG)\,(1-\frac{DD}{DD_{Max}})^2$$

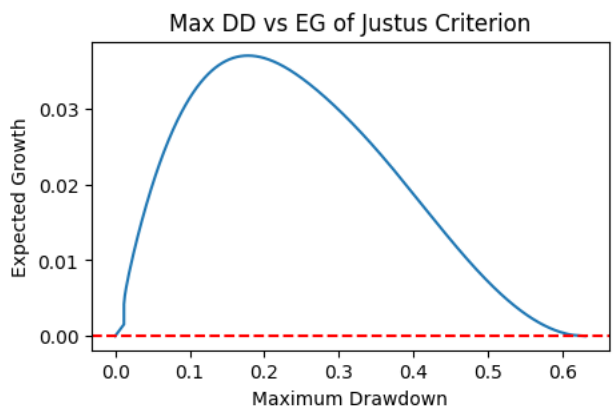


This refinement adjusts our strategy to a more conservative 29% maximum drawdown, with an expected growth of +1.1%, balancing potential returns with the discomfort of losses.

Assessing Growth Utility

Further, to integrate the concept of growth utility, we apply the square root to the growth term [5]:

$$\sqrt{\log(EG)}\,(1-\frac{DD}{DD_{Max}})^2$$



This final calibration, which results in an 18% maximum drawdown and a +0.5% expected growth per trade, epitomizes a conservative yet effective money management strategy.

In-Depth Analysis of the Justus Criterion

In our extensive analysis, we've calculated various outcomes that explore the complex interplay between maximum drawdown and expected growth, under a range of win probabilities and risk-reward scenarios. Our findings reveal that the Justus Criterion's parameters for maximum drawdown and expected growth not only adhere to statistical models but also dovetail with the intuitive risk-reward judgments intrinsic to human decision-making. This intersection of empirical data and innate risk assessment is the hallmark of the Justus Criterion's practicality, presenting a strategy that is as analytically robust as it is intuitively resonant for traders.

Wining Probability \ Risk Reward Ratio	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6	6,5	7	7,5	8	8,5	9	9,5	10
5%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
10%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	28%	22%
15%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	25%	21%	20%	20%	19%	19%	19%	19%	19%
20%	100%	100%	100%	100%	100%	100%	100%	100%	20%	20%	19%	19%	19%	19%	19%	18%	18%	18%	18%	18%
25%	100%	100%	100%	100%	100%	100%	20%	19%	19%	19%	18%	18%	18%	18%	18%	17%	17%	17%	17%	17%
30%	100%	100%	100%	100%	22%	19%	19%	18%	18%	18%	18%	17%	17%	17%	17%	17%	16%	16%	16%	16%
35%	100%	100%	100%	20%	19%	19%	18%	18%	18%	17%	17%	17%	16%	16%	16%	16%	16%	15%	15%	15%
40%	100%	100%	100%	19%	18%	18%	17%	17%	17%	16%	16%	16%	16%	15%	15%	15%	15%	15%	14%	14%
45%	100%	100%	19%	18%	18%	17%	17%	16%	16%	16%	15%	15%	15%	15%	14%	14%	14%	14%	14%	13%
50%	100%	100%	18%	18%	17%	16%	16%	16%	15%	15%	15%	14%	14%	14%	14%	13%	13%	13%	13%	13%
55%	100%	18%	18%	17%	16%	16%	15%	15%	14%	14%	14%	13%	13%	13%	13%	13%	12%	12%	12%	12%
60%	100%	18%	17%	16%	15%	15%	14%	14%	14%	13%	13%	13%	12%	12%	12%	12%	12%	12%	11%	11%
65%	100%	18%	16%	15%	15%	14%	14%	13%	13%	12%	12%	12%	12%	11%	11%	11%	11%	11%	11%	10%
70%	23%	17%	15%	14%	14%	13%	13%	12%	12%	12%	11%	11%	11%	11%	11%					
75%	17%	16%	15%	14%	13%	13%	12%	12%												
80%	17%	15%	14%	13%																
85%	16%	14%																		
90%	13%																			
95%																				

Wining Probability \ Risk Reward Ratio	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6	6,5	7	7,5	8	8,5	9	9,5	10
5%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
10%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
15%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,1%	0,1%	0,1%	0,2%
20%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,1%	0,2%	0,2%	0,3%	0,3%	0,4%	0,5%	0,6%	0,6%
25%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,1%	0,2%	0,3%	0,4%	0,5%	0,6%	0,7%	0,9%	1,0%	1,1%	1,3%	1,4%
30%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,2%	0,4%	0,5%	0,7%	0,9%	1,1%	1,3%	1,5%	1,7%	1,9%	2,1%	2,4%	2,6%
35%	0,0%	0,0%	0,0%	0,0%	0,1%	0,2%	0,4%	0,6%	0,8%	1,1%	1,3%	1,6%	1,9%	2,2%	2,5%	2,8%	3,1%	3,4%	3,8%	4,1%
40%	0,0%	0,0%	0,0%	0,1%	0,2%	0,5%	0,8%	1,1%	1,4%	1,8%	2,2%	2,6%	3,0%	3,4%	3,8%	4,2%	4,7%	5,1%	5,5%	5,9%
45%	0,0%	0,0%	0,0%	0,2%	0,5%	0,9%	1,3%	1,8%	2,3%	2,8%	3,3%	3,8%	4,3%	4,9%	5,4%	6,0%	6,5%	7,0%	7,6%	8,2%
50%	0,0%	0,0%	0,2%	0,5%	1,0%	1,5%	2,1%	2,7%	3,3%	3,9%	4,6%	5,2%	5,9%	6,6%	7,3%	8,0%	8,7%	9,4%	10,0%	10,7%
55%	0,0%	0,0%	0,4%	0,9%	1,5%	2,2%	3,0%	3,7%	4,5%	5,3%	6,1%	6,9%	7,8%	8,6%	9,4%	10,3%	11,1%	11,9%	12,8%	13,6%
60%	0,0%	0,2%	0,7%	1,4%	2,3%	3,1%	4,0%	5,0%	5,9%	6,9%	7,9%	8,9%	9,9%	10,9%	11,9%	12,8%	13,8%	14,8%	15,8%	16,8%
65%	0,0%	0,4%	1,2%	2,1%	3,2%	4,2%	5,4%	6,5%	7,7%	8,8%	10,0%	11,2%	12,3%	13,5%	14,6%	15,8%	17,0%	18,1%	19,2%	20,3%
70%	0,0%	0,7%	1,8%	3,0%	4,3%	5,7%	7,0%	8,4%	9,8%	11,1%	12,6%	13,9%	15,3%	16,7%	18,1%					
75%	0,1%	1,2%	2,6%	4,2%	5,8%	7,5%	9,2%	10,9%												
80%	0,3%	1,9%	3,8%	5,7%																
85%	0,7%	3,0%																		
90%	1,2%																			
95%																				

Crucially, the Justus Criterion holds that the maximum drawdown does not exceed 30% at the limits of profitability for all scenarios and remains below 20% within the plausible winning rates of 35% to 65%. This adherence to commonly accepted trading rules regarding maximum drawdown is both striking and validating.

Further insights gleaned from the criterion reveal that the average risk per trade relative to the total value (capital plus bank) remains consistently below 5%, irrespective of the strategy's profitability. Within the bounds of plausible winning rates, this risk limit tightens to 3%, while hovering around 1% at the edge of profitability — aligning with the general risk management practices of seasoned traders and investors.

Risk Reward Ratio

Wining Probability	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6	6,5	7	7,5	8	8,5	9	9,5	10
5%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
10%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
15%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,1%	0,2%	0,2%	0,2%	0,2%	0,3%	0,3%
20%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,2%	0,2%	0,3%	0,3%	0,4%	0,4%	0,4%	0,5%	0,5%	0,5%	0,6%
25%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,2%	0,3%	0,3%	0,4%	0,5%	0,6%	0,6%	0,7%	0,7%	0,7%	0,8%	0,8%	0,8%	0,9%
30%	0,0%	0,0%	0,0%	0,0%	0,0%	0,3%	0,4%	0,5%	0,6%	0,7%	0,8%	0,9%	0,9%	1,0%	1,0%	1,1%	1,1%	1,1%	1,2%	1,2%
35%	0,0%	0,0%	0,0%	0,0%	0,4%	0,6%	0,7%	0,8%	0,9%	1,0%	1,1%	1,2%	1,2%	1,3%	1,3%	1,4%	1,4%	1,5%	1,5%	1,5%
40%	0,0%	0,0%	0,0%	0,4%	0,7%	0,9%	1,0%	1,2%	1,3%	1,4%	1,4%	1,5%	1,6%	1,6%	1,7%	1,7%	1,7%	1,8%	1,8%	1,8%
45%	0,0%	0,0%	0,4%	0,7%	1,0%	1,2%	1,4%	1,5%	1,6%	1,7%	1,8%	1,8%	1,9%	1,9%	2,0%	2,0%	2,0%	2,1%	2,1%	2,1%
50%	0,0%	0,0%	0,7%	1,1%	1,4%	1,6%	1,7%	1,8%	1,9%	2,0%	2,1%	2,2%	2,2%	2,2%	2,3%	2,3%	2,3%	2,3%	2,4%	2,4%
55%	0,0%	0,4%	1,1%	1,5%	1,7%	1,9%	2,1%	2,2%	2,3%	2,4%	2,4%	2,5%	2,5%	2,5%	2,5%	2,6%	2,6%	2,6%	2,6%	2,6%
60%	0,0%	0,8%	1,5%	1,9%	2,1%	2,3%	2,4%	2,5%	2,6%	2,7%	2,7%	2,7%	2,8%	2,8%	2,8%	2,8%	2,8%	2,8%	2,8%	2,8%
65%	0,0%	1,3%	2,0%	2,3%	2,6%	2,7%	2,8%	2,9%	2,9%	3,0%	3,0%	3,0%	3,0%	3,0%	3,0%	3,0%	3,0%	3,0%	3,0%	3,0%
70%	0,5%	1,8%	2,5%	2,8%	3,0%	3,2%	3,2%	3,3%	3,3%	3,3%	3,4%	3,3%	3,3%	3,3%	3,3%					
75%	0,9%	2,5%	3,1%	3,4%	3,6%	3,7%	3,7%	3,8%												
80%	1,8%	3,2%	3,8%	4,1%																
85%	2,9%	4,4%																		
90%	3,6%																			
95%																				

With respect to profit withdrawals, the Justus Criterion ensures that withdrawals do not surpass 90% under any trading scenario, with a minimum of 10% at the edge of profitability. This disciplined approach to withdrawals further cements the criterion's alignment with prudent trading practices.

Risk Reward Ratio

Wining Probability	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6	6,5	7	7,5	8	8,5	9	9,5	10
5%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
10%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
15%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	2%	10%	16%	20%	24%	28%	31%	34%	37%
20%	0%	0%	0%	0%	0%	0%	0%	0%	4%	17%	23%	28%	33%	37%	40%	43%	46%	48%	50%	52%
25%	0%	0%	0%	0%	0%	0%	12%	21%	28%	34%	39%	43%	47%	50%	52%	55%	57%	59%	60%	62%
30%	0%	0%	0%	0%	3%	18%	28%	35%	41%	46%	50%	53%	56%	59%	61%	63%	64%	66%	67%	69%
35%	0%	0%	0%	3%	21%	32%	40%	46%	51%	55%	58%	61%	63%	65%	67%	69%	70%	71%	73%	74%
40%	0%	0%	0%	21%	34%	43%	50%	55%	59%	62%	65%	67%	69%	71%	72%	73%	75%	76%	77%	77%
45%	0%	0%	15%	33%	44%	52%	57%	61%	65%	68%	70%	72%	73%	75%	76%	77%	78%	79%	80%	80%
50%	0%	0%	28%	43%	52%	59%	63%	67%	70%	72%	74%	76%	77%	78%	79%	80%	81%	82%	82%	83%
55%	0%	15%	39%	51%	59%	65%	69%	72%	74%	76%	78%	79%	80%	81%	82%	83%	83%	84%	85%	85%
60%	0%	28%	48%	58%	65%	70%	73%	76%	78%	79%	81%	82%	83%	84%	84%	85%	86%	86%	87%	87%
65%	0%	39%	56%	65%	70%	74%	77%	79%	81%	82%	83%	84%	85%	86%	86%	87%	87%	88%	88%	89%
70%	12%	49%	63%	70%	75%	78%	80%	82%	83%	84%	85%	86%	87%	87%	88%					
75%	28%	58%	69%	75%	78%	81%	83%	84%												
80%	42%	65%	74%	79%																
85%	55%	72%																		
90%	68%																			
95%																				

This analysis of the Justus Criterion reveals that, beyond its complex calculations for money management, it also offers straightforward, practical rules for everyday trading:

- Maintain Maximum Drawdown below 20% to preserve capital.
- Keep Risk per trade within a cautious 0.5-1% range, capping it at 3%.
- Withdraw 10-30% of profits following each win to balance growth with cash flow.

These practical rules make the Justus Criterion's intricate strategies accessible and applicable for traders at all levels.

General formula

Transitioning from practical application to theoretical formulation, the Justus Criterion encapsulates its strategy within a foundational formula:

$$\sqrt[\alpha]{\log(EG)}\,(1-\frac{DD}{DD_{Max}})^{\beta}$$

Here, α and β are coefficients representing growth utility and loss aversion, respectively. They are fine-tuned to reflect an individual's risk preferences and psychological tendencies. The Justus Criterion adopts a setting where $\alpha = \beta = 2$, striking a harmonious balance that considers both the utility of growth and the psychological impact of losses.

In summary, the Justus Criterion stands as a sophisticated advancement in trading strategy development. It melds the analytical precision of risk management with the nuanced understanding of investor psychology, setting a new paradigm for prudent, growth-focused trading. It exemplifies the synergy of empirical rigor with a profound comprehension of human behavior, offering traders a strategy that is as intuitively acceptable as it is analytically sound.

As we conclude our examination of the Justus Criterion, we recognize its integral role in this research, paving the way for our final reflections on this innovative approach to trading.

5. Conclusion

In conclusion, this paper has charted the evolutionary course of risk management strategies in trading, from the high-risk allure of the YOLO Criterion to the refined complexity of the Justus Criterion. Our journey began with the bold gambles of the YOLO strategy, setting the stage for a more disciplined approach with the Kelly Criterion, which introduced a probabilistic model for optimizing bet sizes to enhance long-term capital growth. The subsequent development of the Tax Criterion marked a significant stride towards harmonizing growth with capital preservation through systematic profit withdrawals.

Progressing further, the Just Criterion brought a dynamic bank/balance ratio into play, adapting trading strategies to the ever-changing financial landscape and amplifying growth opportunities. The Ad'Just Criterion then personalized risk management, allowing traders to tailor their strategies to individual risk appetites and market conditions, underscoring the value of adaptability and personalization in trading.

The Justus Criterion, as the culmination of this evolutionary journey, melds quantitative precision with an understanding of trader psychology, accounting for win probability uncertainties, loss aversion, and growth utility. This criterion represents a holistic approach to trading strategies, acknowledging both the mathematical and human elements of decision-making in the financial markets.

While this paper has not delved into the empirical application of the Justus Criterion in parallel trading environments, preliminary explorations suggest that its principles could be robust across a diversified asset range. This promising avenue, involving multiple assets managed under a shared bank account framework, has shown in initial simulations that the portfolio's total drawdown may not exceed individual asset risk thresholds. These insights pave the way for future research to mathematically validate and potentially integrate the Justus Criterion in multi-asset trading scenarios.

Looking ahead, the broader implications of the Justus Criterion and its adaptability to continuous return assets and comprehensive portfolio management strategies show promise for refining its utility across a range of financial instruments and investment scenarios.

Moreover, the principles underlying the Justus Criterion hold promise for application in fields outside of trading, such as optimizing legal taxation rates to balance treasury revenue and economic vitality, representing a fascinating area of exploration.

In essence, the journey from the YOLO to the Justus Criterion encapsulates a shift from speculative risk-taking to informed, strategic decision-making. Each step in this progression has added a vital piece to the puzzle of optimal trading strategy design, progressively enhancing our understanding of risk management in trading. As we venture into the future, the promise of new methodologies and applications invites us to reimagine the boundaries between economics, psychology, and the dynamic world of financial markets, heralding an era of innovation and interdisciplinary synergy.

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Appendix A: Simplification of the Just Criterion Equation

This appendix details the mathematical process used to simplify the differential equation form of the Just Criterion into its algebraic equation counterpart. We will walk through the necessary algebraic manipulations and calculus operations to transition from the initial complex representation to a more simplified and usable form as presented within the main text of the paper.

We will start by the main differential equation:

$$\frac{dC}{dw} + \mathrm{p}\frac{dA}{dw} + (1 - \mathrm{p})\mathrm{A}\frac{d}{dw}(\ln(\mathrm{C})) = 0$$

Let's simplify C and find the derivative $\frac{dC}{dw}$:

$$C = (1 + (1 - w)bf)^p(1 - f)^{1-p}$$

$$C = (1 + (1 - w)bp - (1 - p))^p \times (1 - p + \frac{(1 - p)}{(1 - w)b})^{1-p}$$

$$C = p^p((1 - w)\ b + 1))^p \times (1 - p + \frac{(1 - p)}{(1 - w)b})^{1-p}$$

$$C = \left(\frac{1 - p}{(1 - w)b}\right)^{1-p} p^p(1 + (1 - w)b)$$

$$C = p\left(\frac{1 - p}{pb}\right)^{1-p}(1 - w)^{p-1} + bp\left(\frac{1 - p}{pb}\right)^{1-p}(1 - w)^p$$

$$\frac{dC}{dw} = -(1 - p)p\left(\frac{1 - p}{pb}\right)^{1-p}(1 - w)^{p-2} - bp^2\left(\frac{1 - p}{pb}\right)^{1-p}(1 - w)^{p-1}$$

$$\frac{dC}{dw} = -\frac{(1-p)p}{r_0}(1 - w)^{p-2} - \frac{bp^2}{r_0}(1 - w)^{p-1} \quad \text{with } r_0 = \left(\frac{1-p}{pb}\right)^{p-1}$$

Now let's calculate $\frac{d}{dw}(\ln(C))$:

$$\ln(C) = (1 - p)(\ln(1 - p) - \ln(1 - w) - \ln(b)) + p\ln(p) + \ln(1 + (1 - w)b)$$

$$\frac{d}{dw}(\ln(C)) = \frac{1 - p}{1 - w} - \frac{b}{1 + (1 - w)b}$$

$$\frac{d}{dw}(\ln(C)) = \frac{(1-p) + (1-p)(1-w)b - (1-w)b}{(1-w)(1+(1-w)b)}$$

$$\frac{d}{dw}(\ln(C)) = \frac{(1-p) - (1-w)bp}{(1-w)(1+(1-w)b)}$$

Now for the derivative of $\frac{dA}{dw}$:

$$\frac{dA}{dw} = \frac{d}{dw}(\frac{w((1-w)pb-(1-p))}{r(1-w)}) = \frac{d}{dw}(\frac{wbp}{r} + \frac{w(1-p)}{r(1-w)})$$

$$\frac{dA}{dw} = \frac{bp}{r} - \frac{(1-p)}{r(1-w)^2} = \frac{bp(1-w)^2-(1-p)}{r(1-w)^2}$$

$$\frac{dA}{dw} = \frac{bp(1-w)^2-(1-p)}{r(1-w)^2}$$

The final equation will be this:

$$\frac{dC}{dw} + \mathrm{p}\frac{dA}{dw} + (1-\mathrm{p})\mathrm{A}\frac{d}{dw}(\ln(\mathrm{C})) = 0$$

$$\frac{(1-p)p}{r_0}(1-w)^{p-2} - \frac{bp^2}{r_0}(1-w)^{p-1} + \frac{bp^2(1-w)^2 - p(1-p)}{r(1-w)^2} - (1-p)\frac{w((1-w)pb-(1-p))((1-p)-(1-w)bp)}{r(1-w)^2(1+(1-w)b)} = 0$$

$$\frac{r}{r_0}(1-p)(1-w)^p + \frac{r}{r_0}bp(1-w)^{1+p} + bp(1-w)^2 - (1-p) - (1-p)\frac{w((1-w)pb-(1-p))((1-p)-(1-w)bp)}{p(1+(1-w)b)} = 0$$

$$\frac{r}{r_0}(1-w)^p - \frac{r}{r_0}\frac{bp}{(1-p)}(1-w)^{1+p} + \frac{bp}{(1-p)}(1-w)^2 - 1 - \frac{w((1-w)pb-(1-p))^2}{p(1+(1-w)b)} = 0$$

$$\frac{r}{r_0}(1-w)^p - \frac{r}{r_0}\frac{1}{j}(1-w)^{1+p} + \frac{1}{j}(1-w)^2 - 1 - \frac{w((1-w)pb-(1-p))^2}{p(1+(1-w)b)} = 0$$

$$\frac{r}{r_0}(1-w)^p - \frac{r}{r_0}\frac{1}{j}(1-w)^{1+p} + \frac{1}{j}(1-w)^2 - 1 - \frac{wpb^2((1-w)-\frac{(1-p)}{pb})^2}{(1+(1-w)b)} = 0$$

$$(1-w)^2 - k(1-w)^{1+p} + jk(1-w)^p - j - (1-p)b\frac{w((1-w)-j)^2}{(1+(1-w)b)} = 0$$

With $r_0 = \left(\frac{1-p}{pb}\right)^{p-1}$, $k = \frac{r}{r_0}$, $j = \frac{(1-p)}{bp}$ and $x = 1-w$ the final equation will be:

$$x^2 - kx^{p+1} + kjx^p - j - (1-p)b\frac{(1-x)(x-j)^2}{1+xb} = 0$$

Appendix B: Python Script

The following Python Script implement the various criteria discussed in this paper, including the Justus Criterion. The code applies the Bisection method for finding optimal tax rates and withdrawal strategies under different risk management scenarios. Comments have been provided to explain the functionality of each segment.

```python
###############################
## Code by Justus L. Ludovic ##
## Feb 2024                  ##
###############################

import random
import matplotlib.pyplot as plt
import numpy as np

# p is probability of winning
# b is the risk-reward ratio
# r is the bank/balance ratio

# Kelly criterion for a withdrawal tax rate x
def _f(x,p,b):
    return ((1-x)*p*b-(1-p))/((1-x)*b)

# Equation of the Just Criterion that will be solved by the Bisection method in find_w
def _n(x,p,b,r):
    r0 = ((1-p)/(p*b))**(p-1)
    k = r/r0
    j = (1-p)/(p*b)

    return (1-x)**2-k*(1-x)**(1+p)+k*j*(1-x)**p-j-(1-p)*b*x*(x-(1-j))**2/(1+(1-x)*b)

# The expected growth of Bank B from tax rate x, it will be used on the second Bisection method in find_w
def _bb(x,p,b,r):
    try :
        F = _f(x,p,b)
        C = (1+(1-x)*b*F)**p*(1-F)**(1-p)
        B = (r*C+x*b*F)**p*(r*C)**(1-p)

        return B
    except :
        return -np.inf

# Find the optimal tax rate according to the Adjust factor d using the Bisection method two times
def find_w0(p,b,r,d):
    x1 = 1-(1-p)/(p*b)
    x0 = -1000

    while x1-x0 > 0.0000001 :
        if _n((x1+x0)/2,p,b,r) >= 0 :
            x0 = (x1+x0)/2
        else:
            x1 = (x1+x0)/2
    w = x1

    if d < 1 and r > 0:
        x1 = 1-(1-p)/(p*b)
        x0 = w

        mx = _bb(w,p,b,r)
        mn = _bb(1-(1-p)/(p*b),p,b,r)

        limit = mn*(mx/mn)**d

        while x1-x0 > 0.0000001 :
            if _bb((x1+x0)/2,p,b,r) - limit >= 0 :
                x0 = (x1+x0)/2
            else:
                x1 = (x1+x0)/2
        w = x1

    return max(w,0)

# Find the optimal withdrawal when the maximum amount risk is reached
def find_wp(f,b,p):
    return 1-(1-p)/(b*(p-f))

# Find the maximum drawdown and expected growth for an adjust fraction d
```

```python
def find_maxdd_growth(p,b,d):
    balance = 0.01
    bank = 1-balance

    dd0 = -1
    dd1 = -2

    for i in range(10000):
        if p*b-(1-p)<=0:
            maxdd = 0

        r = bank/balance
        w = find_w0(p,b,r,d)
        f = _f(w,p,b)
        risk = balance*f

        total_old = balance + bank

        balance = balance + risk*(1-w)*b*p - risk*(1-p)
        bank = bank + risk*w*b*p

        total = balance + bank
        balance = balance / total
        bank = bank / total

        maxdd = (balance + risk*b*(1-w))/(1 + risk*b)

        if abs(maxdd-dd0)<0.000001 or abs(maxdd-dd1)<0.000001 and i > 1000 :
            break

        dd1 = dd0
        dd0 = maxdd

    balance_g = maxdd
    bank_g = 1-maxdd

    r_g = bank_g/balance_g
    w_g = find_w0(p,b,r_g,d)
    f_g = _f(w_g,p,b)
    risk_g = balance_g*f_g

    balance_g = (balance_g + risk_g*(1-w_g)*b)**p*(balance_g - risk_g)**(1-p)
    bank_g = (bank_g+risk_g*w_g*b)**p*bank_g**(1-p)

    growth = balance_g + bank_g

    return maxdd,growth

# Find the Adjust fraction d for a personalized maximum drawdown using the Bisection method
def find_d(p,b,maxdd):
    x1 = 1
    x0 = 0

    while x1-x0 > 0.00001 :
        n = find_maxdd_growth(p,b,(x1+x0)/2)[0] - maxdd
        if n <= 0 :
            x0 = (x1+x0)/2
        else:
            x1 = (x1+x0)/2
    d = x1

    return d

# The Justus Criterion function
# Find the Justus Ad'Just fraction d, maximum drawdown and expected growth accoring to the coeficcients of growth utility Alpha and loss aversion Beta
def find_justus_d(p,b, alpha, beta):
    x1 = 1
    x0 = 0

    maxdd, growth = find_maxdd_growth(p,b,1)

    while x1-x0 > 0.0001 :
        dd, growth = find_maxdd_growth(p,b,(x1+x0)/2)
        value0 = np.log(growth)**(1/alpha)*(1-dd/maxdd)**beta

        dd, growth = find_maxdd_growth(p,b,(x1+x0)/2 + 0.0001)
        value1 = np.log(growth)**(1/alpha)*(1-dd/maxdd)**beta

        n = -(value1 - value0)/0.0001

        if n <= 0 :
            x0 = (x1+x0)/2
        else:
            x1 = (x1+x0)/2

    d = x1

    return d, dd, growth
```

```python
# Simulation Function of the Justus Criterion
def simulate(capital, bank, p, b, maxdd, maxrisk, n):
    capital_log = [capital]
    bank_log = [bank]

    if maxdd <= 0 :
        d, maxdd, growth = find_justus_d(p,b,2,2) # Apply Justus Criterion with Alpha and Beta equals to 2
    else :
        d = find_d(p,b,maxdd) # Find Ad'Just Fraction
        maxdd, growth = find_maxdd_growth(p,b,d) # Getting the growth rate

    both = [capital + bank] # List to log total value of both capital and bank
    dd = [1] # List to log total value drawdow
    bankdd = [1-maxdd] # List to log bank ratio relative to drawdown

    win = [1/(b+1)]*100 #Starting as the break even winning rate for 100 past trades

    # Simulation
    for i in range(n):
            # The probability calculated from the last 100 trades
            p0 = sum(win[-100:])/len(win[-100:])

            # The bank/balance ratio r, just tax rate w, and kelly fraction f
            r = bank_log[-1]/capital_log[-1]
            w = find_w0(p0,b,r,d)
            f = _f(w,p0,b)

            # Risk in dollar
            risk = f*capital_log[-1]

            # If risk is greater than the max dollar risk apply the Ad'Just Max formula
            if risk > maxrisk  and maxrisk > 0 :
                risk = maxrisk
                f = risk/capital_log[-1]
                w = max(w,find_wp(f,b,p0))
                if w < 0 :
                    w = 0

            if random.random() < p : # If the trade is a win
                capital_log.append(capital_log[-1] + risk * b * (1-w))
                bank_log.append(bank_log[-1] + risk * b * w)
                win.append(1) # Loging the trade as win
            else : # If the trade is a loss
                capital_log.append(capital_log[-1] - risk)
                bank_log.append(bank_log[-1])
                win.append(0) # Loging the trade as loss

                # In case of a loss after wins, this part of the code withdrawal excessive capital to maintain the
capital ratio as the maximum drawdown limit
                max_wealth = max(both)
                if 1-bank_log[-1]/max_wealth >= maxdd and max_wealth*(1-maxdd) <= (bank_log[-1] + capital_log[-1]):
                    total = (bank_log[-1] + capital_log[-1])
                    bank_log[-1] = max_wealth*(1-maxdd)
                    capital_log[-1] = total - bank_log[-1]

            both.append(bank_log[-1] + capital_log[-1]) # Log the Total value of both capital + bank
            dd.append(both[-1]/max(both)) # Log the drawdown of both capital + bank
            bankdd.append(bank_log[-1]/max(both)) # Log the bank level relative to drawdown

    ##########################
    ## PLOTING OF THE RESULT ##
    ##########################

    # Set up a figure and specify its overall size
    fig = plt.figure(figsize=(10, 6))  # Width, height in inches

    # First plot (1 row, 2 columns, first subplot)
    ax1 = fig.add_subplot(2, 2, (1, 2))  # Spanning first row
    ax1.plot(both, label='Capital Growth')
    ax1.plot(bank_log, label='Bank Growth')
    ax1.set_xlabel('Number of Trades')
    ax1.set_ylabel('Balance')
    ax1.set_title('Capital + Bank Growth')
    ax1.legend()

    # Second plot (2 rows, 2 columns, third subplot)
    ax2 = fig.add_subplot(2, 2, 3)  # Positioned on bottom left
    ax2.plot(both, label='Capital + Bank Growth')
    ax2.plot(bank_log, label='Bank Growth')
    ax2.set_xlabel('Number of Trades')
    ax2.set_ylabel('Balance')
    ax2.set_title('Log Scale')
    ax2.set_yscale('log')

    # Third plot (2 rows, 2 columns, fourth subplot)
    ax3 = fig.add_subplot(2, 2, 4)  # Positioned on bottom right
    ax3.plot(dd, label='Capital + Bank Drawdown')
    ax3.plot(bankdd, label='Bank Level')
    ax3.set_xlabel('Number of Trades')
    ax3.set_ylabel('Drawdown')
```

```
    ax3.set_title('Drawdown')
    ax3.axhline(y=0, color='r', linestyle='--')
    ax3.axhline(y=1, color='r', linestyle='--')
    ax3.axhline(y=1-maxdd, color='r', linestyle='--')

    # Adjust layout to prevent overlap
    plt.tight_layout()

    # Display the entire figure with all subplots
    plt.show()

    ################################

####################################
########### SIMULATION ############
####################################

################
#### INPUT #####
################

capital = 10000 # Initial Capital
bank = 0 # Initial Bank

p = 0.5 # Winning Probability
b = 2 # Risk Reward Ratio

maxdd = 0 # Adjust Maximum Drawdown, 0 = Justus Criterion
maxrisk = 100000 # Maximum Dollar Risk

n = 1000 # Number of trades

####################
#### Simulation ####
####################

simulate(capital, bank, p, b, maxdd, maxrisk, n)

#################
## END OF CODE ##
#################
```