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New Product-Type Estimators for the Population Mean Using Quartiles of the Auxiliary Variable

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Abstract**—**In this paper, we suggest new product-type estimators for the population mean of the variable of interest exploiting the first or the third quartile of the auxiliary variable. We obtain mean square error equations and the bias for the estimators. We study the properties of these estimators using simple random sampling (SRS) and ranked set sampling (RSS) methods. It is found that, SRS and RSS produce approximately unbiased estimators of the population mean. However, the RSS estimators are more efficient than those obtained using SRS based on the same number of measured units for all values of the correlation coefficient.

Keywords**—**Product estimator; auxiliary variable; simple random sampling, extreme ranked set sampling.

I. Introduction

LET *X* and *Y* denotes the auxiliary variable and the variable of interest respectively, with means μ_X, μ_Y, variances σ_X^2, σ_Y^2 respectively and correlation coefficient ρ. Let $\left(X_{ij}, Y_{ij}\right)$, $i=1,2,...,m$ in the *j*th cycle $j=1,2,...,n$ denote the measurements of *X* and *Y*. Assuming the population mean μ_X of the auxiliary variable *X* is known, the classical product estimator $\hat{\bar{Y}}_{CSRS}$ for μ_Y using SRS is defined as

$$\hat{\bar{Y}}_{CSRS} = \bar{Y}_{SRS} \frac{\bar{X}_{SRS}}{\mu_X}. \quad (1)$$

where,

$$\bar{X}_{SRS} = \frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m} X_{i1j} \text{ and } \bar{Y}_{SRS} = \frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m} Y_{i1j}.$$

Singh and Tailor (2003) suggested a modified product estimator $\hat{\bar{Y}}_{MSRS}$ for μ_Y using SRS from a sample of size *m* as given by

$$\hat{\bar{Y}}_{MSRS} = \bar{Y}_{SRS}\left(\frac{\bar{X}_{SRS}+\rho}{\mu_X+\rho}\right). \quad (2)$$

For more details see Pandey and Dubey [5], Singh and Tailor [6], Upadhyaya and Singh [9] and Cochran [1]. McIntyre [3] was first to suggest the RSS method as a cost efficient alternative to SRS method. Takahasi and Wakimoto [8] independently introduce the same method with necessary mathematical theory. Samawi and Muttlak [6] suggested using RSS to estimate the population ratio. Muttlak [4] suggested using median ranked set sampling (MRSS) to estimate the population mean. Jemain and Al-Omari [2] suggested multistage median ranked set sampling (MMRSS) method for estimating the population mean.

In this paper, we have suggested new product-type estimators for estimating the population mean μ_Y assuming the knowledge of the first or third quartile of the auxiliary variable *X*. We compare the properties of the estimators under SRS and RSS methods.

This paper is organized as follows: in Section 2.1, we present the product-type estimators using SRS with their properties. In Section 2.2 we present the RSS product-type estimators with their properties. In Section 3 simulation study is conducted to investigate the performance of the suggested estimators under RSS with respect to SRS based on the same number of measured units. Conclusion is presented in Section 4.

II. The Suggested Estimators

Let $\left(X_1,Y_1\right),\left(X_2,Y_2\right),...,\left(X_m,Y_m\right)$ be a bivariate random sample with pdf $f(x,y)$, cdf $F(x,y)$, with means μ_X, μ_Y, variances σ_X, σ_Y and correlation coefficient ρ. In this paper we assume that the ranking is performed on the variable *X* to estimate the mean of the variable of interest *Y*. Let $\left(X_{11j},Y_{11j}\right)$, $\left(X_{12j},Y_{12j}\right)$, $...,\left(X_{1mj},Y_{1mj}\right)$, $\left(X_{21j},Y_{21j}\right)$, $\left(X_{22j},Y_{22j}\right)$, $...,\left(X_{2mj},Y_{2mj}\right),...,$ $\left(X_{m1j},Y_{m1j}\right),\left(X_{m2j},Y_{m2j}\right),...,\left(X_{mmj},Y_{mmj}\right)$, $i=1,2,...$ *m*, $j=1,2,...,n$ be *m* independent random samples each of size *m*.

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A. The SRS product-type estimators

The product-type estimators of the population mean μ_Y using SRS are given by

$$\hat{\bar{Y}}_{SRS1} = \bar{Y}_{SRS}\left(\frac{\bar{X}_{SRS}+q_1}{\mu_X+q_1}\right)$$

and

$$\hat{\bar{Y}}_{SRS3} = \bar{Y}_{SRS}\left(\frac{\bar{X}_{SRS}+q_3}{\mu_X+q_3}\right), \tag{3}$$

where q_1 and q_3 are the first and third quartiles of the auxiliary variable X respectively, and where $\bar{X}_{SRS} = \frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m} X_{i1j}$ and $\bar{Y}_{SRS} = \frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m} Y_{i1j}$. Using Taylor expansion as

$$\begin{aligned} h\left(\bar{X}_{SRS},\bar{Y}_{RSS}\right) &\cong h\left(\mu_X,\mu_Y\right) + \left(\bar{X}_{SRS}-\mu_X\right)\left.\frac{\partial(a,b)}{\partial a}\right|_{\substack{\bar{X}=\mu_X\\ \bar{Y}=\mu_Y}} \\ &+ \left(\bar{Y}-\mu_Y\right)\left.\frac{\partial(a,b)}{\partial b}\right|_{\substack{\bar{X}=\mu_X\\ \bar{Y}=\mu_Y}} + \frac{1}{2}\left(\bar{X}_{SRS}-\mu_X\right)^2\left.\frac{\partial^2(a,b)}{\partial a^2}\right|_{\substack{\bar{X}=\mu_X\\ \bar{Y}=\mu_Y}} \\ &+ \frac{1}{2}\left(\bar{Y}_{SRS}-\mu_Y\right)^2\left.\frac{\partial^2(a,b)}{\partial b^2}\right|_{\substack{\bar{X}=\mu_X\\ \bar{Y}=\mu_Y}} \\ &+ \left(\bar{X}_{SRS}-\mu_X\right)\left(\bar{Y}-\mu_Y\right)\left.\frac{\partial^2(a,b)}{\partial a\partial b}\right|_{\substack{\bar{X}=\mu_X\\ \bar{Y}=\mu_Y}} \end{aligned} \tag{4}$$

where, $h\left(\bar{X}_{SRS},\bar{Y}_{RSS}\right) = \hat{\bar{Y}}_{SRSk}$, $k = 1,3$, and $h\left(\mu_X,\mu_Y\right) = \mu_Y$, the estimators in (3) to the first degree of approximation can be written as

$$\hat{\bar{Y}}_{SRSk} \cong \bar{Y}_{SRS} + T\left(\bar{X}_{SRS}-\mu_X\right), \tag{5}$$

where $T = \frac{\mu_Y}{\mu_X+q_k}$. The bias and the MSE of the estimator, respectively, are

$$\begin{aligned} \text{Bias}\left(\hat{\bar{Y}}_{SRSk}\right) &= E\left(\hat{\bar{Y}}_{SRSk}\right) - \mu_Y \\ &\cong E\left(\bar{Y}_{SRS} + T\left(\bar{X}_{SRS}-\mu_X\right)\right) - \mu_Y = 0, \end{aligned} \tag{6}$$

and

$$\begin{aligned} \text{Var}\left(\hat{\bar{Y}}_{SRSk}\right) &\cong T^2\text{Var}\left(\bar{X}_{SRS}\right) + \text{Var}\left(\bar{Y}_{SRS}\right) \\ &+ 2T\text{Cov}\left(\bar{X}_{SRS},\bar{Y}_{SRS}\right) \end{aligned}. \tag{7}$$

Using the two relations

$$\text{Cov}\left(\bar{X}_{SRS},\bar{Y}_{SRS}\right) = \beta\text{Var}\left(\bar{X}_{SRS}\right), \tag{8}$$

and

$$\text{Var}\left(\bar{Y}_{SRS}\right) \cong \beta^2\text{Var}\left(\bar{X}_{SRS}\right) + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right), \tag{9}$$

where

$\beta = \rho\frac{\sigma_Y}{\sigma_X}$, $\text{Var}\left(\bar{X}_{SRS}\right) = \frac{\sigma_X^2}{m}$, and $\text{Var}\left(\bar{Y}_{SRS}\right) = \frac{\sigma_Y^2}{m}$,

the variance of $\hat{\bar{Y}}_{SRSk}$ can be written as

$$\text{Var}\left(\hat{\bar{Y}}_{SRSk}\right) \cong \frac{\sigma_X^2}{m}\left(T+\beta\right)^2 + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right). \tag{10}$$

Note that the Variance and the MSE of the estimators are the same since the estimators are approximately unbiased.

B. RSS product estimator

Let $\left(X_{i(1:m)j},Y_{i[1:m]j}\right)$, $\left(X_{i(2:m)j},Y_{i[2:m]j}\right),\ldots,$ $\left(X_{i(m:m)j},Y_{i[m:m]j}\right)$ be the order statistics of X_{i1j}, X_{i2j}, ..., X_{imj} and the judgment order of $Y_{i1j},Y_{i2j},\ldots,Y_{imj}$, $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$. The RSS involves randomly selecting m^2 units from the population. These units are randomly allocated into *m* sets, each of size *m*. The *m* units of each sample are ranked visually or by any inexpensive method with respect to the variable of interest. From the first set of *m* units, the smallest unit is measured. From the second set of *m* units, the second smallest unit is measured. The process is continued until from the *m*th set of *m* units the largest unit is measured. Repeating the process *n* times yields a set of size *mn* from the initial nm^2 units. Then $\left(X_{1(1:m)j},Y_{1[1:m]j}\right),\left(X_{2(2:m)j},Y_{2[2:m]j}\right),\ldots,$ $\left(X_{m(m:m)j},Y_{m[m:m]j}\right)$ denote the RSS of size *m*. The RSS product-type estimator of the population mean of *Y* from a sample of size *m* is defined as

$$\hat{\bar{Y}}_{RSS1} = \bar{Y}_{RSS}\left(\frac{\bar{X}_{RSS}+q_1}{\mu_X+q_1}\right)$$

and

$$\hat{\bar{Y}}_{RSS3} = \bar{Y}_{RSS}\left(\frac{\bar{X}_{RSS}+q_3}{\mu_X+q_3}\right), \tag{11}$$

where

$\bar{X}_{RSS} = \frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m} X_{i(i:m)j}$ and $\bar{Y}_{RSS} = \frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m} Y_{i[i:m]j}$.

Using (4), the estimator given in (11) can be written as

$$\hat{\bar{Y}}_{RSSk} \cong \bar{Y}_{RSS} + T\left(\bar{X}_{RSS}-\mu_X\right). \tag{12}$$

The bias and the MSE of (12), respectively, are given by

$$\begin{aligned} \text{Bias}\left(\hat{\bar{Y}}_{RSSk}\right) &\cong E\left(\hat{\bar{Y}}_{RSSk}\right) - \mu_Y \\ &\cong E\left(\bar{Y}_{RSS} + T\left(\bar{X}_{RSS}-\mu_X\right)\right) - \mu_Y = 0, \end{aligned} \tag{13}$$

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and

$$\mathrm{Var}\left(\hat{\hat{Y}}_{RSSk}\right) \cong T^2\mathrm{Var}\left(\bar{X}_{RSS}\right) + \mathrm{Var}\left(\bar{Y}_{RSS}\right) + 2T\mathrm{Cov}\left(\bar{X}_{RSS}, \bar{Y}_{RSS}\right). \quad (14)$$

where,

$$\mathrm{Var}\left(\bar{X}_{RSS}\right) = \frac{\sigma_X^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{(i:m)} - \mu_X\right)^2$$

and

$$\mathrm{Var}\left(\bar{Y}_{RSS}\right) = \frac{\sigma_Y^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{[i:m]} - \mu_Y\right)^2 .$$

Using the two relations

$$\mathrm{Cov}\left(\bar{X}_{RSS}, \bar{Y}_{RSS}\right) = \beta \mathrm{Var}\left(\bar{X}_{RSS}\right), \quad (15)$$

and

$$\mathrm{Var}\left(\bar{Y}_{RSS}\right) \cong \beta^2 \mathrm{Var}\left(\bar{X}_{RSS}\right) + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right), \quad (16)$$

the variance is given by

$$\begin{aligned}\mathrm{Var}\left(\hat{\hat{Y}}_{RSSk}\right) &\cong \mathrm{Var}\left(\bar{X}_{RSS}\right)\left(T+\beta\right)^2 + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right) \\ &= \left(T+\beta\right)^2\left(\frac{\sigma_X^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{(i:m)} - \mu_X\right)^2\right) \\ &\quad + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right)\end{aligned} \quad (17)$$

The Efficiency of $\hat{\hat{Y}}_{RSSk}$ with respect to $\hat{\hat{Y}}_{SRSk}$ for estimating the population mean μ_Y is defined as:

$$eff\left(\hat{\hat{Y}}_{SRSk}, \hat{\hat{Y}}_{RSSk}\right) = \frac{\mathrm{Var}\left(\hat{\hat{Y}}_{SRSk}\right)}{\mathrm{Var}\left(\hat{\hat{Y}}_{RSSk}\right)}$$

$$\cong \frac{\frac{\sigma_X^2}{m}\left(T+\beta\right)^2 + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right)}{\left(T+\beta\right)^2\left(\frac{\sigma_X^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{(i:m)} - \mu_X\right)^2\right) + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right)}$$

$$\cong \frac{\sigma_X^2\left(T+\beta\right)^2 + \sigma_Y^2\left(1-\rho^2\right)}{\left(T+\beta\right)^2\left(\sigma_X^2 - \frac{1}{m}\sum_{i=1}^{m}\left(\mu_{(i:m)} - \mu_X\right)^2\right) + \sigma_Y^2\left(1-\rho^2\right)}$$

$$\cong \frac{\sigma_X^2\left(T+\beta\right)^2 + \sigma_Y^2\left(1-\rho^2\right)}{\left(T+\beta\right)^2\sigma_X^2 + \sigma_Y^2\left(1-\rho^2\right) - \left(T+\beta\right)^2\frac{1}{m}\sum_{i=1}^{m}\left(\mu_{(i:m)} - \mu_X\right)^2}$$

$$\cong \frac{1}{1 - \frac{\left(T+\beta\right)^2\frac{1}{m}\sum_{i=1}^{m}\left(\mu_{(i:m)} - \mu_X\right)^2}{\sigma_X^2\left(T+\beta\right)^2 + \sigma_Y^2\left(1-\rho^2\right)}}. \quad (18)$$

Since $\left(T+\beta\right)^2$, $\frac{\beta^2}{\left(\mu_X + q_k\right)^2}$ and $\frac{1}{m}\sigma_Y^2\left(1-\rho^2\right)$ are fixed in the numerator and denominator and since $\mathrm{Var}\left(\bar{X}_{RSS}\right) < \mathrm{Var}\left(\bar{X}_{SRS}\right)$ (Takahasi and Wakimoto, 1968), therefore $eff\left(\hat{\hat{Y}}_{SRSk}, \hat{\hat{Y}}_{RSSk}\right) > 1$. Implies that $\hat{\hat{Y}}_{RSSk}$ is more efficient than $\hat{\hat{Y}}_{SRSk}$ based on the same number of measured units.

III. SIMULATION STUDY

In this section, simulation study is conducted to investigate the performance of $\hat{\hat{Y}}_{SRSk}$ and $\hat{\hat{Y}}_{RSSk}$ methods for estimating the population mean where the ranking was performed on the variable *X*. The samples were generated from bivariate normal distribution with parameters $\mu_X = 2$, $\mu_Y = 4$, $\sigma_X^2 = \sigma_Y^2 = 1$ and $\rho = \pm 0.99, \pm 0.90, \pm 0.80, \pm 0.70, \pm 0.50, \pm 0.25$.

Based on 60,000 replications, the efficiency and bias values of $\hat{\hat{Y}}_{SRSk}$ and $\hat{\hat{Y}}_{RSSk}$ are obtained and the results for $m = 2,3,4,5,6$ based on the knowledge of q_1 are presented in Table 1 and the results are summarized in Table 2 when q_3 is known. The efficiency of $\hat{\hat{Y}}_{RSSk}$ with respect to $\hat{\hat{Y}}_{SRSk}$ is obtained using Equation (18). In this section the ranking is performed on the variable *X*, however the whole process can be repeated while the ranking is performed on the variable *Y*.

TABLE I

THE EFFICIENCY AND BIAS VALUES OF $\hat{\hat{Y}}_{RSSk}$ WITH RESPECT TO $\hat{\hat{Y}}_{SRSk}$ FOR $m = 2,3,4,5,6$ WITH DIFFERENT VALUES OF ρ WHEN q_1 IS KNOWN

ρ		$m=2$	$m=3$	$m=4$	$m=5$	$m=6$
0.99	Eff	1.482	1.926	2.406	2.879	3.263
	Bias RSS	0.089	0.051	0.033	0.024	0.014
	Bias SRS	0.151	0.098	0.075	0.059	0.048
0.90	Eff	1.445	1.858	2.303	2.647	2.970
	Bias RSS	0.093	0.048	0.028	0.020	0.018
	Bias SRS	0.148	0.084	0.067	0.050	0.041
0.80	Eff	1.429	1.816	2.162	2.486	2.724
	Bias RSS	0.087	0.042	0.025	0.017	0.010
	Bias SRS	0.123	0.089	0.063	0.045	0.037
0.70	Eff	1.379	1.747	2.044	2.319	2.619
	Bias RSS	0.079	0.036	0.024	0.012	0.010
	Bias SRS	0.111	0.064	0.053	0.038	0.037
0.50	Eff	1.352	1.623	1.871	2.071	2.216
	Bias RSS	0.045	0.026	0.016	0.013	0.009
	Bias SRS	0.070	0.049	0.041	0.030	0.025
0.25	Eff	1.290	1.497	1.680	1.817	1.952
	Bias RSS	0.026	0.011	0.013	0.009	0.000
	Bias SRS	0.034	0.023	0.021	0.015	0.014
-0.99	Eff	1.796	2.244	2.687	2.872	2.896
	Bias RSS	-0.101	-0.052	-0.032	-0.021	-0.015
	Bias SRS	-0.148	-0.102	-0.075	-0.059	-0.049
-0.90	Eff	1.287	1.372	1.428	1.431	1.454
	Bias RSS	-0.092	-0.048	-0.027	-0.020	-0.014
	Bias SRS	-0.134	-0.091	-0.067	-0.054	-0.046
-0.80	Eff	1.186	1.279	1.299	1.332	1.345
	Bias RSS	-0.082	-0.045	-0.024	-0.017	-0.012
	Bias SRS	-0.012	-0.082	-0.060	-0.048	-0.041
-0.70	Eff	1.163	1.253	1.297	1.337	1.346
	Bias RSS	-0.075	-0.038	-0.019	-0.013	-0.011
	Bias SRS	-0.105	-0.067	-0.054	-0.046	-0.036
-0.50	Eff	1.160	1.268	1.308	1.374	1.394
	Bias RSS	-0.048	-0.030	-0.025	-0.012	-0.010
	Bias SRS	-0.073	-0.054	-0.050	-0.027	-0.025
-0.25	Eff	1.196	1.315	1.421	1.485	1.547
	Bias RSS	-0.027	-0.011	-0.011	-0.005	-0.002
	Bias SRS	-0.043	-0.024	-0.020	-0.016	-0.010

TABLE II

THE EFFICIENCY AND BIAS VALUES OF $\hat{\hat{Y}}_{RSSk}$ WITH RESPECT TO $\hat{\hat{Y}}_{SRSk}$ FOR $m = 2,3,4,5,6$ WITH DIFFERENT VALUES OF ρ WHEN q_3 IS KNOWN

ρ		$m=2$	$m=3$	$m=4$	$m=5$	$m=6$
0.99	Eff	1.480	1.918	2.377	2.865	3.244
	Bias RSS	0.072	0.040	0.024	0.013	0.012
	Bias SRS	0.116	0.072	0.057	0.039	0.035
0.90	Eff	1.420	1.826	2.228	2.627	2.910
	Bias RSS	0.063	0.034	0.020	0.013	0.011
	Bias SRS	0.096	0.060	0.050	0.041	0.034
0.80	Eff	1.387	1.747	2.080	2.372	2.564
	Bias RSS	0.063	0.030	0.019	0.015	0.008
	Bias SRS	0.088	0.066	0.040	0.035	0.032
0.70	Eff	1.376	1.651	1.944	2.159	2.334
	Bias RSS	0.053	0.024	0.012	0.010	0.005
	Bias SRS	0.080	0.053	0.041	0.028	0.023
0.50	Eff	1.301	1.508	1.700	1.875	1.983
	Bias RSS	0.039	0.023	0.011	0.010	0.004
	Bias SRS	0.047	0.038	0.026	0.023	0.020
0.25	Eff	1.215	1.406	1.518	1.614	1.644
	Bias RSS	0.018	0.008	0.007	0.003	0.000
	Bias SRS	0.028	0.019	0.013	0.010	0.007
-0.99	Eff	1.668	2.005	2.140	2.160	2.197
	Bias RSS	-0.072	-0.036	-0.022	-0.014	-0.011
	Bias SRS	-0.106	-0.071	-0.053	-0.042	-0.036
-0.90	Eff	1.147	1.151	1.123	1.120	1.110
	Bias RSS	-0.065	-0.034	-0.020	-0.012	-0.008
	Bias SRS	-0.098	-0.064	-0.048	-0.037	-0.033
-0.80	Eff	1.087	1.066	1.069	1.073	1.076
	Bias RSS	-0.057	-0.029	-0.017	-0.010	-0.008
	Bias SRS	-0.089	-0.058	-0.044	-0.034	-0.028
-0.70	Eff	1.043	1.046	1.068	1.060	1.062
	Bias RSS	-0.051	-0.027	-0.014	-0.010	-0.008
	Bias SRS	-0.072	-0.050	-0.036	-0.032	-0.023
-0.50	Eff	1.062	1.079	1.112	1.119	1.121
	Bias RSS	-0.036	-0.021	-0.012	-0.006	-0.004
	Bias SRS	-0.054	-0.041	-0.027	-0.019	-0.017
-0.25	Eff	1.121	1.164	1.202	1.223	1.238
	Bias RSS	-0.020	-0.009	-0.006	-0.006	0.001
	Bias SRS	-0.032	-0.020	-0.016	-0.010	-0.011

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Based on the results in Table 1 and 2, the following remarks can be concluded:

1). For small values of ρ, a gain in efficiency is obtained based on the suggested estimators using RSS with respect to SRS for all cases considered in this study.

2). The efficiency of the RSS estimators is increasing in the sample size. For example, for $\rho = 0.80$ assuming q_1 is known, for $m = 2,3,4,5,6$ the efficiencies are 1.429, 1.816, 2.162, 2.486 and 2.724 respectively.

3). The efficiencies obtained when q_1 is known are seems to be larger than those obtained when q_3 is known. For example for $m = 4$ and $\rho = 0.70$, the efficiency for q_1 is 2.044 while for q_3 the efficiency is 1.944.

4). When $\rho < 1$, the bias is negative while for $\rho > 1$ the bias is positive.

The efficiency of $\hat{\bar{Y}}_{RSS1}$ and $\hat{\bar{Y}}_{RSS3}$ is decreasing in ρ. For example, for $m = 3$ and q_3, the efficiency for $\rho = 0.99, 0.90, 0.80, 0.70, 0.50, 0.25$, respectively are 1.918, 1.826, 1.747, 1.651, 1.508 and 1.406.

IV. CONCLUSIONS

In this paper, we suggested new product-type estimators for the population mean are suggested using SRS and RSS methods providing that the first or third quartile of the auxiliary variable is known. We obtained the bias and MSE equations of the suggested estimators. The results showed that the estimators are approximately unbiased for the first degree of approximation and the RSS estimators are more efficient than SRS estimators. However, for small values of the correlation coefficient the efficiency still greater than 1.

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