

# Multi-Objective Optimization of Gas Turbine Power Cycle

Mohsen Nikaein

**Abstract**—Because of importance of energy, optimization of power generation systems is necessary. Gas turbine cycles are suitable manner for fast power generation, but their efficiency is partly low. In order to achieving higher efficiencies, some propositions are preferred such as recovery of heat from exhaust gases in a regenerator, utilization of intercooler in a multistage compressor, steam injection to combustion chamber and etc. However thermodynamic optimization of gas turbine cycle, even with above components, is necessary. In this article multi-objective genetic algorithms are employed for Pareto approach optimization of Regenerative-Intercooling-Gas Turbine (RIGT) cycle. In the multi-objective optimization a number of conflicting objective functions are to be optimized simultaneously. The important objective functions that have been considered for optimization are entropy generation of RIGT cycle ( $N_s$ ) derives using Exergy Analysis and Gouy-Stodola theorem, thermal efficiency and the net output power of RIGT Cycle. These objectives are usually conflicting with each other. The design variables consist of thermodynamic parameters such as compressor pressure ratio ( $R_p$ ), excess air in combustion (EA), turbine inlet temperature (TIT) and inlet air temperature ( $T_0$ ). At the first stage single objective optimization has been investigated and the method of Non-dominated Sorting Genetic Algorithm (NSGA-II) has been used for multi-objective optimization. Optimization procedures are performed for two and three objective functions and the results are compared for RIGT Cycle. In order to investigate the optimal thermodynamic behavior of two objectives, different set, each including two objectives of output parameters, are considered individually. For each set Pareto front are depicted. The sets of selected decision variables based on this Pareto front, will cause the best possible combination of corresponding objective functions. There is no superiority for the points on the Pareto front figure, but they are superior to any other point. In the case of three objective optimization the results are given in tables..

**Keywords**—Exergy, Entropy Generation, Brayton Cycle, Design Parameters, Optimization, Genetic Algorithm, Multi-Objective.

## I. INTRODUCTION

**I**N most real-world problems, several goals must be satisfied simultaneously in order to obtain an optimal solution. The multiple objectives are typically conflicting and no commensurable, and must be satisfied simultaneously. For example, we might want to be able to maximize the output shaft power of a Turbo shaft engine while minimizing the fuel consumption. Actually, multi-objective optimization is very different than the single-objective optimization. In single objective optimization, one attempts to obtain the best design or decision, which usually the global minimum or the global maximum depending on the optimization problem is that of

minimization or maximization. In multiple objective optimization, there may not exist one solution which is best (global minimum or maximum) with respect to all objectives. In multi-objective optimization problem, there exist a set of solutions which are superior to the rest of solution in the search space when all objectives are considered but are inferior to other solution in the space in one or more objectives. These solutions are known as Pareto-optimal solutions or no dominated solutions. Since none of the solution in the no dominated set is absolutely better than any other, any one of them is an acceptable solution [1].

There are many methods to solve multi-objective problems. In this paper we use the Non-dominated Sorting Genetic Algorithm (NSGA-II). NSGA-II proposed in Srinivas and Deb [2]. In this paper, an optimal set of design variables in a gas turbine power plant, namely, compressor pressure ratio ( $R_p$ ), excess air in combustion(EA), turbine inlet temperature (TIT or  $T_6$ ),and inlet air temperature( $T_0$ ) are used by Pareto approach to multi objective optimization. Our considerable objective functions are net output power of cycle ( $W_{net}$ ), cycle thermal efficiency ( $\eta_T$ ) and cycle entropy generation ( $N_s$ ). our goal is to optimize this objective functions, with regarding suitable practical constraints, using NSGA-II. Procedure for Paper Submission

## II. MULTI-OBJECTIVE OPTIMIZATION

Multi-objective optimization, which is also called multicriteria optimization or vector optimization, has been defined as finding a vector of decision variables satisfying constraints to give acceptable values to all objective functions [3,4]. In general, it can be mathematically defined as:

find the vector  $X^* = [x_1^*, x_2^*, \dots, x_n^*]^T$  to optimize:

$$F(X) = [f_1(x), f_2(x), \dots, f_k(x)]^T \quad (1)$$

Subject to m inequality constraints

$$g_i(X) \leq 0, \quad i = 1, \dots, m \quad (2)$$

and p equality constraints

$$h_j(X) = 0, \quad i = 1, \dots, p \quad (3)$$

Where  $X^* \in \mathcal{R}^n$  is the vector of decision or design variables,

and  $F(X) \in \mathcal{R}^k$  is the vector of objective functions, which must each be either minimized or maximized. However, without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on Pareto approach can be conducted using some definitions:

M.Nikaein. is with the National Iranian Gas Company ,Bid Boland Gas Refinery, Omidiyeh-P.O.Box161, Iran, (corresponding author to provide phone: +989166714530; fax: +986712232450; e-mail: mohsen\_nikaein@yahoo.com).

- Definition of Pareto dominance

A vector  $U = [u_1, u_2, \dots, u_k] \in \mathcal{R}^k$  is dominant to vector  $V = [v_1, v_2, \dots, v_k] \in \mathcal{R}^k$  (denoted by  $U < V$ ) if and only if  $\forall i \in \{1, 2, \dots, k\}, u_i \leq v_i \wedge \exists j \in \{1, 2, \dots, k\} : u_j < v_j$ .

In other words, there is at least one  $u_j$  which is smaller than  $v_j$  whilst the remaining  $u$ 's is either smaller or equal to corresponding  $v$ 's.

- Definition of Pareto optimality

A point  $X^* \in \Omega$  ( $\Omega$  is a feasible region in  $\mathcal{R}^n$  satisfying Equations (2) and (3)) is said to be Pareto optimal (minimal) with respect to all  $X \in \Omega$  if and only if  $F(X^*) < F(X)$ . Alternatively, it can be readily restated as

$$\forall i \in \{1, 2, \dots, k\}, \forall X \in \Omega - \{X^*\} \\ f_i(X^*) \leq f_i(X) \wedge \exists j \in \{1, 2, \dots, k\} : f_j(X^*) < f_j(X)$$

In other words, the solution  $X^*$  is said to be Pareto optimal (minimal) if no other solution can be found to dominate  $X^*$  using the definition of Pareto dominance.

-Definition of a Pareto set

For a given MOP, a Pareto set  $P^*$  is a set in the decision variable space consisting of all the Pareto optimal vectors  $P^* = \{X \in \Omega \mid \nexists X' \in \Omega : F(X') < F(X)\}$ . In other words, there is no other  $X'$  as a vector of decision variables in  $\Omega$  that dominates any  $X \in P^*$ .

- Definition of a Pareto front

For a given MOP, the Pareto front  $Pf^*$  is a set of vector of objective functions which are obtained using the vectors of decision variables in the Pareto set  $P^*$ , that is  $Pf^* = \{f_1(X), f_2(X), \dots, f_k(X) : X \in P^*\}$ . In other words, the Pareto front  $Pf^*$  is a set of the vectors of objective functions mapped from  $P^*$ .

Different algorithms have been widely used for multi objective optimization because of their natural properties suited for these types of problems. The NSGA-II is one of these algorithms. In order to show this algorithm more clearly, some basics of NSGA-II are represented. In Fig. 1 demonstrated now selects individuals from the entire population  $R_t$  to construct the next parent population  $R_{t+1}$ . The entire population  $R_t$  is simply the current parent population  $P_t$  plus its offspring population  $Q_t$  which is created from the parent population  $P_t$  by using usual genetic operators. The selection is based on non-dominated sorting procedure which is used to classify the entire population  $R_t$

according to increasing order of dominance [1].

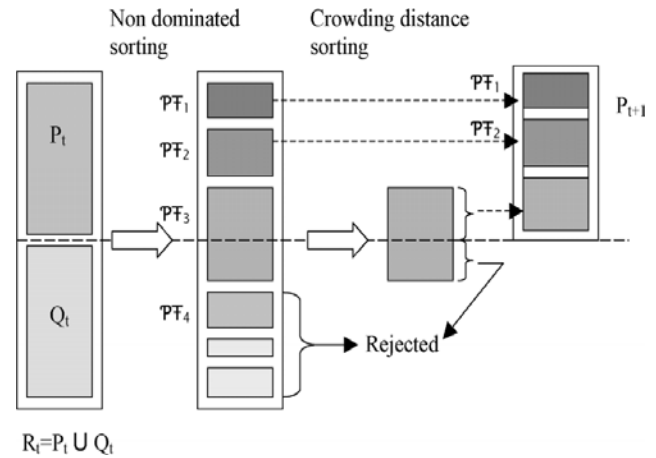


Fig. 1 Basics of NSGA-II procedure [1]

Thus, the best Pareto fronts from the top of the sorted list is chosen to create the new parent population  $P_{t+1}$  which is half the size of the entire population  $R_t$ . So, it should be noted that all the individuals of a certain front cannot be modified in the new parent population because of space, as shown in Figure. 1. To choose an exact number of individuals of that particular front, a crowded comparison operator is used in NSGA-II to find the best solutions to complete the new parent population. The crowded comparison procedure is based on density estimation of solutions surrounding a particular solution in a population or front. So, the solutions of a Pareto front are first sorted in each objective direction in the ascending order of that objective value. The crowding distance is then assigned equal to the half of the perimeter of the enclosing hyper box. Other objectives are sorted too and the overall crowding distance is calculated as the sum of the crowding distances from all objectives. The less crowded non-dominated individuals of that particular Pareto front are then selected to fill the new parent population. It is important to know that in a two-objective Pareto optimization, if the solutions of a Pareto front are sorted in a decreasing order of importance to one objective, these solutions are then automatically ordered in an increasing order of importance to the second objective. In other words, the hyper-boxes surrounding an individual solution remain unchanged in the objective-wise sorting procedure of the crowding distance of NSGA-II in the two-objective Pareto optimization problem. However, in multi-objective Pareto optimization problem with more than two objectives, such sorting procedure of individuals based on each objective in this algorithm will cause different enclosing hyper boxes. Therefore, the overall crowding distance of an individual computed in this way may not exactly reflect the true measure of diversity or crowding property for the multi-objective Pareto optimization problems with more than two objectives.

### III. THE GAS TURBINE POWER PLANT

A schematic of RISIGT cycle is given in figure 1. The system consists of a two-stage intercooled air compressor, a regenerator, a combustion chamber and a gas turbine.

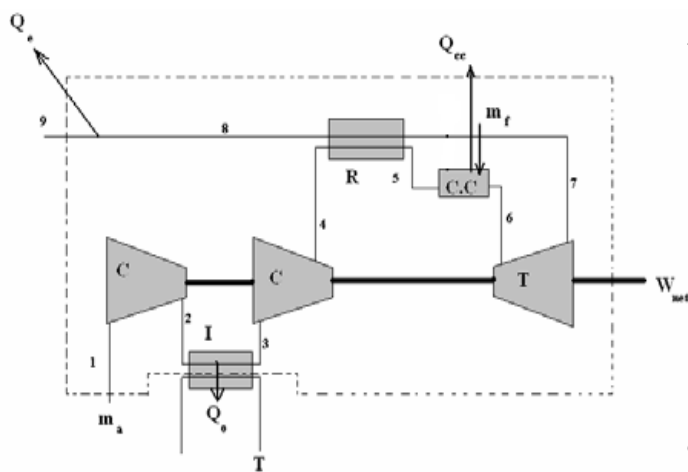


Fig. 2 RISIGT cycle [5]

The incoming air has a pressure of 1.013 bars. Turbine and compressor have an isentropic efficiency of 87 and 85 percent, respectively. The regenerative heat exchanger has an effectiveness of 75%. Combustion chamber adiabatic efficiency is 98%. The pressure drop through the air preheater is 4% of the inlet pressure for both flow streams and through the combustion chamber is 3% of the inlet pressure. It is 2% for intercooler. The fuel (natural gas, type C, C1.5H5), is injected at environment temperature and pressure slightly more than environment pressure. In our cycle, overall compressor pressure ratio is  $R_p = P_4/P_1$  and for each stage, pressure ratios are  $RP_1$  and  $RP_2$ , respectively.

Temperature of hot air exiting from first stage reduces to compressor inlet air temperature, due to heat extraction in intercooler. We know that for two-stage compressor, minimum work consumption occurs when two pressure ratios are equal. So pressure ratio can be written as:

$$R_{p_2} = R_{p_1} = \left( \frac{R_p}{1-d} \right)^{0.5} \quad (4)$$

### IV. ASSUMPTIONS

Our assumptions are:

- Air, combustion product and gaseous fuel are ideal gas with temperature dependent  $C_p = C_p(T)$ .
- Fuel is natural gas with  $C_{1.5}H_5$  chemical formula.
- Pressure drops in regenerator, intercooler and combustion chamber are considered.
- Compressor and turbine isentropic efficiencies and regenerator effectiveness are  $\eta_c$ ,  $\eta_t$  and  $\epsilon$ , respectively.
- Combustion chamber is not adiabatic, and combustion efficiency is  $\eta_b$
- Gaseous fuel injected to combustion chamber at its

incoming flow temperature and pressure.

g) our optimization criterion is minimizing entropy generation and maximizing cycle output power and thermodynamic efficiency.

h) we use Gouy-Stodola theorem to achieve entropy generation in cycle.

### V. OPTIMIZATION PROBLEM

Our goal, as mentioned before, is to

- Minimize  $N_s$  ( $R_p, TIT, EA, T_0$ )
- Maximize  $\eta$  ( $R_p, TIT, EA, T_0$ )
- Maximize  $W_{net}$  ( $R_p, TIT, EA, T_0$ )
- Simultaneously.

### VI. CONSTRAINTS

In each engineering problem, some constraints are exerted on problem from environment, processes and available sources, which must be considered because of acceptance of results. These limitations call Constraint functions or Constraints. They show relation between design variables and constant parameters. These relations write in equal or nonequal and linear or nonlinear form. In our study, according to selection of  $R_p, TIT, EA, T_0$  as design parameters, suitable practical constraints must exert on objective function. These constraints are selected regarding to responsible references and sources. Our linear constraints are:

- Compression ratios between 3 to 15 are used at modern gas turbine cycles. Higher amount of this parameter is used for propulsive gas turbine cycles. In common power stations, compression ratio is bounded between 11 to 16. So, for considering wide range of compression ratios, we select it as follow:  $1 \leq rp \leq 50$

- Constraint of maximum temperature of cycle, is metallurgical nor thermodynamically. Presently maximum turbine inlet temperature is about 1250 to 1340 0C. In modern gas turbine cycles, this temperature is about 1500 0C. Therefore:

$$1000 \leq TIT \leq 1700 \text{ K}$$

- Selected domain for excess air is according to reference [6] is:  $1 \leq EA \leq 4$

- For inlet air temperature, according to climatic conditions, we have:  $263 \leq T_0 \leq 323 \text{ K}$

This objective function is limited with two nonlinear constraints, too. First, in order to ease in natural displacement of exhaust gases (produced in combustion process) in stacks due to inequality of densities, and ecological considerations, we have [7]:  $Exh$  (Non Lin. Fun. of Des Par).  $\geq T_0(23)$  Also, because of presence of some compositions such as Nitrogen, Sulfur and etc in combustion productions, and for avoiding formation of corrosive materials such as sulfuric acid, nitric acid, etc, we must restrain formation of water drops in cycle exhaust. For this purpose we consider this constraint as [8, 9]:  $Exh$  (Non Lin. Fun. of Des Par).  $\geq T_{dewpoint}$  which  $T_{dewpoint}$  is the dew point of combustion products. Finally,

according to these constraints, desired nonlinear objective function is optimized using genetic algorithm in MATLAB. Results are shown in form of diagrams and tables.

### VII. RESULTS

The results of the single-objective optimizations are summarized in Table 1.

TABLE I  
 VALUES OF DECISION VARIABLES AND OBJECTIVE FUNCTIONS

$N_s = 0.0683$	$W_{net} = 723.0595$
$R_p = 36.7464$	$R_p = 49.6249$
$T_0 = 263.0094$	$T_0 = 263.05325$
$T_6 = 1700$	$T_6 = 1699.2$
$EA = 4$	$EA = 1.0029$
$\eta_T = 47.47\%$	
$R_p = 36.1870$	
$T_0 = 263.0074$	
$T_6 = 1699.5$	
$EA = 1.003$	

Some Pareto fronts of each pair of two objectives have been shown through Figures 3-5.

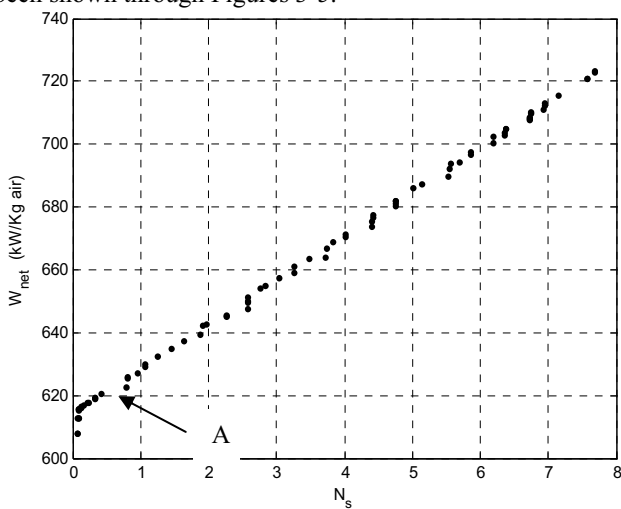


Fig. 3 Pareto front of net power output and entropy generation in 2-objective optimization.

Figure 3 demonstrates changes of dimensionless entropy generation ( $N_s$ ) with net power output. According to this figure, the result curve ( $W_{net}$  vs  $N_s$ ) is nearly linear and depending on the problem, designer selects an optimum point. Maximum and minimum amount of  $N_s$  and  $W_{net}$  are (0.0707, 7.675) and (608.1, 723.5), respectively, which satisfies single objective optimization.

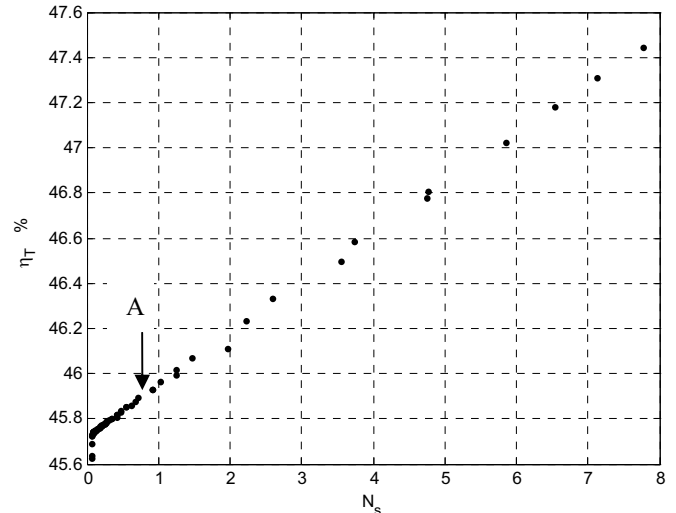


Fig. 4 Pareto front of thermal efficiency and entropy generation in 2-objective optimization.

Changes in entropy generation with thermal efficiency are shown in figure 4. According to this figure, due to increase in  $N_s$ , thermal efficiency has a few changes. So, the first points are the best for design. As it shown in figures 3 and 4, if cycle entropy generation was important, it's not prefer to use A and more. Objective functions in this point are 0.71, 622.5, and 45.91, respectively for cycle entropy generation, net power output and thermal efficiency.

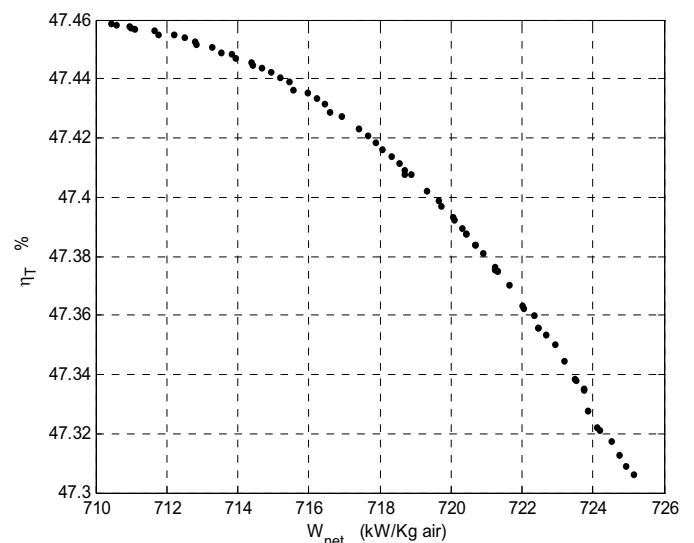


Fig. 5 Pareto front of thermal efficiency and net power output in 2-objective optimization

Figure 5 shows variation of thermal efficiency and net power output. Interval variations are (47.31, 47.46) and (7140.4, 725.1) for thermal efficiency and net power output, respectively.

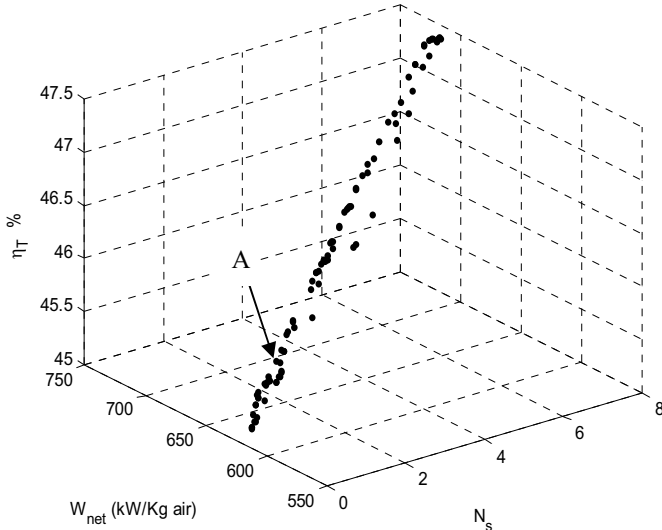


Fig. 6 Pareto front of thermal efficiency, net power output and entropy generation in 3-objective optimization

Figure 6 shows changes in entropy generation, net power output and thermal efficiency, which are optimized simultaneously. Change interval for entropy generation, net power output and thermal efficiency are (0.0731, 7.8061), (595.0422, 724.6081), (45.1607, 47.45), respectively. Point A is preferred as a best point for design. Our constraints are:

TABLE II

VALUES OF DECISION VARIABLES FOR THREE OBJECTIVE OPTIMIZATION

$28.443 < R_p < 49.927$	$263.063 < T_0 < 264.059$
$1698.5 < T_6 < 1700$	$1.0014 < EA < 4$

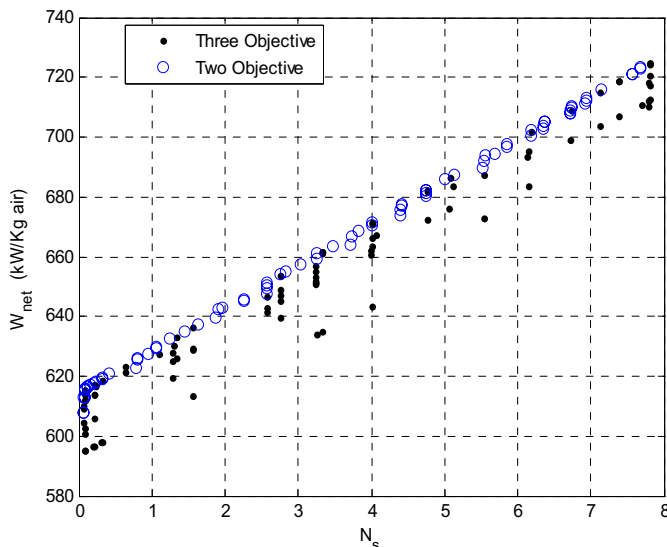


Fig. 7 Pareto front of net power output and entropy generation in 2-objective optimization

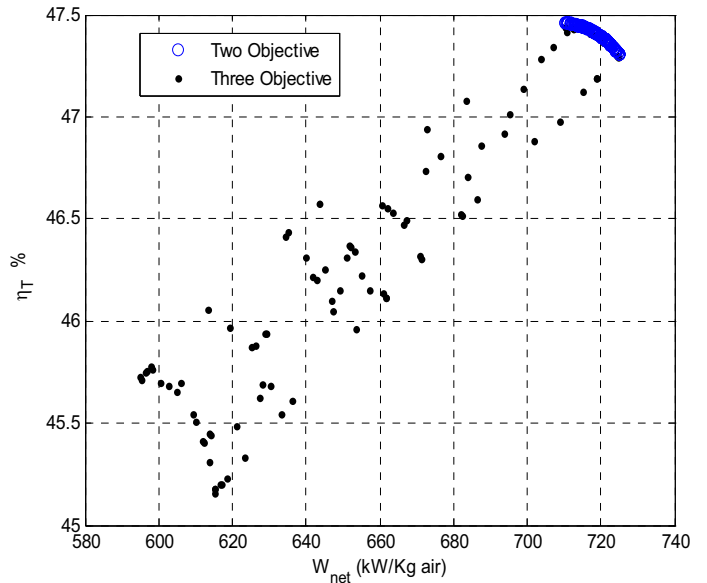


Fig. 8 Pareto front of thermal efficiency and net power output in 2-objective optimization

In figures 7 and 8, results from optimization with 2 and 3 objective functions are compared with together. In figure 7, objective functions are net power output and entropy generation. In figure 8, these are net power output and thermal efficiency. According to these figures, we can satisfy the results optimization of three objective function conditions. Also, as it shown, results of two objective function problems are the boundary of three objective function optimization.

### VIII. CONCLUSION

The work presented in this paper provided a provided a multi objective GA (non-dominated sorting genetic algorithm, NSGAI) to obtain pareto based optimization of the performance of a Brayton Cycle. Applying the exergy analysis and Goua-Stodola theorem, three objective functions, namely entropy generation of RIGT cycle ( $N_s$ ), thermal efficiency and the net output power were determined in terms of four design variables (Compressor pressure ratio, Excess air in combustion, turbine inlet temperature and inlet air temperature). Simultaneous optimization of three outputs revealed some interesting features among optimal objective functions and decision variables involved in the thermodynamic cycle of proposed system that would have not been obtained without the use of a multi-objective optimization approach. It was also demonstrated that two extreme points in the pareto included those of single objective optimization results. Further it has been shown that the results of three objective optimization include those of two objective optimization in terms of pareto frontiers and provide consequently more choices for optimal design.

REFERENCES

- [1] S. Farahat, E. Khorasani Nejad, S. M. Hoseini Sarvari , Thermodynamic Optimization of Turbohaft Engine using Multi-Objective Genetic Algorithm, World Academy of Science, Engineering and Technology 56 2009, pp 782-788.
- [2] N. Srinivas, K. Deb, "Multi-Objective Optimization using Nondominated Sorting in Genetic Algorithm," Evolutionary Computation, Vol. 2, No. 3, pp. 221-248, 1994.
- [3] K. Atashkari, N. Nariman-zadeh, A. Pilchi, A. Jamali, "Thermodynamic Pareto Optimization of Turbojet Engine Using Multi-Objective Genetic Algorithm," International Journal of Thermal Sciences, 44, PP. 1061-1071, 2005.
- [4] A. Osyezka, "Multi criteria optimization for engineering design," in: J. S. Gero(Ed.), Design Optimization, Academic Press, New York, 1985, pp. 193–227.
- [5] M.Nikaein, "Thermodynamic optimization of Brayton cycle using exergy analysis with genetic algorithm", IEHT Journal, Vol. 1, PP 63-68, 2009