Rescaled Range Analysis of ELF Natural Electromagnetic Noise from Antarctica

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Abstract— We present the results of a statistical rescaled range (R/S) analysis of natural electromagnetic noise in the extremely low frequency (ELF) band. The Hurst exponent was derived from the records of two horizontal magnetic antennas taken in Antarctica with a sample frequency of 512 Hz, to guarantee enough resolution with the small number of samples in the analysis. The study shows that ELF radio noise is a persistent random process, maintaining its value between 0.75 and 0.85 for the different series analyzed. A numerical simulation was performed, in which each discharge was modeled by the radiated field of a wire antenna excited by a Gaussian pulse, demonstrating a way of inferring lightning rates from R/S analysis.

1. INTRODUCTION

Fractals are uncommon mathematical objects defined in an imprecise way that shows auto-resemblance, i.e., one part is similar to the whole. The auto-resemblance concept is equivalent to scale invariance. Fractals are interesting because many natural phenomena have auto-resemblance. Traditional geometry does not account for the properties of these shapes. The mathematician Benoît Mandelbrot, known for his work with fractals, defines a fractal set as a collection having a fractal dimension higher than its topological dimension. The Hurst exponent, which is involved in R/S calculations, relates to the fractal dimension [1].

Natural electromagnetic noise in the ELF band produced by electrical storms on Earth can be studied in terms of the Hurst exponent by employing R/S analysis. This technique was developed by the engineer Harold Edwin Hurst in order to study the Nile floods, with the aim of optimizing water retention. The method is fully described in [2, 3].

In this work we present the R/S analysis of three records of both (North-South and East-West) horizontal components of the magnetic field, taken between January and February 2008 in Antarctica (where the anthropogenic noise should be low), employing the magnetotelluric method [4].

2. R/S ANALYSIS

R/S analysis starts from a digital series of 2^N samples, which in our case is a magnetic field time series. For each value of the index i ($1 \leq i \leq N$) we can define a partition of the series, which consists of splitting it into data blocks of $M = 2^i$ length. The average is then calculated from each partition i for each data block j:

$$\langle x \rangle_{i,j} = \frac{1}{M} \sum_{l=1}^{M} x_{(j-1)\cdot M+l}, \tag{1}$$

to obtain the sum (integral) of the time series with the average value taken

$$y_j(k) = \sum_{l=1}^{k} \left(x_j(l) - \langle x \rangle_{i,j} \right). \tag{2}$$

The range and standard deviation is also calculated from each partition i for each data block j:

$$R_{i,j} = \max_{l \leq k \leq M} (y_j(k)) - \min_{l \leq k \leq M} (y_j(k)) \tag{3}$$

$$S_{i,j} = \sqrt{\frac{1}{M}\sum_{l=1}^{M}(x_j(l) - \langle x \rangle_{i,j})^2}. \tag{4}$$

The average value of $R_{i,j}$ and $S_{i,j}$ can be calculated for each partition i, and $Z[M]$ is defined as the quotient between them:

$$Z[M] \equiv \frac{R_M}{S_M} = \frac{\langle R_{i,j} \rangle}{\langle S_{i,j} \rangle}. \tag{5}$$

The Hurst exponent is defined from the empiric relation between the coefficient Z and the number of samples in the data block by following a power relation which indicates scale invariance:

$$Z = \left(\frac{M}{2}\right)^{Hu} \tag{6}$$

Hu being the Hurst exponent (also called K by H. E. Hurst and later H by B. Mandelbrot). R/S analysis tries to find a power relation between Z and the number of samples, as illustrated in Equation (6). By representing $\log_2(Z(M))$ against $I = \log_2(M)$ we obtain a straight line whose slope determines the Hurst exponent. The Hurst exponent takes values in the interval (0.7, 0.8) for many geophysical processes [5, 6]. Due to the complexity of natural electromagnetic phenomena and of those involving different scales in general [7], it is convenient to define different Hurst exponents for different time windows, i.e., for different numbers of samples.

We can think of a time series as the sum of discontinuities, tendencies, periodic components and a stochastic component. The last of these would account for all the effects not included in those listed previously. One important fact to ascertain about a time series is whether its behavior is persistent, random or anti-persistent. If neighboring samples are not correlated then the stochastic component is random and the Hurst exponent takes values of $Hu = 0.5$. If the correlation is positive then the local fluctuations are lower than the average fluctuations and the behavior is persistent. If the correlation is negative then the neighbor values tend to be far apart and the behavior is anti-persistent [3]. In summary, if $0.5 < Hu < 1$ then the series is persistent and if $0 < Hu < 0.5$ then the series is anti-persistent.

The time series employed in this study corresponds to a sample frequency of 512 Hz and the number of samples is $2^{18} = 262144$, which corresponds to roughly 9 minutes of recording. The measurements were taken at three different sites, called S_{22}, S_{23}, and S_{24}. We obtained the Z coefficient as a function of the number of samples in the data block, as can be seen in Figure 1.

The Z coefficient shows a common behavior for the six series at values of $I \leq 6$. At around $I = 6$ we can observe a change in the slope and a major difference in the Z values. In order to obtain the Hurst exponent in different time scales a lineal adjustment has been made, by using earlier and later points for each adjustment. The results can be seen in Figure 2. For the interval $I = 3$–5, the Hurst exponent falls between 0.75–0.85, depending on the component and the site. For a time scale of $I = 6$–9, the exponents take values around 0.5, which correspond to random

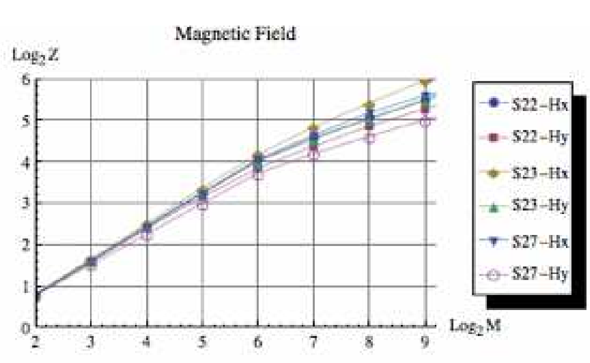

Figure 1: $\log(Z)$ against $I = \log(M)$ representation for the horizontal components of the magnetic field at three different sites.

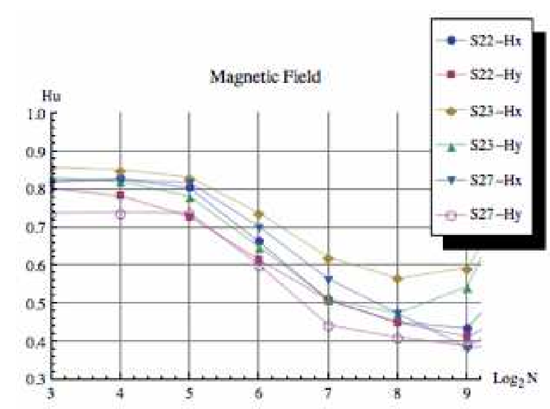

Figure 2: Hu against $I = \log(M)$ representation for the horizontal components of the magnetic field at three different sites.

behavior. When the number of samples increases, i.e., for $I > 9$, the Hurst exponent goes beyond unity (not shown in the figure). This happens because as the number of samples increases we have fewer data blocks to average.

R/S analysis is sensitive to the internal structure of the noise and could therefore be a tool for finding its different sources and the specific processes involved in its generation. The statistical treatment of the time series is not hard to implement; the difficulty here is in establishing the connection between the results and the underlying physical processes. In [5], a possible relation between global lightning activity (i.e., the global lightning rate) and the number of samples in which Hu abandons its persistent behavior (see Figure 2) is indicated. This relation can be expressed as:

$$r = \frac{f_s}{m_k} \tag{7}$$

where r is the average number of lightning strokes per second, f_s is the sample frequency and m_k is the number of samples in which Hu starts to decrease towards 0.5 (knee of the curve). From the results depicted in Figure 2 we can establish a global lightning rate of 16–32 strokes per second, because the sample frequency is 512 Hz. This result is in agreement with other observations [8].

In order to corroborate the empirical law expressed in Equation (7), a numerical simulation has been performed. The field generated by a stroke has been substituted by the response of a wire dipole antenna to a Gaussian pulse excitation. Hallén's equation has been solved for the spectral content by using the Method of Moments (MoM) to obtain the distribution of charge and current in the antenna. From these distributions we can obtain the temporal fields by using the Inverse Fourier Transform, and by applying Jefimenko's expressions we can obtain the electric and magnetic fields at any point in the space [9]. We have taken the electric field measured in the equatorial plane at a distance of five times the length of the antenna. This is a resonant structure and the radiated field presents resonances (see Figure 3), like the electromagnetic noise in the Earth-ionosphere cavity in the ELF band.

Each "stroke" of our numerical simulation is composed of 774 samples and the "natural electromagnetic noise" is obtained by superposing strokes with a fixed time interval in between, giving a final signal composed of 2^{16} samples. The lightning rate is selected according to the number of samples inserted between two discharges. For this study, three different signals were created, with a distance between discharges of 16, 64 and 128 samples, which correspond to 1/16, 1/64 and 1/128 strokes per second respectively (we are taking $f_s = 1\,\mathrm{Hz}$).

R/S analysis has been applied to these signals. In Figure 4, we can observe the behavior of $\log_2 Z$ as a function of $I = \log_2 N$. The legend of the figure indicates the number of samples inserted between strokes and represents, as previously stated, the inverse of the lightning rate. So the curve labeled as 16 presents a constant value for Hu until $I = 4$, where the knee appears and its value changes abruptly to 0. As stated in Equation (7), the lightning rate must be 1/16, which coincides with its theoretical value. The same happens for the other two synthetic signals, where the knee appears at $I = 6$ and $I = 7$, which means rates of 1/64 and 1/128 lightning per second respectively.

The signals recorded in Antarctica present similar lightning rates and we cannot differentiate between them regarding the location of the knee. It would be necessary to record signals at maximum and minimum times of lightning activity in order to see the differences with the method presented.

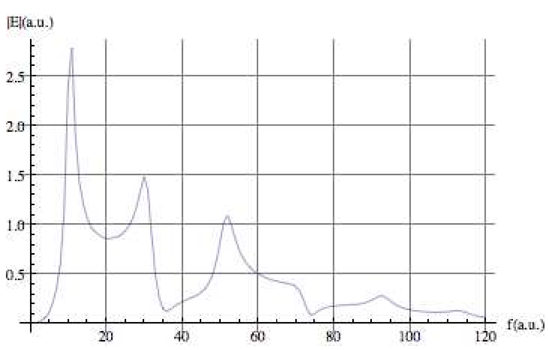

Figure 3: Electric field spectrum radiated by a wire antenna, measured at its equatorial plane.

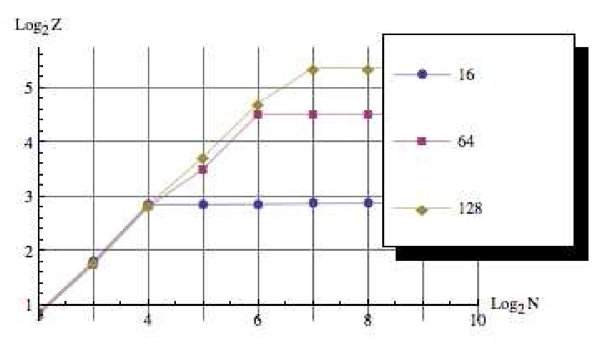

Figure 4: R/S analysis applied to the numerical simulation.

3. CONCLUSION

In this study, the R/S method has been applied to ELF-band (512 Hz sample frequency) electromagnetic noise, measured in Antarctica during January and February 2008. The Hurst exponent obtained shows the persistent behavior of the process for the scale between 2^2 and 2^5, and random behavior between 2^6 and 2^9. The number of samples where the transition occurs seems to be related to the lightning rate in the atmosphere, as shown in a numerical simulation.

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REFERENCES

1. Feder, J., *Fractals*, Plenum Press, New York, 1988.
2. Hurst, H. E., R. P. Black, and Y. M. Simaika, *Long-term Storage: An Experimental Study*, Constable, London, 1965.
3. Turcotte, D. L., *Fractals and Chaos in Geology and Geophysics*, Cambridge University Press, New York, 1997.
4. Cagniard, L., "Basic theory of the magnetotelluric method of geophysical prospecting," *Geophysics*, Vol. 18, No. 3, 605–635, 1953.
5. Nickolaenko, A. P., C. Price, and D. D. Iudin, "Hurst exponent derived for natural terrestrial radio noise in Schumann resonance band," *Geophysics. Res. Lett.*, Vol. 27, No. 19, 3185–3188, 2000.
6. Nickolalenko, A. P. and M. Hayakawa, *Resonances in the Earth-ionosphere Cavity*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2002.
7. Mandelbrot, B. B. and J. R. Wallis, "Robustness of the rescaled range R/S in the measurement of noncyclic long run statistical dependence," *Water Resour. Res.*, Vol. 5, 967–988, 1969.
8. Christian, H. J., R. J. Blackslee, D. J. Boccippio, W. L. Boeck, D. E. Buechler, K. T. Driscoll, S. J. Goopdman, J. M. Hall, W. J. Koshakl, D. M. Mach, and M. F. Stewart, "Global frequency and distribution of lightning as observed by the optical transient detector (OTD)," *Proc. of 11th Conference on Atmospheric Electricity*, 726, Guntersville, Alabama, USA, June 7–11, 1999.
9. Jackson, J. D., *Classical Electrodynamics*, 3rd Edition, Wiley University Press, New York, 1999.