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Auction Theory: Bidder's Perspective in a Public Out-Cry English Auction

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***Abstract*—**This paper provides an overview of auction theory literature. We present a general review on literature of various auctions and focus ourselves specifically on an English auction. We are interested in modelling bidder's behavior in an English auction environment. And hence, we present an overview of the New Zealand wool auction followed by a model that would describe a bidder's decision making behavior from the New Zealand wool auction. The mathematical assumptions in an English auction environment are demonstrated from the perspective of the New Zealand wool auction.

***Keywords*—**Bidder, English auction, New Zealand, Wool

I. Introduction

THIS paper aims to present an overview of auction theory literature. We begin with a brief history of the auction followed by the types of auctions practiced nowadays. In the second section, we present the review of literature on auction theory highlighting the English auction environment. The third section deals with a brief introduction of the New Zealand wool auction and its relation to the New Zealand Wool Industry (NZWI). In the fourth section, the mathematical assumptions in an English auction environment are demonstrated taking into account the New Zealand wool auction. Further, an illustration of a bidder's decision making behavior in the New Zealand wool auction is presented in a model. Finally, we summarize the paper and propose future work.

Literally, the word auction is derived from 'augere' meaning "to increase". When we go to the history of auction we can see the practice was started in Babylon from 500 BC [1]. In the modern world, auctions are used primarily for transactions of goods and services. Auctions help in a way to form the price and make a contribution of empirical significance on the exchange of ownership of goods. The study of auction is interesting to the traders because the value of the item is uncertain in an auction. Further, the parties involved (buyer and seller) do not know the exact information of the item. However, among the two parties one is much more familiar with the relevant information of the item than the other. The buyer does not know the details of the auction and hence auctions are said to be a game of incomplete information. Hayek explained that the auction environment illustrates the problem of utilizing knowledge which is not given to anyone in its totality [12]. An established fact is that the uncertainty of the value of the items under transaction is the beauty of the auction which allows competitive bidding. The areas of transactions where auctions allow theory and its implications to be employed are: houses, cars, agricultural products, art and antiques. Further, the transaction range is becoming wider with the advent of Information and Communication Technology (ICT), for example, in general e-commerce and more specifically is *eBay* and *trademe* online auctions. There are four main types of auctions in practice; English, Dutch, first-price sealed-bid and second-price sealed-bid [9]. These auctions are practiced elsewhere for the transaction purpose of goods and services. English auction methods have been used in trading the arts and antiques, the Dutch auction is being used in the trading of cut flowers in the Netherlands, fish in Israel and tobacco in Canada. First-price sealed-bid auctions are primarily used by governments in tenders for procurement. On the other hand, second-price sealed-bid auctions have significant theoretical importance; however, they are seldom used in practice.

Though the practices of auction principles have been used since the Babylonian era (193 A.D.), attention became widespread from 1996 once William Vickery was awarded the Nobel Prize in Economics for his contribution to auction theory [1]. The paper by Vickery entitled "Counterspeculation, Auctions, and Competitive Sealed Tenders" [8] is the widely recommended paper to read by auction theorists [1], [4]. The paper developed some special cases of Revenue Equivalence Theorem (RET). The second price sealed-bid auction is sometimes called the Vickery auction after his name. The English auction is widely used, and the New Zealand wool auction is an example of public out-cry English auction.

II. Literature focusing on various auctions

A. English Auction

English auctions are the ascending type auction where the auctioneer starts from a minimum price. In an English auction, the auctioneer begins by asking for bids at a low price and then gradually raises the price until only one willing buyer remains. The price increases along with the bidders bidding. Once, only one bidder is there to bid for a certain type of sale item he or she becomes the owner of that item. All the bidders who want to bid

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in an English auction environment have their own value for the item. This independent-private-value (IPV) is not known to other bidders. On the other hand, they would be in a common value (CV) environment if the bidders have common information of the item to be auctioned. Further, CV is possible without having common information once the bidders enter the real auction environment and know the value of other bidders. In the real- world auction environment, CV is known to all Participating bidders. Still their respective independent-private-value towards the item is unknown. We can assume that IPV and CV follow some probability distribution. The real-world auction situations are likely to contain aspects of both models simultaneously. The IPV and CV are identically and independently drawn values from a common probability distribution. The IPV and CV of one bidder are statistically independent to other bidders.

Among the various literature on auction theory, [1]-[16] are relevant to this study. The references [1]-[7], covered many aspects of auction. For example, [1] is a collection of journal articles written from 1999 to 2003 and published in various journals. This collection of articles is an extraordinary presentation of auction theory in a non-technical manner. In other words, this book is a thorough review of auction theory. On the other hand, [2] focuses on how to put auction theory to work. Milgrom presented the mathematical overview, in particular the tools of ordinary demand theory that includes the envelope theorem and the comparative static techniques [2]. A detailed overview of single object and multiple object auction models is provided in [3]. The discussed models were illustrated by proven propositions. Reference [4] is primarily focused on the beginner reader in auction theory. This book presented auction mechanism design along with the values of auction which are independent private value, common value and affiliated value. A detailed explanation of common value is covered in [5]. The success story of a booming online local New Zealand auction*, trademe*, is depicted in [6]. With a different perspective, [7] is primarily focused on how auctions play a role in forming the social construction of value.

The journal articles [8]-[12] and [14]-[16] bring the auction theory into the picture with various illustrations of assumptions and their relaxations. As recommended by many auction theorists, [8] and [9] are still essential reading [1], [4]. Reference [8] analyses the special cases of RET while [9] surveys the developments in the theory of bidding mechanisms and discusses the relevance of the theoretical results for auctions in practice. Mathematical models covering all four types of auctions are presented in [10]. A focus on asymmetric English auctions is presented in [11]. This study introduced two different conditions (average crossing and cyclic crossing) and contrasted them in an English auction environment. On the other hand, [12] described an auction as an incomplete and imperfect information scenario which is not given to anyone in its totality. The possibility of re-auctioning is presented by a model in [14].

As reviewed above, there are many theoretical studies carried out focusing on an English auction environment. The following studies primarily highlighted the different aspects of an English auction environment from the applications perspective. For example, reference [15] focused on the inference with an incomplete model of English auctions. This study explored an incomplete model based on two assumptions: bidders neither bid more than their valuations nor let an opponent win at a price they would be willing to beat. The proposed approach was applied to the evaluation of U.S. Forest Service timber auctions which successfully addressed a policy debate regarding reserve prices in timber auctions.

A final price prediction model in the auction environment has recently been studied taking into account the stochastic process [16]. This study proposed stochastic models and tested them with the auction data from *eBay*. In particular, simulation based model and Markov process model were developed and compared for the prediction of final price.

B. Other Auctions

As described previously, the theoretical literature [1]-[5] cover the aspects of other auction environments too. We are interested in the English auction and in particular New Zealand wool auction, hence, we are not presenting other auction environments and their applications.

III. NEW ZEALAND WOOL AUCTION: AN ENGLISH PUBLIC OUT-CRY AUCTION

The New Zealand Wool Industry (NZWI) is contributing over a billion New Zealand dollars per year to the New Zealand economy. Being a major contributor to the economy, there is a high significance of the study of wool trading systems like auctions in New Zealand. The New Zealand wool auction is a public out-cry auction which has been running for the last 150 years. The New Zealand wool auction is a selling avenue of New Zealand Wool Clip (NZWC). A total of 60% of NZWC is traded via auction. The auction runs in the English auction principle. The procedure of auction operation and participation of the bidder is similar to the online English auction. However, the difference is that the bidders' are physically present there to make a bid for particular New Zealand wool clip of interest to their business. The NZWC is produced through the combined efforts of some 20,000 individual farmers. During any one twelve month period, each farmer produces a variety of wool types which come forward at times that are dictated by weather and necessary farming practices. There are over 3000 types of wool produced in New Zealand. This reflects also the different breeds, age, and place of origin on the sheep's body. Wool is further separated and differentiated by various types of possible contamination from vegetable matter to cotts. It is also produced at different locations throughout the country. The diversity inherited in the NZWC resembles a complex system. Being diverse and complex in nature, the bidders' in the wool auction follow the traditional ways of bidding instead of any automated system. The live scenario of the New Zealand wool auction shows that though the bidding procedure is traditional, the bidders are getting real time feedback via their offices on some major factors like currency fluctuations during the hours. An example of the Christchurch wool auction centre, the only centre in the South island of New Zealand, shows that around forty bidders are registered for the bidding purpose. However, all of the registered bidders are not actively involved in buying the wool via auction. This shows that the number of bidders during the auction hours is uncertain and hence it follows the independent –private- value (IPV) model and common value (CV) model like in other English auction environments. However, in practice, the major market is controlled by quite a few exporters. Taking the whole New Zealand wool export market into perspective, the statistics show that New Zealand exports 20% of the world wool market [13]. The top ten export destinations of NZWC are shown in Table I.

IV. Mathematical assumptions in an English auction: bidder's perspective from New Zealand wool auction

In any auction environment, we consider that Revenue Equivalence Theorem (RET) has a significant role. RET in general gives the message that the seller can expect equal profits on average from all the standard as well as non-standard types of auctions [1]. As RET is a fundamental basis for auction theory, we can assume that if the bidders are risk neutral the seller would not benefit from such risk neutral bidders. However, on the other hand, if the bidders are risk averse the seller would expect more benefit from such bidders. The risk averse bidders are more volatile than the risk neutral bidders. Further, as discussed in section II, every bidder may have to

TABLE I
TOP TEN EXPORT DESTINATION OF NEW ZEALAND WOOL CLIP

Country	Percentage
China	25
UK	13
India	10
Italy	9
Belgium	8
Germany	6
Australia	5
Japan	4
Turkey	3
USA	3

follow two models simultaneously during the time of bidding.

In this paper, we try to follow as closely as possible the standard notation of auction theory papers. Among the various assumptions, we would like to present two assumptions which are relevant to the New Zealand wool auction. The first one is whether RET holds true in the case of the uncertain number of bidders in an English auction environment. And, secondly, what the expected price of the bidder in any English auction environment is.

The suitability of RET in the case of uncertain numbers of bidders which is relevant to the New Zealand wool auction is presented.

We follow the proof based on [3],

Let $\mathbb{N} = \{1, 2, ..., N\}$ denote the set of potential bidders and let $A \subseteq \mathbb{N}$ be the set of actual bidders. Let us consider that an actual bidder $i \in A$ and let p_n denote the probability that any participating bidder assigns to the event that he is facing n other bidders. Thus, bidder i assigns probability p_n that the number of actual bidder is $n+1$. The probabilities p_n do not depend on the identity of the bidder or on his value. It is also important that the set of actual bidders does not depend on the realized values. As long as bidders hold the same beliefs about the likelihood of meeting different numbers of rivals in an auction environment, the proposition holds that the expected payment of a bidder with value zero is zero, and yields the same expected revenue to the seller provided the values drawn are independently and identically distributed.

Let us consider a standard auction a and a symmetric and increasing equilibrium β of the auction. It should be noted here that the bidders are unsure about the number of rivals they have to face, β does not depend on n. Let us assume that the expected payoff of a bidder with value x who bids $\beta(z)$ -strategic bid- instead of the equilibrium bid $\beta(x)$. The probability that he faces n other bidders is p_n. In this case, the bidder wins if $Y_1^{(n)}$, the highest of n values drawn from F, is less than z and the probability of this event is $G^{(n)}(z) = F(z)^n$. The overall probability that he will win when he bids $\beta(z)$ is therefore,

$$G(z) = \sum_{n=0}^{N-1} p_n G^{(n)}(z) \tag{1}$$

His expected payoff from bidding $\beta(z)$ when his value is x is then

$$\prod^{a}(z,x) = G(z)x - m^{a}(z) \tag{2}$$

This shows that in the situation of uncertain number of bidders RET also holds true.

Now, we would like to present the expected price of the bidder in an English auction environment. Let us take a case of three bidders having actual private types, $x_1 = a, x_2 = b, x_3 = c,$ respectively with $a < b < c$, after [4], The implicit assumption is that the price starts at zero and rises continuously. The first player to drop out is player 1 at price p_1 such that:

$$E[V_1 | x_1 = a = X_2 = X_3] = p_1 \tag{3}$$

Prior to bidder 1 dropping out, the expected values of bidders 2 and 3 were given, respectively, by:

$$E[V_2 | x_2 = b = X_1 = X_3] \tag{4}$$

and

$$E[V_3 | x_3 = c = X_1 = X_2]. \tag{5}$$

After bidder 1 drops out, these expected values are revised in the following way:

$$E[V_2 \mid x_2 = b, E[V_2 \mid X_1 = X_2 = X_3 = a] = p_1] \quad (6)$$

and

$$E[V_3 \mid x_3 = c, E[V_3 \mid X_1 = X_2 = X_3 = a] = p_1] \quad (7)$$

The price keeps rising until the final price p_2 and the winner is determined as follows:

$$E[V_2 \mid x_2 = b = X_3, p_1] = p_2 \quad (8)$$

The value of the object to player 3 prior to knowing p_2 is

$$E[V_3 \mid x_3 = c = X_2, p_1]. \quad (9)$$

Bidder 3 wins the auction at price p_2.

If we generalize for n players, the expected price in an English auction environment is,

$$P^E = E[E[V_1 \mid X_1 = y_1 = Y_2, Y_3, ..., Y_n] \mid X_1 = x > Y_1] \quad (10)$$

Now, we will present a model. The model presented in Fig. 1 is flexible enough to accommodate the decision making behaviors of a bidder in any New Zealand wool auction. Two stages of the model can be categorized: *prior to auction* and *during and after auction*

As explained in [10], the English auction has many variants. The New Zealand wool auction can also be considered as the Japanese version of the English auction. It is idealized as follows: prior to auction day, the bidders are given the opportunity to inspect the sample of wool types going to be auctioned. Further, the bidders are provided catalogues of wool types going to be auctioned in the auction center for the day. In the catalogues, the information like wool quality factors (Diameter, color, vegetable matter, length of the wool staple) is provided. The prior observation would help to do the requirement analysis for the bidder. Prior to auction, the bidders take into account many factors. Likewise, during and after auction the bidders take into consideration the factors. Among the factors, three major factors play a role in both situations which are 'physical parameters of wool', 'wool quality category' and 'number of bales in stock and types'. These factors are common to the bidders in competition. The model generated from these factors is common to all the bidders. In auction theory terms, the distribution generated by such common factors is common value (CV) model. From the figure, it can be seen that the bidder with the requirement analysis in their mind competes in the auction. This leads us to assume that like other English auction types, the bidders have CVs for the wool types they are going to buy. With their CV, they compete in the auction environment and the bidder chooses whether to be active at the start price p_0.

In addition to this, particularly for the wool auction, five specific factors, which are independent to the bidders, determine the price of the wool in an auction. The five factors are different to all the bidders and not known to each other. In other words, we can say that the price of the wool in an auction is a function of these five factors:

Price of the wool in any auction = f (Stock held, forward sales position, available wool types in specific auction, currency factor, seasonality of supply).

These five factors are the major constituents that contribute to make the independent-private-value (IPV) model different to each bidder. With the IPV and CV in their minds, bidders participate in the auction environment. IPV and CV are simultaneously used by the respective participating bidders. As the auctioneer raises the price, bidders drop out one by one. However, any bidder can come back in up to the final knock of the hammer, finalizing the sale.

V. Conclusion

Auction theory with a highlight of an English auction environment is presented. A model showing the decision making behavior of bidders from the New Zealand wool auction shows that the price of wool type in an auction is the function of independent private values and common values of the bidders. This study will help us to make an extension of the functioning model that will help the bidders to make efficient decisions in the wool auction environment. Such semi-automatic tools will contribute to high returns in wool trading.

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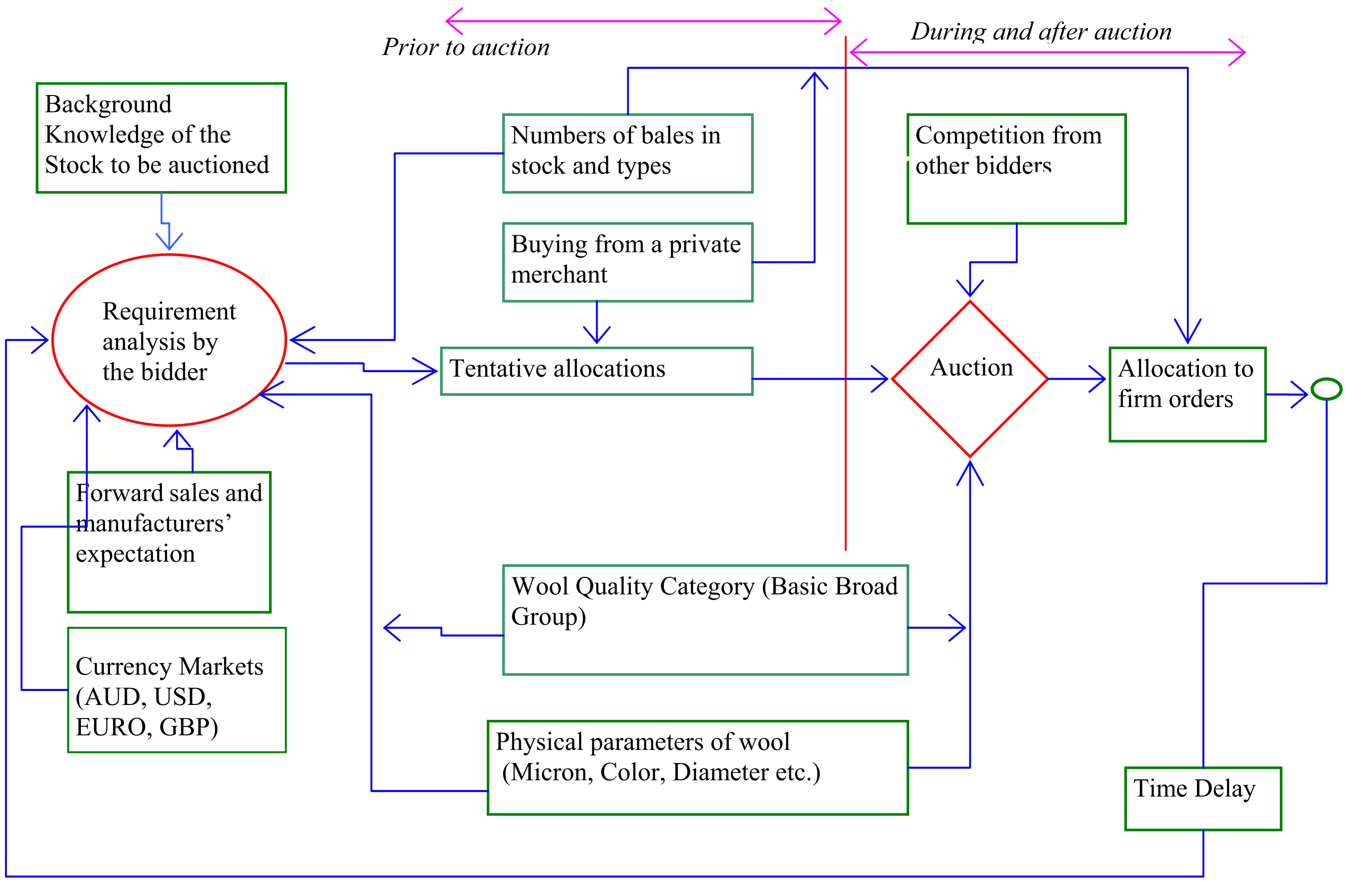

Fig. 1 A model showing the decision making behaviors of a bidder in the New Zealand wool auction environment.

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