Closed form optimal solution of a tuned liquid column damper responding to earthquake

A. Farshidianfar, P. Oliazadeh

Abstract**—**In this paper the vibration behaviors of a structure equipped with a tuned liquid column damper (TLCD) under a harmonic type of earthquake loading are studied. However, due to inherent nonlinear liquid damping, it is no doubt that a great deal of computational effort is required to search the optimum parameters of the TLCD, numerically. Therefore by linearization the equation of motion of the single degree of freedom structure equipped with the TLCD, the closed form solutions of the TLCD-structure system are derived. To find the reliability of the analytical method, the results have been compared with other researcher and have good agreement. Further, the effects of optimal design parameters such as length ratio and mass ratio on the performance of the TLCD for controlling the responses of a structure are investigated by using the harmonic type of earthquake excitation. Finally, the Citicorp Center which has a very flexible structure is used as an example to illustrate the design procedure for the TLCD under the earthquake excitation.

Keywords**—**Closed form solution, Earthquake excitation, TLCD.

I. INTRODUCTION

THE current trend towards structure with increasing height and advanced construction techniques has led to lighter and highly flexible civil engineering structures in many urban areas, such as Petronas Twin Tower in Malaysia, and Shanghai World Financial Center and Jin Mao Building in China [1]. These structures are vulnerable to dynamic loads, such as wind gusts, ocean waves and earthquakes. It is thus necessary to find a cost effective solution for suppressing the vibration of structures. Among many varieties of control devices [2], [3] the tuned liquid column damper (TLCD) as a energy absorbing device which does not require an external power source for operation and utilizes the motion of the structure to develop the control forces is a good candidate. A TLCD consists of tube like containers filled with liquid (commonly water) where energy is dissipated by the movement of the liquid through an orifice. It can provide the same level of vibration suppression as a conventional tuned mass damper (TMD) systems [4], [5] but with following advantages:

1) The TLCD parameters (frequency and mass) can be easily tuned by adjusting the height of the liquid in the tube.

2) The required level of damping can be readily achieved and controlled through the orifice.

3) It can utilize as a water storage facility at the top of the buildings for an emergency such as fire.

4) Easy installation and little maintenance needed.

Moreover, TLCD can dissipate energy in vertical direction [6] similar to horizontal motion of structures [7]; also it can reduce rotational and pitching vibration of structures [8], [9].

In recent years, there have been several studies under taken on the evaluation of TLCD performance in suppressing the vibration of structures under wind excitations [10-12] and relatively few studies have been made on seismic performance of TLCDs [13], [14].

The intention of this paper is to find a closed form solution of a TLCD-structure system under a harmonic type of earthquake excitation. Due to inherent nonlinear damping, iterations is generally required in order to obtain the frequency domain response of a structure equipped with a TLCD. It would be quite a time consuming task to carry out a detailed design of the mass damper. Therefore to facilitate design of the damper, the analytical formulas of the TLCD-structure responses are derived and compared with the results of other researcher [7]. After that the effects of design parameters (mass and length ratio) on the performance index and the geometry of the TLCD is discussed and finally the Citicorp Center is used as an example to illustrate the design procedure for the TLCD under the harmonic type of earthquake excitation.

II. EQUATION OF MOTION

Let it be assumed that the motion of a building modeled as a single degree of freedom (SDOF) system, is to be mitigated using tuned liquid column damper (TLCD). A structure-TLCD system under ground motion is shown in Fig. 1. $r = A_v/A_h$ is the cross sectional area ratio of the mass damper. It can be uniform ($r = 1$) which corresponds to the tuned liquid column damper and non-uniform ($r \neq 1$) which corresponds to the liquid column vibration absorber (LCVA) [15], where A_v and A_h are the vertical and horizontal column cross sectional area, respectively. In consideration of dynamic equilibrium condition and the interaction between the structure and the liquid column in TLCD, the equation of motion of a structure equipped with the TLCD for lateral vibration control under earthquake loading expressed as

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$$M\left(\ddot{X}+\ddot{X}_g\right)+\rho A_h L_e\left(\ddot{X}+\ddot{X}_g\right)+ C\dot{X}+KX+\rho A_h r L_h \ddot{Y}=0 \quad (1)$$

$$\rho A_h r L_e \ddot{Y}+\tfrac{1}{2}\rho A_h r^2 \eta\left|\dot{Y}\right|\dot{Y}+2\rho A_h g r Y+ \rho A_h r L_h\left(\ddot{X}+\ddot{X}_g\right)=0 \quad (2)$$

where M,C,K are the structural mass, damping and stiffness constant; X is the lateral displacement of the structure; Y is the motion of the liquid surface inside the TLCD; $\ddot{X}_g$ is the absolute acceleration of ground motion; L_h and L_v are the horizontal and vertical column length, respectively; ρ is the liquid density in the TLCD; g is the acceleration due to gravity; $L_e=2L_v+rL_h$ is defined as the effective length which means the length of an equivalent uniform liquid column having the same circular frequency, if r = 1 (for TLCD), then $L_e=L=2L_v+rL_h$ that represents the total length of the TLCD.

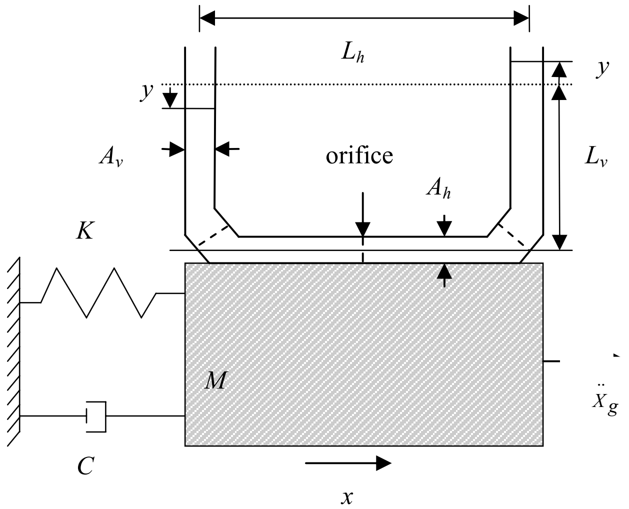

Fig. 1 structure equipped with a tuned liquid column damper (TLCD)

III. NONDIMENSIONALIZATION

It is convenient to work with dimensionless position and time, according to $x=\dfrac{X}{L_h}$; $y=\dfrac{Y}{L_h}$; $\tau=\dfrac{t}{T_d}$. It is easily observed that the natural frequency of a TLCD is $\omega_d=\sqrt{\dfrac{2g}{L_e}}$, and accordingly the natural period is $T_d=2\pi\sqrt{\dfrac{L_e}{2g}}$. In such a way "1" and "2" are rewritten in the following form

$$\mu n y''+(1+\mu)x''+4\pi\zeta\gamma x'+4\pi^2\gamma^2 x=-(1+\mu)x_g'' \quad (3)$$

$$y''+\tfrac{1}{2}rn\eta\left|y'\right|y'+4\pi^2 y+nx''=-nx_g'' \quad (4)$$

where the notation prime *(")* stands for differentiation with respect to the scaled time τ, and the following abbreviation were introduced

$\mu=\dfrac{\rho A_h L_e}{M}$ is the mass ratio of the liquid column to the structure.

$\zeta=\dfrac{C}{2M\omega_s}$ is the damping ratio of the structure.

$\omega_s=\sqrt{\dfrac{K}{M}}$ is the natural frequency of the structure.

$\gamma=\dfrac{\omega_s}{\omega_d}$ is the frequency ratio of the structure versus TLCD and for the TLCD (r = 1) $n=\dfrac{L_h}{L}$.

IV. EQUIVALENT DAMPING

Under a harmonic type of earthquake excitation, the inherent nonlinear damping of liquid motion $\frac{1}{2}rn\eta\left|y'\right|y'$, could be replaced by a linear equivalent damping term $\left(\dfrac{4}{3\pi}\right)\rho A_h r^2\eta\varphi_Y\omega\dot{Y}$, using the equivalent linearization technique that minimizes the error between these two expressions [7]. Consequently after non-dimensionalization of "4", it can rewritten as

$$y''+\frac{8}{3}rn\eta\varphi_y k y'+4\pi^2 y+nx''=-nx_g'' \quad (5)$$

where

$k=\dfrac{\omega}{\omega_d}$ is the non-dimensional excitation frequency; and

$\varphi_y=\dfrac{\varphi_Y}{L_h}$ is the non-dimensional steady state response amplitude of liquid displacement.

V. ANALYTICAL SOLUTION TO HARMONIC LOADING

In this section, closed form solution of TLCD-structure system is derived. By substituting the excitation term with $x_g''=x_{g0}''e^{i2\pi k\tau}$ and the responses with complex harmonic functions $x=x_0 e^{i2\pi k\tau}$ and $y=y_0 e^{i2\pi k\tau}$ into "3" and "5", the frequency response functions of both the structure and the liquid motion of the TLCD under harmonic force earthquake excitation can be obtained as

$$x_0=\Psi_{xx_g''}x_{g0}'' \quad (6)$$

$$y_0=\Psi_{yx_g''}x_{g0}'' \quad (7)$$

in which $\Psi_{xx_g''}$ and $\Psi_{yx_g''}$ represent the frequency response functions of x and y induced by x_g''.

$$\Psi_{xx_g''}=\frac{b_0-b_2k^2+b_1ki}{(A_0+A_1\varphi_y)+i(A_2+A_3\varphi_y)} \quad (8)$$

$$\Psi_{yx''_g} = \frac{b_3 + b_4 ki}{(A_0 + A_1\varphi_y) + i(A_2 + A_3\varphi_y)} \quad (9)$$

where

$$\begin{aligned} b_0 &= -(1+\mu) \\ b_1 &= -\frac{4}{3\pi} krn\eta\varphi_y(1+\mu) \\ b_2 &= -(1+\mu) + \mu n^2 \\ b_3 &= -n\gamma^2 \\ b_4 &= -2\zeta\gamma n \\ A_0 &= 4\pi^2(1-k^2)\left[\gamma^2 - k^2(1+\mu)\right] - 4\pi^2 k^2 \mu n^2 \\ A_1 &= -\frac{32}{3}\pi k^3 rn\eta\zeta\gamma;\ A_2 = 8\pi^2 k\zeta\gamma(1-k^2) \\ A_3 &= \frac{16}{3}\pi k^2 rn\eta\left[\gamma^2 - k^2(1+\mu)\right] \end{aligned} \quad (10)$$

By using the relation $\varphi_y = |y_0|$ and substituting of "8"-"10" into square of the absolute value on both sides of "6" and "7" lead to

$$|x_0|^2 = \frac{\left[(b_0 - b_2 k^2)^2 + (b_1 k)^2\right]\cdot {x''_{g0}}^2}{(A_0 + A_1 \times |y_0|)^2 + (A_2 + A_3 \times |y_0|)^2} \quad (11)$$

$$|y_0|^2 = \frac{\left[(b_3)^2 + (b_4 k)^2\right]\cdot {x''_{g0}}^2}{(A_0 + A_1 \times |y_0|)^2 + (A_2 + A_3 \times |y_0|)^2} \quad (12)$$

The amplitude $|y_0|$ can actually be obtained by solving a polynomial equation which is constituted by rearranging "12" as

$$C_0|y_0|^4 + C_1|y_0|^3 + C_2|y_0|^2 + C_3 = 0 \quad (13)$$

where

$$\begin{aligned} C_0 &= A_1^2 + A_3^2 \\ C_1 &= 2\left[(A_0 \cdot A_1) + (A_2 \cdot A_3)\right] \\ C_2 &= A_0^2 + A_2^2 \\ C_3 &= -\left[(b_3)^2 + (b_4 k)^2\right]\cdot {x''_{g0}}^2 \end{aligned} \quad (14)$$

Consequently the frequency response functions $\Psi_{xx''_g}$ and $\Psi_{yx''_g}$ and the amplitude $|x_0|$ can be obtained by substituting the solution of $|y_0|$ into "8", "9" and "11", respectively.

The closed form solutions obtained from this analytical procedure are then verified through a comparison with the numerical results obtained from [7]. Three cases demonstrated in TableI, Table II and Table III. The TLCD-structure parameters are the same for all three cases and they are as follows:

$\zeta = 0.01,\ \mu = 0.01;\ n = 0.6;\ x''_g = 0.01;\ \eta = 10\ and\ \gamma = 1$,

while each case uses the cross sectional ratio, r, equal to 1, 2 and 0.5, respectively. It can be seen that the results from closed form solution match very well with those from [7].

TABLE I
COMPARISON WITH THE NUMERICAL RESULTS

$k = \frac{\omega}{\omega_d}$	$\lvert x_0 \rvert$ (Present Paper)	$\lvert x_0 \rvert$ (Reference [7])	Error(%)
0.8	0.0007358	0.0007240	1.6
0.85	0.0009779	0.0009593	1.9
0.9	0.0015259	0.0014866	2.6
1.1	0.0012660	0.0012790	1.0
1.2	0.0005771	0.0005782	0.19

TABLE II
COMPARISON WITH THE NUMERICAL RESULTS

$k = \frac{\omega}{\omega_d}$	$\lvert x_0 \rvert$ (Present Paper)	$\lvert x_0 \rvert$ (Reference [7])	Error(%)
0.8	0.0007359	0.0007240	1.6
0.85	0.0009782	0.0009592	2.0
0.9	0.0015315	0.0014846	3.1
1.1	0.0012715	0.0012728	0.1
1.2	0.0005772	0.0005782	0.2

TABLE III
COMPARISON WITH THE NUMERICAL RESULTS

$k = \frac{\omega}{\omega_d}$	$\lvert x_0 \rvert$ (Present Paper)	$\lvert x_0 \rvert$ (Reference [7])	Error(%)
0.8	0.0007359	0.0007240	1.6
0.85	0.0009779	0.0009592	1.9
0.9	0.0015245	0.0014860	2.6
1.1	0.0012645	0.0012771	1.0
1.2	0.0005772	0.0005782	0.17

VI. OPTIMIZATION OF TLCD

The optimal design aims at minimizing the structural response. To find the optimal parameters, the peak amplitude of the normalized structural and liquid responses over all possible frequencies $(k \in \Re)$ are used as the performance index (*P.I.*) [7] and they can be find respectively as

$$P.I_x = Max([x_0]_{norm}) = Max\left([x_0] \big/ x_{p(original)} \right) \quad (15)$$

$$P.I_y = Max([y_0]_{norm}) = Max\left([y_0] \big/ x_{p(original)} \right) \quad (16)$$

where $x_{p(original)}$ represents the structural response without installing the TLCD device, which can be expressed by [7]

$$x_{p(original)} = \frac{x''_g}{4\pi^2\gamma^2} \cdot \frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad (17)$$

Hence, according to "15", a smaller $P.I_x$ represents a better performance. It should be emphasized that the value $P.I_y$ must be checked, if the liquid surface displacement exceeds the length of vertical liquid column.

VII. Optimal Design Parameters for a TLCD

A. *Optimum mass ratio*

Fig. 2 shows the variation of performance index to the mass ratio μ, for three different structural damping ratios. Fig. 2 represents that the larger μ results in better control performance but the geometry of the TLCD is affected by the mass ratio and becomes larger. Since the mass of the structure is constant and increasing the mass ratio results in increasing the mass of the TLCD. Hence practically it is impossible to use μ ≥ 3% and it is better to use more than one TLCD in such conditions which is called multiple tuned liquid column damper (MTLCD).

B. *Optimum length ratio*

Fig. 3 shows the variation of performance index with respect to the length ratio, L_h/L, and μ. It is observed that the larger μ and L_h/L result in the better control performance in general. Accordingly, it is advantageous to increase L_h/L as much as possible. However, L_h/L more than a certain threshold value destroys the basic characteristics of TLCDs. Because the liquid surface displacement will also become larger, so larger displacement greater than the vertical column should be avoided in the design of a TLCD.

VIII. A Design Example

Fig. 4 provides a flowchart that proposed the design procedure for the TLCD and can be directly applied in this example. To illustrate the design procedure of the damper, the Citicorp Center [16] is taken as an example structure in this study. A TLCD is to be installed at the top of the building to for abating the vibration induced by the harmonic type of earthquake excitation. The mass, stiffness and damping constants of this building are, $1.8\times10^7\ N.s^2/m$, $1.82\times10^7\ N/m$ and $0.36\times10^6\ N.s/m$ $(\zeta = 0.01)$ respectively. These properties represented the first mode of natural frequency. The step by step procedure for TLCD design are stated as follows:

1) The first step of designing a TLCD is to select a proper mass ratio. The larger mass ratio, results on better control performance but by increasing the mass ratio, the geometry of the TLCD becomes larger and may require a stronger supporting system at the top of the structure, which would increase the installation cost. In this example, a mass ratio of 0.025 is selected and with a horizontal length ratio, n, chosen as 0.6, the tuning frequency ratio $\gamma = \frac{\omega_s}{\omega_d}$ of 1 is selected. Therefore, the total length, L, the horizontal length, L_h and the vertical length, L_v, of the TLCD, can be obtained as follows

$$\omega_s = \sqrt{\frac{K}{M}} = 1.006\ rad/s$$

$$\omega_d = \omega_s = 1.006\ rad/s$$

$$\sqrt{\frac{2g}{L}} = 1.006,\ L = 19.4\ m$$

$$n = \frac{L_h}{L}$$

$$0.6 = \frac{L_h}{L},\ L_h = 11.6\ m$$

$$L = L_h + 2rL_v\ ;\ r = 1\ (for\ the\ TLCD)$$

$$L_v = 3.88\ m$$

2) By $\mu = \frac{\rho A_h (L_h + 2L_v)}{M}$ and knowing that the density of the water is about 997 kg/m^3, a cross section A=23.26 m^2 is thus determined. This huge cross section is due to the large

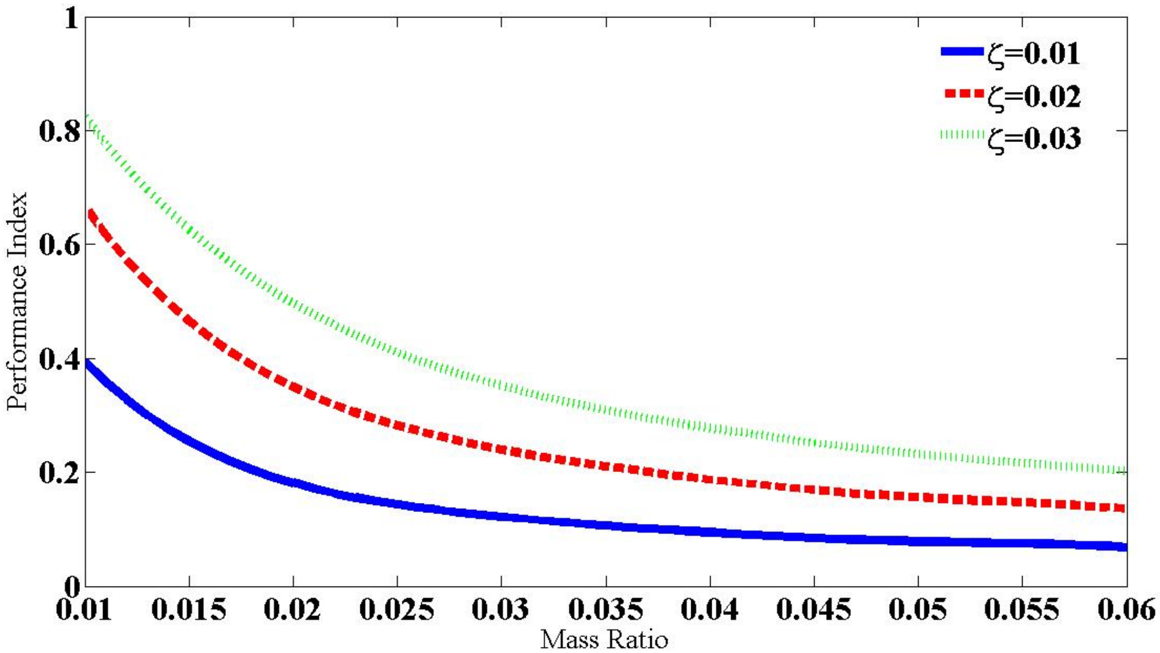

Fig. 2 Variation of performance index with respect to the mass ratio

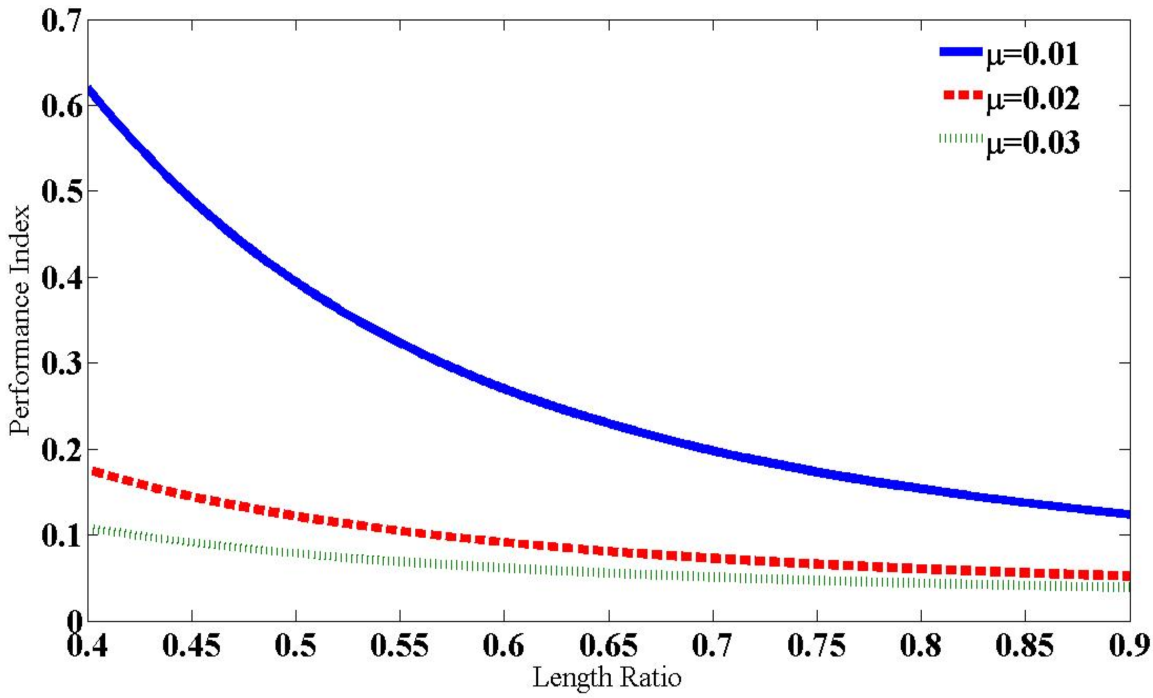

Fig. 3 Variation of performance index with respect to the length ratio

first modal mass and the mass ratio that is chosen. One possible solution to create a space for such a huge TLCD is to divide it into few smaller TLCDs with identical configurations.

3) The last step is to check if the liquid surface displacement exceeds the vertical column length. After choosing η=10, k=0.8, $x_g'' = 0.2$ and also knowing that ζ=0.01, the amplitude $|y_0|$ can be evaluated from "13", which is $|y_0| = 0.025\ m = 2.5\ cm$. Since the vertical column length of the TLCD is $L_v = 3.88\ m$, which is much more than $|y_0|$, this design is feasible.

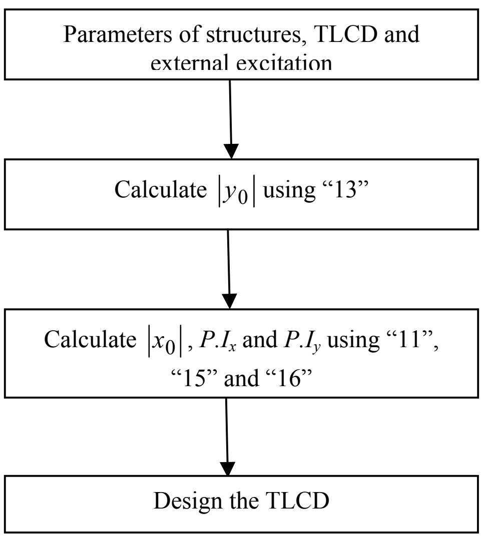

Fig. 4 Flowchart of designing the TLCD

IX. Conclusion

The main objective of this paper is to develop some analytical formulas and closed form solution for determining the optimum parameters of the tuned liquid column damper in suppressing horizontal vibration of structures under a harmonic type of earthquake excitation. Using Lagrange equation, an unsteady and non-uniform flow equation for the TLCD were investigated. By using the peak amplitude of the normalized structural and liquid responses over all possible frequencies $(k \in \Re)$ as the performance index, the optimal parameters of the TLCD can be investigated. The analytical formulas were compared with [7]. The comparison indicated that the formula is quite accurate for the TLCD designs for damped structure and also decreases the time needed for the TLCD design procedure. Then the optimal parameters of a TLCD such as length ratio and mass ratio are obtained by using the harmonic type of earthquake excitation and the following conclusions can be made:

1. By increasing the mass ratio the displacement variance of the structure reduced. However a larger mass ratio does not always guarantee superior results in general and it is impossible to use $\mu \geq 3\%$.

2. The performance of the TLCD also depends on the length ratio. It is shown, by increasing the length ratio, the better results in control performance can be achieved. However, a TLCD with very large length ratio does not work properly; thus, there is a threshold for length ratio of a TLCD.

Finally, the Citicorp Center, a flexible skyscraper is used as an example to illustrate the design procedure for the TLCD under the harmonic type of earthquake excitation.

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