Chua's Circuit Regulation Using a Nonlinear Adaptive Feedback Technique

Abolhassan Razminia, and Mohammad-Ali Sadrnia

***Abstract*—**Chua's circuit is one of the most important electronic devices that are used for Chaos and Bifurcation studies. A central role of secure communication is devoted to it. Since the adaptive control is used vastly in the linear systems control, here we introduce a new trend of application of adaptive method in the chaos controlling field. In this paper, we try to derive a new adaptive control scheme for Chua's circuit controlling because control of chaos is often very important in practical operations. The novelty of this approach is for sake of its robustness against the external perturbations which is simulated as an additive noise in all measured states and can be generalized to other chaotic systems. Our approach is based on Lyapunov analysis and the adaptation law is considered for the feedback gain. Because of this, we have named it NAFT (Nonlinear Adaptive Feedback Technique). At last, simulations show the capability of the presented technique for Chua's circuit.

***Keywords*—**Chaos, Adaptive control, Nonlinear control, Chua's circuit.

I. Introduction

CHUA'S circuit (Fig. 1) is a nonlinear electronic circuit that is the object of much scientific research activities. This circuit contains four linear elements (two capacitors, one inductor, and one resistor) and a nonlinear resistor, called *Chua's diode* [1], which can be built using off-the-shelf op-amps.

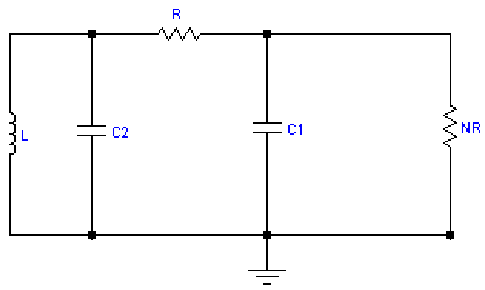

Fig. 1 Schematic description of Chua's circuit

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Since Chua's circuit is endowed with an unusually rich repertoire of nonlinear dynamical phenomena, it has become a *universal paradigm* for chaos [2].

This circuit can generate even more phenomena that are chaotic. Also it is *canonical* in the sense that its vector field is *topologically conjugate* (i.e. qualitatively equivalent) to a large class of 3-D vector fields [2].

The Chua's circuit (a third-order autonomous, dissipative electrical circuit) has been investigated thoroughly at the experimental, numerical and analytical levels by many researchers.

Anyway using KVL and KCL, the equations that describe the nonlinear dynamics of Chua's circuit is as follows:

$$\begin{aligned} C_1\dot{v}_1 &= \frac{1}{R}(v_1 - v_2) - f(v_1) \\ C_2\dot{v}_2 &= \frac{1}{R}(v_1 - v_2) + i_3 \\ L\dot{i}_3 &= -v_2 \end{aligned} \tag{1}$$

where the nonlinear term $f(v_1)$ is :

$$f(v_1) = m_0 v_1 + \frac{1}{2}(m_1 - m_0)\{|v_1 + B_P| - |v_1 - B_P|\} \tag{2}$$

B_p is the break point of the nonlinear function f Fig. 2 (theoretically B_{p1} assumed be enough large).

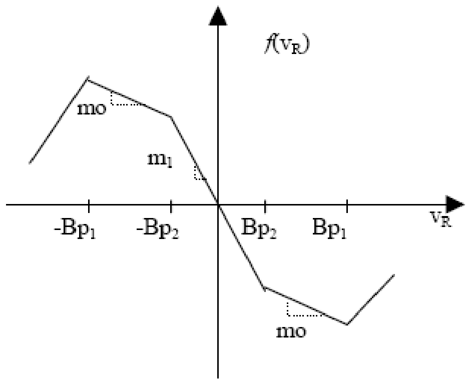

Fig. 2 Nonlinear term in Chua's circuit

The practical diagram of this circuit is depicted in Fig. 3 which is simulated using EWB software.

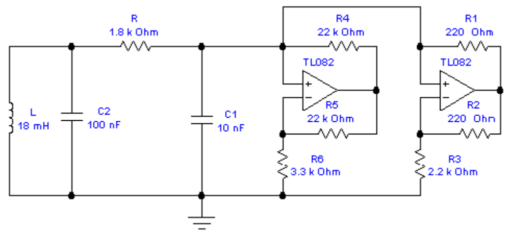

Fig. 3 EWB simulation of Chua

Chua's circuit surprisingly exhibits approximately all features, which is usual in the chaos, bifurcation and fractal studies. For different values of the elements used in the implementation of this circuit, we can see different chaotic motion. For example using the quantities shown in Fig. 3 it is easy to derive a strange attractor as depicted in Fig. 4.

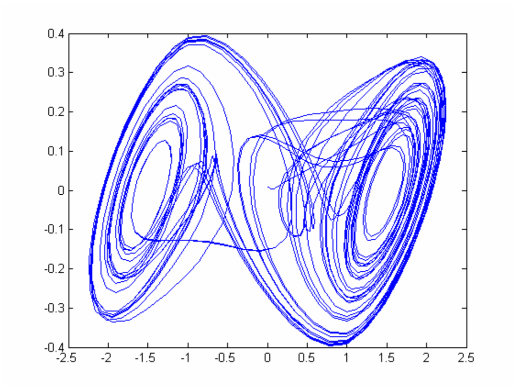

Fig. 4 Phase portrait of v1-v2 for Chua's circuit

II. Problem Statement

Up to now many control techniques have proposed for controlling systems that exhibit chaos. A complete list of references can be found in [3], [4]. Each method has its advantages and disadvantages. However here we want to present a method that is applicable in a vast variety of chaotic systems.

Many publications consider the possibility of applying the methods of adaptation to the control of chaotic processes, which is not surprising because in many physical applications the parameters of the controlled plant are unknown and the information about the model structure (for example, dimensionality of the system equations or the form of the nonlinear characteristics) more often than not is incomplete.

Three classes of control problem are usually considered in chaos control methodologies:

-stabilization

-chaotization

-synchronization

The problems of *stabilization* of the unstable periodic solution (orbit) arise in suppression of noise and vibrations of various constructions, elimination of harmonics in the communication systems, electronic devices, and so on. The purpose of this paper is "stabilization" of Chua's circuit using a nonlinear adaptive technique. In other words, the control objective can be stated as:

$$\lim_{t\to\infty}(x_i(t)) = 0 \tag{3}$$

where x_i (i =1 , 2, 3) is the Chua's circuit state variables; i.e. v_1, v_2, i_3. That is the control aim is push the states of the given system to zero.

III. Adaptive Control Methodology

The majority of works make use of the methods of direct or indirect (identification-based) adaptive parametric control. The system model is, thus, parameterized, that is, comes to:

$$\dot{x} = F(x,\theta,u) \tag{4}$$

where θ is the vector of the unknown parameters. The control law is also set down in the parametric form:

$$u = U(x,\xi) \tag{5}$$

where $\xi = \Phi(\theta)$ that is, the vector of controller parameters is defined through the vector of parameters of system (4) (this format is called as Indirect STR). Controller (5) is usually designed using the reference model or the methods of linearization by feedback. However since our objective in this work is stabilization to zero the reference model is easily zero dynamics.

The historical motivation for this method was come from [5] in which the authors consider a discrete time system as the plant and have used an approximate linearization about an operating point.

Here based on what published recently in [6] we generalize that method and modify it to achieve an adaptive feedback gain which strongly guaranty the robustness of the chaotic system especially Chua's circuit. In addition, it is allowed to be nonautonomous system. Thus, we consider the uncontrolled general form of (4) as:

$$\dot{x} = F(x(t),t) \tag{6}$$

where $x \in \Re^n$ and nonlinear $F(.)$ are state variables and well-defined nonlinear function which describe the system dynamics , respectively. . One of the most important assumption in the following formulation is that the nonlinear function $F(.)$ satisfy the Lipschitz condition:

$$\|F(x)-F(y)\| \le \rho\|x-y\| \tag{7}$$

where ρ is a positive number. Note that here we use the infinity-norm for the notation $\|.\|$; i.e.

$$\|x\| = \sup_{1\le i\le n}|x_i| \tag{8}$$

Inserting the control signal $u(x,t)$ into the system equation we expect the asymptotical stability. Thus the control signal varies the dynamics as follows:

$$\dot{x} = F(x(t),t) + u(x,t) \tag{9}$$

Note that our technique is schematically as shown in Fig. 5.

Control and adaptation law can be considered as follows:

$$u_i = -\beta_i x_i - \beta_i F_i(x) \tag{10}$$

$$\dot{\beta}_i = \gamma_i(x_i^{\,4} + x_i^{\,3}F_i(x)) \tag{11}$$

As can be seen from the above equations (10),(11) the control law is applied to each state separately.

Now we claim that this control law asymptotically stabilizes the Chua's circuit into its equilibrium point. Using the following Lyapunov function, it is straightforward to prove the asymptotical stability of the controlled system:

$$V = \frac{1}{4}\sum_{i=1}^{n} x_i^{\,4} + \frac{1}{2}\sum_{i=1}^{n}\frac{1}{\gamma_i}(\beta_i - L_i)^2 \tag{12}$$

where $L_i(1\le i\le n)$ are constants that relate with the Lipschitz constant ρ as follows:

$$\sum_{j=1}^{n}|1-L_j|\rho < L_i \tag{13}$$

Direct differentiation it is easy to see that:

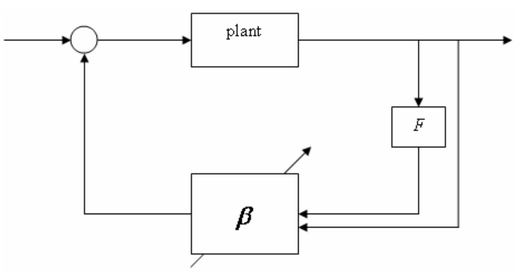

Fig. 5 Control methodology used in Chua's circuit

$$\dot{V} \le 0 \tag{14}$$

Now using Barbalat's lemma we conclude that $\dot{V} = 0$ iff $x = 0$. Thus it can be stated that the Chua's circuit is globally asymptotically stable about its equilibrium point that is the chaotic orbits of uncontrolled Chua's circuit can be stabilized to the origin using NAFT. This is a theoretical support for the following simulation based on the adaptive controller.

IV. Historical and Bibliographical Description

The chaotic nature of Chua's circuit was first observed by Matsumoto in 1983 using computer simulations [7], following the instructions of Chua, who had invented this circuit and had explained its operating principles to Matsumoto moments before he was rushed to a hospital for a major surgery, and who did not participate in the early phases of this research. In acknowledging his subsidiary role as a computer programmer, Matsumoto had named this circuit *Chua's circuit* [7], [8].

The first experimental Chua's circuit which confirms the presence of chaos was due to Zhong and Ayrom in 1984 [9], [10]. A second experimental circuit was reported by Matsumoto shortly after [11] and was designed by *Tokunaga*, who is also responsible for obtaining all of the experimental result presented in that paper. The global bifurcation landscape of Chua's circuit [12] was obt ained by *Komuro* and a team of students of Matsumoto. The colorful bifurcation landscape in this paper [12] was drawn by a professional artist.

The first rigorous proof of the chaotic nature of Chua's circuit was given in [13], where the authors proved that there exists some parameters (α,β) such that Chua's circuit satisfies *Shil'nikov*'s theorem and, therefore, has infinitely many Horseshoe maps [14]. Although the authors in [8] are listed in alphabetical order as a compromise to Matsumoto's tradition of ordering his name first in earlier publications on Chua's circuit, the rigorous proof of the main theorem is due to Komuro. However, since the limiting Cantor set from a Horseshoe map is not an attractor, this result does not imply that the double scroll Chua's attractor is directly related to the chaotic phenomena associated with the Horseshoe map. This unsatisfactory situation has now been resolved by a recent proof that a 2-D geometrical model of Chua's circuit gives rise to a *double Horseshoe map*, which generates strange attractors [15]. Another milestone was achieved in 1990 when a canonical circuit was discovered which is qualitative equivalent to a 21-parameter family C of continuous odd-symmetric piecewise-linear vector fields [16].

Inspired by a question posed by professor J. Neirynck in 1991 on whether this canonical circuit is unique, a systematic search has since been completed by several researchers, including A. Huang and Lj. Kocarev, where many more distinct canonical circuits has been found. The universal Chua's circuit presented in this paper is therefore only one among many qualitatively equivalent circuits in the class C^*.

V. Simulations Results

In this section, the numerical simulations for Chua's circuit are presented. The simulations are executed for the quantities which are depicted on Fig. 3. The results can be seen in the attached figures.

VI. Conclusion

In this paper a new nonlinear adaptive control of chaos applied to the well-known Chua's circuit is presented. The main approach is considering an adaptation law in the feedback path that is tuned via a nonlinear function based on Lyapunov analysis. The robustness of the introduced method can be seen from the attached graphs. As can be realized from the above text the motivation is from some previous tasks. However, the novelty is its robustness against the parameter variation and noisy measurements. In addition, complete references for the interested readers are brought at the end of this task.

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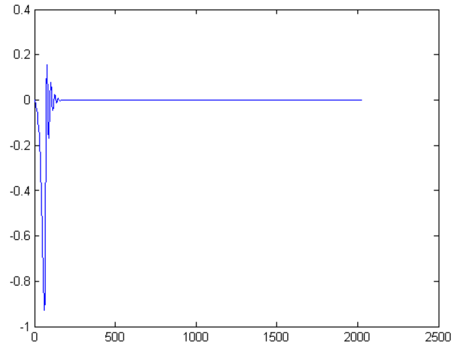

Fig. 6 i_3 response of controlled Chua's circuit without noise and perturbation

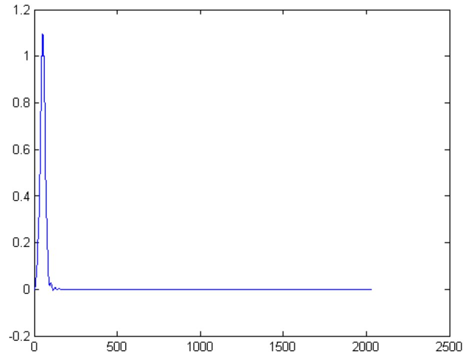

Fig. 7 v_1 response for chua's circuit without noise and perturbation

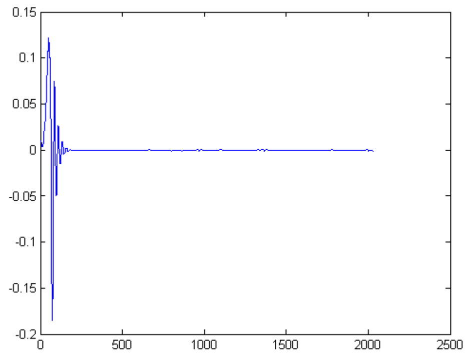

Fig. 8 v_2 response for chua's circuit without noise and perturbation

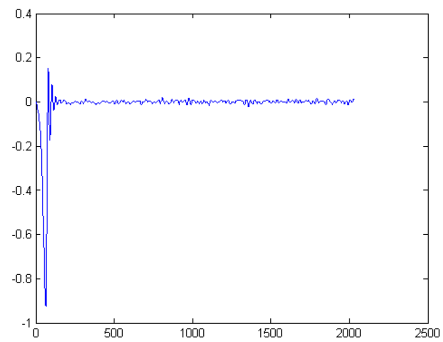

Fig. 9 i_3 response of controlled chua's circuit with noise and perturbation using NAFT

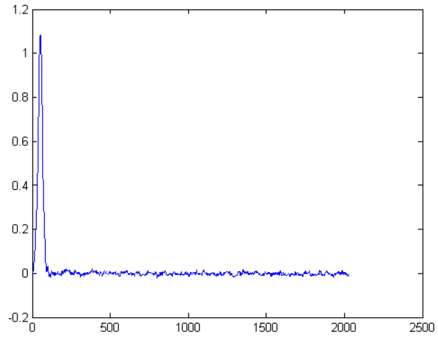

Fig. 10 v_1 response for chua's circuit with noise and perturbation using NAFT

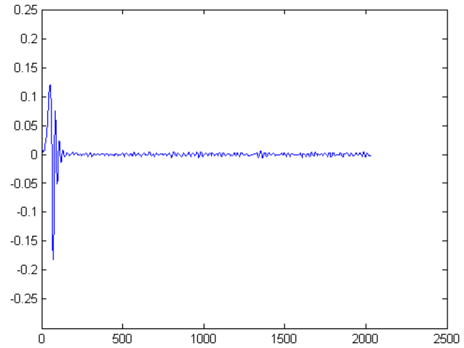

Fig. 11 v_2 response for chua's circuit with noise and perturbation using NAFT

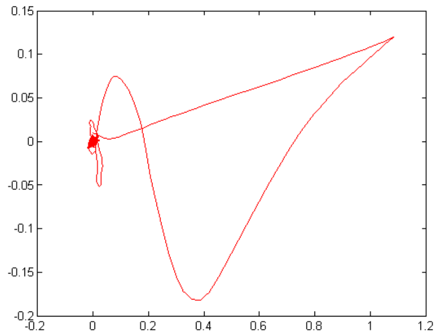

Fig. 12 Phase portrait of controlled Chua's circuit using NAFT

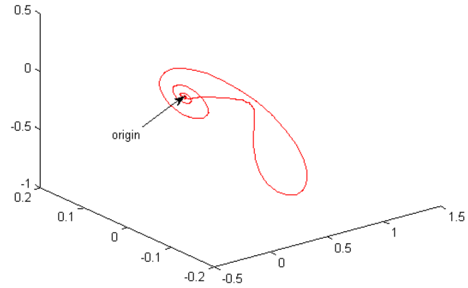

Fig. 13 3D phase portrait of controlled Chua's circuit using NAFT