pth moment exponential stability of stochastic recurrent neural networks with distributed delays

Zixin Liu, Jianjun Jiao Wanping Bai

Abstract—In this paper, the issue of pth moment exponential stability of stochastic recurrent neural network with distributed time delays is investigated. By using the method of variation parameters, inequality techniques, and stochastic analysis, some sufficient conditions ensuring pth moment exponential stability are obtained. The method used in this paper does not resort to any Lyapunov function, and the results derived in this paper generalize some earlier criteria reported in the literature. One numerical example is given to illustrate the main results.

Keywords—Stochastic recurrent neural networks, pth moment exponential stability, distributed time delays.

I. Introduction

DURING the past few decades, recurrent neural networks (RNNs) such as Hopfield neural networks (HNNs), cellular neural networks (CNNs) and other networks have been well investigated since they play an important role in classification of patterns, associative memories, optimization, etc [1,3−8,11,21,23−25]. It should be pointed out that time delays are commonly encountered in real systems due to the finite switching speed of neurons and amplifiers, and they are one of the important source of oscillation and instability. Hence, it is necessary and important to discuss the delayed RNNs models. Up to now, many results on stability of neural networks with constant delays (see [1,3,5,21]) or time-varying delays (see [4,6,8,23−25,29]) have been developed. However, a real system is usually affected by external perturbations which in many cases are of great uncertainty and hence may be treated as random. As pointed out by Haykin [10] that in real nervous systems, synaptic transmission is a noisy process brought on by random fluctuations form the release of neurotransmitters and other probabilistic causes, therefore, stochastic effects should be taken into account. In [13,14], Liao and Mao investigated the mean square exponential stability and instability problem of CNNs. In [2], the authors continued their research to discuss almost sure exponential stability for a class of stochastic CNNs with discrete delays by using the non-negative semi-martingale convergence theorem. In [17], several Razumikihin-type theorems on exponential stability were established for stochastic functional differential equations. In [18], Mao investigated robustness of exponential stability of stochastic system with small time lag. In [15], the problem of exponential stability of stochastic delayed interval systems was considered via Razumikihin-type theorems. In [22], Wan and Sun investigated mean square exponential stability of a

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class of stochastic delayed HNNs via the method of variation parameters and inequality techniques. The method used in [22] does not resort to any Lyapunov function, the activation functions are not required to be differentiable or monotonic, and the connection matrices are not to be symmetric.

Motivated by the method used in [22], the main aim of this paper is to further investigate the pth moment exponential stability problem of a class of stochastic RNNs with distributed time delays. Similar to the method used in [22], several sufficient conditions are derived to guarantee pth moment exponential stability. The results obtained in this paper generalize some previous results obtained in the literature cited therein, and which will be shown by the one numerical example provided later.

The rest of the paper is arranged as follows. In Section 2, a class of stochastic distributed time delayed RNNs models are presented, then some necessary notations and assumptions and several lemmas to be used will be given later. The pth moment exponential stability condition, several useful extensions are given in Section 3. One numerical example is provided in Section 4 to demonstrate the validity of the main results. The conclusions are given in Section 5.

II. Preliminaries

Notations. The notations are used in our paper except where otherwise specified. For $A, B \in R^n, A \le B(A > B)$ means that each pair of corresponding elements of A and B satisfies the inequality $\le, (>)$. In particular, A is called a nonnegative matrix if $A \ge 0$; E(.) stands for the mathematical expectation operator; T represents the transpose of the matrix; $|\cdot|$ denotes the Euclidean norm; $\|\cdot\|$ denotes a vector or a matrix norm; The notation $\|\cdot\|^p$ is used to denote a vector norm defined by $\|\cdot\|^p = \sum_{i=1}^{n} |x_i|^p$; I denotes the identity matrix and $\rho(\cdot)$ denotes the spectral radius of a square matrix.

In [22], Wan and Sun investigated the following stochastic HNNs model

$$
\begin{cases}
dx_i(t) = [-c_i x_i(t) + \sum_{j=1}^{n} a_{ij} \int_{-\infty}^{t} k_{ij}(t-s) \\
\qquad \times f_j(x_j(s))ds]dt + \sum_{j=1}^{n} \sigma_{ij}(x_j(t))d\omega_j(t) \\
x_i(t) = \eta_i(t), t \le 0.
\end{cases}
\tag{1}
$$

In this paper, a generalized stochastically perturbed neural network model with *distributed time* delays will be considered, which is defined by the following state equations

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$$\begin{cases} dx_i(t) = [-c_i x_i(t) + \sum_{j=1}^{n} a_{ij} \int_{-\infty}^{t} k_{ij}(t-s)f_j(x_j(s))ds \\ \quad + \sum_{j=1}^{n} b_{ij} \int_{-\infty}^{t} k_{ij}(t-s)g_j(x_j(s-\tau_j))ds]dt \\ \quad + \sum_{j=1}^{n} \sigma_{ij}(x_j(t-\tau_j))d\omega_j(t) \\ x_i(t) = \eta_i(t), t \le 0, \end{cases} \quad (2)$$

or

$$dx(t) = [-Cx(t) + A\int_{-\infty}^{t} K(t-s)f(x(s))ds$$
$$+ B\int_{-\infty}^{t} K(t-s)g(x(s-\overline{\tau}))ds]dt + \sigma(x(t-\overline{\tau}))d\omega(t), (3)$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in R^n$ is the state vector associated with the neurons; $C = diag(c_i) > 0$, $c_i > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and the external stochastic perturbations; $A = (a_{ij})_{n \times n}$ and $B = (b_{ij})_{n \times n}$ represent the connection weight matrix and delayed connection weight matrix, respectively; f_i and g_i are activation functions, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T \in R^n$, $g(x(t)) = (g_1(x_1(t-\tau_1)), g_2(x_2(t-\tau_2)), \ldots, g_n(x_n(t-\tau_n)))^T \in R^n$, where $\tau_i > 0$ is transmission delay. Moreover, $\omega(t) = (\omega_1(t)), \omega_2(t)), \ldots, \omega_n(t))^T$ is a n-dimensional standard Brown motion defined on a complete probability space (Ω, F, P) with a natural filtration $F_{t \ge 0}$(i.e., $F_t = \sigma\{\omega(x) : -\infty \le s \le t\}$) and $\sigma : R^+ \times R^n \to R^{n \times n}, \sigma = (\sigma_{ij})_{n \times n}$ is the diffusion coefficient matrix. The initial conditions for system (3) are given in the form

$$x(t) = \phi(t), -\infty \le t \le 0, \quad (4)$$

where $\phi \in L_{F_0}^p([-\infty, 0], R)$, here $L_{F_0}^p([-\infty, 0], R)$ is the family of all F_0 measurable $C([-\infty, 0], R^n)-$valued random variable satisfying that $\sup_{-\infty \le t \le 0} E|\phi(t)|^p < \infty$, $C([-\infty, 0], R^n)$ denotes the family of all continuous R^n-valued functions $\phi(t)$ on $[-\infty, 0]$ with the norm $\|\phi\|^p = \sup_{-\infty \le t \le 0} |\phi(t)|^p$. Throughout this paper, the following standard hypothesis are needed

(H_1) Both $f_i(x)$ and $g_i(x)$ satisfy the Lipschitz condition. That is, for each $i = 1, 2, \ldots, n$, there exist constants $\alpha_i > 0, \beta_i > 0$, such that

$$|f_i(x) - f_i(y)| \le \alpha_i |x - y|, |g_i(x) - g_i(y)| \le \beta_i |x - y|, \forall x, y \in R^n,$$

where α_i, β_i are Lipschitz constants, respectively.

(H_2) $\sigma_{ij}(t, x)$ satisfies the Lipschitz condition, and there are nonnegative constants L_{ij} such that

$$|\sigma_{ij}(t, x) - \sigma_{ij}(t, y)| \le L_{ij}|x - y|, \forall x, y \in R^n$$

(H_3) Assume that $f(0) \equiv 0, g(0) \equiv 0, \sigma(t, 0) \equiv 0$.
(H_4) The kernels k_{ij} for $i, j = 1, 2, \ldots, n$, are real-valued nonnegative continuous functions defined on $[0, \infty)$ and satisfies

$$\int_0^\infty k_{ij}(t)dt = 1, \int_0^\infty e^{\mu t} k_{ij}(t)dt = \overline{k}_{ij} < \infty.$$

for some positive constant μ.

It follows from [9,20] that under the hypothesis $(H_1), (H_2)$ and (H_3), system (3) has one unique global solution. Clearly, system (3) admits the trivial solution $x(t, 0) \equiv 0$.

Definition 2.1 (Mao [16,19]). The trivial solution of system (2), or system (3) is said to be pth moment exponentially stable if there exists a pair of positive constants λ and C such that

$$E\|x(t, t_0, \phi)\|^p \le CE\|\phi\|^p e^{-\lambda(t-t_0)}, t \ge t_0,$$

holds for any t_0 and $\phi \in L_{F_{t_0}}^p([-\infty, 0] : R^n)$. Especially when $p = 2$, it is called to be exponentially stable in mean square.

In this paper, we always set $t_0 = 0$. In what follows, we need introduce several lemmas which will be used in section 3.

Lemma 2.1 (*Holder inequality*).*Assume that there exist two continuous functions $f(x), g(x)$ and a set Ω, p and q satisfying $1/q + 1/p = 1$, for any $p > 0, q > 0$, if $p > 1$, then the following inequality holds.*

$$\int_\Omega |f(x)g(x)|dx \le (\int_\Omega |f(x)|^p dx)^{1/p} (\int_\Omega |g(x)|^q dx)^{1/q}.$$

Lemma 2.2[29] *Assume that there exist constants $a_k \ge 0, k = 1, 2, \ldots, n, p$ and q satisfying $1/q + 1/p = 1$, for any $p > 0, q > 0$, if $p > 1$, then the following inequality holds.*

$$(\sum_{k=1}^n a_k)^p \le n^{p-1} \sum_{k=1}^n a_k^p.$$

Lemma 2.3[22] *Set $\omega(t) = (\omega_1(t)), \omega_2(t)), \ldots, \omega_n(t))^T$ is a n-dimensional standard Brown motion defined on a complete probability space (Ω, F, P), then the following formula holds*

$$E(\int_0^t f_i(s)d\omega_i(s) \int_0^t f_j(s)d\omega_j(s)) = E\int_0^t f_i(s)f_j(s)d\langle \omega_i, \omega_j \rangle_s.$$

where $\langle \omega_i, \omega_j \rangle_s = \delta_{ij}t$, is cross-variations, δ_{ij} is correlation coefficient.

Lemma 2.4 (*Horn[12]*).*If $M \ge 0$ and $\rho(M) < 1$, then $(I - M)^{-1} \ge 0$, where I denotes the identity matrix and $\rho(M)$ denotes the spectral radius of a square matrix M.*

For convenience, in this paper, we always set $p \ge 2$ when discuss the pth moment exponential stability of system (2) or system(3), and denotes $\tau = \max\{\tau_i, 1 \le i \le n\}$.

III. MAIN RESULTS

For system (2), by the method of variation parameter, for all $t \ge 0, i = 1, 2, \ldots, n$, we have

$$x_i(t) = \int_0^t e^{-c_i(t-s)} \sum_{j=1}^n \sigma_{ij}(s, x_j(s-\tau_j))d\omega_j(s) + e^{-c_i t}x_i(0)$$
$$+ \int_0^t e^{-c_i(t-s)}(\sum_{j=1}^n a_{ij} \int_{-\infty}^s k_{ij}(s-v)f_j(x_j(v)dv)ds$$
$$+ \int_0^t e^{-c_i(t-s)}(\sum_{j=1}^n b_{ij} \int_{-\infty}^s k_{ij}(s-v)f_j(x_j(v-\tau_j)dv)ds$$
$$= I_{1i} + I_{2i} + I_{3i} + I_{4i} \quad (5)$$

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From lemma2.2, when n=4, the following inequality holds

$$|x_i(t)|^p \le 4^{p-1}(|I_{1i}|^p + |I_{2i}|^p + |I_{3i}|^p + |I_{4i}|^p)$$

this means that for all $t \ge 0$,

$$e^{\lambda t}E|x_i(t)|^p \le e^{\lambda t}4^{p-1}E(|I_{1i}|^p + |I_{2i}|^p + |I_{3i}|^p + |I_{4i}|^p).$$

Denote $G_j(t) = \sup_{-\infty < \theta \le t} E|x_j|^p e^{\lambda \vartheta}$ where $\lambda \le \min\{c_i\}, i = 1, 2, \ldots, n$. In order to get the pth moment exponential stable theorem, we first established some important results as follows.

Lemma 3.1 *For I_{1i}, the following inequality holds*

$$e^{\lambda t}E|I_{1i}|^p \le e^{\lambda \tau}n^{p-1}(2c_i)^{1-\frac{p}{2}}(c_i - 1)^{-1}\sum_{j=1}^{n}L_{ij}^2 G_j(t).$$

Proof. By lemma 2.3, it yields that

$$
\begin{aligned}
& e^{\lambda t} E |I_{1i}|^p \\
& = e^{\lambda t}E\{|\int_0^t e^{-c_i(t-s)}\sum_{j=1}^{n}\sigma_{ij}(s, x_j(s-\tau_j))d\omega_j(s)|^p\} \\
& = e^{\lambda t}E\{|\sum_{j=1}^{n}\int_0^t e^{-c_i(t-s)}\sigma_{ij}(s, x_j(s-\tau_j))d\omega_j(s)|^p\} \\
& \le e^{\lambda t}E\{\sum_{j=1}^{n}|\int_0^t e^{-c_i(t-s)}\sigma_{ij}(s, x_j(s-\tau_j))d\omega_j(s)|\}^p \\
& \le e^{\lambda t}n^{p-1}\sum_{j=1}^{n}E|\int_0^t e^{-c_i(t-s)}\sigma_{ij}(s, x_j(s-\tau_j))d\omega_j(s)|^p \\
& \le e^{\lambda t}n^{p-1}\sum_{j=1}^{n}E\{\int_0^t e^{-c_i(t-s)}|\sigma_{ij}(s, x_j(s-\tau_j))|d\omega_j(s)\}^p \\
& = e^{\lambda t}n^{p-1}\sum_{j=1}^{n}E\{[\int_0^t e^{-c_i(t-s)}|\sigma_{ij}(s, x_j(s-\tau_j))|d\omega_j(s)]^2\}^{p/2} \\
& = e^{\lambda t}n^{p-1}\sum_{j=1}^{n}E\{\int_0^t e^{-2c_i(t-s)}\sigma_{ij}^2(s, x_j(s-\tau_j)ds\}^{p/2} \\
& \le e^{\lambda t}n^{p-1}\sum_{j=1}^{n}L_{ij}^2 E\{\int_0^t e^{-2c_i(t-s)}x_j^2(s-\tau_j)ds\}^{p/2} \\
& \le e^{\lambda t}n^{p-1}\sum_{j=1}^{n}L_{ij}^2 E\{[\int_0^t e^{-2c_i(t-s)}ds]^{p/2-1} \\
& \quad \times [\int_0^t e^{-2c_i(t-s)}|x_j(s-\tau_j)|^p ds]\} \\
& = e^{\lambda t}n^{p-1}\sum_{j=1}^{n}L_{ij}^2 E\{[\frac{1-e^{-2c_i t}}{2c_i}]^{p/2-1} \\
& \quad \times [\int_0^t e^{-2c_i(t-s)}|x_j(s-\tau_j)|^p ds]\} \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
& \le e^{\lambda t}n^{p-1}(\frac{1}{2c_i})^{p/2-1}\sum_{j=1}^{n}L_{ij}^2 E\{\int_0^t e^{-2c_i(t-s)}|x_j(s-\tau_j)|^p ds\} \\
& = n^{p-1}(\frac{1}{2c_i})^{p/2-1}\sum_{j=1}^{n}L_{ij}^2 E\{\int_0^t e^{(\lambda-2c_i)(t-s)}e^{\lambda \tau_j}e^{\lambda(s-\tau_j)} \\
& \quad \times |x_j(s-\tau_j)|^p ds\} \\
& \le n^{p-1}(\frac{1}{2c_i})^{p/2-1}\sum_{j=1}^{n}L_{ij}^2 E\{\sup_{0 \le s \le t}e^{\lambda(s-\tau_j)}|x_j(s-\tau_j)|^p \\
& \quad \times \int_0^t e^{(\lambda-2c_i)(t-s)}ds e^{\lambda \tau}\} \\
& \le e^{\lambda \tau}n^{p-1}(\frac{1}{2c_i})^{p/2}\sum_{j=1}^{n}L_{ij}^2 E\{\sup_{0 \le s \le t}e^{\lambda(s-\tau_j)}|x_j(s-\tau_j)|^p \\
& \quad \times \frac{1-e^{\lambda-2c_i}}{2c_i-\lambda}\} \\
& \le e^{\lambda \tau}n^{p-1}(\frac{1}{2c_i})^{p/2-1}\frac{1}{2c_i-\lambda}\sum_{j=1}^{n}L_{ij}^2 \\
& \quad \times E\{\sup_{0 \le s \le t}e^{\lambda(s-\tau_j)}|x_j(s-\tau_j)|^p\} \\
& \le e^{\lambda \tau}n^{p-1}(\frac{1}{2c_i})^{p/2-1}\frac{1}{c_i-\lambda}\sum_{j=1}^{n}L_{ij}^2 \\
& \quad \times E\{\sup_{0 \le s \le t}e^{\lambda(s-\tau_j)}|x_j(s-\tau_j)|^p\} \\
& \le e^{\lambda \tau}n^{p-1}(\frac{1}{2c_i})^{p/2-1}\frac{1}{c_i-\lambda}\sum_{j=1}^{n}L_{ij}^2 \\
& \quad \times E\{\sup_{-\infty \le s \le t}e^{\lambda(s-\tau_j)}|x_j(s-\tau_j)|^p\} \\
& = e^{\lambda \tau}n^{p-1}(\frac{1}{2c_i})^{p/2-1}\frac{1}{c_i-\lambda}\sum_{j=1}^{n}L_{ij}^2 E\{\sup_{-\infty \le \theta \le t-\tau_j}e^{\lambda \theta}|x_j(\theta)|^p\} \\
& \le e^{\lambda \tau}n^{p-1}(\frac{1}{2c_i})^{p/2-1}\frac{1}{c_i-\lambda}\sum_{j=1}^{n}L_{ij}^2 E\{\sup_{-\infty \le \theta \le t}e^{\lambda \theta}|x_j(\theta)|^p\} \\
& = e^{\lambda \tau}n^{p-1}(2c_i)^{1-\frac{p}{2}}(c_i-\lambda)^{-1}\sum_{j=1}^{n}L_{ij}^2 G_j(t), \qquad (7)
\end{aligned}
$$

which complete the proof.

Lemma 3.2 *For I_{2i}, the following inequality holds*

$$e^{\lambda t}E|I_{2i}|^p \le c_i^{-\frac{p}{q}}\frac{1}{c_i-\lambda}[\sum_{j=1}^{n}|a_{ij}|^q|\alpha_j|^q]^{\frac{p}{q}}\sum_{j=1}^{n}\bar{k}_{ij}G_j(t).$$

Proof. By lemma 2.1 and lemma 2.2, it yields that

$$
\begin{aligned}
& e^{\lambda t}E|I_{2i}|^p \\
& = e^{\lambda t}E|\int_0^t e^{-c_i(t-s)}(\sum_{j=1}^{n}a_{ij}\int_{-\infty}^{s}k_{ij}(s-v)f_j(x_j(v))dv)ds|^p \\
& \le e^{\lambda t}E\{\int_0^t e^{-c_i(t-s)}(\sum_{j=1}^{n}|a_{ij}|\int_{-\infty}^{s}k_{ij}(s-v)|f_j(x_j(v))|dv)ds\}^p \\
& = e^{\lambda t}E\{\int_0^t e^{\frac{-c_i(t-s)}{q}}e^{\frac{-c_i(t-s)}{p}}(\sum_{j=1}^{n}|a_{ij}| \\
& \quad \times \int_{-\infty}^{s}k_{ij}(s-v)|f_j(x_j(v))|dv)ds\}^p \\
& \le e^{\lambda t}E\{[\int_0^t e^{-c_i(t-s)}ds]^{\frac{p}{q}}[\int_0^t e^{-c_i(t-s)}(\sum_{j=1}^{n}|a_{ij}| \qquad (8)
\end{aligned}
$$

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$$\times \int_{-\infty}^{s} k_{ij}(s-v)|f_j(x_j(v))|dv)^p ds]\}$$

$$= e^{\lambda t}E\{[\frac{1-e^{-c_it}}{c_i}]^{\frac{p}{q}}[\int_0^t e^{-c_i(t-s)}(\sum_{j=1}^n |a_{ij}|$$

$$\times \int_{-\infty}^{s} k_{ij}(s-v)|f_j(x_j(v)|dv)^p ds]\}$$

$$\leq e^{\lambda t}(c_i)^{-\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}(\sum_{j=1}^n |a_{ij}|$$

$$\times \int_{-\infty}^{s} k_{ij}(s-v)|f_j(x_j(v))|dv)^p ds\}$$

$$\leq e^{\lambda t}(c_i)^{-\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}(\sum_{j=1}^n |a_{ij}||\alpha_j|$$

$$\times \int_{-\infty}^{s} k_{ij}(s-v)|x_j(v)|dv)^p ds\}$$

$$\leq e^{\lambda t}(c_i)^{-\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}(\sum_{j=1}^n |a_{ij}|^q|\alpha_j|^q)^{\frac{p}{q}}$$

$$\times \sum_{j=1}^n (\int_{-\infty}^{s} k_{ij}(s-v)|x_j(v)|dv)^p ds\}$$

$$= e^{\lambda t}(c_i)^{-\frac{p}{q}}(\sum_{j=1}^n |a_{ij}|^q|\alpha_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}$$

$$\times \sum_{j=1}^n (\int_{-\infty}^{s} k_{ij}(s-v)|x_j(v)|dv)^p ds\}$$

$$= e^{\lambda t}(c_i)^{-\frac{p}{q}}(\sum_{j=1}^n |a_{ij}|^q|\alpha_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}$$

$$\times \sum_{j=1}^n (\int_{-\infty}^{s} k_{ij}^{\frac{1}{q}}(s-v)k_{ij}^{\frac{1}{p}}(s-v)|x_j(v)|dv)^p ds\}$$

$$\leq e^{\lambda t}(c_i)^{-\frac{p}{q}}(\sum_{j=1}^n |a_{ij}|^q|\alpha_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}$$

$$\times \sum_{j=1}^n \{[\int_{-\infty}^{s} k_{ij}(s-v)dv]^{\frac{p}{q}}[\int_{-\infty}^{s} k_{ij}(s-v)|x_j(v)|^p dv]\}ds\}$$

$$= e^{\lambda t}(c_i)^{-\frac{p}{q}}(\sum_{j=1}^n |a_{ij}|^q|\alpha_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}$$

$$\times \sum_{j=1}^n [\int_{-\infty}^{s} k_{ij}(s-v)|x_j(v)|^p dv]ds\}$$

$$= e^{\lambda t}(c_i)^{-\frac{p}{q}}(\sum_{j=1}^n |a_{ij}|^q|\alpha_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{-(c_i-\lambda)(t-s)}$$

$$\times \sum_{j=1}^n [\int_{-\infty}^{s} e^{-\lambda t}k_{ij}(s-v)e^{\lambda s}|x_j(v)|^p dv]ds\}$$

$$= (c_i)^{-\frac{p}{q}}(\sum_{j=1}^n |a_{ij}|^q|\alpha_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{-(c_i-\lambda)(t-s)}$$

$$\times \sum_{j=1}^n [\int_{-\infty}^{s} e^{\lambda(s-v)}k_{ij}(s-v)e^{\lambda v}|x_j(v)|^p dv]ds\}$$

$$\leq (c_i)^{-\frac{p}{q}}(\sum_{j=1}^n |a_{ij}|^q|\alpha_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{-(c_i-\lambda)(t-s)}$$

$$\times \sum_{j=1}^n [\sup_{-\infty \leq v \leq s} e^{\lambda v}|x_j(v)|^p \int_{-\infty}^{s} e^{\lambda(s-v)}k_{ij}(s-v)dv]ds\}$$

$$= (c_i)^{-\frac{p}{q}}(\sum_{j=1}^n |a_{ij}|^q|\alpha_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{-(c_i-\lambda)(t-s)}$$

$$\times \sum_{j=1}^n [\bar{k}_{ij}\sup_{-\infty \leq v \leq s} e^{\lambda v}|x_j(v)|^p]ds\}$$

$$\leq (c_i)^{-\frac{p}{q}}(\sum_{j=1}^n |a_{ij}|^q|\alpha_j|^q)^{\frac{p}{q}}$$

$$\times \sum_{j=1}^n E\{\bar{k}_{ij}\sup_{-\infty \leq v \leq t} e^{\lambda v}|x_j(v)|^p[\int_0^t e^{-(c_i-\lambda)(t-s)}ds]\}$$

$$= (c_i)^{-\frac{p}{q}}(\sum_{j=1}^n |a_{ij}|^q|\alpha_j|^q)^{\frac{p}{q}}$$

$$\times \sum_{j=1}^n E\{\bar{k}_{ij}\sup_{-\infty \leq v \leq t} e^{\lambda v}|x_j(v)|^p[\frac{1-e^{-(c_i-\lambda)t}}{c_i-\lambda}]\}$$

$$\leq c_i^{-\frac{p}{q}}\frac{1}{c_i-\lambda}[\sum_{j=1}^n |a_{ij}|^q|\alpha_j|^q]^{\frac{p}{q}}\sum_{j=1}^n \bar{k}_{ij}G_j(t). \qquad (9)$$

This completes the proof.

Lemma 3.3 *For I_{3i}, the following inequality holds*

$$e^{\lambda t}E|I_{3i}|^p \leq e^{\lambda \tau}c_i^{-\frac{p}{q}}\frac{1}{c_i-\lambda}[\sum_{j=1}^n |b_{ij}|^q|\beta_j|^q]^{\frac{p}{q}}\sum_{j=1}^n \bar{k}_{ij}G_j(t).$$

Proof. Similar to the proof of the lemma3.2, we can obtain

$$e^{\lambda t}\ E\ |I_{3i}|^p$$

$$= e^{\lambda t}E|\int_0^t e^{-c_i(t-s)}(\sum_{j=1}^n b_{ij}$$

$$\times \int_{-\infty}^{s} k_{ij}(s-v)g_j(x_j(v-\tau_j))dv)ds|^p$$

$$\leq e^{\lambda t}c_i^{-\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}(\sum_{j=1}^n |b_{ij}||\beta_j|$$

$$\times \int_{-\infty}^{s} k_{ij}(s-v)|x_j(v-\tau_j)|dv)^p ds\}$$

$$\leq e^{\lambda t}c_i^{-\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}(\sum_{j=1}^n |b_{ij}|^q|\beta_j|^q)^{\frac{p}{q}}$$

$$\times \sum_{j=1}^n [\int_{-\infty}^{s} k_{ij}(s-v)|x_j(v-\tau_j)|dv]^p ds\}$$

$$= e^{\lambda t}c_i^{-\frac{p}{q}}(\sum_{j=1}^n |b_{ij}|^q|\beta_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}$$

$$\times \sum_{j=1}^n [\int_{-\infty}^{s} k_{ij}(s-v)|x_j(v-\tau_j)|dv]^p ds\}$$

$$= e^{\lambda t}c_i^{-\frac{p}{q}}(\sum_{j=1}^n |b_{ij}|^q|\beta_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}$$

$$\times \sum_{j=1}^n [\int_{-\infty}^{s} k_{ij}(s-v)^{\frac{1}{q}}k_{ij}(s-v)^{\frac{1}{p}}|x_j(v-\tau_j)|dv]^p ds\}$$

$$\leq e^{\lambda t}c_i^{-\frac{p}{q}}(\sum_{j=1}^n |b_{ij}|^q|\beta_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}$$

$$\times \sum_{j=1}^n [\int_{-\infty}^{s} k_{ij}(s-v)dv]^{\frac{p}{q}}[\int_{-\infty}^{s} k_{ij}(s-v)|x_j(v-\tau_j)|^p dv]ds\}$$

$$= e^{\lambda t}c_i^{-\frac{p}{q}}(\sum_{j=1}^n |b_{ij}|^q|\beta_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{-c_i(t-s)}$$

$$\times [\sum_{j=1}^n \int_{-\infty}^{s} k_{ij}(s-v)|x_j(v-\tau_j)|^p dv]ds\}$$

$$= c_i^{-\frac{p}{q}}(\sum_{j=1}^n |b_{ij}|^q|\beta_j|^q)^{\frac{p}{q}}E\{\int_0^t e^{(-c_i-\lambda)(t-s)} \qquad (10)$$

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$\times [\sum_{j=1}^n \int_{-\infty}^s e^{\lambda(s-v)} k_{ij}(s-v) e^{\lambda(v-\tau_j)} |x_j(v-\tau_j)|^p e^{\tau_j} dv] ds\}$

$\leq c_i^{-\frac{p}{q}} (\sum_{j=1}^n |b_{ij}|^q |\beta_j|^q)^{\frac{p}{q}} e^{\lambda\tau} E\{\int_0^t e^{(-c_i-\lambda)(t-s)}$

$\times \sum_{j=1}^n [\sup_{-\infty \leq v \leq s} e^{\lambda(v-\tau_j)} |x_j(v-\tau_j)|^p$

$\times \int_{-\infty}^s e^{\lambda(s-v)} k_{ij}(s-v) dv] ds\}$

$= c_i^{-\frac{p}{q}} (\sum_{j=1}^n |b_{ij}|^q |\beta_j|^q)^{\frac{p}{q}} e^{\lambda\tau} E\{\int_0^t e^{(-c_i-\lambda)(t-s)}$

$\times \sum_{j=1}^n [\overline{k}_{ij} \sup_{-\infty \leq v \leq s} e^{\lambda(v-\tau_j)} |x_j(v-\tau_j)|^p ds\}$

$\leq c_i^{-\frac{p}{q}} (\sum_{j=1}^n |b_{ij}|^q |\beta_j|^q)^{\frac{p}{q}} e^{\lambda\tau} \sum_{j=1}^n E\{\overline{k}_{ij}$

$\times \sup_{-\infty \leq v \leq t} e^{\lambda(v-\tau_j)} |x_j(v-\tau_j)|^p \cdot (\int_0^t e^{(-c_i-\lambda)(t-s)} ds)\}$

$= c_i^{-\frac{p}{q}} (\sum_{j=1}^n |b_{ij}|^q |\beta_j|^q)^{\frac{p}{q}} e^{\lambda\tau} \sum_{j=1}^n E\{\overline{k}_{ij}$

$\times \sup_{-\infty \leq v \leq t} e^{\lambda(v-\tau_j)} |x_j(v-\tau_j)|^p \cdot (\frac{1-e^{\lambda-c_i}}{c_i-\lambda})\}$

$= c_i^{-\frac{p}{q}} (\sum_{j=1}^n |b_{ij}|^q |\beta_j|^q)^{\frac{p}{q}} e^{\lambda\tau} \sum_{j=1}^n E\{\overline{k}_{ij}$

$\times \sup_{-\infty \leq v \leq t-\tau_j} e^{\lambda\theta} |x_j(\theta)|^p \cdot (\frac{1-e^{\lambda-c_i}}{c_i-\lambda})\}$

$\leq c_i^{-\frac{p}{q}} (\sum_{j=1}^n |b_{ij}|^q |\beta_j|^q)^{\frac{p}{q}} e^{\lambda\tau} \sum_{j=1}^n E\{\overline{k}_{ij}$

$\times \sup_{-\infty \leq v \leq t} e^{\lambda\theta} |x_j(\theta)|^p \cdot (\frac{1-e^{\lambda-c_i}}{c_i-\lambda})\}$

$\leq c_i^{-\frac{p}{q}} (\sum_{j=1}^n |b_{ij}|^q |\beta_j|^q)^{\frac{p}{q}} e^{\lambda\tau} \sum_{j=1}^n E\{\overline{k}_{ij}$

$\times \sup_{-\infty \leq v \leq t} e^{\lambda\theta} |x_j(\theta)|^p \cdot (\frac{1}{c_i-\lambda})\}$

$= e^{\lambda\tau} c_i^{-\frac{p}{q}} \frac{1}{c_i-\lambda} [\sum_{j=1}^n |b_{ij}|^q |\beta_j|^q]^{\frac{p}{q}} \sum_{j=1}^n \overline{k}_{ij} G_j(t).$ (11)

This complete the proof.

Theorem 3.1 *If $\rho(C^{-1}(D_1D_2\overline{K} + D_1D_3\overline{K} + D_4L)) < 1$ and $C^{-1}(D_1D_2\overline{K} + D_1D_3\overline{K} + D_4L) \geq 0$, then system (2) is pth moment exponentially stable, where*

$\overline{K} = (\overline{k}_{ij})_{n\times n}, L = (L_{ij}^2)_{n\times n},$

$D_1 = diag(d_1, d_2, \ldots, d_n), D_2 = diag(a_1, a_2, \ldots, a_n),$

$D_3 = diag(b_1, b_2, \ldots, b_n), D_4 = diag(m_1, m_2, \ldots, m_n),$

and $a_i = [\sum_{j=1}^n |a_{ij}|^q |\alpha_j|^q]^{\frac{p}{q}}, d_i = 4^{p-1} c_i^{-\frac{p}{q}},$ $b_i = [\sum_{j=1}^n |b_{ij}|^q |\beta_j|^q]^{\frac{p}{q}}, m_i = 4^{p-1} n^{p-1} (2c_i)^{1-\frac{p}{2}}.$

Proof. From lemma3.1, lemma3.2, and lemma3.3, the following inequality holds for all $t \geq 0$

$e^{\lambda t} E |x_i(t)|^p$

$\leq 4^{p-1}\{E|x_i(0)|^p + e^{\lambda\tau} n^{p-1} (2c_i)^{1-\frac{p}{2}} (c_i-\lambda)^{-1}$

$\times \sum_{j=1}^n L_{ij}^2 G_j(t) + c_i^{-\frac{p}{q}} \frac{1}{c_i-\lambda} [\sum_{j=1}^n |a_{ij}|^q |\alpha_j|^q]^{\frac{p}{q}} \sum_{j=1}^n \overline{k}_{ij} G_j(t)$

$+ e^{\lambda\tau} c_i^{-\frac{p}{q}} \frac{1}{c_i-\lambda} [\sum_{j=1}^n |b_{ij}|^q |\beta_j|^q]^{\frac{p}{q}} \sum_{j=1}^n \overline{k}_{ij} G_j(t)\}.$ (12)

This means that for all $t \geq 0$

$G_i(t) \leq 4^{p-1}\{E|x_i(0)|^p + e^{\lambda\tau} n^{p-1} (2c_i)^{1-\frac{p}{2}} (c_i-\lambda)^{-1}$

$\times \sum_{j=1}^n L_{ij}^2 G_j(t) + c_i^{-\frac{p}{q}} \frac{1}{c_i-\lambda} [\sum_{j=1}^n |a_{ij}|^q |\alpha_j|^q]^{\frac{p}{q}} \sum_{j=1}^n \overline{k}_{ij} G_j(t)$

$+ e^{\lambda\tau} c_i^{-\frac{p}{q}} \frac{1}{c_i-\lambda} [\sum_{j=1}^n |b_{ij}|^q |\beta_j|^q]^{\frac{p}{q}} \sum_{j=1}^n \overline{k}_{ij} G_j(t)\}.$ (13)

Namely,

$G_i(t) \leq 4^{p-1}\{E|x_i(0)|^p + \frac{1}{c_i-\lambda}\{c_i^{-\frac{p}{q}} [\sum_{j=1}^n |a_{ij}|^q |\alpha_j|^q]^{\frac{p}{q}}$

$+ e^{\lambda\tau} c_i^{-\frac{p}{q}} [\sum_{j=1}^n |b_{ij}|^q |\beta_j|^q]^{\frac{p}{q}}\} \sum_{j=1}^n \overline{k}_{ij} G_j(t)$

$+ e^{\lambda\tau} n^{p-1} (2c_i)^{1-\frac{p}{2}} \sum_{j=1}^n L_{ij}^2 G_j(t)\}.$ (14)

Thus, we have

$G(t) \leq 4^{p-1} E|x(0)|^p + (C-\lambda I)^{-1}(D_1D_2\overline{K} + e^{\lambda\tau} D_1D_3\overline{K} + e^{\lambda\tau} D_4L)G(t),$ (15)

where

$G(t) = (G_1(t), G_2(t), \ldots, G_n(t))^T,$

$E|x(0)|^p = (E|x_1(0)|^p, E|x_2(0)|^p, \ldots, E|x_n(0)|^p)^T.$

Since

$\rho(C^{-1}(D_1D_2\overline{K} + D_1D_3\overline{K} + D_4L)) < 1,$

$C^{-1}(D_1D_2\overline{K} + D_1D_3\overline{K} + D_4L) \geq 0,$

by using lemma2.4, it yields that

$(I - C^{-1}(D_1D_2\overline{K} + D_1D_3\overline{K} + D_4L))^{-1} \geq 0.$

Hence, there exists a sufficiently small positive constant $\alpha < \lambda$ such that

$(I - (C-\alpha I)^{-1}(D_1D_2\overline{K} + e^{\alpha\tau} D_1D_3\overline{K} + e^{\alpha\tau} D_4L))^{-1} \geq 0$

Set $(I - (C-\alpha I)^{-1}(D_1D_2\overline{K} + e^{\alpha\tau} D_1D_3\overline{K} + e^{\alpha\tau} D_4L))^{-1} = M(\alpha)$, we have

$E|x(t)|^p \leq 4^{p-1} M(\alpha) E|x(0)|^p e^{-\lambda t}.$

Therefore, for all $i = 1, 2, \ldots, n$ we have

$E|x_i(t)|^p \leq 4^{p-1} e^{-\lambda t} \sum_{j=1}^n M_{ij}(\alpha) \sum_{j=1}^n E|x_j(0)|^p.$

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Namely,

$$\sum_{i=1}^{n} E|x_i(t)|^p \leq 4^{p-1} e^{-\lambda t} (\sum_{i=1}^{n} \sum_{j=1}^{n} M_{ij}(\alpha)) \sum_{j=1}^{n} E|x_j(0)|^p.$$

That is

$$E\|x(t)\|^p \leq 4^{p-1} (\sum_{i=1}^{n} \sum_{j=1}^{n} M_{ij}(\alpha)) E\|x(0)\|^p e^{-\lambda t}.$$

This complete the proof.

Notice that $\rho(A) \leq \|A\|$ for any $A \in R^{n \times n}$, in which $\|\cdot\|$ is an arbitrary matrix norm. Moreover, for any matrix norm and any nonsingular matrix S, a matrix norm $\|A\|_S$ can be given by $\|A\|_S = \|S^{-1}AS\|$. For the convenience of calculation, in general, taking $S = diag\{s_1, \ldots, s_n\} > 0$. Therefore, corresponding to the matrix norm widely applied as the row norm, column norm and Frobenius norm, one can obtain the following sufficient conditions to guarantee $\|A\|_S < 1$, respectively.

Corollary 3.1. *The system(2) is pth moment exponentially stable if there exists positive constants $\xi_1, \xi_2, \ldots, \xi_n$ such that one of following inequalities holds*

$$(1) \sum_{j=1}^{n} c_i^{-1} \frac{\xi_i}{\xi_j} [c_i^{-\frac{p}{q}} (a_i + b_i)\overline{k}_{ij} + m_i L_{ij}] \leq 4^{1-p}, \ 1 \leq i \leq n. \quad (16)$$

$$(2) \sum_{i=1}^{n} c_i^{-1} \frac{\xi_i}{\xi_j} [c_i^{-\frac{p}{q}} (a_i + b_i)\overline{k}_{ij} + m_i L_{ij}] \leq 4^{1-p}, \ 1 \leq j \leq n. \quad (17)$$

Corollary 3.2. *Under hypothesis $(H_1) - (H_3)$, if $\rho(C^{-1}(D_1 D_2 \overline{K} + D_1 D_3 \overline{K} + D_4 L)) < 1$ and $C^{-1}(D_1 D_2 \overline{K} + D_1 D_3 \overline{K} + D_4 L) \geq 0$, then system(2) is exponentially stable in mean square, where D_1, D_2, D_3, D_4 are defined as the same in theorem3.1.*

IV. NUMERICAL EXAMPLE

Consider the following stochastic neural network with continuously distributed delays.

$$\begin{cases} dx_1(t) = [-2x_1(t) + 0.01 \int_{-\infty}^{t} e^{t-s} f_1(x_1(t)) \\ \qquad + 0.01 \int_{-\infty}^{t} e^{t-s} f_2(x_2(t)) - 0.01 \int_{-\infty}^{t} e^{t-s} g_1(x_1(t - \tau_1)) \\ \qquad - 0.01 \int_{-\infty}^{t} e^{t-s} g_2(x_2(t - \tau_2))]dt \\ \qquad + (L_{11} x_1(t) d\omega_1(t) + L_{12} x_2(t) d\omega_2(t)) \\ dx_2(t) = [-2x_2(t) + 0.01 \int_{-\infty}^{t} e^{t-s} f_1(x_1(t)) \\ \qquad + 0.01 \int_{-\infty}^{t} e^{t-s} f_2(x_2(t)) + 0.01 \int_{-\infty}^{t} e^{t-s} g_1(x_1(t - \tau_1)) \\ \qquad + 0.01 \int_{-\infty}^{t} e^{t-s} g_2(x_2(t - \tau_2))]dt \\ \qquad + (L_{21} x_1(t) d\omega_1(t) + L_{22} x_2(t) d\omega_2(t)) \end{cases}$$

$$(18)$$

,

where $f(x(t)) = \arctan x(t)$, $g(x(t)) = \frac{e^{x(t)} - e^{-x(t)}}{e^{x(t)} + e^{-x(t)}}$. Set $p = 3$, one can easily check that this equation satisfies Assumptions $(H_1) - (H_4)$. Set $\lambda = 0.5$, we have

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, A = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix}, B = \begin{bmatrix} -0.01 & -0.01 \\ 0.01 & 0.01 \end{bmatrix},$$

$$K = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}, \alpha_i = \beta_i = 1, i = 1, 2.$$

For this example, the criteria obtained in [22, 29] can not be used to determine the stability, but the criteria obtained in this paper are valid. Set $L = \begin{bmatrix} 0.001 & 0.001 \\ 0.001 & 0.001 \end{bmatrix}$, one can obtain $D_1 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, D_2 = D_3 = \begin{bmatrix} (0.002)^2 & 0 \\ 0 & (0.002)^2 \end{bmatrix}, D_4 = \begin{bmatrix} 32 & 0 \\ 0 & 32 \end{bmatrix}$. Hence $C^{-1}(D_1 D_2 \overline{K} + D_1 D_3 \overline{K} + D_4 L) =$

$$\begin{bmatrix} 8 \times (0.002)^2 + 0.016 & 8 \times (0.002)^2 + 0.016 \\ 8 \times (0.002)^2 + 0.016 & 8 \times (0.002)^2 + 0.016 \end{bmatrix}$$

Since $\Delta_i \geq 0, i = 1, 2$, where Δ_i is the principal minor of the above matrix, then $C^{-1}(D_1 D_2 \overline{K} + D_1 D_3 \overline{K} + D_4 L) \geq 0$, and one can easily get the eigenvalue $\lambda_1 = 0, \lambda_2 = 16 \times (0.002)^2 + 0.032 < 1$, thus, $\rho(C^{-1}(D_1 D_2 \overline{K} + D_1 D_3 \overline{K} + D_4 L)) < 1$. It follows from Theorem 3.1 that system (18) is exponentially stable in 3th moment.

V. CONCLUSIONS

By using the method of variation parameter, inequality technique, and stochastic analysis, the problem on pth moment exponential stability of a class of stochastic neural networks with distributed time delays is investigated. Without assuming the bounded, monotonicity and differentiability of the output functions, some sufficient conditions to guarantee the pth moment exponential stability are derived. The results established in this paper generalize some previous criteria obtained in the literature cited therein. Numerical example shows that the criteria obtained in paper are valid.

REFERENCES

[1] S. Arik, V. Tavsanoglu. *Equilibrium analysis of delayed CNNs*, IEEE Trans Circuits Syst. 1998, 45: 168-171.
[2] S. Blythea, X. Mao, and X. Liao. *Stability of stochastic delay neural networks*. J. Franklin Inst, 2001, 338: 481-495.
[3] J. Cao. *New results concerning exponential stability and periodic solutions of delayed cellular neural networks*. Phys. Lett. A 2003, 307: 136-147.
[4] J. Cao, J.Wang. *Global asymptotic stability of a general class of recurrent neural networks with time-varying delays*. IEEE Trans. Circuits Syst, 2003, 50: 34-44.
[5] J. Cao, J. Wang. *Global asymptotic and robust stability of recurrent neural networks with time delays*, IEEE Trans. Circuits Syst, 2005, 52: 417-426.
[6] T. Chen,W. Lu, and G. Chen, *Dynamical behaviors of a large class of general delayed neural networks*, Neural Comput, 2005, 17 :949-968.
[7] L.O. Chua, L.Yang. *Cellular neural networks: theory*. IEEE Trans. Circuits Syst, 1988, 35: 1257-1272.

World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:4, No:7, 2010

[8] T. Ensari, S. Arik. *Global stability of a class of neural networks with time-varying delay*. IEEE Trans. Circuits Syst., 2005, 52: 126-130.

[9] A. Friedman. *Stochastic Differential Equations and Applications*. Academic Press, NewYork, 1976

[10] S. Haykin. *Neural Networks*. Prentice-Hall, Englewood Cliffs, NJ, 1994.

[11] J. Hopfield. *Neurons with graded response have collective computational properties like those of two-stage neurons*. Proc. Nat. Acad. Sci. USA., 1984, 81: 3088-3092.

[12] R. Horn, C. Johnson. *Matrix Analysis*. Cambridge University Press, London, 1985.

[13] X. Liao, X. Mao. *Stability of stochastic neural networks*, Neural Parallel Sci. Comput., 1996, 4: 205-224.

[14] X. Liao, X. Mao. *Exponential stability and instability of stochastic neural networks*. Stochast. Anal. Appl., 1996, 14: 165-185.

[15] X. Liao, X. Mao. *Exponential stability of stochastic delay interval systems*. Syst. Control. Lett., 2000, 40: 171-181.

[16] X. Mao. *Exponential Stability of Stochastic Differential Equations*. Marcel Dekker, NewYork, 1994.

[17] X. Mao. *Razumikihin-type theorems on exponential stability of stochastic functional differential equations*. Stochast. Proc. Appl., 1996, 65: 233-250.

[18] X. Mao. *Robustness of exponential stability of stochastic differential delay equations*. IEEE Trans. Automat. Control., 1996, 41 :442-447.

[19] X. Mao. *Stochastic Differential Equations and Applications*. Horwood Publication, Chichester, 1997.

[20] S. Mohammed. *Stochastic Functional Differential Equations*. Longman Scientific and Technical, 1986.

[21] S. Mohamad, K. Gopalsamy. *Exponential stability of continuous-time and discrete-time cellular neural networks with delays*. Appl. Math. Comput., 2003, 135: 17-38.

[22] L.Wan, J. Sun. *Mean square exponential stability of stochastic delayed Hopfield neural networks*. Phys. Lett. A., 2005, 343: 306-318.

[23] H. Zhao. *Global exponential stability and periodicity of cellular neural networks with variable delays*. Phys. Lett. A., 2005, 336: 331-341.

[24] J. Zhang. *Globally exponential stability of neural networks with variable delays*. IEEE Trans. Circuits Syst., 2003, 50: 288-291.

[25] Z. Zeng, J. Wang, and X. Liao. *Global exponential stability of a general class of recurrent neural networks with time-varying delays*. IEEE Trans. Circuits Syst., 2003, 50: 1353-1358.

[26] J. Cao, Q. Song. *Stability in Cohen-Grossberg type BAM neural networks with time-varying delays*. Nonlinearity, 2006, 19: 1601-1617.

[27] J. Cao, J. Lu. *Adaptive synchronization of neural networks with or without time-varying delays*. Chaos, 2006, 16: 013133.

[28] J. Cao et al., *Global point dissipativity of neural networks with mixed time-varying delays*. Chaos, 2006, 16: 013105.

[29] Y. Sun,J. Cao. *pth moment exponential stability of stochastic recurrent neural networks with time-varying delays*. Nonlinear Analysis: RealWorld Applications, 2007, 8: 1171-1185.