

# Nonlinear Observer Design and Sliding Mode Control of Four Rotors Helicopter

H. Bouadi, and M. Tadjine

**Abstract**—In this paper; we are interested in dynamic modelling of quadrotor while taking into account the high-order nonholonomic constraints as well as the various physical phenomena, which can influence the dynamics of a flying structure. These permit us to introduce a new state-space representation and new control scheme. We present after the development and the synthesis of a stabilizing control laws design based on sliding mode in order to perform best tracking results. It ensures locally asymptotic stability and desired tracking trajectories. Nonlinear observer is then synthesized in order to estimate the unmeasured states and the effects of the external disturbances such as wind and noise. Finally simulation results are also provided in order to illustrate the performances of the proposed controllers.

**Keywords**—Dynamic modelling, nonholonomic constraints, Sliding mode, Nonlinear observer.

## I. INTRODUCTION

UNMANNED aerial vehicles (UAV) have shown a growing interest thanks to recent technological projections, especially those related to instrumentation. They made possible the design of powerful systems (mini drones) endowed with real capacities of autonomous navigation at reasonable cost.

Despite the real progress made, researchers must still deal with serious difficulties, related to the control of such systems, particularly, in the presence of atmospheric turbulences. In addition, the navigation problem is complex and requires the perception of an often constrained and evolutionary environment, especially in the case of low-altitude flights.

Nowadays, the mini-drones invade several application domains [3]: safety (monitoring of the airspace, urban and interurban traffic); natural risk management (monitoring of volcano activities); environmental protection (measurement of air pollution and forest monitoring); intervention in hostile sites (radioactive workspace and mine clearance), management of the large infrastructures (dams, high-tension lines and pipelines), agriculture and film production (aerial shooting).

In contrast to terrestrial mobile robots, for which it is often possible to limit the model to kinematics, the control of aerial robots (quadrotor) requires dynamics in order to account for gravity effects and aerodynamic forces.

In this paper, authors propose a control-law based on the choice of a stabilizing Lyapunov function ensuring the desired tracking trajectories along  $(X, Z)$  axis and roll angle. However, they do not take into account nonholonomic constraints. ; do not take into account frictions due to the aerodynamic torques nor drag forces or nonholonomic constraints. They proposed firstly a control-law based on backstepping and secondly sliding mode controller based upon backstepping approach in order to stabilize the complete system (i.e. translation and orientation). In [1], authors take into account the gyroscopic effects and show that the classical model-independent PD controller can stabilize asymptotically the attitude of the quadrotor aircraft. Moreover, they used a new Lyapunov function, which leads to an exponentially stabilizing controller based upon the  $PD^2$  and the compensation of coriolis and gyroscopic torques. While in [2] the authors develop a PID controller in order to stabilize altitude.

Others papers; presented the sliding mode and high-order sliding mode respectively like an observer [6] and [7] in order to estimate the unmeasured states and the effects of the external disturbances such as wind and noise.

In this paper, based on the vectorial model form presented in [2] we are interested principally in the modelling of quadrotor to account for various parameters which affect the dynamics of a flying structure such as frictions due to the aerodynamic torques, drag forces along  $(X, Y, Z)$  axis and gyroscopic effects which are identified in [2] for an experimental quadrotor and for high-order nonholonomic constraints. Consequently, all these parameters supported the setting of the system under more complete and more realistic new state-space representation, which cannot be found easily in the literature being interested in the control laws synthesis for such systems.

Then, we present a control technique based on the development and the synthesis of a stabilizing control laws by sliding mode approach ensuring locally asymptotic stability and desired tracking trajectories expressed in term of the center of mass coordinates along  $(X, Y, Z)$  axis and yaw angle, while the desired roll and pitch angles are deduced from nonholonomic constraints unlike to . However, the synthesis of nonlinear observer becomes necessary in order to estimate unmeasured states and the effects of additive uncertainties.

Finally all the control laws synthesized are highlighted by simulations which gave results considered to be satisfactory.

## II. MODELLING

### A. Quadrotor Dynamic Modelling

The quadrotor have four propellers in cross configuration. The two pairs of propellers (1,3) and (2,4) as described in Fig. 2, turn in opposite directions. By varying the rotor speed, one can change the lift force and create motion. Thus, increasing or decreasing the four propeller's speeds together

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generates vertical motion. Changing the 2 and 4 propeller's speed conversely produces roll rotation coupled with lateral motion. Pitch rotation and the corresponding lateral motion; result from 1 and 3 propeller's speed conversely modified. Yaw rotation is more subtle, as it results from the difference in the counter-torque between each pair of propellers.

Let  $E(O, X, Y, Z)$  denote an inertial frame, and  $B(o', x, y, z)$  denote a frame rigidly attached to the quadrotor as shown in Fig. 2.

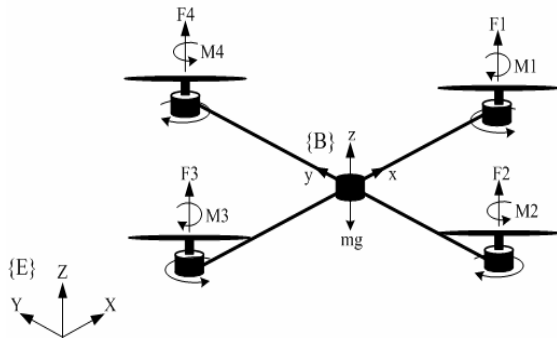


Fig. 1 Quadrotor configuration

We will make the following assumptions:

- The quadrotor structure is rigid and symmetrical.
- The center of mass and  $o'$  coincides.
- The propellers are rigid.
- Thrust and drag are proportional to the square of the propellers speed.

Under these assumptions, it is possible to describe the fuselage dynamics as that of a rigid body in space to which come to be added the aerodynamic forces caused by the rotation of the rotors.

Using the formalism of Newton-Euler, the dynamic equations are written in the following form:

$$\begin{cases} \dot{\xi} = v \\ m\ddot{\xi} = F_f + F_t + F_g \\ \dot{R} = RS(\Omega) \\ J\dot{\Omega} = -\Omega \wedge J\Omega + \Gamma_f - \Gamma_a - \Gamma_g \end{cases} \quad (1)$$

$\xi$  is the position of the quadrotor center of mass with respect to the inertial frame.  $m$  is the total mass of the structure and  $J \in \mathbb{R}^{3 \times 3}$  is a symmetric positive definite constant inertia matrix of the quadrotor with respect to  $B$ .

$$J = \begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix} \quad (2)$$

$\Omega$  is the angular velocity of the airframe expressed in  $B$ :

$$\Omega = \begin{pmatrix} 1 & 0 & -\sin \theta \\ 0 & \cos \phi & \cos \theta \sin \phi \\ 0 & -\sin \phi & \cos \phi \cos \theta \end{pmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (3)$$

in the case when the quadrotor performs many angular motions of low amplitude  $\Omega$  can be assimilated to  $[\dot{\phi} \ \dot{\theta} \ \dot{\psi}]^T$ .

$R$  is the homogenous matrix transformation.

$$R = \begin{pmatrix} C\theta C\psi & C\psi S\theta S\phi - S\psi C\phi & C\psi S\theta C\phi + S\psi S\phi \\ C\theta S\psi & S\psi S\theta S\phi + C\psi C\phi & S\psi S\theta C\phi - C\psi S\phi \\ -S\theta & S\phi C\theta & C\phi C\theta \end{pmatrix} \quad (4)$$

Where  $C$  and  $S$  indicate the trigonometric functions **cos** and **sin** respectively.  $S(\Omega)$  is a skew-symmetric matrix; for a given vector  $\Omega = [\Omega_1 \ \Omega_2 \ \Omega_3]^T$  it is defined as follows:

$$S(\Omega) = \begin{pmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{pmatrix}$$

$F_f$  is the resultant of the forces generated by the four rotors

$$F_f = \begin{pmatrix} \cos \phi \cos \psi \sin \theta + \sin \phi \sin \psi \\ \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\ \cos \phi \cos \theta \end{pmatrix} \sum_{i=1}^4 F_i \quad (5)$$

$$F_i = K_p \omega_i^2 \quad (6)$$

where  $K_p$  is the lift coefficient and  $\omega_i$  is the angular rotor speed.

$F_t$  is the resultant of the drag forces along  $(X, Y, Z)$  axis

$$F_t = \begin{pmatrix} -K_{ftx} & 0 & 0 \\ 0 & -K_{fyy} & 0 \\ 0 & 0 & -K_{ftz} \end{pmatrix} \xi \quad (7)$$

such as  $K_{ftx}$ ,  $K_{fyy}$  and  $K_{ftz}$  are the translation drag coefficients.

$F_g$  is the gravity force.

$$F_g = [0 \ 0 \ -mg]^T \quad (8)$$

$\Gamma_f$  is the moment developed by the quadrotor according to the body fixed frame. It is expressed as follows:

$$\Gamma_f = \begin{bmatrix} d(F_3 - F_1) \\ d(F_4 - F_2) \\ K_d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{bmatrix} \quad (9)$$

$d$  is the distance between the quadrotor center of mass and the rotation axis of propeller and  $K_d$  is the drag coefficient.

$\Gamma_a$  is the resultant of aerodynamics frictions torques.

$$\Gamma_a = \begin{bmatrix} K_{fax} & 0 & 0 \\ 0 & K_{fay} & 0 \\ 0 & 0 & K_{faz} \end{bmatrix} \Omega^2 \quad (10)$$

$K_{fax}, K_{fay}$  and  $K_{faz}$  are the frictions aerodynamics coefficients.

$\Gamma_g$  is the resultant of torques due to the gyroscopic effects.

$$\Gamma_g = \sum_{i=1}^4 \Omega \wedge J_r \begin{bmatrix} 0 \\ 0 \\ (-1)^{i+1} \omega_i \end{bmatrix} \quad (11)$$

Such as  $J_r$  is the rotor inertia.

Consequently the complete dynamic model which governs the quadrotor is as follows:

$$\begin{cases} \ddot{\phi} = \frac{1}{I_x} \{ \dot{\theta} \dot{\psi} (I_y - I_z) - K_{fax} \dot{\phi}^2 - J_r \bar{\Omega} \dot{\phi} + dU_2 \} \\ \ddot{\theta} = \frac{1}{I_y} \{ \dot{\phi} \dot{\psi} (I_z - I_x) - K_{fay} \dot{\theta}^2 + J_r \bar{\Omega} \dot{\theta} + dU_3 \} \\ \ddot{\psi} = \frac{1}{I_z} \{ \dot{\phi} \dot{\theta} (I_x - I_y) - K_{faz} \dot{\psi}^2 + K_d U_4 \} \\ \ddot{x} = \frac{1}{m} \{ (C\phi S\theta C\psi + S\phi S\psi) U_1 - K_{fx} \dot{x} \} \\ \ddot{y} = \frac{1}{m} \{ (C\phi S\theta S\psi - S\phi C\psi) U_1 - K_{fy} \dot{y} \} \\ \ddot{z} = \frac{1}{m} \{ (C\phi C\theta) U_1 - K_{fz} \dot{z} \} - g \end{cases} \quad (12)$$

With  $U_1, U_2, U_3$  and  $U_4$  are the control inputs of the system which are written according to the angular velocities of the four rotors as follows:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{pmatrix} K_p & K_p & K_p & K_p \\ -K_p & 0 & K_p & 0 \\ 0 & -K_p & 0 & K_p \\ K_d & -K_d & K_d & -K_d \end{pmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (13)$$

and

$$\bar{\Omega} = (\omega_1 - \omega_2 + \omega_3 - \omega_4)$$

### B. Nonholonomic Constraints

Taking into account nonholonomic constraints for our system is of major importance as are in compliance with physical laws and define the coupling between various states of the system.

From the equations of the translation dynamics (12) we can extract the expressions of the high-order nonholonomic constraints:

$$\begin{cases} \tan \theta = \frac{\left( \ddot{x} - \frac{K_{fx}}{m} \dot{x} \right) \cos \psi + \left( \ddot{y} - \frac{K_{fy}}{m} \dot{y} \right) \sin \psi}{\ddot{z} + g - \frac{K_{fz}}{m} \dot{z}} \\ \sin \phi = \frac{-\left( \ddot{x} - \frac{K_{fx}}{m} \dot{x} \right) \sin \psi + \left( \ddot{y} - \frac{K_{fy}}{m} \dot{y} \right) \cos \psi}{\sqrt{\left( \ddot{x} - \frac{K_{fx}}{m} \dot{x} \right)^2 + \left( \ddot{y} - \frac{K_{fy}}{m} \dot{y} \right)^2 + \left( \ddot{z} + g - \frac{K_{fz}}{m} \dot{z} \right)^2}} \end{cases} \quad (14)$$

### C. Rotor Dynamic

The rotor is a unit constituted by D.C-motor actuating a propeller via a reducer. The D.C-motor is governed by the following model [9]:

$$\dot{\omega}_i = bV_i - \beta_0 - \beta_1 \omega_i - \beta_2 \omega_i^2 \quad (15)$$

$$i \in [1, 4]$$

with:

$$\beta_0 = \frac{C_s}{J_r}, \beta_1 = \frac{k_e k_m}{r J_r}, \beta_2 = \frac{k_r}{J_r} \text{ and } b = \frac{k_m}{r J_r}$$

and:

$V$  : motor input.

$k_e, k_m$  : electrical and mechanical torque constant respectively.

$k_r$  : load constant torque.

$r$  : motor internal resistance.

$J_r$  : rotor inertia.

$C_s$  : solid friction.

### III. CONTROL OF THE QUADROTOR

The choice of this method is not fortuitous considering the major advantages it presents:

- It ensures Lyapunov stability.
- It ensures the robustness and all properties of the desired dynamics.
- It ensures the handling of all system nonlinearities.

The model (12) developed in the first part of this paper can be rewritten in the state-space form:

$\dot{X} = f(X) + g(X, U) + \delta$  and  $X = [x_1 \dots x_{12}]^T$  is the state vector of the system such as:

$$\underline{X} = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, x, \dot{x}, y, \dot{y}, z, \dot{z}]^T \quad (16)$$

From (12) and (16) we obtain the following state representation:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a_1 x_4 x_6 + a_2 x_2^2 + a_3 \bar{\Omega} x_4 + b_1 U_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = a_4 x_2 x_6 + a_5 x_4^2 + a_6 \bar{\Omega} x_2 + b_2 U_3 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = a_7 x_2 x_4 + a_8 x_6^2 + b_3 U_4 \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = a_9 x_8 + U_x \frac{U_1}{m} \\ \dot{x}_9 = x_{10} \\ \dot{x}_{10} = a_{10} x_{10} + U_y \frac{U_1}{m} \\ \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = a_{11} x_{12} + \frac{C x_1 C x_3}{m} U_1 - g \end{cases} \quad (17)$$

$$\begin{cases} U_2 = \frac{1}{b_1} \{-k_1 \text{sign}(S_\phi) - a_1 x_4 x_6 - a_2 x_2^2 - a_3 x_4 \bar{\Omega} + \ddot{\phi}_d + \lambda_1 e_2\} \\ U_3 = \frac{1}{b_2} \{-k_2 \text{sign}(S_\theta) - a_4 x_2 x_6 - a_5 x_4^2 - a_6 x_2 \bar{\Omega} + \ddot{\theta}_d + \lambda_2 e_3\} \\ U_4 = \frac{1}{b_3} \{-k_3 \text{sign}(S_\psi) - a_7 x_2 x_4 - a_8 x_6^2 + \ddot{\psi}_d + \lambda_3 e_5\} \\ U_x = \frac{m}{U_1} \{-k_4 \text{sign}(S_x) - a_9 x_8 + \ddot{x}_d + \lambda_4 e_8\} \quad / U_1 \neq 0 \\ U_y = \frac{m}{U_1} \{-k_5 \text{sign}(S_y) - a_{10} x_{10} + \ddot{y}_d + \lambda_5 e_{10}\} \quad / U_1 \neq 0 \\ U_1 = \frac{m}{C\phi C\theta} \{-k_6 \text{sign}(S_z) - a_{11} x_{12} + \ddot{z}_d + \lambda_6 e_{12} + g\} \end{cases} \quad (20)$$

$$\begin{cases} a_1 = \left( \frac{I_y - I_z}{I_x} \right), a_2 = \frac{-K_{fix}}{I_x}, a_3 = \frac{-J_r}{I_x} \\ a_4 = \left( \frac{I_z - I_x}{I_y} \right), a_5 = \frac{-K_{fix}}{I_y}, a_6 = \frac{J_r}{I_y} \\ a_7 = \left( \frac{I_x - I_y}{I_z} \right), a_8 = \frac{-K_{fix}}{I_z}, a_9 = \frac{-K_{fix}}{m}, a_{10} = \frac{-K_{fix}}{m}, a_{11} = \frac{-K_{fix}}{m} \\ b_1 = \frac{d}{I_x}, b_2 = \frac{d}{I_y}, b_3 = \frac{1}{I_z} \end{cases} \quad (18)$$

$$\begin{cases} U_x = C x_1 S x_3 C x_5 + S x_1 S x_5 \\ U_y = C x_1 S x_3 S x_5 - S x_1 C x_5 \end{cases} \quad (19)$$

The state representation of the system under this form has never been developed before.

From high-order nonholonomic constraints developed in (14), roll ( $\phi$ ) and pitch ( $\theta$ ) angles depend not only on the yaw angle ( $\psi$ ) but also on the movements along ( $X, Y, Z$ ) axis and their dynamics. However the adopted control strategy is summarized in the control of two subsystems; the first relates to the position control while the second is that of the attitude control as shown it below the synoptic scheme:

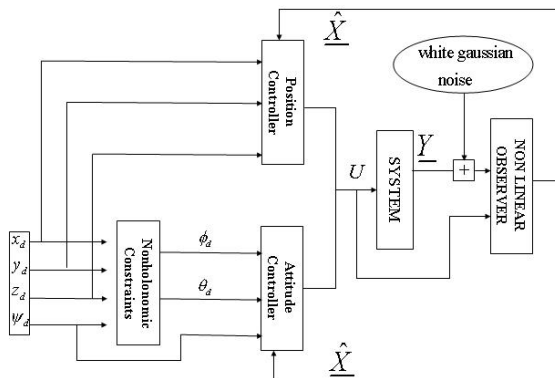


Fig. 2 Synoptic scheme of the proposed controller

In this section, the purpose is to design a sliding mode controller. The basic sliding mode controller design procedure in our case is performed in two steps. Firstly, the choice of sliding surface ( $S$ ) according to the tracking error, while the second step, consist into the design of a Lyapunov function which can satisfy the necessary sliding condition ( $S\dot{S} < 0$ ).

### Proof:

Let us choose the sliding surfaces given by:

$$\begin{cases} S_\phi = e_2 + \lambda_1 e_1 \\ S_\theta = e_4 + \lambda_2 e_3 \\ S_\psi = e_6 + \lambda_3 e_5 \\ S_x = e_8 + \lambda_4 e_7 \\ S_y = e_{10} + \lambda_5 e_9 \\ S_z = e_{12} + \lambda_6 e_{11} \end{cases} \quad (21)$$

Such as:

$$\lambda_i > 0 \text{ and } \begin{cases} e_i = x_{id} - x_i \\ e_{i+1} = \dot{e}_i \end{cases} \quad i \in [1, 11] \quad (22)$$

We assume that:

$$V(S_\phi) = \frac{1}{2} S_\phi^2 \quad (23)$$

If  $\dot{V}(S_\phi) < 0$ , so  $S_\phi \dot{S}_\phi < 0$  then, the necessary sliding condition is verified and Lyapunov stability is guaranteed.

The chosen law for the attractive surface is the time derivative of (21) satisfying ( $S_\phi \dot{S}_\phi < 0$ ):

$$\begin{aligned} \dot{S}_\phi &= -k_1 \text{sign}(S_\phi) \\ &= \ddot{x}_{1d} - \dot{x}_2 + \lambda_1 \dot{e}_1 \\ &= -a_1 x_4 x_6 - a_2 x_2^2 - a_3 \bar{\Omega} x_4 - b_1 U_2 + \ddot{\phi}_d + \lambda_1 (\dot{\phi}_d - x_2) \\ U_2 &= \frac{1}{b_1} \{-k_1 \text{sign}(S_\phi) - a_1 x_4 x_6 - a_2 x_2^2 - a_3 \bar{\Omega} x_4 + \ddot{\phi}_d + \lambda_1 e_2\} \end{aligned} \quad (24)$$

The same steps are followed to extract  $U_3, U_4, U_x, U_y$  and  $U_1$ .

### IV. OBSERVER DESIGN

Consider the model system (17) and denote  $\hat{X}$  the estimate of the state vector (16). The observer model is a copy of the original system, which has corrector gains functions of estimation errors; so:

$$\hat{\Sigma} : \begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \Lambda_1(z_1) \\ \dot{\hat{x}}_2 = a_1 \hat{x}_4 \hat{x}_6 + a_2 \hat{x}_2^2 + a_3 \hat{x}_4 \bar{\Omega} + b_1 U_2 + \Lambda_2(z_2) \\ \dot{\hat{x}}_3 = \hat{x}_4 + \Lambda_3(z_3) \\ \dot{\hat{x}}_4 = a_4 \hat{x}_2 \hat{x}_6 + a_5 \hat{x}_4^2 + a_6 \hat{x}_2 \bar{\Omega} + b_2 U_3 + \Lambda_4(z_4) \\ \dot{\hat{x}}_5 = \hat{x}_6 + \Lambda_5(z_5) \\ \dot{\hat{x}}_6 = a_7 \hat{x}_2 \hat{x}_4 + a_8 \hat{x}_6^2 + b_3 U_4 + \Lambda_6(z_6) \\ \dot{\hat{x}}_7 = \hat{x}_8 + \Lambda_7(z_7) \\ \dot{\hat{x}}_8 = a_9 \hat{x}_8 + \frac{U_1}{m} U_x + \Lambda_8(z_8) \\ \dot{\hat{x}}_9 = \hat{x}_{10} + \Lambda_9(z_9) \\ \dot{\hat{x}}_{10} = \hat{a}_{10} \hat{x}_{10} + \frac{U_1}{m} U_y + \Lambda_{10}(z_{10}) \\ \dot{\hat{x}}_{11} = \hat{x}_{12} + \Lambda_{11}(z_{11}) \\ \dot{\hat{x}}_{12} = a_{11} \hat{x}_{12} + \frac{\cos x_1 \cos x_3}{m} U_1 - g + \Lambda_{12}(z_{12}) \end{cases} \quad (25)$$

The estimation error dynamics are given by:

$$\begin{cases} \dot{z}_1 = z_2 - \Lambda_1 \\ \dot{z}_2 = a_1 \Delta_{x_4 x_6} + a_2 \Delta_{x_2^2} + a_3 \Delta_{x_4} \bar{\Omega} - \Lambda_2 \\ \dot{z}_3 = z_4 - \Lambda_3 \\ \dot{z}_4 = a_4 \Delta_{x_2 x_6} + a_5 \Delta_{x_4^2} + a_6 \Delta_{x_2} \bar{\Omega} - \Lambda_4 \\ \dot{z}_5 = z_6 - \Lambda_5 \\ \dot{z}_6 = a_7 \Delta_{x_2 x_4} + a_8 \Delta_{x_6^2} - \Lambda_6 \\ \dot{z}_7 = z_8 - \Lambda_7 \\ \dot{z}_8 = a_9 z_8 - \Lambda_8 \\ \dot{z}_9 = z_{10} - \Lambda_9 \\ \dot{z}_{10} = a_{10} z_{10} - \Lambda_{10} \\ \dot{z}_{11} = z_{12} - \Lambda_{11} \\ \dot{z}_{12} = a_{11} z_{12} - \Lambda_{12} \end{cases} \quad (26)$$

$$\text{with: } \begin{cases} \Delta_{x_i x_j} = x_i x_j - \hat{x}_i \hat{x}_j \\ \Delta_{x_i^2} = x_i^2 - \hat{x}_i^2 \\ \Delta_{x_i} = x_i - \hat{x}_i \end{cases} \quad (27)$$

$$\text{and: } \begin{cases} z_i = y_i - \hat{y}_i & \text{if } i \text{ impair} \\ z_i = x_i - \hat{x}_i & \text{if } i \text{ pair} \end{cases} \quad (28)$$

The considered outputs of our system are:

$$\underline{Y} = [x_1, x_3, x_5, x_7, x_9, x_{11}]^T \quad (29)$$

In order to calculate the corrector gains, it is necessary that the estimation errors dynamics be stable so, let us choose the Lyapunov function given by:

$$V(z_1, z_2) = \frac{1}{2}(z_1^2 + z_2^2)$$

$$\begin{aligned} \dot{V}(z_1, z_2) &= z_1 \dot{z}_1 + z_2 \dot{z}_2 \\ &= z_1(z_2 - \Lambda_1) + z_2(a_1 \Delta_{x_4 x_6} + a_2 \Delta_{x_2^2} + a_3 \Delta_{x_4} \bar{\Omega} - \Lambda_2) \end{aligned}$$

The necessary condition to get a Lyapunov stability is  $\dot{V}(z_1, z_2) \leq 0$ , for this :

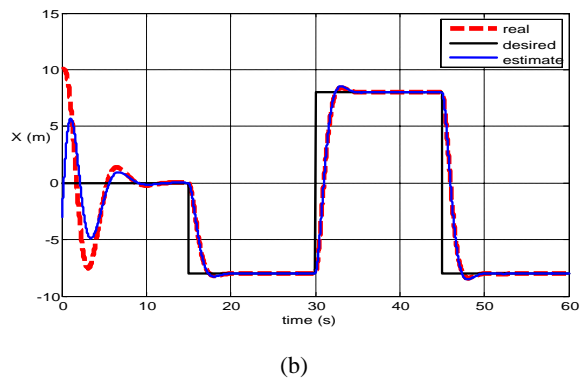
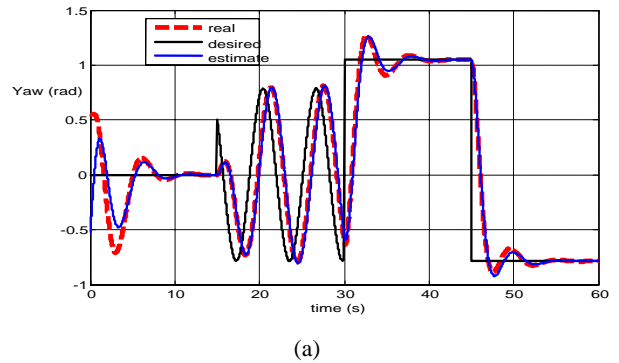
$$\begin{cases} \Lambda_1(z_1) = z_2 + k_1 z_1 \\ \Lambda_2(z_2) = a_1 \Delta_{x_4 x_6} + a_2 \Delta_{x_2^2} + a_3 \Delta_{x_4} \bar{\Omega} + k_2 z_2 \end{cases} \quad \text{with: } (k_1, k_2) \in \mathfrak{R}^{+2} \quad (30)$$

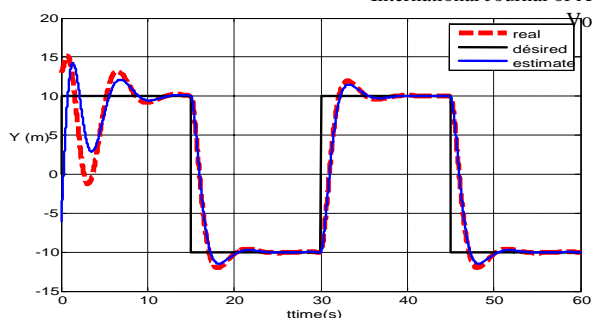
The same steps are followed to extract others corrector gains:

$$\begin{cases} \Lambda_3(z_3) = z_4 + k_3 z_3 \\ \Lambda_4(z_4) = a_4 \Delta_{x_2 x_6} + a_5 \Delta_{x_4^2} + a_6 \Delta_{x_2} \bar{\Omega} + k_4 z_4 \\ \Lambda_5(z_5) = z_6 + k_5 z_5 \\ \Lambda_6(z_6) = a_7 \Delta_{x_2 x_4} + a_8 \Delta_{x_6^2} + k_6 z_6 \\ \Lambda_7(z_7) = z_8 + k_7 z_7 \\ \Lambda_8(z_8) = a_9 z_8 + k_8 z_8 \\ \Lambda_9(z_9) = z_{10} + k_9 z_9 \\ \Lambda_{10}(z_{10}) = a_{10} z_{10} + k_{10} z_{10} \\ \Lambda_{11}(z_{11}) = z_{12} + k_{11} z_{11} \\ \Lambda_{12}(z_{12}) = a_{11} z_{12} + k_{12} z_{12} \end{cases} \quad (31)$$

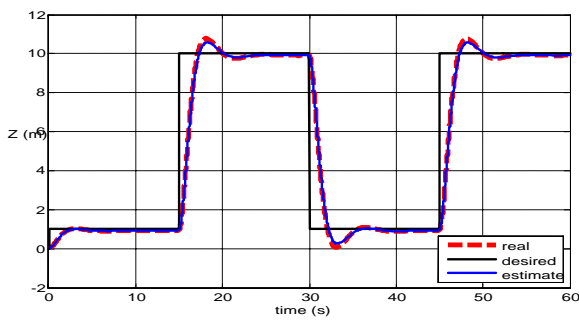
## V. SIMULATION RESULTS

The simulation results are obtained based on the following real parameters [8]:



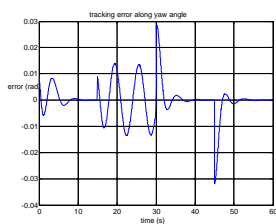


(c)

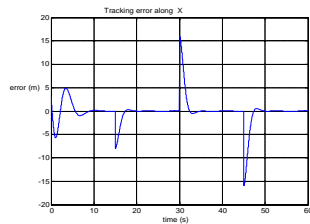


(d)

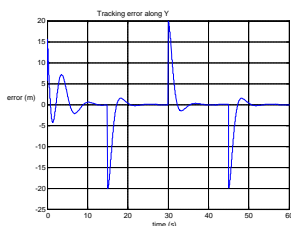
Fig. 3 Tracking simulation results of desired trajectories along yaw angle ( $\psi$ ) and ( $X, Y, Z$ ) axis



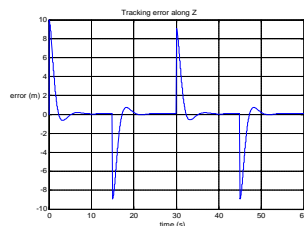
(a)



(b)

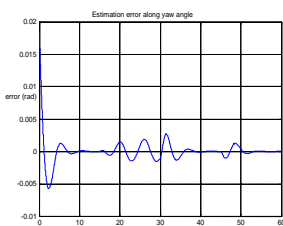


(c)

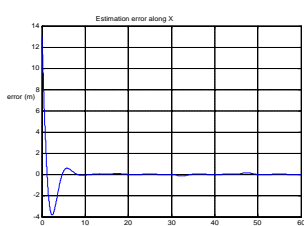


(d)

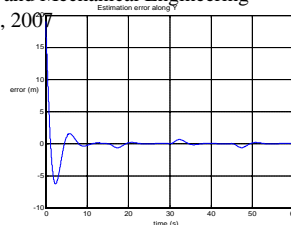
Fig. 4 Tracking errors according yaw ( $\psi$ ) angle and ( $X, Y, Z$ ) respectively



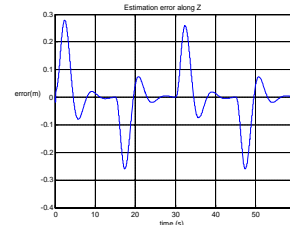
(a)



(b)



(c)



(d)

Fig. 5 Estimation errors according yaw ( $\psi$ ) angle and ( $X, Y, Z$ ) respectively

## V. CONCLUSION

In this paper, we presented stabilizing control laws synthesis by sliding mode. Firstly, we start by the development of the dynamic model of the quadrotor taking into account the different physics phenomena and the high-order nonholonomic constraints imposed to the system motions; this says these control laws allowed the tracking of the various desired trajectories expressed in term of the center of mass coordinates of the system in spite of the complexity of the proposed model. After we are interested to the development of a nonlinear observer in order to be able to estimate unmeasured states and the effects of external additive disturbances like wind and noise. As prospects we hope to develop other control techniques and other kinds of nonlinear observer in order to improve the performances and to ensure good navigation of such systems in evolutionary and constrained environment.

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