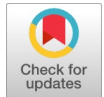


Vibration Analysis and Motion Control Method for an Under-Actuated Tower Crane

Roberto P. L. Caporali



Abstract: In this paper, we developed a solution for controlling a tower crane thought as a no-rigid system, and therefore able to have deformation and, during the motion, vibrations. Particularly, large tower cranes show high structural dynamics. Under external excitations, the payload tends to sway around its vertical position and this motion is coupled to the resulting dynamic vibration of the crane structure. These induced vibrations may cause instability and serious damage to the crane system. Furthermore, the energy stored in the flexible structure of a tower crane causes vibrations in the structure during the acceleration and deceleration of slewing movements. A crane operator perceives these vibrations as an unstable speed of the boom. Such behavior involves the control of the crane, particularly precise positioning and manual control of the crane movement at low pivoting speed. We define an Elastic model of the Slewing crane and analyze the bending and Torsional elasticity of the Tower, and the Jib Elasticity. With an approximated method, we calculate the natural wavelenghts of the crane structure in the slewing direction. We consider the tower crane as a nonlinear under-actuated system. The motion equations are obtained considering both the normal vibration modes of the tower crane and the sway of the payload. An elastic model of the Slewing crane is achieved, modeling the crane jib as an Euler-Bernoulli beam. Even the payload dynamic is considered, developing an Anti-sway solution by the equation of the movement. We define an iterative calculation of the sway angles and obtain the corresponding velocity profiles, implementing two kinds of solution: an input-shaping control in open-loop, to be used with automatic positioning, and a "command smoothing" method in open-loop, used for reducing the sway of the payload with the operator control. These solutions lead to a reduction of the vibrations of the crane structure. As a consequence, the tower crane is not subject to the strong horizontal and vertical oscillations during the motion of the elastic structure.

Keywords: Slewing Control, Tower Crane, Vibrational Analysis, Finite Element Method, Anti-Sway.

I. INTRODUCTION

An analysis of the elastic structure of a Tower crane is obtained in this work. We consider a Tower crane like an elastic system during the slewing movement.

Because the Tower crane structure is not rigid, a resulting dynamic vibration of the crane structure is generated also without the presence of the payload. Besides, the payload swings around its vertical position. These induced vibrations

sum up to the payload swaying and may cause dangerous instability. Such behavior makes difficult the control of the crane, particularly precise positioning and manual control of the crane movement at low pivoting speed. Ramli et al. [1] gave an exhaustive literature review relative to published works related to crane solutions.

Cranes are under-actuated systems because they have a lower number of actuators than degrees of freedom. That is the result of decreasing the cost and the complexity. However, that means high difficulty to crane control, due to their complex dynamics.

In the last few years, different works have been carried out relative to the sway control of a slewing crane. We can classify these works into open and closed-loop control methods. Feedback controls (closed-loop) mainly focus on disturbance while open-loop controls have the main goal of reducing unwanted payload oscillations for defined reference or input signals. Some recent papers relative to the closed-loop control are [2]-[3]-[4]-[5]-[6]-[7]-[8]-[9]-[10]-[11]-[12]-[13]-[14]-[15]-[16]-[17]-[18]-[19]. Each of these works corresponds to different control methods, for example, Adaptive Output Feedback Control, Observer-Based Nonlinear Control, Observer Design for Non-Linear Systems, Robust adaptive boundary control, and Lyapunov Functions-based adaptive control.

Instead, as regards open-control works are concerned, we can cite [20]-[21]-[22]-[23]-[24]-[25]-[26]-[27]-[28]-[29]-[30]-[31], as well as the works of the author of the present paper [32]-[33]. We can also consider the works [50]-[51]-[52].

Also, some Patents and Thesis have been developed regarding the control of the slewing of a tower crane. We can cite [34]-[35]-[36]-[37]-[38]. Some of which can be associated with important companies in the Crane sector. Specifically, torque control of the crane's rotation movement was defined with an open-loop system [34]. Nevertheless, the torque control of the jib rotation allows efficient control of the vibrations of the crane structure, but does not allow precise positioning. The positioning is delegated only to the skill of the operator. We must consider that a tower crane behaves like a spring during the pivoting movement. The energy emitted by the engine results in a torsion of the tower and the boom. The energy stored in the mechanical system can generate vibrations of the crane. These vibrations add to the pendulum oscillations due to the payload movement must be taken into account to avoid dangerous excitations of resonance frequencies.

To date, a limited number of papers have considered the fundamental contribution of tower crane vibrations to the stability of the slewing movement.

Manuscript received on 17 December 2023 | Revised Manuscript received on 23 December 2023 | Manuscript Accepted on 15 January 2024 | Manuscript published on 30 January 2024.

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Because of under-actuation, the dynamic model of the tower crane requires dynamic feedback linearization with the inclusion of some control inputs and the associated time derivatives in an extended state vector and in the form of additional state variables. The solution to the closed-loop feedback control problem for tower cranes through global linearization-based techniques requires complicated state-space model transformations. Ghazwani et al. [39] defined a Failure Analysis of Tower Crane using FEM. Nevertheless, they studied the tower crane's stability during cyclones, without considering the slewing of the crane during the work phase.

In the work of Ju et al. [40] a perturbation approximation and the assumption of small pendulum angles are applied to the non-linear system. The tower crane is modeled using the finite element method. The limit of this work is given by the high non-linearity of the crane system.

In a recent paper [41], Liu et al. describe and simulate dynamic models to understand tower crane's dynamic characteristics and vibration features under compound working conditions. This paper introduces two mathematical models by D'Alembert's principle and corresponding model verification.

Cao et al. [42] developed a mathematical model of a tower crane with a long flexible boom. Both the vertical deformation and the horizontal deformation of the boom (thought of as an Euler-Bernoulli boom) are considered in the model. The Lagrangian method and the considered mode method are combined to formulate the rigid-flexible coupling model in order to describe the motion equations. In these works, a solution to the swaying of the payload is not given.

Also, the author of this paper developed a work [43] on the vibration of a Tower crane during the slewing movement. Nevertheless, although an anti-sway solution was developed in that work, the limit was that its applicability might be limited for cranes with large jibs. This effectively also applies to the previously cited works. Instead, in their relevant paper, Rauscher et al. [44] developed a modal approach for the slewing control based on a distributed-mass model. The crane jib is considered an Euler-Bernoulli beam. The partial differential equations are discretized using finite differences and a modal order reduction is developed to describe the slewing dynamics as a low-order model to use for anti-swaying control of the payload. A possible limit of this work is given by the high number of discretization nodes. This number should be chosen high enough to precisely model the elastic jib dynamics, hence representing a high-order ODE. Therefore, they had to perform a complex modal order reduction.

The solution to the closed-loop feedback control problem for tower cranes through global linearization-based techniques requires complicated state-space model transformations. However, there exist results on nonlinear optimal (H-infinity) control of tower cranes which are based on approximate linearization and which avoid transformations of this system's state-space description. This is the case considered, for example, in the work of Rigatos et al. [45].

On the contrary, in this paper we propose a different and practical approach to the analysis of the vibration during the Tower crane slewing. We, first, define an elastic analysis of

the deformation of the Tower and the Jib, determining the approximated normal modes of vibration. However, unlike [44], in our Finite Element Analysis, we model the Jib with 3 fundamental zones. These zones are modeled as 3 beams with distributed weight that can be bent, connected with 3 nodes where the concentrated weights are located. This method, relative only to the elastic analysis of the Jib deformation is described in a detailed way in the recent paper of the author [46]. In this way the system is greatly simplified and streamlined in its complex calculations; it has been verified that the error made when considering a large number of nodes is negligible and the method is much simpler to determine.

The analysis of the System Stability, for the anti-Sway method relative to a Slewing Crane, was implemented and a stability proof was made by the author in a recent paper [47]. Open loop control methods would require an exactly known dynamic model of the controlled system, while they can be easily destabilized because of exogenous perturbations and model uncertainty. Because of under-actuation, the dynamic model of the tower crane requires dynamic feedback linearization with the inclusion of some control inputs, and the associated time derivatives are defined in the form of additional state variables. Obviously, the solution of the closed-loop feedback control problem, for tower cranes, through global linearization-based techniques requires complicated state-space model transformations. However, there exist results on nonlinear optimal (H-infinity) control of tower cranes which are based on approximate linearization and avoid transformations of this system's state-space description (see in example [45]).

Furthermore, based on the work [32] of the author of this paper, the control of the payload sway is implemented in open-loop, using the information on the frequency of the payload swaying. In this way, it is possible to define a Tower crane control method that takes into account the vibrational analysis. In fact, two solutions are distinguished: with command smoothing, when the system is controlled by an operator, and with Input-shaping, controlled by an automatic command, using the frequencies corresponding to the first vibrational normal modes.

The advantageous novelty of this method is given by the fact that the eigenfrequencies can be calculated in real-time, using a supervisor PC, which sends the data of the same eigen-frequencies to the PLC that controls the Jib crane. In fact, the method uses a low number of discretization nodes and the calculations can be implemented in real-time. Therefore, these data are according to the variable position on the Jib of the trolley and payload, so that the PLC determines the most correct anti-vibration speed profiles.

We organized this work in the following way. In Section II the dynamical model of the Jib crane is represented. The tower crane is modeled as an elastic system, deriving from the finite element method. The bending and torsional elasticity of the Tower, and the bending elasticity of the Jib are taken into account, considering the jib as a continuous beam.

From Lagrange equations, the two dynamical equations relative to the two sway angles are obtained. In Section III, an implementation of the method is presented using data representative of a real case. We obtain a strong reduction of the vibration mode and of the payload swaying. The two possible control solutions are presented. At the end, in Section IV concluding remarks are defined.

II. ELASTIC MODEL OF THE SLEWING CRANE

The crane system can be described as a vertical column, the Tower, a flexible Jib, a trolley system, a hoisting line, and a payload. φ and l represent the slew angle and the length of the hoisting line respectively. Sway angles are generated by the system operates: they are the tangential sway φ_t , and the radial sway φ_r . In this study, the payload is considered as a point mass and the payload shows the behavior of a pendulum. A geometric description of the Tower Crane system is given in Fig.1.

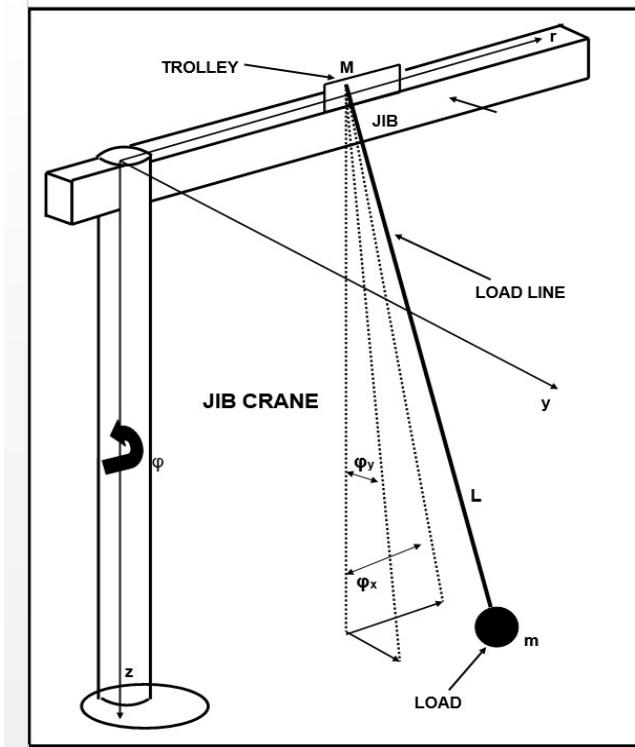


Fig. 1. Geometric Description of the Tower Crane System

A. Bending Elasticity of the Vertical Tower

Towers with long jibs exhibit both torsional and bending elasticities. We can neglect the bending movement in the jib direction since it slightly regards the slewing control. Therefore, we consider the bending movement perpendicular to the jib direction. Let's assume that the tower stiffness is high enough so that only its first eigenfrequency is taken into account and approximated by a spring-mass system. We will define the stiffness k_b and mass m_T of the spring-mass system approximating the bending motion such that it exhibits the same deflection and the same first eigenfrequency as a homogeneous Euler-Bernoulli beam. It will have a constant second moment of inertia J_T , an elastic modulus E , a length l_T and a linear density μ_T for the mass per length.

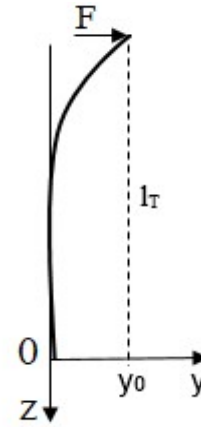


Fig. 2. Bending of the Vertical Tower Perpendicular to the Jib Direction

With reference to Fig.2, based on the theory of bending deformation of a thin beam, we obtain the elastic deflection y_0 with load F concentrated on the end of the beam:

$$y_0 = \frac{Fl_T^3}{3EJ_T} \tag{1}$$

We will apply the Euler-Bernoulli theory, by ignoring the effects of shear deformation and rotatory inertia; it is thus a special case of Timoshenko–Ehrenfest beam theory. The Euler–Bernoulli equation describes the relationship between the beam's deflection w and the applied load q :

$$\frac{d}{dy} \left(EJ_T \frac{d^2w}{dy^2} \right) = q \tag{2}$$

As a consequence, by the Euler-Bernoulli theory we obtain, applying the boundary conditions (fixed beam at $y=0$) for the first natural (eigen)frequency of vibration. These eigenfrequencies are obtained considering the free vibrations of a cantilevered beam and applying the boundary conditions. The first natural (eigen)frequency of vibration was:

$$\omega_0 = \frac{3.5160}{l_T^2} \sqrt{\frac{EJ_T}{\mu_T}} \tag{3}$$

We obtain, using the spring-mass model, the stiffness k_b and mass m_T modeling the bending motion:

$$k_b = \frac{F}{y_0} = \frac{3EJ_T}{l_T^3}; m_T = \frac{k_b}{\omega_1^2} = \frac{3\mu_T l_T}{(3.5160)^2} \tag{4}$$

B. Torsional Elasticity of the Vertical Tower

The torsional movement of the vertical Tower can be obtained using the model of a rotating slim body with J_T as a moment of Inertia, linked to the ground with a spring having stiffness k_T . For shafts of uniform cross-section unrestrained against warping, the torsion is:

$$T = \frac{J_T}{l_T} G\varphi \tag{5}$$

where T is the torsion, J_T is the Tower torsional moment, G is the shear modulus, and φ is the torsion angle.



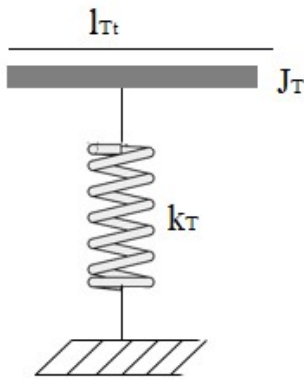


Fig.3. Equivalent Mass-spring for the Torsion of the Vertical Tower

From the equation for the first natural self-frequency of the vibrational rotating system (with boundary condition given by a fixed slim beam at $y=0$), we can deduce the expressions of the equivalent J_T and k_T . We obtain:

$$k_T = \frac{GJ_T}{l_T}; J_T = (0.405)\rho_T l_T \Gamma_T \quad (6)$$

where T_t is the torsion, J_T is the torsional moment of the Tower, G is the shear modulus, Γ_T is polar momentum, ρ_T is the mass density and φ is the torsion angle. Hence, for a slim beam, we obtain the approximated expression for the equivalent length l_T (see [44]).

C. Jib Elasticity: Bending Vibrations of the thin Beam using the Finite Element Method

As regards the analysis of the elastic deformations of the Jib (referring to the books [48] and [49] we will make the hypothesis of approximating the Jib to a thin beam, with a load concentrated in well-defined nodes. We will apply the Euler-Bernoulli theory, not considering the effects of shear deformation and rotary inertia.

Reference is made to Fig.4 which represents the jib as a single continuous beam that is clamped onto an elastic tower. The flexing effect is strongly emphasized to highlight the elasticity of the jib.

The method used is essentially a discretization technique to derive approximate solutions of the system's equation of motion when the displacement $v(x,t)$ is obtained as a linear combination of prescribed functions multiplied by the unknown functions. The degree of correctness of the method lies in the goodness of the prescribed shape functions and their number. In a paper recently published [46], we described in depth this method, applied to the case in question of the bending of the Slewing Crane. The methodology followed through the finite element method is defined and summarized in the following points.

1. The structure of the Jib is divided into a number of elements of finite size. The elements are joined to each other by knots. In our system, each node corresponds to a relevant point of the thin flexible beam with which the Jib is schematized: either it is the fixing point to the ground (knot 0, via the vertical Tower) or that of the final end of the Jib (knot 2), or those where the concentrated masses are defined (knots 1 and 3). Therefore, with reference to Fig.4, we set:

$$L_1 \equiv x_{Tr}; L_2 \equiv L - l_{cj} - x_{Tr}; L_3 \equiv l_{cj} \quad (7)$$

$$\left. \begin{aligned} M_1 &\equiv m_{Tr} + m_L + (\mu \cdot x_{Tr}) \\ M_2 &\equiv \mu \cdot (L - l_{cj} - x_{Tr}) \\ M_3 &\equiv m_{cj} + (\mu \cdot l_{cj}) \end{aligned} \right\} \quad (8)$$

where M_1, M_2, M_3 are the mass corresponding to the positions 1, 2, 3 including the distributed masses respectively on the elements 0-3, 0-1, 1-2, being μ the mass per length unit on the Jib. l_{cj} is the length of the counter-jib, x_{Tr} is the trolley position on the x-axes on the Jib, L is the total length of the Jib.

2. A given number of d.o.f. is associated with each node. To study the bending vibrations of the beam, each node i was associated with a displacement v_i along the y-axis and a rotation φ_i around the z-axis.

3. A set of functions (shape functions) are constructed for each beam element between 2 nodes, such that each has a unit value in one degree of freedom and zero in all others. A third-order polynomial is used which describes the flexural deformation of the finite element written as a function of the x coordinate perpendicular to the direction of the bent beam. From here on out, we will use a single underscore to define one-dimensional vectors and a double lowercase underscore for two-dimensional matrices. Since x is the coordinate that describes the direction perpendicular to the beam in the horizontal plane, the aforementioned polynomial is given, for each single node i , by an expression in which the temporal (t) and spatial (x) dependencies are separated:

$$v(x, t) = \underline{p}(x) \cdot \underline{\alpha}(t) \quad (9)$$

$$\underline{p}(x) = [1 \quad x \quad x^2 \quad x^3] \quad (10)$$

$$\underline{\alpha}^T(t) = [\alpha_0 \quad \alpha_1 \quad \alpha_2 \quad \alpha_3] \quad (11)$$

being $\underline{\alpha}(t)$ the vector of the time coefficients. It turns out like this:

$$\varphi_z(x, t) \equiv \frac{d}{dx} v(x, t) = [0 \quad 1 \quad 2x \quad 3x^2] \cdot \underline{\alpha}(t) \quad (12)$$

For the single beam element between nodes i and j we will then obtain:

$$\begin{aligned} v_{ij}(x, t) &= \underline{p}_{ij}(x) \cdot \underline{\alpha}_{ij}(t) \Rightarrow \underline{p}_{ij}(x) \cdot \underline{A}_{ij}^{-1}(x) \cdot v_{ij}(t) \\ &\equiv \underline{N}_{ij}(x) \cdot v_{ij}(t) \end{aligned} \quad (13)$$

so we arrive at defining and can calculate the matrices of the shape functions $\underline{N}_{ij}(x)$ relating to the individual ij elements with the \underline{A} is the Coefficient matrix. as:

$$\underline{N}_{ij}(x) \equiv \underline{p}_{ij}(x) \cdot \underline{A}_{ij}^{-1}(x) \quad (14)$$

4. Once the shape functions $\underline{N}_{ij}(x)$ of an element ij are known, these are replaced in the expression of the kinetic energy and the potential energy to obtain the mass and stiffness matrices of each finite element.

In the following we will study only the bending vibrations of the beam (Fig. 4); therefore the expressions to use for the kinetic energy T and the elastic potential energy U , relative to the single beam element ij , will be (referring to the books [48] and [49]):

$$T_{ij} = \frac{1}{2} \int_i^j \mu \left(\frac{\partial v(x,t)}{\partial t} \right)^2 dx; \quad (15)$$

$$U_{ij} = \frac{1}{2} \int_i^j EI_z \left(\frac{\partial^2 v(x,t)}{\partial x^2} \right)^2 dx$$

where μ is the mass per unit length. But, since T and U can also be expressed by:

$$T_{ij} = \frac{1}{2} \{ \dot{v}_{ij}(t) \}^T [\underline{\underline{M}}_{ij}] \{ \dot{v}_{ij}(t) \}; \quad (16)$$

$$U_{ij} = \frac{1}{2} \{ v_{ij}(t) \}^T [\underline{\underline{K}}_{ij}] \{ v_{ij}(t) \}$$

Therefore, by comparing eq. (15) with eq.(16) we obtain the expressions of the element ij of the mass matrix $\underline{\underline{M}}$ and of the stiffness matrix $\underline{\underline{K}}$, which can thus be calculated for all individual beam elements:

$$\underline{\underline{M}}_{ij} = \mu \int_i^j \{ \underline{\underline{N}}_{ij}(t) \}^T \{ \underline{\underline{N}}_{ij}(t) \} dx; \quad (17)$$

$$\underline{\underline{K}}_{ij} = EI_z \int_i^j \left\{ \frac{d^2 \underline{\underline{N}}_{ij}(t)}{dx^2} \right\}^T \left\{ \frac{d^2 \underline{\underline{N}}_{ij}(t)}{dx^2} \right\} dx$$

5. The kinetic and potential energies of each element are added to obtain the energies of the complete system and then the so-called "assembly" of the mass $\underline{\underline{M}}$ and stiffness matrices $\underline{\underline{K}}$ is carried out. Let $v(x,t)$ be the vector containing all the d.o.f. of the 3-element beam considered:

$$\{ \underline{\underline{v}}(t) \}^T = [v_0 \quad \phi_0 \quad v_1 \quad \phi_1 \quad v_2 \quad \phi_2] \quad (18)$$

It can be related to the vectors containing the d.o.f. of each single finite element using the transformation matrix $\underline{\underline{\Gamma}}$:

$$\underline{\underline{v}}_{ij} = \underline{\underline{\Gamma}}_{ij} \underline{\underline{v}} \quad (19)$$

The total kinetic energy is given by the sum of the individual kinetic energies of the element, according to the relation:

$$T = \sum_{ij=0}^2 T_{ij} = \frac{1}{2} \int_i^j \{ \dot{v} \}^T \sum_{ij=0}^2 \left([\underline{\underline{\Gamma}}_{ij}]^T [\underline{\underline{M}}_{ij}] [\underline{\underline{\Gamma}}_{ij}] \right) \{ \dot{v} \} \quad (20)$$

$$\equiv \frac{1}{2} \{ \dot{v} \}^T [\underline{\underline{M}}] \{ \dot{v} \}$$

$$U = \sum_{ij=0}^2 U_{ij} = \frac{1}{2} \int_i^j \{ v \}^T \sum_{ij=0}^2 \left([\underline{\underline{\Gamma}}_{ij}]^T [\underline{\underline{K}}_{ij}] [\underline{\underline{\Gamma}}_{ij}] \right) \{ v \} \quad (21)$$

$$\equiv \frac{1}{2} \{ v \}^T [\underline{\underline{K}}] \{ v \}$$

From these relations, it is thus possible to calculate the expressions of the total mass $\underline{\underline{M}}$ and stiffness $\underline{\underline{K}}$ matrices. In order to assemble both structural and payload dynamics as one system, the masses of the counter-jib ballast m_{cj} , the tower approximation body m_T , and the trolley m_{Tr} are added to the distributed mass matrix.

III. PAYLOAD DYNAMICS AND ANTI-SWAY CONTROL

Regarding the dynamics of the payload we will refer to the works of the author of this paper [32] and [43]. The slewing crane is defined, as regards the motion, by:

- 1) a rotating Jib;
 - 2) a translating trolley on the jib;
 - 3) a load-cable having length L ;
 - 4) a payload of mass m_L attached to the trolley with mass m_{Tr} .
- Since the specific variable dimension of possible payloads is usually unknown, the payload is considered as a mass point. Besides, we consider the mass of the rope small as regards the payload, and therefore it is neglected. The dynamics of the generalized slewing crane is represented by a system with five independent degrees of freedom, described by five Lagrangian coordinates q_i :

$q_1 = r$: the radial position of the mass center of the trolley on the axis x .

$q_2 = \varphi$: slewing angle (rotation around the z axis).

$q_3 = l$: hoisting position of the crane (along the z axis).

$q_4 = \varphi_x$: sway angle tangential to the radial direction.

$q_5 = \varphi_y$: sway angle normal to the radial direction.

By the Lagrange function:

$$L = T - U \quad (22)$$

where T is the Kinetic Energy of the crane system, U is the Potential Energy, and applying the generalized Lagrange equations:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \quad (23)$$

we obtain the system of equations in the generalized coordinates q_i .

In the Lagrange equations, the generalized non-conservative Forces Q_i are introduced relative to the dissipative forces. These forces represent:

- 1) the components, on the axes x and y , of air resistance forces acting on both the payload and the hoisting system,
- 2) the components of the wind force and the components of the forces responsible for the rotation movement damping.

The Kinetic energy of the slewing crane is given the sum of the corresponding terms for the rigid slewing tower T_T , for the jib T_B , for the trolley T_R , and the payload T_L . Therefore, the Kinetic energy T of the crane system is obtained as:

$$T = T_T + T_B + T_R + T_L \quad (24)$$

We focus our interest relative to the Lagrangian coordinates ϕ_x and ϕ_y since φ , r and l are obtained by the generated profiles that the Plc. From Lagrange equations (23), we obtain the system of the following two equations:

$$\left\{ \begin{array}{l} \ddot{r} - l \dot{\varphi}_y \dot{\varphi} + l \ddot{\varphi}_x - (r + l \varphi_x) \dot{\varphi}^2 \\ + 2 (\dot{l} \dot{\varphi}_x - l \dot{\varphi}_y \dot{\varphi} - \dot{l} \varphi_y \dot{\varphi}) + g \varphi_x \end{array} \right\} \quad (26)$$

$$- (\dot{l}^2 \varphi_x) / l = F_x / m_L$$

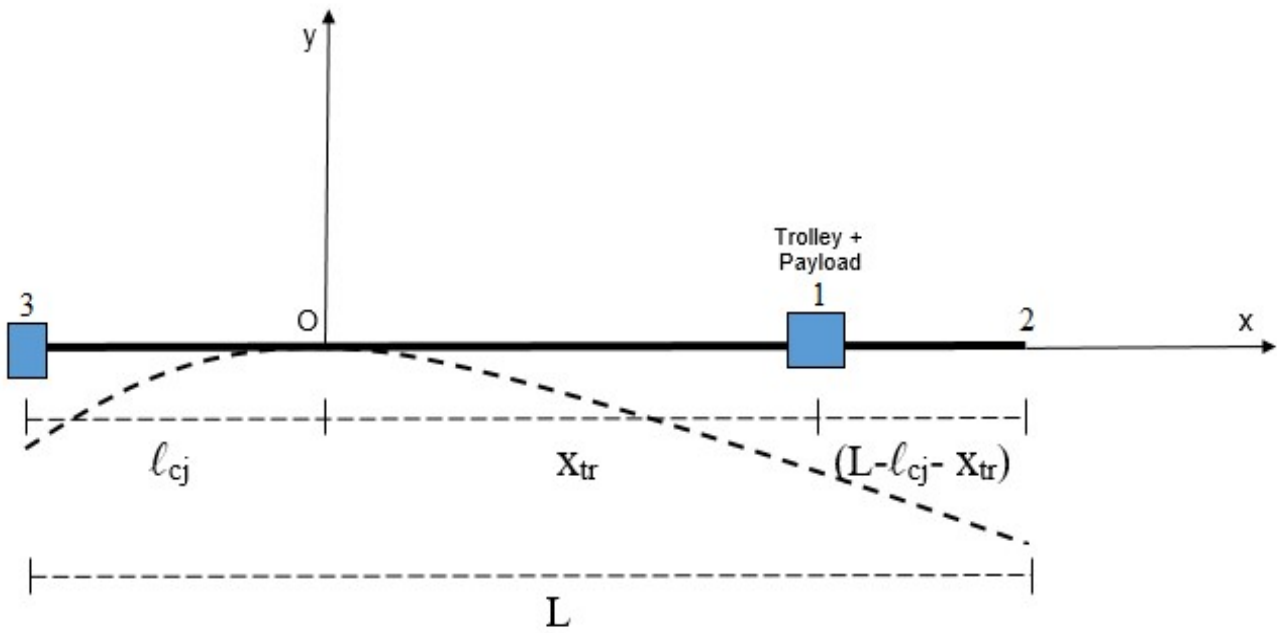


Fig. 4 Finite Elements Distribution on the Jib

$$\left\{ \begin{array}{l} (r + l\varphi_x) \ddot{\varphi} + l\ddot{\varphi}_y - l\varphi_y \dot{\varphi}^2 \\ + 2(l\dot{\varphi}_y + r\dot{\varphi} + l\dot{\varphi}_x \dot{\varphi} + l\varphi_x \dot{\varphi}) + g\varphi_y \end{array} \right\} - \frac{(j^2 \varphi_y)}{l} = \frac{F_y}{m_L} \quad (25)$$

where: m_L is the Load mass; g is the gravity acceleration. F_x and F_y are the generalized, non-conservative, dissipative and active forces working on the system. Particularly, they represent the components, on the axes x and y , of air resistance forces acting on the payload and on the hoisting system, the components of the wind force, the components of the forces due to the damping of the rotation movement and due to the structure's elasticity of the hoisting system.

We used an iterative method, where the calculated values of the sway angles and of the sway angle velocities are used to compute the correction at the trolley and slewing velocity. These corrections are added to the set-point velocities, obtaining the speed profiles for the drives.

Focusing, for example, on eq. (25), φ_y and $\dot{\varphi}_y$ are obtained with an iterative procedure, at any time t , in the following way:

$$\ddot{\varphi}_{y,t} = \frac{1}{l_t} \left\{ \begin{array}{l} -g \sin \varphi_{y,t} - r_t \ddot{\varphi}_t \cos \varphi_{y,t} \\ + \varphi_t^2 \cdot (l_t \sin \varphi_{y,t}) \cos \varphi_{y,t} \\ + (l_t - k_f) \cdot \dot{\varphi}_{y,t} + k l_t \sin \varphi_{y,t} \\ + [(k_{w,y} \cdot r_t \dot{\varphi}_t) / m_l] \end{array} \right\} \quad (27)$$

$$\dot{\varphi}_{y,t} = \dot{\varphi}_{y,t-1} + \ddot{\varphi}_{y,t-1} \cdot \Delta t \quad (28)$$

$$\varphi_{y,t} = \varphi_{y,t-1} + \dot{\varphi}_{y,t-1} \cdot \Delta t \quad (29)$$

In these equations, we have:

- 1) $\varphi_{y,t}$ and $\varphi_{y,t-1}$ are the sway angle along the y direction respectively at the time t and at the previous time $t-1$,
- 2) $\dot{\varphi}_{y,t}$ and $\dot{\varphi}_{y,t-1}$ are the velocity of the sway angle along the y direction respectively at the time t and at the previous time $t-1$,
- 3) $\ddot{\varphi}_{y,t}$ and $\ddot{\varphi}_{y,t-1}$ are the acceleration of the sway angle along the x direction respectively at the time t and at the previous time $t-1$.

The same considerations can be applied to the direction x .

To the initial instant, the values of the sway angle φ_y the velocity of the sway angle $\dot{\varphi}_y$ and the acceleration of the sway angle $\ddot{\varphi}_y$ are equal to zero, because the pendulum is in quiet conditions. Therefore, for $t=0$, we have:

$$\varphi_{y,0} = \dot{\varphi}_{y,0} = \ddot{\varphi}_{y,0} = 0 \quad (30)$$

We obtain (see [32]) the correction $\Delta \dot{\varphi}$ of the angular velocity of the movement along the axis y according to the following equation:

$$\Delta \dot{\varphi} = K_{0,\phi} \cdot \varphi_y + K_{1,\phi} \cdot \dot{\varphi}_y \quad (31)$$

- 1) $K_{0,\phi}$, $K_{1,\phi}$ are the observer gains applied at φ_y , $\dot{\varphi}_y$.
- 2) $\Delta \dot{\varphi}$ is the correction added to the velocity set-point $\dot{\varphi}_{set}$ along the axis y .

Therefore, the reference of velocity $\dot{\varphi}_{ref}$ as an input to the inverter driving the motor relative to the axis y is the angular velocity set is the velocity set, given by:

$$\dot{\varphi}_{ref} = \dot{\varphi}_{set} + \Delta\dot{\varphi} \quad (32)$$

The same considerations can be applied along the axis x (trolley rotation movement). The length l has to be considered in order to compute the total Period time T and the corresponding angular frequency ω of the pendulum describing the sway of the load attached to the crane. They will be equal, to a first approximation, neglecting the cable elasticity, to:

$$T_{Load} = 2\pi\sqrt{\frac{l}{g}} \quad ; \quad \omega_{Load} = \sqrt{\frac{g}{l}} \quad (33)$$

The latter typically represents the fundamental frequency in the vibrational system constituted by the Slewing Tower Crane, since it corresponds to the longest period T among the normal modes of vibration.

IV. VIBRATIONAL MODES OF THE OVERALL SYSTEM

Ultimately we obtain four eigenmodes with the slowest eigenfrequencies of the overall system. Typically, using the most usual experimental data, we will obtain that the first mode is characterized by the payload oscillation, while the second one by the effect of the torsional motion of the tower on the deformation of the Jib. The third eigenmode is relative to a deflection due to the tower bending while the fourth, like all the further modes, is characterized by the perpendicular deformation of the jib along the x-axis.

The definition of a state-space model for the flexible crane would not be free of modeling error since one would have to consider only a small number of vibrations in this model and would have to truncate higher-order vibration modes. This is the prize paid for turning the infinite-dimensional model of the flexible crane into a finite-dimensional one. As a consequence of that, we will consider only the modes with frequencies lower than the cutoff frequency of the drive used for the definition of the slewing trajectory setting.

In order to obtain the Tower slewing control mode able to not generate vibrations, we must calculate the overall natural wavelength of the crane structure in the slewing direction. To this end, we will make a fundamental approximation: given that each described vibrational mode (modes that contribute to the generation of vibration in the slewing direction) can be represented by a wave of defined frequency, we will hypothesize that these modes have periods whose ratio between them is a rational number. This can be thought true at first approximation. As a consequence, the period of the overall natural wave is the sum of the component periods. From this, it is possible to obtain the overall natural frequency of the system. So we will have for the overall period T_{tot} :

$$T_{tot} = T_{Load} + T_{bendT} + T_{torsT} + \sum T_{bendJ} \quad (34)$$

where T_{tot} is the period of the overall natural wave. T_{Load} is the period of the payload sway. T_{bendT} is the period of the

Tower bending wave, T_{torsT} is the period of the Tower torsion wave. $\sum T_{bendJ}$ is the sum of the period of the Jib bending waves considering all modes with frequencies lower than the cutoff frequency of the used slewing drive.

There are a variety of signals that satisfy the requirement for cancellation of oscillations at a given frequency of a system, the simplest signal being represented by two equal pulses with an offset in time. This signal must provide the shortest acceleration and deceleration ramps, which is one of the most important criteria for the operator. The desired velocity reference profile is generated by convolution of the arbitrary velocity command originating from the operator with the frequency-cancellation signal canceling vibrations at the overall natural frequency of the crane structure. The result of this convolution operation is the velocity reference signal, which does not excite vibrations at the overall natural frequency of the system, thus allowing smooth slewing movement of the Jib. We will consider two cases: Jib slewing control with Input-shaping, and Jib slewing with a load anti-sway profile.

V. RESULT AND DISCUSSION

We used Codesys V3.5 SP7, written in Structured Language (SL), in order to simulate the time evolution of the motion equations. An example of the used crane parameters to model the system is defined in Table.1.

Table I: Exemplary Crane Parameters

Symbol	Parameter	Value	Unit
$V_{Ref,Sl}$	Slewing Speed reference	30.0	Hz
$V_{Ref,Tr}$	Trolley Speed reference	40.0	Hz
$V_{MaxRef,Sl}$	Slewing Max Speed reference	50.0	Hz
$V_{MaxRef,Tr}$	Trolley Max Speed reference	50.0	Hz
$\dot{\varphi}$	Angular Rotation Velocity	0.1	Rad/s
Acc_{Sl}	Slewing Acc. Ramp Set	4.0	s
Dec_{Sl}	Slewing Dec. Ramp Set	4.0	s
Acc_{Tr}	Trolley Acc. Ramp Set	3.0	s
Dec_{Tr}	Trolley Dec. Ramp Set	3.0	s
μ	Linear density (Mass per length unit)	100	kg/m
E	Elastic Modulus	$210 \cdot 10^9$	Pa
I_z	Second Moment of Inertia	$5 \cdot 10^{-3}$	m^4
J_T	Tower Moment of Inertia	$5 \cdot 10^3$	N/m^2
k_T	Stiffness of the Tower Torsion Elasticity	$5 \cdot (17.5)^6$	Nm/rad
k_B	Stiffness of the Tower Bending Elasticity	$110 \cdot 10^3$	N/m
K_v	Stiffness matrix element along v def.	0.1	N/m
l	Cable length	1-50	m
L	Length of the Jib	60	m
l_T	Length of the Tower	60	m
l_{CJ}	Length of the counter-jib	12	m
X_{Tr}	Variable position of the Trolley on the Jib	30	m
m_{CJ}	Counter-Jib mass	$5 \cdot 10^3$	kg
m_L	Load mass	$2 \cdot 10^3$	kg
m_T	Tower mass	$7 \cdot 10^3$	kg
m_{Tr}	Trolley mass	$6 \cdot 10^2$	kg

A. Jib Slewing Control with Input-Shaping

Input shaping is one of the most common open-loop feedforward control techniques that can be applied in real-time. Input Shaping is easier to implement than time-optimal control systems. It does not require the feedback mechanisms of closed-loop. This technique was used by many researchers for effective feed-forward control of minimizing the motions that are induced by the vibrations or the oscillations of flexible structures such as cranes. By using this technique, the system's vibration is obtained by the convolution of the command input signal with a sequence of impulses, based on the natural frequencies of the system. Namely, we can say: if two numbers of $T/2$ delayed accelerating pulses of constant magnitude 'k' are applied to a linear pendulum the sway will be zero at the end. We will apply this technique in the case of Jib slewing in order to cancel the vibrations.

Therefore, once we obtained the first eigenmodes with the slowest eigenfrequencies for the overall system, we are able to deduce, as described in Subsection IV, the values of the periods T_i relating to the different main modes of vibration. From these individual values of T_i , using eq. (34) it is possible to calculate the period T_{Tot} . Once this parameter has been obtained, based on the ramps established for the Acceleration and Deceleration of the Slewing Jib, the movement profile necessary to cancel the vibrations of the slewing movement is established. As a consequence, we can calculate the Input-shaping control using the period T_{Tot} .

In Fig.5, we can observe the Velocity reference profile of the Slewing $\dot{\phi}$ obtained using the Input-shaping control. The Slewing speed reference is 30 Hz and the ramp set is equal to 9.0 s. At the end of period T the effect of canceling the main vibrations of the system is obtained. In Fig.6, the same Velocity reference profile of the Slewing $\dot{\phi}$ with the same 30 Hz of the previous profile, but the ramp set is faster, equal to 6.0 s. In this way, the internal period time where the velocity is constant results, as a consequence, superior to the previous case: that is in order to have the same total period.

The fundamental disadvantage of the Input shaping control of the sway (and consequent crane vibration) is that the speed profile must be terminated (i.e. the Period T_{Tot} must be completed) for the anti-vibration effect to be correctly implemented. Therefore, the operator cannot define a succession of manual commands before the period is over. This makes this kind of control essentially effective in the case of automatic positioning.

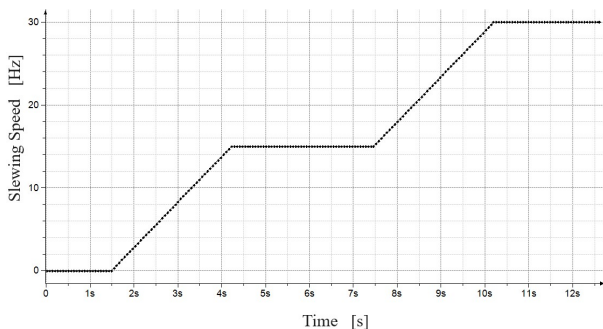


Fig. 5. Velocity reference profile of the Slewing $\dot{\phi}$ using the Input-shaping control for the Anti vibration system. That is in

correspondence to a command relative to a Slewing movement. The specific values are: Slewing speed reference = 30 Hz, ramp set = 9.0 s, cable length = 30 m.

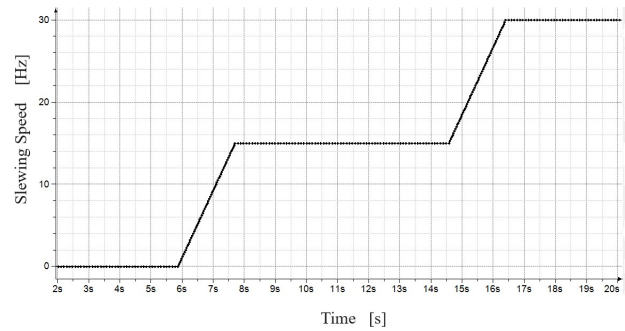


Fig. 6. Velocity reference profile of the Slewing $\dot{\phi}$ using the Input-shaping control for the Anti vibration system. That is in correspondence to a command relative to a Slewing movement. The specific values are: Slewing speed reference = 30 Hz, ramp set = 6.0 s, cable length = 30 m.

B. Jib Slewing with a Load Anti-Sway Profile

In Fig.7, we see the Slewing speed reference and the corresponding normal sway angle. They correspond to a Slewing speed reference is 30 Hz and a ramp set equal to 7.0 s. It is visible how, when the velocity profile reaches its target asymptotically, the sway becomes zero. In Fig.8, we describe some subsequent commands (3 commands) by the operator, in order to reach different velocity targets for the slewing movement, before reaching the velocity profile target. It is possible to see that the corresponding normal sway angle, when the target speed is stabilized, goes to zero.

Therefore, we see that, with this method, we obtain the damping of the sway, also varying many times the velocity set. That is obtained also if the previous command, and corresponding oscillation, are again in progress. This effect allows the operator to change often the target velocity with his command (impulse command). The disadvantage of this method is that, being asymptotic, a typically longer anti-sway ramp is generated than that obtained with the "input-shaping" method.

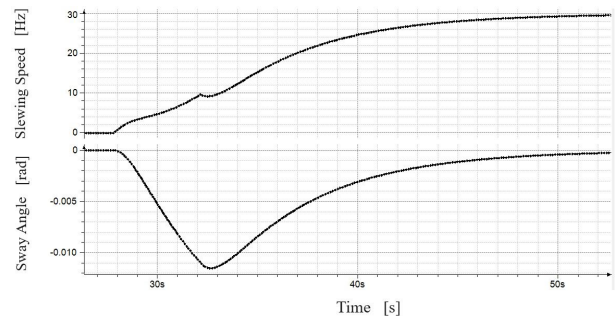


Fig. 7. Velocity reference profile of the Slewing $\dot{\phi}$ and corresponding perpendicular sway angle ϕ_y using the Load Anti-sway profile. That is in correspondence to a command relative to a Slewing movement. The specific values are: Slewing speed reference = 30 Hz, ramp set = 7.0 s, cable length = 30 m.

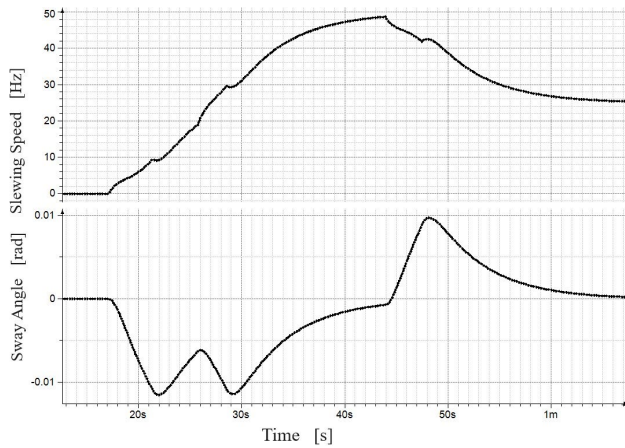


Fig. 8. Velocity reference profile of the Slewing $\dot{\phi}$ and corresponding perpendicular sway angle ϕ_y using the Load Anti-sway profile. That is in correspondence to a succession of commands relative to different Slewing speed references. The specific values are: initial Slewing speed reference = 30 Hz, ramp set = 7.0 s, cable length = 30 m.

VI. CONCLUSION

This work describes a method for controlling a Tower crane taking into account the structure elasticity. An Elastic Model of the Slewing Crane is described, considering the Bending Elasticity and the Torsional Elasticity of the vertical Tower, and the Bending vibrations of the thin beam, calculated using a finite element method.

The normal modes of the vibration are obtained considering the jib as a continuous beam; we used a simplified method in order to compute the normal modes and the corresponding period. It is shown how the structure undergoes deformations and how the deformations can be reduced with the use of an anti-sway method. We developed two kinds of solution: an input-shaping control in open-loop, to be used with automatic positioning, and a “command smoothing” method in open-loop to use in manual control. This allows for the stabilization and correct positioning of the payload for cranes with large jibs. The novelty of this method is that the eigenfrequencies can be calculated in real time during the operational work, using a supervisor PC, which sends the data of the same eigenfrequencies to the PLC that controls the Jib crane. In fact, the method uses a low number of discretization nodes and the calculations can be implemented in real-time. In the future, a feedback closed-loop control will be implemented for considering exogenous perturbations in the present vibrational model. The present model must be applied in real cases in order to verify experimentally the obtained theoretical results.

DECLARATION STATEMENT

Funding	No, I did not receive.
Conflicts of Interest	No conflicts of interest to the best of our knowledge.
Ethical Approval and Consent to Participate	No, the article does not require ethical approval and consent to participate with evidence.
Availability of Data and Material	Not relevant.
Authors Contributions	I am only the sole author of the article.

REFERENCES

- L. Ramli, Z. Mohamed, A. Abdullahi, HI Jaafar and IM Lazim, “Control strategies for crane systems: A comprehensive review”, *Mechanical Systems and Signal Processing*, 2017. DOI:10.1016/j.ymssp.2017.03.015. <https://doi.org/10.1016/j.ymssp.2017.03.015>
- D. Kruk and M. Sulowicz, “AHRS Based Anti-Sway Tower Crane Controller”, *20th International Conference on Research and Education in Mechatronics*, 2019. DOI: 10.1109/REM.2019.8744117 <https://doi.org/10.1109/REM.2019.8744117>
- M. Zhang, Y. Zhang, H. Ouyang, C. Ma and X. Cheng. “Modeling and Adaptive Control for Tower Crane Systems with Varying Cable Lengths”, *Proceedings of the 11th International Conference on Modelling, Identification and Control*, 2019. DOI: 10.1007/978-981-15-0474-7-21. https://doi.org/10.1007/978-981-15-0474-7_21
- H. Chen, Y. Fang, N. Sun. “A tower crane tracking control method with swing suppression”, *Conference: 2017 Chinese Automation Congress*, 2017. DOI: 10.1109/CAC.2017.8243407. <https://doi.org/10.1109/CAC.2017.8243407>
- Y. Wu, N. Sun, H. Chen and Y. Fang. “Adaptive Output Feedback Control for 5-DOF Varying-Cable-Length Tower Cranes With Cargo Mass Estimation”. *IEEE Transactions on Industrial Informatics*, 2020. DOI: 10.1109/REM.2019.8744117. <https://doi.org/10.1109/REM.2019.8744117>
- T. Yang, N. Sun, H. Chen and Y. Fang. “Observer-Based Nonlinear Control for Tower Cranes Suffering From Uncertain Friction and Actuator Constraints with Experimental Verification”, *IEEE Transactions on Industrial Informatics*, 2020. DOI:10.1109/TIE.2020.2992972. <https://doi.org/10.1109/TIE.2020.2992972>
- N. Sun, Y. Wu, H. Chen and Y. Fang. “Anti-swing Cargo Transportation of Under-actuated Tower Crane Systems by a Nonlinear Controller Embedded With an Integral Term”, *IEEE Transactions on Automation Science and Engineering*, 2019, 1387-1398, 16-3. DOI:10.1109/TASE.2018.2889434. <https://doi.org/10.1109/TASE.2018.2889434>
- N. Sun, Y. Fang, H. Chen, B. lu and Y. Fu. “Slew Translation Positioning and Swing Suppression for 4-DOF Tower Cranes with Parametric Uncertainties: Design and Hardware Experimentation”, *IEEE Transactions on Industrial Electronics*, 2016; 1, 63-10. DOI:10.1109/TIE.2016.2587249.
- P. Srivastava, A. Pal Singh, T. Srivastava, N.M. Singh. “Observer Design for Non-Linear Systems”, *International Journal of Applied Engineering*, 2013; 8-8, 957-967. <https://doi.org/10.1109/TIE.2016.2587249>
- W. He, M. Tingting, H. Xiuyu, S. Sam Ge. “Unified iterative learning control for flexible structures with input constraints”, *Automatica*, 2018; 96, 146. DOI: 10.1016/j.automatica.2018.06.051. <https://doi.org/10.1016/j.automatica.2018.06.051>
- W. He, S. Sam Ge, B. Voon Ee How, Y. Sang Choo and K. Hong. “Robust adaptive boundary control of a flexible marine riser with vessel dynamics”, *Automatica*, 2011; 47, 722. DOI:10.1016/j.automatica.2011.01.064. <https://doi.org/10.1016/j.automatica.2011.01.064>
- Y.J. Liu and S. Tong. “Barrier Lyapunov Functions-based adaptive control for a class of nonlinear pure-feedback systems with full state constraints”, *Automatica*, 2015; 64, 70. DOI:10.1016/j.automatica.2015.10.034 <https://doi.org/10.1016/j.automatica.2015.10.034>
- G. Tysse, A. Cibicik, L. Tingelstad and O. Egeland. “Lyapunov-based damping controller with nonlinear MPC control of payload position for a knuckle boom crane”, *Automatica*, 2022; 140. DOI:10.1016/j.automatica.2022.110219. <https://doi.org/10.1016/j.automatica.2022.110219>
- J. Schatz and R. Caverly. “Passivity-Based Adaptive Control of a 5-DOF Tower Crane”, *2021 IEEE Conference on Control Technology and Applications*, 2021; 1109-1114. DOI:10.1109/CCTA48906.2021.9659155. <https://doi.org/10.1109/CCTA48906.2021.9659155>
- B. Wen-Wen, H. Ren. “Horizontal Positioning and Anti-swinging Control Tower Crane Using Adaptive Sliding Mode Control”, *2018 Chinese Control and Decision Conference*, 2018; 140. DOI:10.1109/CCDC.2018.84078209.

Vibration Analysis and Motion Control Method for an Under-Actuated Tower Crane

16. V. Pham, H. Cuong, H. Dong, H.T. Nguyen, L. Tuan, "Adaptive fractional-order fast terminal sliding mode with fault-tolerant control for underactuated mechanical systems: Application to tower cranes", *Automation in Construction*, 2021; 123. DOI:10.1016/j.autcon.2020.103533. <https://doi.org/10.1016/j.autcon.2020.103533>
17. Z. Liu, N. Sun, Y. Wu, X. Xin and Y. Fang Y. "Nonlinear Sliding Mode Tracking Control of Underactuated Tower Cranes", *International Journal of Control, Automation and Systems*, 2020. DOI:19. 10.1007/s12555-020-0033-5. <https://doi.org/10.1007/s12555-020-0033-5>
18. N. Sun, Y. Wu, H. Chen and Y. Fang. "Antiswing Cargo Transportation of Underactuated Tower Crane Systems by a Nonlinear Controller Embedded With an Integral Term", *IEEE Transactions on Automation Science and Engineering*, 2019; 1-12. DOI:10.1109/TASE.2018.2889434. <https://doi.org/10.1109/TASE.2018.2889434>
19. L. Shu-Guang, Z. Long and Chao-Qi. "An ADRC-based Positioning and Anti-swing Control for Tower Crane", *Conference: 2021 China Automation Congress*, 2021; 7880-7884. DOI:10.1109/CAC53003.2021.9728161. <https://doi.org/10.1109/CAC53003.2021.9728161>
20. F. Schlagenhaupt and W.E. Singhose, "Comparison of Sway Reduction Controllers for Construction Cranes", *IEEE 14th International Conference on Control and Automation*, 2018; 1101-1106. DOI:10.1109/ICCA.2018.8444294. <https://doi.org/10.1109/ICCA.2018.8444294>
21. A. Alfadhli and E. Khorshid. "Payload Oscillation Control of Tower Crane Using Smooth Command Input", *Journal of Vibration and Control*, 2021. DOI: 10.1177/10775463211054640. <https://doi.org/10.1177/10775463211054640>
22. S. Iles, J. Matusko and F. Kolonic. "Sequential distributed predictive control of a 3D tower crane", *Control Engineering Practice*, 2018; 22-35, 79. DOI: 10.1016/j.conengprac.2018.07.001. <https://doi.org/10.1016/j.conengprac.2018.07.001>
23. J. Wang, B. Chen, H. Ouyang and L. Xu. "Study on an Embedded Monitoring and Control System of Tower Crane", *Proceedings of the 4th International Conference on Sustainable Energy and Environmental Engineering*, 2016. DOI: 10.2991/icseee-15.2016.126. <https://doi.org/10.2991/icseee-15.2016.126>
24. R. Capkova, A. Kozakova and M. Minar, "Experimental Modelling and Control of a Tower Crane in the Frequency domain", *Journal of Mechanical Engineering*, 2019, vol. 69, no. 3, pp. 17-26. DOI: 10.2478/scjme-2019-0025. <https://doi.org/10.2478/scjme-2019-0025>
25. D. Blackburn, J. Lawrence, J. Danielson, W.E. Singhose, T. Kamoi and A. Taura. "Radial-motion assisted command shapers for nonlinear tower crane rotational slewing", *Control Engineering Practice*, 2010; 523-531, 18. DOI:10.1016/j.conengprac.2010.01.014. <https://doi.org/10.1016/j.conengprac.2010.01.014>
26. L. Shehata and A. El-Badawy. "Anti-sway control of a tower crane using inverse dynamics", *2nd International Conference on Engineering and Technology*, 2014. DOI:10.1109/ICEngTechnol.2014.7016747. <https://doi.org/10.1109/ICEngTechnol.2014.7016747>
27. J.H. Montonen, N. Nevaranta, M. Niemelä and T. Lindh." Comparison of Extrainsensitive Input Shaping and Swing-Angle-Estimation-Based Slew Control Approaches for a Tower Crane", *Applied Sciences*. 2022, Vol.12. pp. 5945. DOI: 10.3390/app12125945. <https://doi.org/10.3390/app12125945>
28. M. Zhang, Y.F. Zhang, X. Cheng X. "Model-Free Adaptive Integral Sliding Mode Control for 4-DOF Tower Crane Systems", *IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, 2019; 708-713. DOI:10.1109/AIM.2019.8868534. <https://doi.org/10.1109/AIM.2019.8868534>
29. M. Zhang, Y.F. Zhang, H. Ouyang, C. Ma and X. Cheng, "Adaptive integral sliding mode control with payload sway reduction for 4-DOF tower crane systems", *Nonlinear Dynamics*, 2020, Vol. 99. DOI:10.1007/s11071-020-05471-3. <https://doi.org/10.1007/s11071-020-05471-3>
30. Z. Liu, T. Yang, N. Sun and Y. Fang. "An Antiswing Trajectory Planning Method With State Constraints for 4-DOF Tower Cranes: Design and Experiments", *IEEE Access*, 2019; 1-1. DOI:10.1109/ACCESS.2019.2915999. <https://doi.org/10.1109/ACCESS.2019.2915999>
31. G. Li, X. Ma, Z. Li and Y. Li. "Time-Polynomial-Based Optimal Trajectory Planning for Double-Pendulum Tower Crane With Full-State Constraints and Obstacle Avoidance". *IEEE/ASME Transactions on Mechatronics*, 2022, PP. 1-14. DOI: 10.1109/tmech.2022.3210536. <https://doi.org/10.1109/TMECH.2022.3210536>
32. Roberto P. L. Caporali, "Iterative Method for controlling the sway of a payload on tower (slewing) cranes using a command profile approach", *International Journal of Innovative Computing, Information and Control*, 2018, Vol. 14, 4, 1169-1187. DOI: 10.24507/ijic.14.04.1169. <https://doi.org/10.24507/ijic.14.04.1169>
33. Roberto P. L. Caporali, "Anti-Sway control on a Harbor Crane system using a Command Smoothing iterative method", *International Journal of Innovative Research & Development*, 2022, Vol. 11, 7, 21-31. DOI: 10.24940/ijird/2022/v11/i7/JUL22007. <https://doi.org/10.24940/ijird/2022/v11/i7/JUL22007>
34. C. Juraszek, "Method for controlling the Slewing movement of the rotary part of a Tower Crane", *United States Patent*, Assignee Manitowoc Crane, US 8,235,230 B2, 2012.
35. Y. Yunhua and Y. Yao. "Tower crane, swing control system and swing control method", *China Patent*, CN103274299A; 2013.
36. W. Devesse. "Slew Control Methods for Tower Cranes", *Master of Science Thesis*, Sweden, commissioned by ABB, 2012.
37. C. Juraszek, "Method for controlling the Slewing movement of the rotary part of a Tower Crane", *United States Patent*, Assignee Manitowoc Crane, US 8,235,230 B2, 2012.
38. R.P.L. Caporali. "Method and Device to control in open-loop the sway of pay-load for Slewing Cranes", *European Patent*, EP2896590, 2015.
39. M.H. Ghawzani, A.H. Alnujaie, M.L. Chandravanshi, D. Deepak, C. Singh, M. Kumar, Y.S. Choo and F.S. Cui , "Failure Analyses of Tower Crane using FEM and theoretical studies", *Journal of Mechanical Design and Vibration*, 2019, Vol. 7, No. 1, 33. DOI:10.53370/001c.36800. <https://doi.org/10.53370/001c.36800>
40. F. Ju, Y.S. Choo and F.S.Cui. "Dynamic response of tower crane induced by the pendulum motion of the payload", *International Journal of Solids and Structures*, 2006; 376-389, 43. DOI:10.1016/j.ijsolstr.2005.03.078. <https://doi.org/10.1016/j.ijsolstr.2005.03.078>
41. F. Liu, J. Yang, J. Wang and C. Liu. "Swing Characteristics and Vibration Feature of Tower Cranes under Compound Working Condition", *Hindawi Shock and Vibration*, 2021. DOI:10.1155/2021/8997396. <https://doi.org/10.1155/2021/8997396>
42. X. Cao, Y. Yang, W. Wang and Gu Z. "Rigid-Flexible Coupling Dynamic Modeling of a Tower Crane with Long Flexible Boom", *International Conference on Mechanical Design*, 2018; 39-57. DOI:10.1007/978-981-10-6553-8-4. https://doi.org/10.1007/978-981-10-6553-8_4
43. R.P.L. Caporali, "Anti-sway method for reducing vibrations on a tower crane structure", *International Journal of Nonlinear Sciences and Numerical Simulation*, 2021, DOI: 10.1515/ijnsns-2021-0046. <https://doi.org/10.1515/ijnsns-2021-0046>
44. F. Rauscher and O. Sawodny, "An Elastic Jib Model for the Slewing Control of Tower Cranes", *Science Direct IFAC PapersOnLine*, 2017, Vol. 50-1, pp.9796-9801. DOI: 10.1016/j.ifacol.2017.08.886. <https://doi.org/10.1016/j.ifacol.2017.08.886>
45. G. Rigatos, M. Abbaszadeh and J. Pomares, "Nonlinear optimal control for the 4-DOF underactuated robotic tower crane", *Autonomous Intelligent Systems*, 2022. DOI:10.1007/s43684-022-00040-4. <https://doi.org/10.1007/s43684-022-00040-4>
46. R.P.L. Caporali, "Vibration Normal Modes of a Jib Crane modeled as an Euler-Bernoulli boom using FEM", *International Journal of Basic Sciences and Applied Computing*, 2023, DOI:10.35940/ijbsac.D0509.1210423. <https://doi.org/10.35940/ijbsac.D0509.1210423>
47. R.P.L. Caporali, "Analysis of the System Stability for an anti-Sway method relative to Harbour Cranes", *International Journal of Recent Engineering Research and Development*, 2022, ISSN: 2455-8761.
48. M. Petyt, *Finite Element Vibration Analysis*. Cambridge University Press, 1990, ch. 3-11. <https://doi.org/10.1017/CBO9780511624292>
49. G. Diana, F. Cheli, *Dinamica e Vibrazione dei Sistemi*. Torino, UTET, 1993.
50. Small Composition-X System of Air Flow & Solar with Novel SWAY Translate to Enhance the SWAY Quality. (2019). In *International Journal of Innovative Technology and Exploring Engineering* (Vol. 9, Issue 2S4, pp. 573-577). <https://doi.org/10.35940/ijitee.bl203.1292s419>

51. Kasim, M. A. B., Hanafi, S. R. B. M., & Suki, N. M. (2019). Relevance of Technology Acceptance Model for the Implementation of Value Added Tax (VAT) In the United Arab Emirates (UAE): Evidence of Distinctive Behavioral Connections. In International Journal of Recent Technology and Engineering (IJRTE) (Vol. 8, Issue 3, pp. 6357–6365). <https://doi.org/10.35940/ijrte.c5784.098319>
52. Balate, A. M., & Patil, H. R. M. (2020). Assessment of Response Reduction Factor of Flat Slab Structures by Pushover Analysis. In International Journal of Engineering and Advanced Technology (Vol. 9, Issue 6, pp. 157–163). <https://doi.org/10.35940/ijeat.fl355.089620>

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