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Application of Set Intersecting Graph in Number Theory

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Abstract:

In this paper, we offer evidence for a fascinating equation found in number theory, where the unanticipated usage of set intersection graphs demonstrates its practicality. Set intersection graphs are graphs formed from sets, with nodes representing sets and edges indicating a non-empty intersection between two sets. More specifically, we illustrate that the expression 3^n - 2^(n+1) + 1 is consistently an even number.

Keywords — **Graph theory, Graph, Set intersecting graph, Number theory**

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I. INTRODUCTION

This section presents the foundational definition of a graph.

DEFINITION:

A graph G = (V, E) is a mathematical structure defined as a set V of vertices and a set E of edges. The edges in E are composed of pairs of vertices from V. To denote the vertex set and edge set of graph G, V(G) and E(G) are commonly used, respectively.

DEFINITION:

A graph G=(V,E) is called an intersection graph for a finite family F of a non-empty set if there is a one-to-one correspondence between F and V such that two sets in F have non-empty intersection if and only if their corresponding vertices in V are adjacent.

II. MAIN RESULT

THEOREM: 3^{N}-2^{N+1}+1 IS EVEN

Proof: We create an intersection graph where each node represents a non-empty set in the set [n]={1,2,…,n}, namely, as an element in 2[n]- Ø. An edge exists between two nodes if the sets they represent have a non-empty intersection. For every set A, the count of edges originating from node represented by A in the graph is as follows:

$$\sum_{|A|=1}^{n} (2^{n-|A|} - 1) * \binom{n}{|A|} = \sum_{\{i=1\}}^{\{n\}} (2^{\{n-i\}} - 1) \binom{n}{i}$$

$$= \sum_{\{i=1\}}^{n} 2^{n-i} * \binom{n}{i} - \binom{n}{i}$$

$$= (1+2)^{n} - 2^{n} - 2^{n} + 1$$

$$= 3^{n} - 2^{n+1} + 1$$

Now, as the above calculation adds up the degrees of all the nodes and according to the handshaking lemma, this sum is equal to twice the number of edges in the graph, resulting in an even number.

III. CONCLUSION

In conclusion, we have effectively employed the set intersection graph to demonstrate a minor result in number theory. Looking ahead, our goal is to investigate additional correlations among various disciplines within mathematics.

REFERENCES

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